What does a root locus plot depict?

- The root locus plot depicts how the closed loop poles of the system varies in relation to some system parameter. It is useful for designing higher orders systems (order > 2) because it give us a simple way to pick values of s depending on the system design specification. We can also use the value of s to calculate the gain that would achieve a desired specification.

What must be done to a transfer function before it's root locus can be graphed?

- You must get the closed loop transfer function of the system in order to graph its root locus.

What is the significance of the gain K?

- The gain of the system determines how the poles of the closed loop transfer function move along the root locus plot. Understanding that different values of K effects the system's transient response and stability. We can use the root locus to find a value of K that makes the system stable and meet specifications like overdamping or underdamping the system, %OS and settling time.

How can a root locus plot be used to design a controller?

- A control system has a controller which takes the input to the system, manipulates it and then outputs that value into the plant of the system. The root locus plots depict the performance of a uncompensated system at various poles. You would use this information to understand how much the system needs to be compensated to yield a desired value. Finding the dominant pole location in the uncompensated system's root locus plot for both he imaginary and real parts. We can use these information to find where we can place a zero to design our controller. Overall, it is about understanding what information the uncompensated system's root locus tells us and by placing zero's (following the PID controller rules) we can design a system that yields a desired output.

Imagine we have a partially finished root locus plot where only the pole and zero locations have been plotted. What are the rules for completing the root locus plot using pencil and paper? (Hint! Your textbook has this information!)

1. **Real- axis segments:** The root locus is to the left of an odd number of pole or zero. You would draw a line between a pole → zero but have to make sure that the line is not left of an even number of pole or zero
    a. If # of poles = # zeros, each pole goes to a zero
    b. If the # of poles > # zeros, the poles with no zero pairs go to infinity.
    c. If the # of poles < # zeros, the zeros with no pole's pairs come from infinity.
2. **Starting and ending points:** The root locus starts at the finite and infinite poles and ends at the finite and infinite zeros.
    a. If two poles converge on each other, the break out into the complex plane moving as complex conjugate and they return to the real axis if there is an unpaired zero and if not, they go off to infinite.
3. **Behavior at infinity:** The root locus approaches straight lines as asymptotes as the locus approaches infinity.

a. You find the angle and number of asymptotes using the equations below.

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}}$$

$$\theta_a = \frac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}}$$

where k = 0, ±1, ±2, ±3 and the angle is given in radians with respect to the positive extension of the real axis.

Drawing the root locus Question 2:

Equation 1:

$$G(s) = \frac{1}{(s+5)(s+9)}$$

Poles at -5, -9

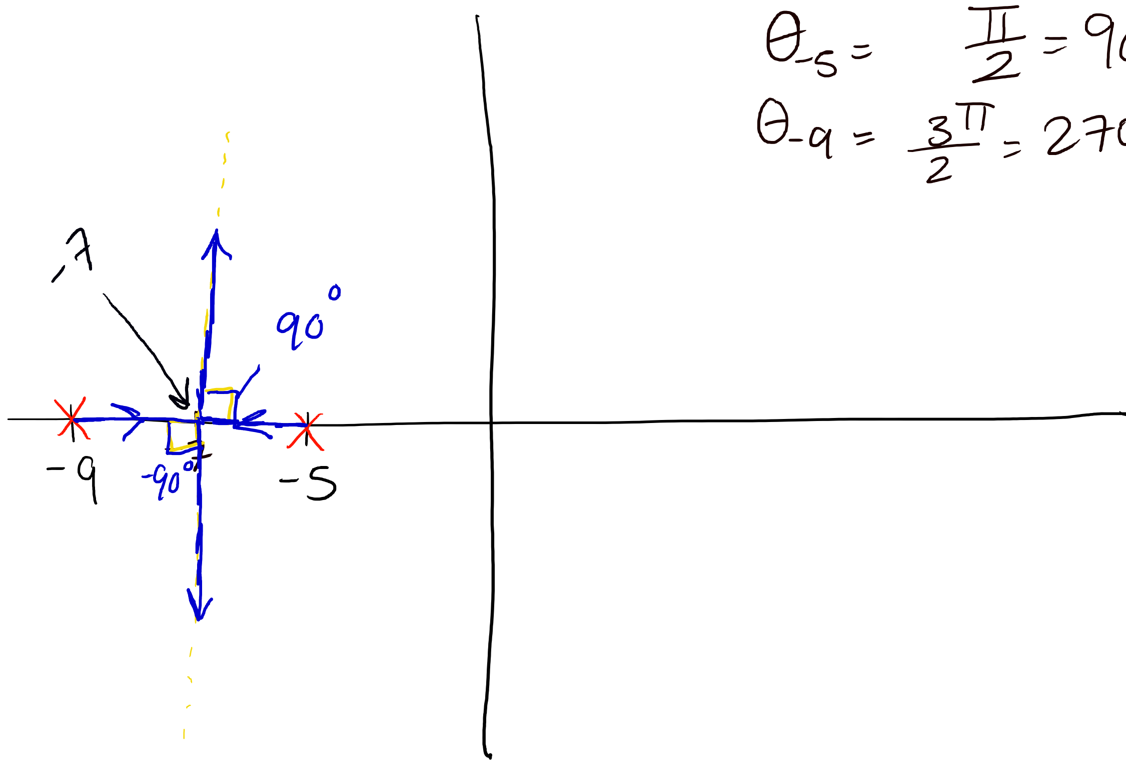
no zeros

$$\text{Centroid } (\sigma_a) = \frac{-9-5}{2} = -7$$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

$$\theta_{-5} = \frac{\pi}{2} = 90^\circ$$

$$\theta_{-9} = \frac{3\pi}{2} = 270^\circ = -90^\circ$$



Equation 2:

$$G(s) = \frac{(s-4)(s-7)}{(s+2)(s+5)(s+12)}$$

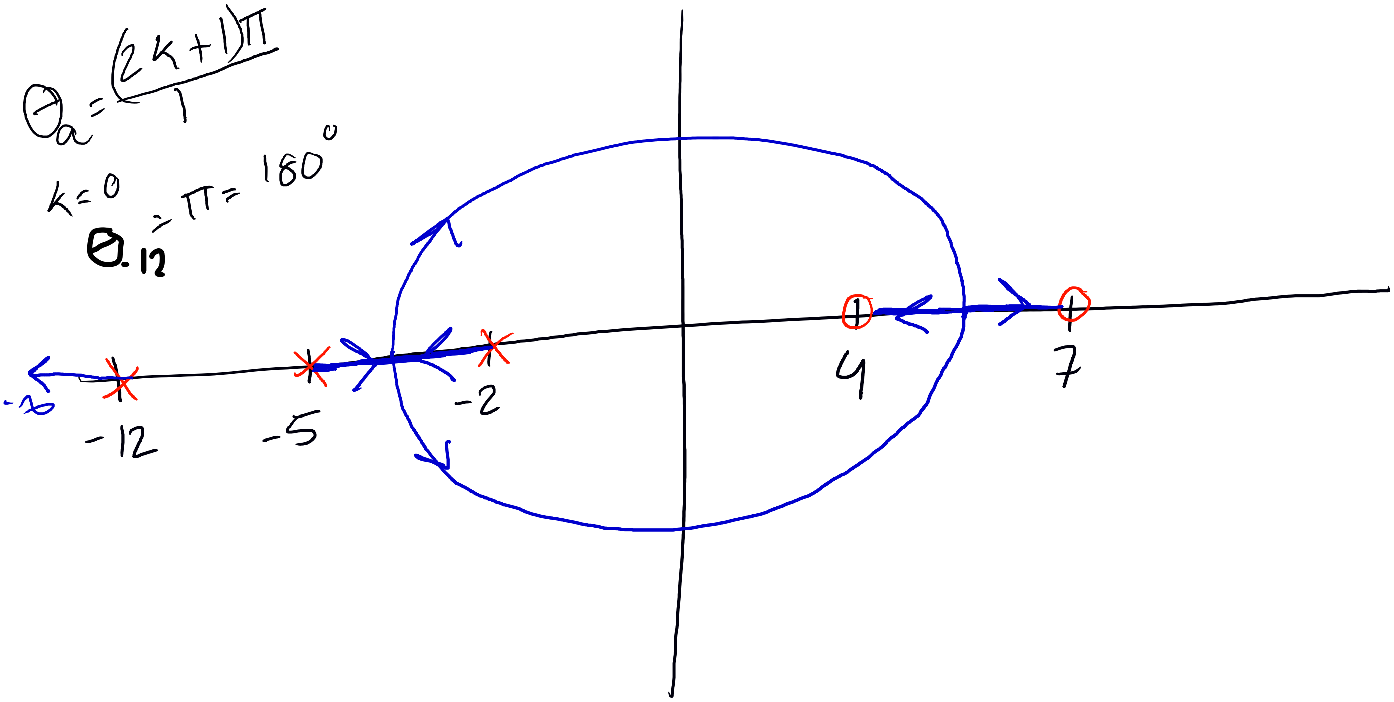
$(s-4)(s-7) \nearrow$ zeros $= +4, +7$

$(s+2)(s+5)(s+12) \rightarrow$ Poles $= -2, -5, -12$

$$\theta_a = \frac{(2k+1)\pi}{1}$$

$k = 0$

$\theta_{12} = \pi = 180^\circ$



Equation 3:

$$G(s) = \frac{(s+7)}{(s+8)(s+9)(s+3)^2} = \frac{(s+7)}{(s+8)(s+9)\underbrace{(s+3)(s+3)}_{\text{take it like this}}}$$

$$\sigma_a = \frac{(-3-3-9}{3} = -5$$

$$\theta_{-3} = \frac{(2(0)+1)\pi}{3} = \frac{\pi}{3} = 60^\circ$$

$$\theta_{-3} = -60^\circ \text{ (conjugate)}$$

$$\theta_{-9} = \frac{2(1)+1}{3} = \pi = 180^\circ$$