

# **Team 448**

## **Problem B: FIFA Penalty Kicks**

### **Abstract:**

In this paper we find out the initial ball velocity and spins what will result a successful shot to an upper corner from the penalty mark. For this investigation, we have assumed many notations for calculating the values for best shoot. We have calculated with many factors for successful goal. For this, a player needs to calculate the velocity and spin for the ball to lift the ball to the upper corner. Initial ball velocity can be many with the respect to the angle of kicking the ball. By considering some important factor the initial ball velocity and spin is being calculated.

**CONTENTS**

---

1	Nomenclature / Notations used .....	2
2	Introduction.....	3
3	Assumptions and Approximation.....	3
4	Theoretical analysis .....	4
4.1	Projectile motion .....	4
4.2	Velocity measurement with considering no spins.....	6
4.3	Velocity measurement with considering spins.....	7
4.3.1	Magnus Effect .....	7
4.3.2	Bernoulli's Effect – Spinning ball in an airflow .....	8
5	Calculations.....	11
5.1	Velocity measurement with considering no spins.....	11
5.2	Data Table.....	12
5.3	Velocity measurement with considering spins.....	13
6	Code .....	18
6.1	Velocity measurement with no spins .....	18
7	Conclusion .....	18
7.1	Velocity measurement with no spins .....	18
7.2	Velocity measurement with considering spins.....	18
8	Reference .....	19

## 1 NOMENCLATURE / NOTATIONS USED

Serial No.	Notation	Meaning
1	$\theta$	Angle
2	$a_x$	Acceleration on X axis
3	$a_y$	Acceleration on Y axis
4	$V_{xo}$	Velocity component is toward to X axis
5	$V_{yo}$	Velocity component is toward to Y axis
6	$V_o$	Initial Ball Velocity
7	$t$	time
9	$F$	Force
10	$m$	Mass of object
11	$R$	Radius of curvature
12	$r$	Radius of football

## 2 INTRODUCTION

---

The study of physics is incredibly fascinating; it regularly makes significant contributions to the advancement of our way of life, some of which we simply take for granted. Even though many who participate in sports may not be aware of it, it plays a significant role in that world as well. One of them is football, the sport with the largest global following. The FIFA Football World Cup 2022 will be held in Qatar in a few days, thus there is no better opportunity to examine the role performed by physics in the game of football.

FIFA football is famous all over the world for its unique style of playing. In here, sometimes penalty kick is happened. For this only one player can shoot the ball to goal box for purposing of goal. The kick is taken at the penalty mark point. From this place, player can shoot ball at his own technique to make the goal. In here, initial velocity and spin technique can make a player to make successful goal. There are many factors can make problem to make goal which is opposing by the player and the proper spins, velocity and direction. Sometimes, the player kicks the ball too high and that's why there won't happen a goal. For a successful goal, the player needs to focus and make some calculation in mind by which the player will kick the ball toward the goal bar. The ball must enter the goal bar and must be missed by goal keeper who protect the goal of opponent team.

In FIFA penalty kicks problem, we have to determine the velocity and spin which is the shooting attempt to create to have best chance of avoiding the goalkeeper and making the goal. The problem has many chances to make goal. There can be a winning probability to determine the best chances of goal. In this paper, we will try to find the spin and ball velocity. Here we can find those in two stages.

## 3 ASSUMPTIONS AND APPROXIMATION

---

In here, the things we assumed,

The diameter of football is 0.22m

The radius of football is 0.11m

The bar is too thin.

In here the things we used as approximation,

$$\pi = 3.1416$$

$$g = 9.8 \text{ ms}^{-2}$$

## 4 THEORETICAL ANALYSIS

### 4.1 Projectile motion

An item being launched or projected into the air will move in a projectile motion, with the sole acceleration coming from gravity. The trajectory of the item is referred to as the projectile's route. We take into account projectile velocity in two dimensions, such as the motion of a football or another item for which air resistance is minimal.

The most crucial thing to keep in mind in this situation is that movements along perpendicular axes are independent and may thus be studied individually. Breaking two-dimensional projectile motion into movements along the horizontal and vertical axes is the key to understanding it. (This selection of axes is the best option since gravity's vertical acceleration means that there won't be any acceleration along the horizontal axis when air resistance is minimal.)

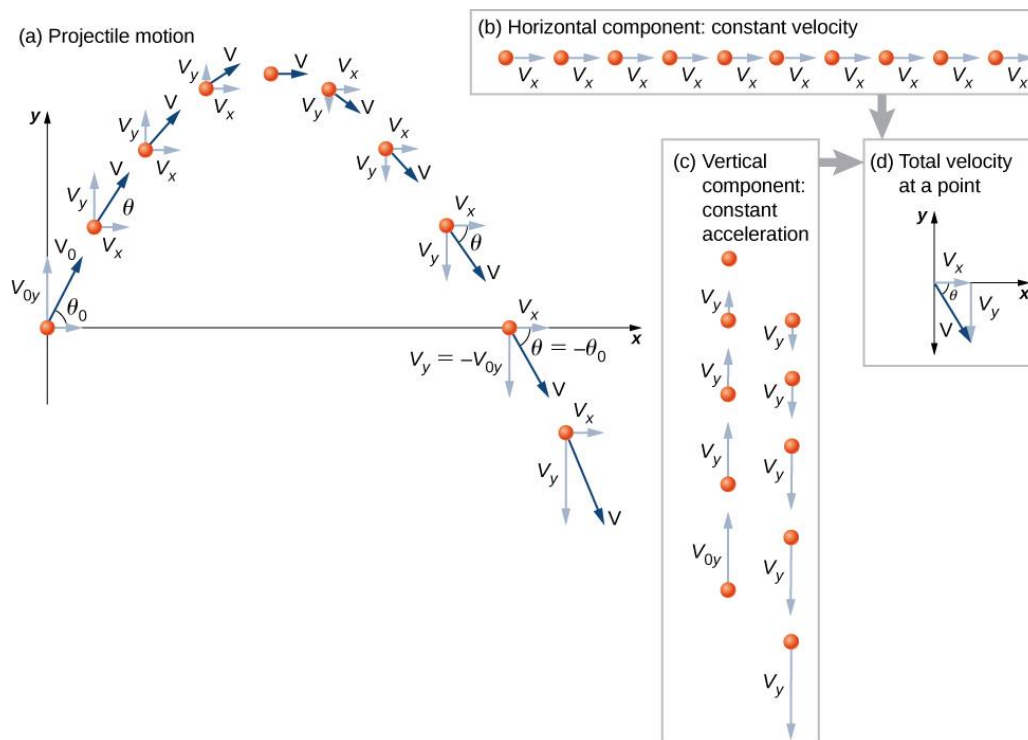


Figure 1: Projectile Motion

If we go for watching any Football, Cricket game, we can notice that the ball first bounces off the ground and returns to the ground in a curved path. This kind of curvature is called transverse motion and the motion path is called Projection. It's a parabola and two-dimensional motion.

**The variables that affect the "ball in the air"**

A football's motion when in the air is influenced by three things:

1. The speed at which it departs the footballer's foot
2. The kicking direction's angle with the horizontal
3. The football's rotation

The ball is moved up and horizontally by the velocity's vertical and horizontal components, respectively. To put it another way, the football turns into a projectile. The horizontal acceleration is practically zero because there is no horizontal force, other from the negligible air resistance. However, the ball accelerates vertically until it reaches the top before decelerating vertically again under the power of gravity.

For this problem the approximation figure is,

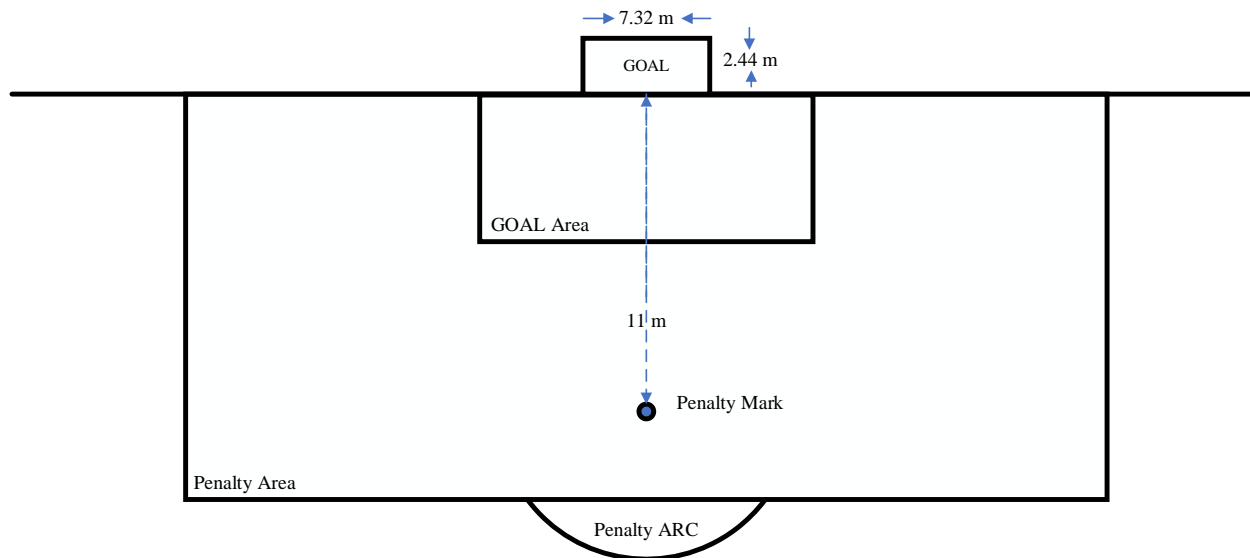


Figure 2: Penalty Box in Football with the data of Problem B

In here we assumed different types of spins and their corresponding characteristics. Those are given below,

## 4.2 VELOCITY MEASUREMENT WITH CONSIDERING NO SPINS

For here,

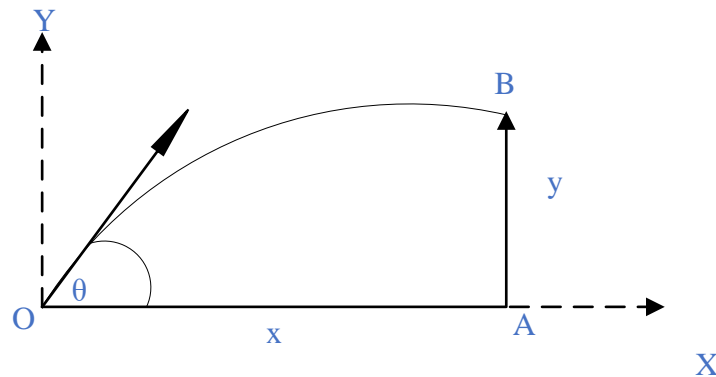


Figure 3: Projectile Motion

Let, a ball is kicked from point O with an angle  $\theta$  and primary velocity  $V_o$ . In here we can divide this velocity into two velocity components.

One velocity component is towards to X axis (OA) which is  $V_{xo} = V_o \cos \theta$

Another velocity component is toward to Y axis which is  $V_{yo} = V_o \sin \theta$

Acceleration in both x axis and y axis,

- $a_x = 0$
- $a_y = -g \sin \theta = -g \sin 90^\circ = -g$

Distance in x axis,

- $x = V_{xo} t$
- $x = (V_o \cos \theta) t$
- $t = \frac{x}{V_o \cos \theta}$

Distance in y axis,

- $y = V_{yo} t - \frac{1}{2} g t^2$
- $y = (V_o \sin \theta) t - \frac{1}{2} g t^2$

and we put t in that equation,

- $y = (V_o \sin \theta) * \left( \frac{x}{V_o \cos \theta} \right) - \frac{1}{2} g \left( \frac{x}{V_o \cos \theta} \right)^2$
- $y = (x \tan \theta) - \left( \frac{g x^2}{2 V_o^2 \cos^2 \theta} \right)$ ; which is similar to a parabolic equation.

From this  $V_o = \pm \frac{1}{\cos\theta} \sqrt{\frac{gx^2}{2(x\tan\theta - y)}}$

For this equation, we have used only positive value because we want to use the forward velocity which is toward the goal. So, the equation for this condition,

$$V_o = \frac{1}{\cos\theta} \sqrt{\frac{gx^2}{2(x\tan\theta - y)}}$$

### 4.3 VELOCITY MEASUREMENT WITH CONSIDERING SPINS

When a football is kicked into the air, it transforms into a projectile and flies in a parabolic trajectory. Football players use free kicks and crosses to move the ball across the field and over opponents. Gravity and air resistance are the only two forces acting on the ball when it is in the air. The ball will arc and fall back to the earth since gravity is always pulling something downward. The ball's velocity in the x direction decreases as it travels through the air due to air resistance slowing it down.

If we notice during kicking any football, we can see that the shooter makes a curve via the ball to make a perfect goal. This is mainly caused by the Magnus effect. So, for making a perfect goal avoiding the goalkeeper, we have to know the concept of Magnus effect. And also, for finding the spin we have to go through the concept of Bernoulli effect. Here it is discussed shortly.

#### 4.3.1 Magnus Effect

Magnus effect is defined as a phenomenon when there is relative motion between the spinning body and the fluid and is mostly characterized by a spinning object traveling through a fluid (gas or liquid). When the Magnus effect occurs, the spinning object's path is typically deflected in an entirely different direction than when it is not spinning. The differential in fluid pressure on the two sides of the rotating item can be used to explain the deflection that takes place.

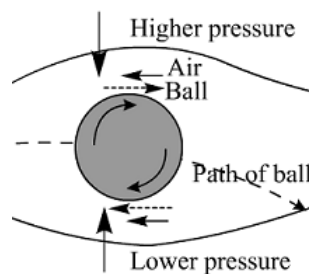


Figure 4: Magnus Effect on a Ball

In layman's terms, a spinning item experiences the development of a sideways force. The Magnus effect is significantly impacted by rotational speed, or we might say that it is dependent on it. The Magnus effect is responsible for both the movement of the cricket ball during traditional swing bowling and a football player's ability to bend the ball into the goal around a 5-person wall.



#### 4.3.2 Bernoulli's Effect – Spinning ball in an airflow

The conservation of energy concept as it applies to moving fluids can be expressed as Bernoulli's equation. One of the most significant and practical equations in fluid mechanics. It relates pressure and speed in an invisible, incompressible flow.

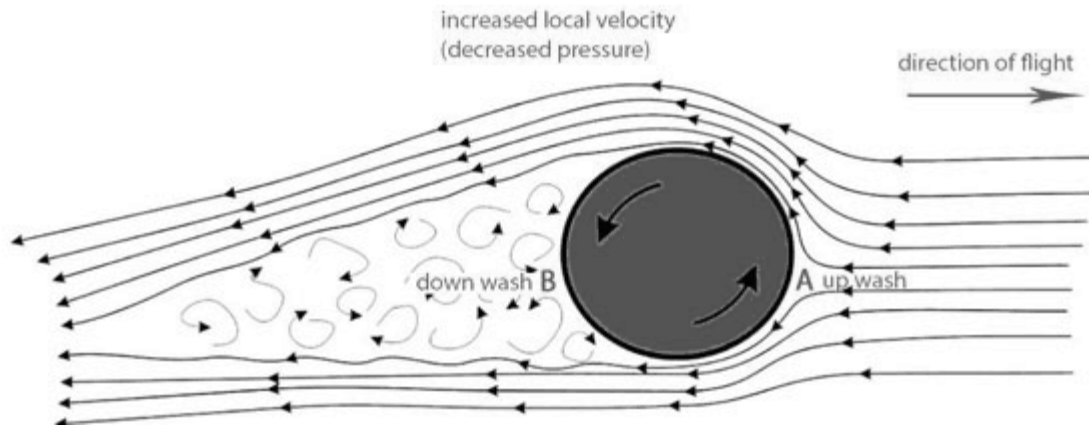


Figure 5: Bernoulli's Effect – Spinning ball in an airflow

According to Bernoulli's principle, faster-moving air exerts less pressure, and therefore the air must exert an upward force on the ball. In fact, in this case, the use of Bernoulli's principle may not be correct. Bernoulli's principle assumes incompressibility of the air, but in reality, the air is easily compressible. But there are more limitations of explanations based on Bernoulli's principle.

A free kick is one of the most thrilling plays in Football. By giving the ball a spin, players may frequently bend the ball's trajectory into the goal. This phenomenon, which football players refer to as “bending” is brought on by aerodynamic forces acting on the ball. Turning an air flow is all that is required to produce lift. A spinning ball turns a flow just like a wing's airfoil does. The specifics of how the power is created are somewhat intricate. The ball is deflected along its flight path as a result of the force acting on it. We have a constant force that is always operating perpendicular to the flow direction (at a right angle), if we ignore the viscous forces on the ball, which slow it down and vary the magnitude and direction of the force. An arc-shaped flight path is what happens as a result.

For spin there are some points to make spins, those are figured below,

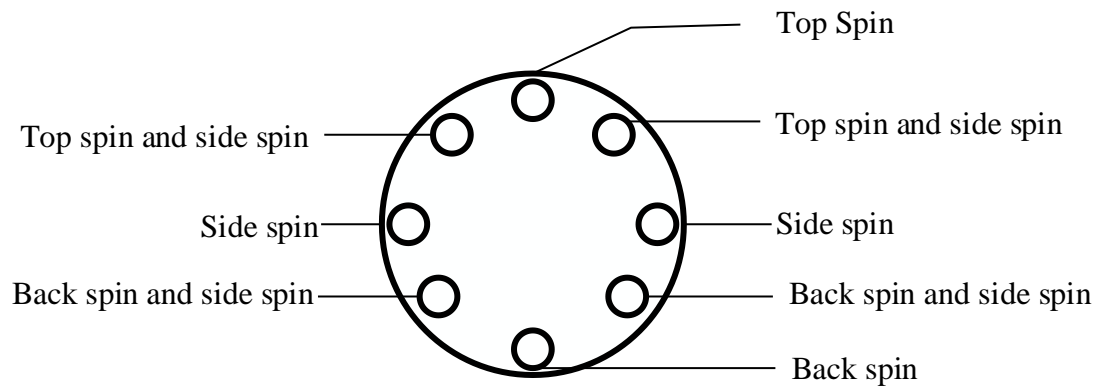


Figure 6: Spin point Kick

For bending a football,

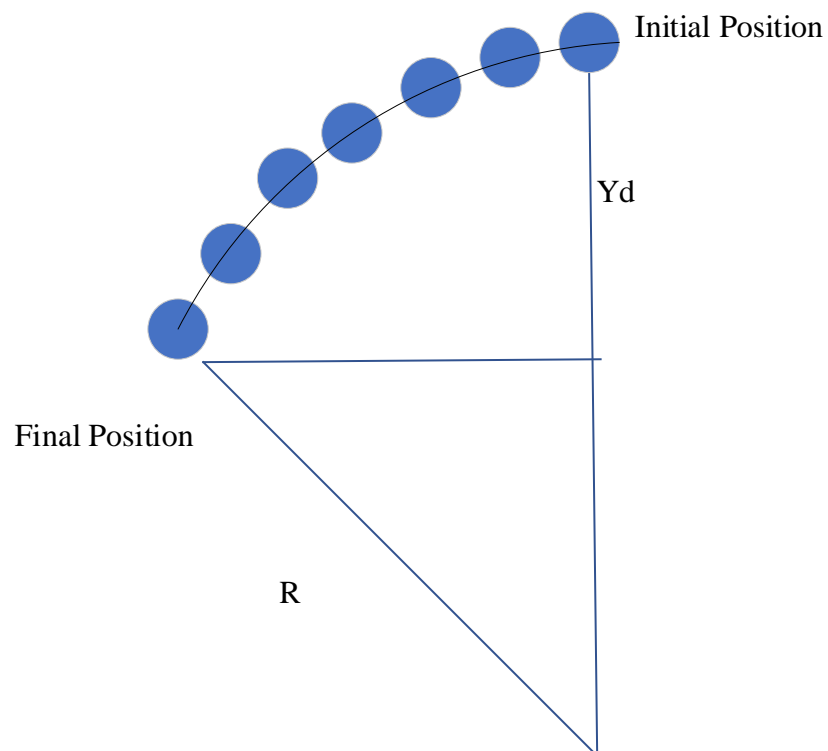


Figure 7: Bending of a football

Using the force on a spinning ball and the mass  $m$  of the ball, Newton's second rule of motion allows us to calculate this acceleration.

$$a = F / m$$

The magnitude of the force  $F$  depends on the radius of the ball  $b$ , the spin of the ball  $s$ , the velocity  $V$  of the kick, the density  $r$  of the air, and an experimentally determined lift coefficient  $Cl$ .

$$F = \frac{16 Cl r s V \pi^2 b^3}{3}$$

Where,

$F$  = force

$Cl$  = lift coefficient

$b$  = radius

$V$  = velocity of the path

$s$  = spin of the ball

$r$  = density of air

From this equation we can find out the spin of ball itself. By,

$$S = \frac{3F}{16 Cl r V \pi^2 b^3}$$

## 5 CALCULATIONS

### 5.1 VELOCITY MEASUREMENT WITH CONSIDERING NO SPINS

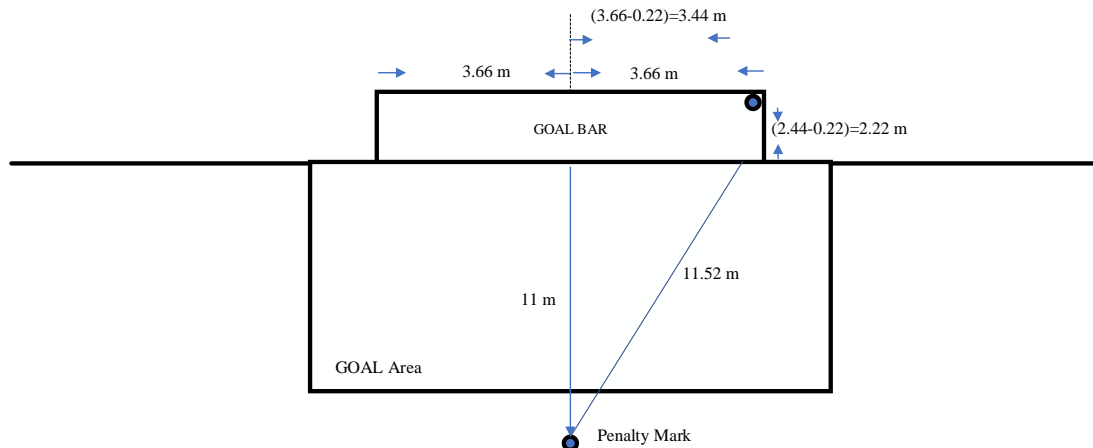


Figure 8: Calculation of Horizontal Axis

In here, we have calculated the projectile and velocity as there is no spin calculations for left upper corner. But for the right upper corner a player just needs to kick the opposite side of the ball.

As we have assumed the diameter of 22 cm. So, if a player wants to make a successful goal, then the goal would go under the goal bar and side bar. As the ball has its own diameter so for projectile,

Vertical distance is  $y = 2.44 - 0.22$  m

$$= 2.22 \text{ m}$$

The football must go maximum this height. Because if the ball passes through 2.44 meter or higher then it won't consider as a goal and if accurate 2.44 meter may cause hit in the goal bar, it can also make difficulties to make goal. So, we eliminate the diameter of the football which is commonly 0.22 meter.

Horizontal distance is  $x = \sqrt{11^2 + ((7.32/2) - 0.22)^2}$

$$= 11.52 \text{ m}$$

For horizontal axis, we have used the upper left corner, we have calculated the horizontal axis by the theory of Pythagoras Theorem. As penalty mark is middle of the goal so we have considered the two variable of Pythagoras theorem as 11 m of height and  $(3.66 - 0.22)$  or 3.44 meter as width. We have eliminated the diameter of common football as we have eliminated in height measurement.

As we have used the equation of  $V_0 = \frac{1}{\cos\theta} \sqrt{\frac{gx^2}{2(x\tan\theta - y)}}$

$$V_0 = \frac{1}{\cos 45} \sqrt{\frac{9.8 * 11.52^2}{2(11.52 \tan 45 - 2.22)}}$$

$$= 11.89 \text{ ms}^{-1}$$

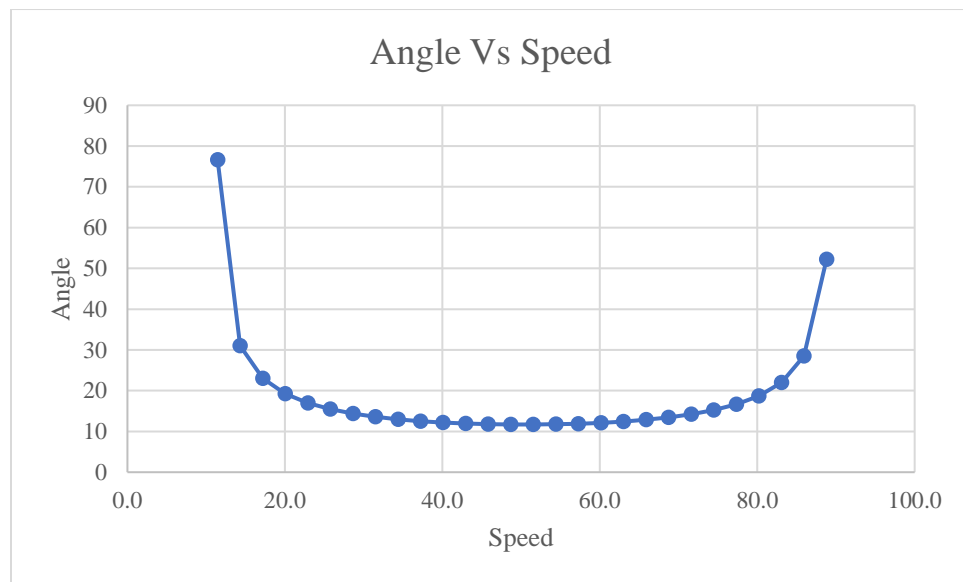
So, for different angle there are different velocity,

## 5.2 DATA TABLE

Angle	Velocity
11.5	76.7
14.3	31.0
17.2	23.0
20.1	19.3
22.9	17.0
25.8	15.5
28.6	14.4
31.5	13.6
34.4	13.0
37.2	12.5
40.1	12.2
43.0	11.9
45.8	11.8
48.7	11.7
51.6	11.7
54.4	11.8
57.3	11.9
60.2	12.1
63.0	12.4
65.9	12.9
68.8	13.4
71.6	14.2
74.5	15.2
77.3	16.6
80.2	18.7
83.1	22.0
85.9	28.5
88.8	52.2

Data Table 1: Data for different angle for different velocity

Graph:



Graph 1: Angle vs Speed Graph

### 5.3 VELOCITY MEASUREMENT WITH CONSIDERING SPINS

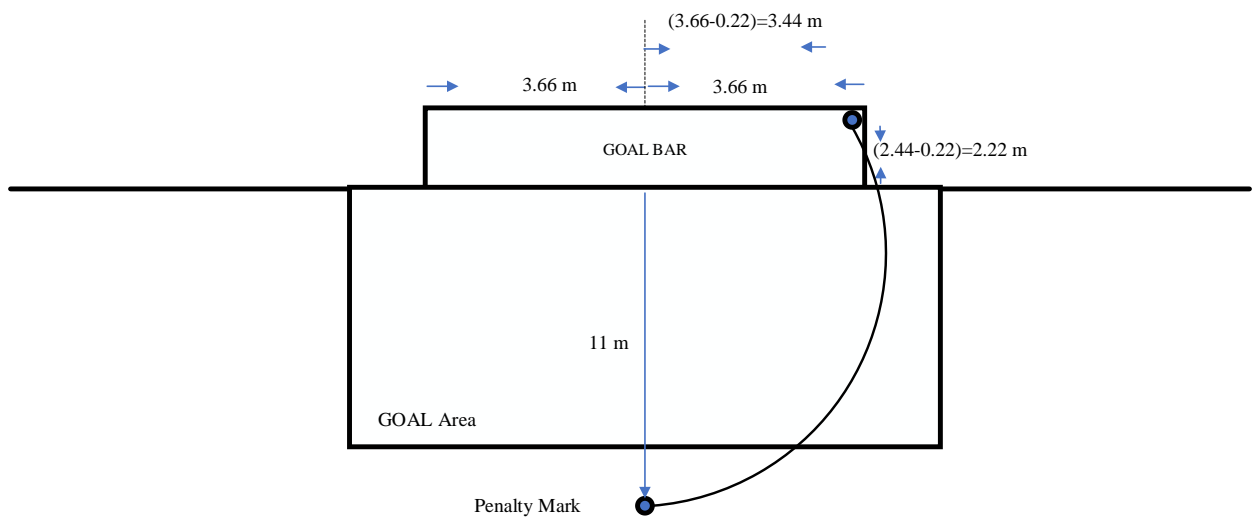


Figure 9: Calculation of Horizontal Axis considering spin

For CL calculation,

(As an experimental and calculated graph from the Reference)

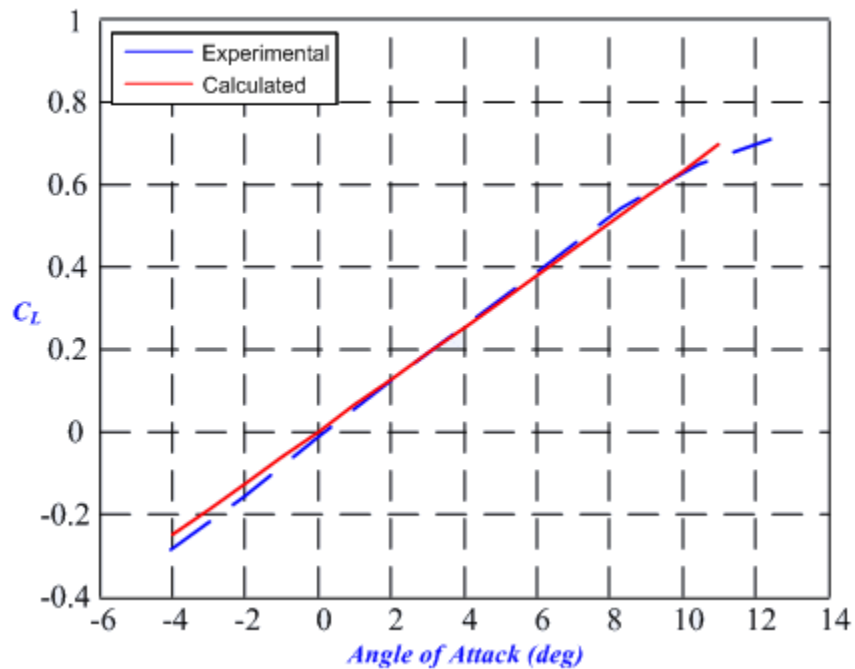


Figure 10: Experimental Data plot

In the lift coefficient,

The slop is  $\tan\theta = .4/6$

$$= 0.0667$$

So, the lift coefficient will be  $= \text{attacking angle} * 0.0667$

$$C_l = 45^\circ * 0.0667 = 0.7853 \text{ rad} * 0.0667 = 0.052$$

We know the volume of a football  $= (4/3) * \pi r^3$

$$= 0.05 \text{ m}^3$$

As we know the equation,

$$S = \frac{3F}{16 C_l r V \pi^2 b^3}$$

$$S = \frac{3 * 0.45 * 9.8}{16 * 0.052 * 1.23 * 0.05 * \pi^2 * 0.11^3}$$

$$= 1968.265 \text{ rpm}$$

In here, we have calculated the projectile and velocity as there is no spin calculations for left upper corner. But for the right upper corner a player just needs to kick the opposite side of the ball.

For finding the second problem solution that is the initial ball velocity and spin should the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal, we will go through 3 stages are given below.

### Stage-I: Ball is going in anticlockwise direction

Here the ball tends to move from lower pressure area to high pressure area. Thus, the ball will go to the left side and we can call it left spin.

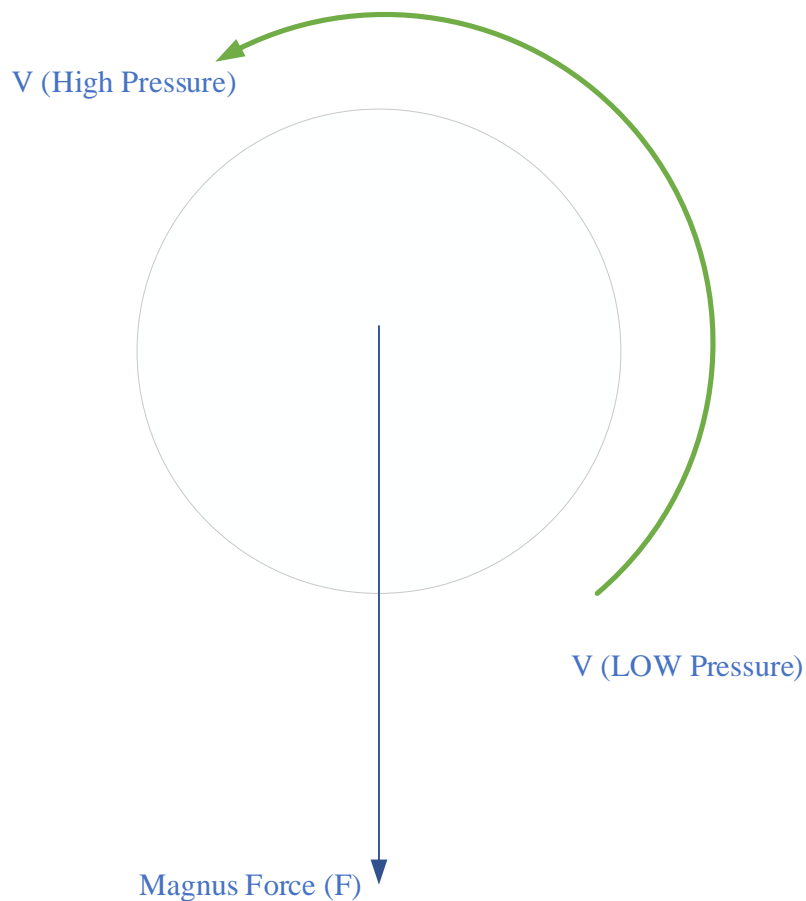


Figure 11: Magnus Force in anticlockwise rotation



**Stage -II: Ball is going in clockwise direction**

Here the ball tends to move from lower pressure area to high pressure area. Thus, the ball will go to the right side and we can call it right spin.

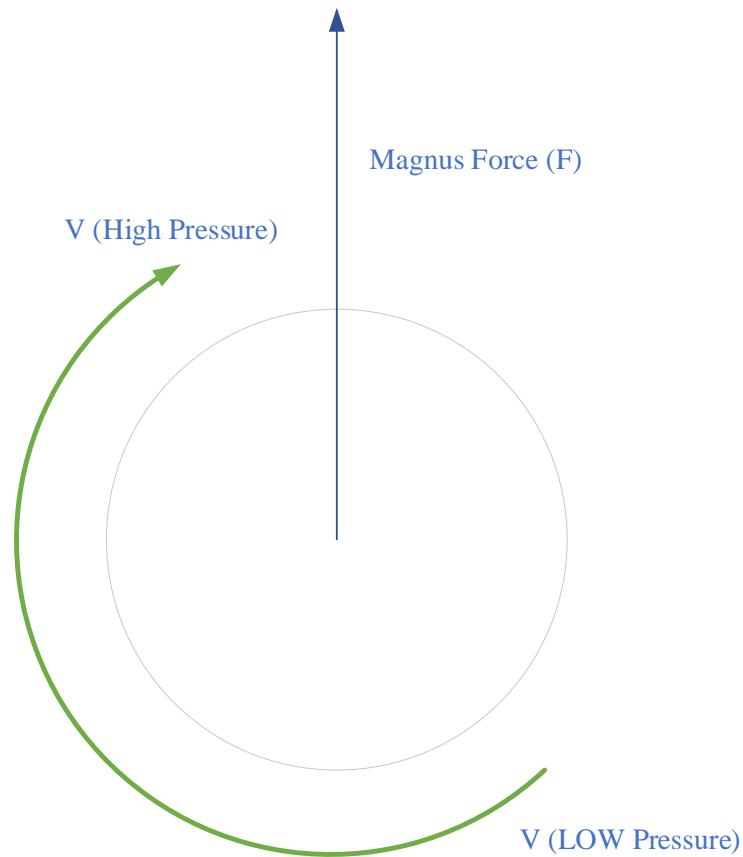


Figure 12: Magnus Force in anticlockwise rotation

### Stage III: Ball is going in straight forward direction

It may be seemed surprised but sometimes kicking the ball straight may be the best chance to avoid goalkeeper and make goal.

Then the initial velocity calculation will be like below,

Here no spin will be in consideration means spin = 0.

$$V_o = \frac{1}{\cos\theta} \sqrt{\frac{gx^2}{2(x\tan\theta - y)}}$$

$$V_0 = \frac{1}{\cos 45} \sqrt{\frac{9.8 * 11^2}{2(11 \tan 45 - 2.22)}}$$

$$= 11.62 \text{ ms}^{-1}$$

### Stage IV:

Here the shooter kick the ball by rolling it on the ground that means the ball won't create any angle to the ground. Thus we apply here general law of motion. This kick may be taken straight , towards right corner, towards left corner , by right spin and left spin.

When the ball go straight, the velocity will be

$$u^2 = v^2 - 2as$$

$$= -2as$$

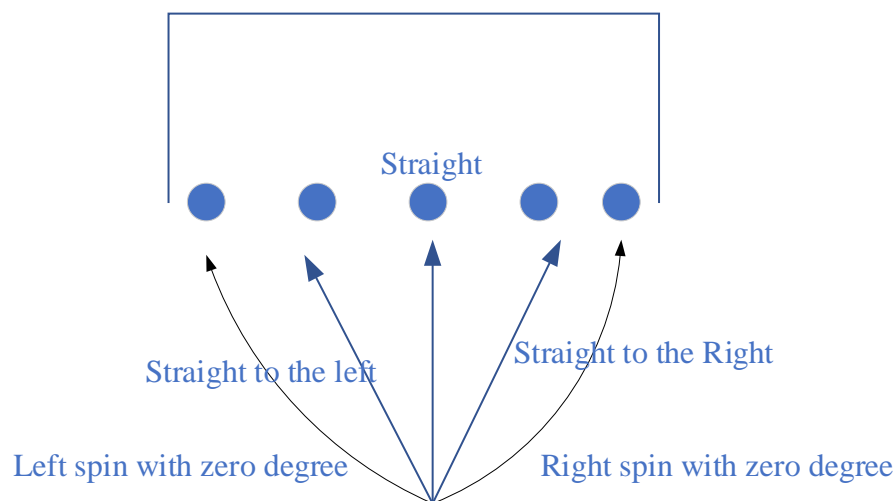


Figure 13: Diagram for case 4

## 6 CODE

---

### 6.1 VELOCITY MEASUREMENT WITH NO SPINS

```
1 clc
2 x=input("Horizontal :");
3 y=input("Vertical :");
4 for i=0:0.05:1.5708
5     v=(1/cos(i))*sqrt((9.8*x*x)/(2*(x*tan(i)-y)));
6     an=(i*180)/3.1416;
7     disp(an);
8     disp(v);
9     disp("    ");
10 end
```

## 7 CONCLUSION

---

### 7.1 VELOCITY MEASUREMENT WITH NO SPINS

For this problem, firstly we have calculated initial velocity without any spin consideration. We have seen that lower angle and higher angle thrown requires higher velocity. In middle ranged angle which is between 0 to 90 degrees, the initial velocity requirement is lower. So, for better chance of goal a player needs to hit the ball at medium ranged angle.

### 7.2 VELOCITY MEASUREMENT WITH CONSIDERING SPINS

For this problem, secondly, we have calculated initial velocity with spin consideration. For this we have found out the spin required to make a successful goal. In here we have found the spin in rpm which is rotating on its own axis. So, for the better chances of goal, spin calculation is necessary to avoid the goal keeper.

In conclusion we can say, there are many ways to attempt to have the best chance of avoiding the goalkeeper and making the goal and it's difficult to find the best possibilities. Playing Football is an art, and the player sometimes play the game with his artistic skill. Sometimes a player can do miracle.

## 8 REFERENCE

---

1. <https://openstax.org/books/university-physics-volume-1/pages/4-3-projectile-motion>
2. <https://www.grc.nasa.gov/www/k-12/airplane/straj.html>
3. [https://link.springer.com/chapter/10.1007/978-3-642-97619-3\\_2#:~:text=The%20ball%20is%20spinning%20with,is%20an%20angle%20of%20rotation](https://link.springer.com/chapter/10.1007/978-3-642-97619-3_2#:~:text=The%20ball%20is%20spinning%20with,is%20an%20angle%20of%20rotation)
4. <https://www.nuclear-power.com/nuclear-engineering/fluid-dynamics/bernoullis-equation-bernoullis-principle/spinning-ball-bernoullis-effect/?fbclid=IwAR1B-ah5t5yLbPZvwKrqDP1pC4li2ReRTU6FaEQ4udkqgLW4VPExQgRphf4>
5. <https://www.vivaxsolutions.com/%20a/the-physics-of-football/>
6. [http://ffden-2.phys.uaf.edu/webproj/211\\_fall\\_2014/Stephen\\_Ringle/stephen\\_ringle/ProjectileMotion.html](http://ffden-2.phys.uaf.edu/webproj/211_fall_2014/Stephen_Ringle/stephen_ringle/ProjectileMotion.html)