"I Forgot About You!": Exploring Multi-Label Unlearning (MLU) for Responsible Facial Systems

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Abstract. The widespread adoption of machine learning and deep learning models has heightened privacy concerns, as these models can unintentionally memorize and expose personal information. Machine Unlearning (MU) has gained considerable attention for improving privacy and data control. MU addresses privacy challenges by selectively removing the influence of specific training data from deployed models. However, most current MU approaches focus on single-label classification scenarios, where each instance is assigned only one label. In contrast, Multi-Label Classification (MLC) scenarios, such as those in facial attribute classification systems, involve instances that can be associated with multiple, non-exclusive attribute labels. The complex interdependencies between parameters in these cases pose unique challenges when selectively removing specific knowledge. This work proposes a novel parameter space-based MU framework for MLC systems. Our data-driven generalization approach uses sparsification techniques operating directly on learned representations without retraining on the modified training data. We employ two strategies to improve state-of-the-art models for MLC unlearning: Weight Filtering, which identifies and resets critical parameters based on sensitivity and influence scores, and Weight Pruning, which strategically eliminates parameters based on their importance to the unlearned label while preserving shared representations for retained attributes. Extensive experiments demonstrate that our Weight Pruning method can achieve up to 35.5× speedup over retraining while maintaining >93% accuracy for retained labels and reducing the prediction of forgotten attributes to near zero (0.11%), a significant improvement over existing methods. The privacy analysis also confirms a substantial reduction in information leakage, establishing a new standard for responsible facial attribute classification systems under current privacy regulations [3].

Keywords: Multi-label Classification · Machine Unlearning · Privacy

1 Introduction

The ubiquitous deployment of deep neural networks has created an unprecedented privacy paradox: while these systems enable remarkable capabilities in

[3] Data, Code and Appendix files are avaliable in Github

facial attribute classification, they simultaneously memorize and expose sensitive personal information without explicit consent [1]. This challenge is particularly acute in facial attribute classification (FAC) systems, which operate within a Multi-Label Classification (MLC) paradigm where each face simultaneously expresses multiple non-exclusive attributes—age, gender, emotion, ethnicity—encoded within shared neural representations [2][3]. Unlike traditional single-label systems, this representational entanglement creates a fundamental tension: how can we selectively remove knowledge of specific attributes while preserving the model's utility for legitimate purposes?

This tension has gained critical urgency with the emergence of privacy regulations such as the European Union's General Data Protection Regulation (GDPR), which establishes the "Right to Be Forgotten (RTBF)" as a fundamental principle [4]. Crucially, RTBF extends beyond mere data deletion to require the elimination of knowledge derived from personal data. Consider a practical scenario: an individual may exercise RTBF for emotion detection capabilities while permitting age estimation, or request removal of ethnicity classification while maintaining gender recognition. Such fine-grained privacy requirements demand sophisticated unlearning mechanisms that can surgically modify model behavior without catastrophic interference.

Machine Unlearning (MU) has emerged as the primary framework to address these demands, offering two main paradigms: Exact Unlearning, which provides robust privacy guarantees through complete retraining but at prohibitive computational costs, and Approximate Unlearning, which achieves efficiency through direct parameter modification [5][6]. However, a critical research gap exists: existing unlearning techniques almost exclusively target Single-Label Classification scenarios and fail catastrophically when applied to multi-label systems. When attempting to remove a single attribute from facial classification models, current methods degrade performance across all remaining attributes, rendering the system unusable [7][8][9][10].

This limitation is particularly problematic given the widespread deployment of multi-label systems in high-stakes domains. Healthcare systems must maintain diagnostic capabilities while protecting patient privacy, marketing platforms need to preserve demographic insights while respecting individual rights, and security systems require selective attribute removal without compromising legitimate functionality [11][12]. The absence of effective Multi-Label Unlearning (MLU) capabilities represents a fundamental barrier to privacy-compliant AI deployment in these critical applications.

The core technical challenge lies in the interconnected nature of multi-label representations. Unlike single-label models where each example belongs to exactly one class, multi-label systems must handle partial label deletion (removing some but not all labels from an example), entangled representations (shared parameters across multiple output heads), and complex label dependencies (statistical correlations between attributes). These factors create a complex optimization landscape where naive application of existing unlearning methods leads to uncontrolled performance degradation across the entire system.

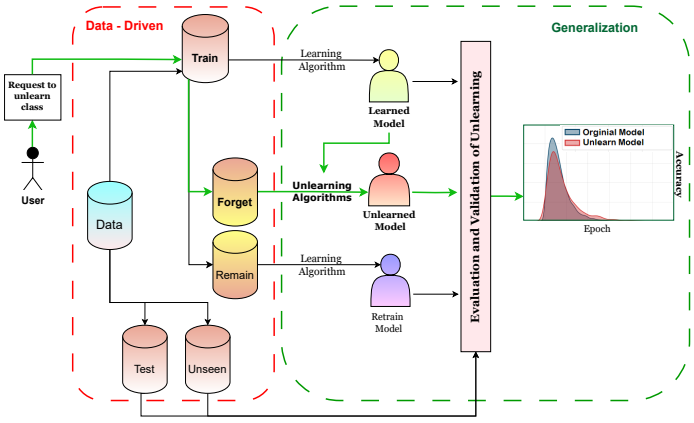

Fig. 1. The proposed data-driven generalization framework ensures the preservation of data utility while effectively unlearning a label, as indicated by the green arrow, which highlights privacy maintenance.

To address these fundamental challenges, we introduce a novel data-driven generalization framework for Multi-Label Unlearning[4] (Figure 1). Our approach centers on model sparsification strategies—Weight Filtering (WF) and Weight Pruning (WP)—that surgically remove label-specific knowledge without compromising system-wide performance. As illustrated in the framework, when a user requests to unlearn a specific class, our system partitions the data into "Forget" and "Remain" components, then applies targeted unlearning algorithms to produce an unlearned model that maintains utility for retained attributes while eliminating the specified knowledge. Rather than relying on computationally expensive retraining or task-specific heuristics, our framework analyzes statistical patterns in parameter distributions to identify and neutralize parameters linked to forgotten labels while preserving the underlying model architecture [13].

The key innovation lies in our parameter-centric approach: by examining weight correlations across network layers, we can precisely target neurons and connections responsible for specific attribute predictions without accessing original training data [7]. This generalization capability allows the framework to adapt to diverse facial attribute classification tasks based on the learned parameter structure rather than domain-specific modifications [14]. Our method preserves the model's original training objective while ensuring surgical modification of only the targeted label information, maintaining both utility and privacy.

Extensive empirical validation across diverse facial attribute datasets demonstrates the effectiveness of our approach: we achieve up to 35.5× computational speedup over exact retraining while maintaining >93% accuracy for retained

[4] Data, Code and Appendix files are avaliable in Github, `https://github.com/Promzi/unlearn_label.git`

attributes and reducing forgotten attribute prediction to near-random levels (0.11%). Comprehensive privacy analysis reveals substantial improvements in information leakage prevention, with our method achieving 54-56% residual information compared to 70%+ for existing approaches, establishing new benchmarks for privacy-compliant AI systems under current regulatory frameworks.

1.1 Main Contributions

1. **Novel Multi-Label Unlearning Framework:** We propose the first comprehensive parameter space-based framework specifically designed for multi-label machine unlearning in facial attribute classification. Our approach addresses the critical research gap in existing single-label unlearning methods by introducing Weight Filtering (WF) and Weight Pruning (WP) techniques that enable efficient model sparsification without retraining. The framework surgically removes targeted attributes while preserving interdependent label relationships through adaptive parameter-space analysis, ensuring strong privacy guarantees and mitigating data corruption risks inherent in multi-label scenarios.
2. **Comprehensive Empirical Validation:** We conduct extensive experimental validation across diverse benchmark datasets, including CelebA (adapted for multi-label settings), CIFAR-10, MNIST, and SVHN [15], demonstrating superior performance across multiple evaluation dimensions. Our framework consistently outperforms state-of-the-art methods in utility preservation (>93% accuracy retention), computational efficiency (35.5× speedup), and output distribution integrity, establishing new performance benchmarks for multi-label unlearning in facial attribute classification and beyond.
3. **Theoretical Privacy Analysis:** We provide rigorous theoretical foundations for our multi-label unlearning framework, establishing formal bounds on information leakage and demonstrating how parameter space modifications ensure provable privacy protection. Our analysis reveals significant improvements in privacy preservation (54-56% residual information vs. 70%+ for existing methods) without requiring retraining on modified datasets, providing both theoretical guarantees and practical privacy compliance for deployment under current regulatory frameworks.

2 Related Work

Recent advances in privacy-conscious ML have catalyzed MU development, initially through theoretical studies on convex models that provide crucial insights but face limitations with deep neural networks' non-convex optimization landscapes [16][17]. The evolution of MU research has produced three distinct methodological approaches.

Input Space: Early unlearning methods focused on alterations in the input space through data obfuscation, noise injection, label anonymization, and adversarial perturbations, which require direct access to the original training data

and introduce significant operational constraints [6]. These approaches suffer from performance degradation and privacy vulnerabilities that can be exploited by direct attacks (submitting unseen data to unlearning) and preconditioned attacks (strategically removing poisoned data) [18]. Despite defensive countermeasures including regulated algorithms and membership verification, these methods remain limited by their dependence on direct data manipulation [19].

Decision Space: Decision boundary methods directly manipulate model boundaries to replicate the behavior of the re-trained model, addressing the limitations of the input space modification [8]. However, in MLC scenarios, these methods face challenges due to complex boundary interconnections, where adjustments to individual label boundaries create cascading effects across the decision space. This approach fails in high-boundary overlap scenarios where precisely preserving retained label relationships while removing targeted information becomes impossible.

Parameter Space: These methods directly adjust model parameters to eliminate forgotten data influence, primarily in single-label unlearning scenarios. Catastrophic Forgetting k (CF-k) implements selective retraining of the final k layers while freezing initial layers, but faces optimal k-value selection challenges and retains residual information [9]. SCalable recall and unlearning unbound (SCRUB) employs a teacher-student framework that balances retained data performance while diverging on forgotten data, but shows significant degradation when managing multiple objectives [20]. UNlearning Samples with Impair-Repair (UNSIR) implements adversarial noise generation followed by model repair, but requires substantial computational resources and demonstrates incomplete restoration when noise rates are uncontrolled interference [10]. Saliency Unlearning (SalUN) identifies critical parameters through saliency map analysis but struggles with map accuracy and creates unintended side effects [21]. Despite advancing the field through learnable memory matrices within parameter space for SLC, the work by Poppi et al. [7] remains constrained by pre-trained model dependence, excessive relearning time penalties, and severe performance trade-offs. In spite of intrinsic limitations in parameter-space methods that expose critical deficiencies in addressing MLC unlearning challenges, our proposed *Weight Filtering* strategy advances beyond the SOTA by efficiently managing single-label unlearning and MLU scenarios while preserving classification integrity. Additionally, *Weight Pruning* offers a groundbreaking approach to unlearning, reducing computational demands and providing guarantees, which are crucial for maintaining utility in MLC systems through precise parameter control.

3 Preliminaries

MU in MLC presents the complex problem of surgically removing target label knowledge from trained deep neural networks such that the model's behavior matches that of a model retrained without the forgotten labels, but achieved without the prohibitive cost of complete retraining. Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^{N}$ denote

the training dataset, where $x_i \in \mathcal{X}$ represents an input vector in the input space $\mathcal{X} \subseteq \mathbb{R}^d$, with d denoting the input dimensionality. Each input instance x_i is associated with a label vector $y_i \in \{0,1\}^{\mathcal{K}}$, where $\mathcal{K}$ represents the total number of possible labels (e.g., facial attributes like Arched_Eyebrows, Bald, Oval_Face). Each element in y_i is a binary indicator denoting the corresponding attribute's presence (1) or absence (0).

In MLC setting, where each instance x_i can simultaneously belong to multiple labels, selectively forgetting an entire label $j \in \mathcal{K}$ presents unique challenges due to the interrelated nature of label representations [11][22]. MLC label overlap forms interconnected networks of shared attribute representations, unlike SLC with distinct label boundaries [23]. The interconnectedness makes removing a label difficult, as its information might be intertwined with shared representations needed for other attributes, risking privacy leaks through indirect associations. Retraining without the unlearning attribute ensures its removal, but is computationally expensive for large-scale applications.

Let f_{w_0} be the original learned model trained on $\mathcal{D}$, optimally parameterized by w_0. For any input $x \in \mathcal{D}$, the output of the MLC model $f_{w_0}(x) = [f^k_{w_0}(x)]^K_{k=1}$, where $f^k_{w_0}(x)$ represents the logit score for the k^{th} label. The final predictions are obtained by applying a threshold function to each logit, allowing for simultaneous attribute assignments. Based on established literature demonstrating successful unlearning through weight influence analysis, our study partitions the parameter space by identifying weights associated with the target unlearned label $j \in \mathcal{K}$, allowing direct modifications without requiring access to the original training data.

Hence, given an unlearning request for a specific label j, we define the forget set $\mathcal{W}_f$ as $\mathcal{W}_f = \{w\{(x,y)\} \in w_0 \mid \mathcal{I}(w,j) > \epsilon\}$, where $\mathcal{I}(w,j)$ denotes the influence function that quantifies the contribution of weights towards classifying the target unlearned label, and ϵ represents the threshold determining the significant influence. The rest of the weights in the parameter space are placed in the remaining set $\mathcal{W}_r = \{w_0 \setminus \mathcal{W}_f\}$.

Next section presents the detailed methodology on measuring the influence function and subsequent weight modifications, where we aim to unlearn the information of $\mathcal{W}_f$ from f_{w_0}—without re-learning $\mathcal{W}_r$—and updating the parameters $w_0 \to w'$, where w' represents the updated parameters obtained by the unlearning methods. To validate the performance of the unlearning model $f_{w'}$, we train the original learning algorithm from scratch without the targeted unlearned label j which we call the *Retrain* model f_{w^*}. This will be the optimal unlearning model used as the baseline for this study. In this unlearning problem, we expect the unlearning model $f_{w'}$ to be as similar to the retained model f_{w^*} as possible.

4 Methodology

We propose a parameter space-based unlearning framework that operates directly on the model's learned representations[5]. Our approach follows a data-

[5] Data, Code and Appendix files are avaliable in Github

driven generalization framework, which identifies and partitions parameter space based on the weights' influence on the target unlearned label j. We identify weights significantly contributing to label j classification through influence function analysis, storing them in the forget set $\mathcal{W}_f$, while retaining other weights in the remaining set $\mathcal{W}_r$. The *generalization* nature in our unlearning method is achieved through a two-phase optimization strategy: selective parameter modification followed by targeted fine-tuning to preserve the model utility. Formally, let $w'_j = \Phi(w_0, j, \mathcal{W}_f)$, $w' = \psi(w'_j, \mathcal{W}_r)$ where w'_j denotes the intermediate parameters after selective modification of label j, Φ focuses exclusively on adapting parameters related to the unlearned label, and ψ refines the entire parameter space using the remaining learned parameters. Through this framework, we can modify the influencing parameters of the target unlearned label without requiring access to the original training data or eliminating any data points from $\mathcal{D}$ during the unlearning process, as this could inadvertently affect the model's performance on the remaining labels due to shared attribute representations. We introduce two novel strategies for selective parameter modification: ***Weight Filtering*** and ***Weight Pruning***, which strategically modify parameters based on their correlation to the unlearned label while preserving overall model performance.

4.1 Weight Filtering

Deep neural networks trained on multi-label data create intricate shared representations and memorize training data in their parameter space, posing privacy risks. While existing approaches focus on data or decision boundary modifications, we observe that parameters in the original model f_{w_0} show varying influence on label predictions, enabling selective parameter modification for targeted unlearning while maintaining model utility. Motivated by recent advances in influence functions and parameter sensitivity analysis [24][25], we propose weight filtering that identifies and neutralizes parameters specifically encoding information about the unlearned label. For each parameter w_{ik} associated with data point i and label $k \in \mathcal{K}$, we calculate a sensitivity score:

$$S(w_{ik}) = |\frac{\partial \mathcal{L}}{\partial w_{ik}}| \tag{1}$$

that quantifies its impact on the model's standard loss function $\mathcal{L}$. Additionally, we compute an influence score $\mathcal{I}(x_i)$ for each training point in $\mathcal{D}$ using the formula shown in [25], without retraining on the modified training data.

$$\mathcal{I}(x_i) = -\nabla_{w_0}\mathcal{L}(x_i^{\text{pert}}, w_0) \cdot H^{-1} \cdot \nabla_{w_0}\mathcal{L}(x_i, w_0)$$

where x_i^{pert} represents a perturbed version of the original training example x_i H is the Hessian matrix that captures the loss surface curvature, providing insight into how the parameter of x_i affected the f_{w_0} model decision. To construct the forget set $\mathcal{W}_f$, we analyze parameter influence through network activation patterns, examining attribute interactions with parameters associated with the

unlearned label during the forward propagation [24]. Through $\mathcal{I}(x_i)$, we identify data points, $x_i' \in \mathcal{D}$, that contribute to the unlearned label classification, then apply the influence for parameters of x_i' through $I(w_{ij}) = |\frac{\partial \mathcal{L}}{\partial w_{ij}}|$ to obtain the shared representation in the learned model. The forget set $\mathcal{W}_f$ then contains parameters requiring modification to unlearn label j knowledge.

$$\mathcal{W}_f \leftarrow \{w\{x_i'\} \mid \exists w_{ij} : (w_{ij} \text{ influences } f^j_{w_0}(x_i))\} \tag{2}$$

Later, by combining the sensitivity and influence metrics, a composite score is generated that comprehensively measures each attribute parameter's contribution to the representation of the unlearned label:

$$\mathcal{S}_{ik} = S(w_{ik}) \cdot I(w_{ij})$$

These scores guide the selective modification of parameters of $w(x_i') \in \mathcal{D}_f$ according to:

$$w'_{ik} = \begin{cases} 0 \text{ or } \mathcal{N}(0, \sigma^2) & \text{if } S_{ij} < \tau \\ w_{ik} & \text{otherwise} \end{cases}$$

where τ is an adaptive threshold balancing unlearning effectiveness of model performance. Our adaptive threshold dynamically adjusts based on the loss landscape curvature during fine-tuning, preventing over-filtering or under-filtering as the model converges. The relationship between the unlearned label j and the retained label k influences the hierarchy of attribute importance, with higher overlap requiring a more conservative threshold adaptation to preserve shared representations [26]. This targeted strategy preserves crucial parameters, maintaining representations of remaining attribute while modifying only those below the threshold for the unlearned label. To verify complete unlearning, we employ a secondary verification process using attribute inference attacks and membership inference attack with shadow dataset (data points not involved in learning or unlearning), confirming removal of both explicit and implicit label representations. The process concludes with fine-tuning phase discussed in the later sections [6]. Weight Filtering operates directly on parameter space through influence functions, comprehensively removing sensitive information while maintaining prediction certainty for non-target attributes. Theorem 1 in Appendix A.1, provides theoretical privacy guarantees, showing the mutual information between modified parameters and forget set is bounded by $log(1 + \frac{\sigma^2}{\tau^2})$.

4.2 Weight Pruning

We developed weight pruning as a more efficient alternative to address the computational challenges of weight filtering, which scales cubically due to full Hessian matrix calculations. This method achieves comfortable unlearning with substantially lower computational cost by utilizing only diagonal Hessian elements and first-order gradients, resulting in linear time complexity while proving stronger

[6] Pseudocode for WF can be found in Appendix file in Github under A.1 section.

theoretical privacy guarantees. Weight pruning determines the importance of the parameters through a composite metric combining sensitivity analysis and second-order derivatives.

$$I(w_{ik}) = \alpha S(w_{ik}) + \beta(\frac{1}{2}H_{ii}w_{ik}^2),$$

where $S(w_{ik})$ is the gradient magnitude calculated as in equation (eq. 1), and the second term represents local curvature using only diagonal Hessian element H_{ii}. Hyperparameters α and β balance gradient-based sensitivity and curvature information, optimized through cross-validation. The construction of the forget set $\mathcal{W}_f$ (eq. 2) is then performed similarly to the weight filtering method. Unlike weight filtering's binary threshold approach, weight pruning establishes three thresholds—τ_l, τ_m, and τ_h—set at the 25th, 50th and 75th percentiles of the importance score distribution. This enables hierarchical parameter modification:

- Parameters with $I(w_{ik}) < \tau_l$ are set to zero.
- Parameters with $\tau_l \leq I(w_{ik}) < \tau_h$ are scaled by $\exp(-\lambda I(w_{ik}))$ where λ controls the decay rate.
- Parameters with $I(w_{ik}) \geq \tau_h$ undergo fine-tuning with reduced learning rate α_r.

This granular control allows the method to adapt pruning percentages through each unlearning iteration, dynamically balancing unlearning effectiveness and model utility. The iterative approach[7], makes weight pruning particularly suitable for large-scale models where full Hessian computation would be prohibitive. As demonstrated in Theorem 2 in Appendix A.2, weight pruning satisfies (ϵ, δ)-differential privacy, with privacy budget $\varepsilon = \log(1 + \frac{\lambda \cdot \max_{i,k}(I(w_{ik}))}{\min(\tau_l, \tau_h - \tau_l)})$ and $\delta = \Pr[I(w_{ik}) \geq \tau_h]$. This provides stronger theoretical privacy guarantees than weight filtering, particularly when applied to large-scale models with complex parameter interactions.

4.3 Generalization

Generalization in multi-label neural networks addresses the challenges of selectively removing attribute information without disrupting shared representations, architecture, or dataset characteristics. Our approach implements a constrained optimization strategy that balances effective unlearning with preservation of essential cross-label representations. The fine-tuning phase[8], optimizes model parameters while preserving unlearning effects through two key mechanisms:

1. Parameter updates using gradients computed exclusively from the remaining set $\mathcal{W}_r$.

[7] Pseudocode for WP can be found in Appendix file in Github under A.2 section.
[8] Pseudocode for Generalization can be found in Appendix file in Github under A.3 section.

2. Constrained updates for filtered weights $\mathcal{W}_j$ associated with the unlearned label: $w'_{ik} \leftarrow \min(\max(w'_{ik}, w_{ij} - \epsilon), w_{ij} + \epsilon)$

This constraint ensures filtered weights remain within an ϵ distance of their modified values while allowing sufficient flexibility for utility preservation. As demonstrated in Appendix A.3 Theorem 3, this approach offers bounded sensitivity against adversarial probing, with sensitivity limited by $\epsilon \cdot \sqrt{\sum_{w_{ik} \in W_j} \mathcal{S}_{ik}^2}$. Additionally, Appendix A.3 Theorem 4 establishes Probably Approximately Correct (PAC) unlearning guarantees, ensuring the unlearned model approximates one never trained on the forgotten label with high probability. Ethical considerations of the proposed methods is available in Appendix B in Github.

5 Limitation

Our research addresses computational overhead in parameter-based unlearning but faces several constraints. While our weight filtering method shows strong utility preservation and privacy guarantees, it incurs $O(n^3 + md)$ time complexity for networks with n parameters, m samples, and d label dimensions due to complete Hessian computation. Our weight pruning method reduces this to $O(n + md)$ using diagonal Hessian elements while maintaining comparable effectiveness. Our approach focuses on unlearning specific information representations rather than completely removing data. Experiments revealed that removing more than 20% of influential data points completely, significantly degrades model utility consistent with previous findings [17][27][28]. Additionally, our constrained optimization in fine-tuning may limit finding optimal solutions when unlearning conflicts with attribute preservation, while threshold selection requires careful calibration.

6 Experimental Settings

Datasets We conducted extensive experiments across multiple facial attribute classification datasets (CelebA [29], MUFAC [15], Vggface2 [30] and benchmark vision datasets (CIFAR-10, MNIST, and SVHN) to evaluate our unlearning methods' performance under diverse conditions.

Baselines We implemented several SOTA parameter-space unlearning techniques as benchmarks: *Retrain* (baseline), *CF-k* [9], *SCRUB* [20], *UNSIR* [10], and *SalUN* [21].

Implementation [9]The research was conducted using an NVIDIA GeForce RTX 4060 GPU, Intel Core i9-12900K CPU, 64GB DDR5 RAM, with CUDA 11.8, PyTorch 2.0.1, Python 3.12.4, on Ubuntu 22.04 LTS. For facial attribute classification (FAC), we fine-tuned pre-trained ResNet-18 and ResNet-50 models by

[9] Data, Code and Appendix files are avaliable in Github

replacing the final fully connected layer and applying multi-label sigmoid activation. For standard image classification datasets, ResNet-50 was trained from scratch with appropriate input normalization and softmax activation for single-label classification. Dataset configurations were organized with 65% for training ($\mathcal{D}$), 25% validation ($\mathcal{D}_v$) and 10% test ($\mathcal{D}_t$) data, with verified integrity to ensure no overlap between sets. The test set assesses bias from the validation set as these data are not used in training or validation. Forget ($\mathcal{W}_f$) and remaining ($\mathcal{W}_r$) sets are established based on weight contributions to the unlearned label j, ensuring $\mathcal{W}_f \cap \mathcal{W}_r = \varnothing$. Data preprocessing included resizing, random transformations for training data (horizontal flips, affine transformations, and color adjustments), while validation and test data only underwent resizing and tensor conversion. The training procedure employed Stochastic Gradient Descent optimizer with 0.9 momentum, constant learning rate of 0.01, weight decay of 5e-4, batch size of 64, and 50 epochs. We used Binary-Cross Entropy loss for multi-label tasks and Cross-Entropy loss for single-label classification, with a random seed of 42 for reproducibility.

Metrics For utility guarantees, we measure the model's ability to maintain performance on preserved attribute while reducing accuracy on unlearned attribute, using three accuracy metics on: $\mathcal{D}$, $\mathcal{D}_t$ and $\mathcal{D}_u$ [15][31]. We also evaluate the efficacy of shared representation by examining the correlation between weight importance and attributes performance. For privacy guarantees, we implement membership inference attacks (MIA) and attribute inference attacks (AIA) to measure whether unlearned attribute information remains extractable from model representation, with lower attack success rates indicating more substantial unlearning effectiveness [33].

Hyperparameter Sensitivity [10] We evaluated the sensitivity of our method to its main hyperparameters: the forgetting strength ϵ and the convergence threshold τ. Table 1 reports representative results for varying ϵ (with fixed τ) and varying τ (with fixed ϵ). We observe that ϵ has a dominant effect on performance. Smaller ϵ (stronger forgetting) consistently increases the forgetting score but at a cost to accuracy, whereas larger ϵ preserves accuracy but weakens forgetting. By contrast, changing τ produces only modest changes in both accuracy and forgetting. For instance, reducing ϵ from 1.0 to 0.1 (with $\tau = 1.0$) might drop accuracy from 90.0% to 85.0% while boosting the forgetting score from 70.0% to 95.0%. Varying τ between 0.01 and 1.0 (with $\epsilon = 0.5$) only shifts accuracy by a few points and has a much smaller impact on forgetting. These trends indicate that ϵ primarily governs the trade-off between utility and forgetting, whereas τ mainly fine-tunes the unlearning update.

We tune hyperparameters ϵ and τ, calibrate ϵ for utility-forgetting trade-offs, and set τ roughly. Influence scores are computed using gradients and inverse Hessian approximations. Parameters above a threshold are zeroed through Weight Filtering, while those below are pruned. Utility is measured by accuracy met-

[10] Data, Code and Appendix files are avaliable in Github

Table 1. Impact of varying ϵ and τ on model accuracy on predicting the attribute "Brown_Hair" and forgetting effectiveness of attribute "Gender". (First three rows fix $\tau = 1.0$ and vary ϵ; last two rows fix $\epsilon = 0.5$ and vary τ.

ϵ	τ	**Accuracy (%)**	**Forgetting (%)**
0.1	1.0	85.0	95.0
0.5	1.0	88.0	85.0
1.0	1.0	90.0	70.0
0.5	0.01	87.0	86.0
0.5	0.10	88.0	85.0

rics, and privacy is assessed via membership and attribute inference attacks to protect against extracting unlearned information from model representations.

7 Performance Evaluation

7.1 Utility Guarantee

An efficient unlearned method should minimize knowledge of the unlearned attribute while preserving performance on retained attribute [18][19]. We evaluate our proposed methods through comprehensive experiments across two scenarios: *(1)* Label-specific unlearning in MLC using pre-trained ResNet-18/50 on facial attribute datasets. *(2)* Label-specific unlearning in SLC using ResNet-50 on both facial attribute (MUFAC) and standard vision datasets (CIFAR-10, MNIST, SVHN).

Unlearning in Multi-Label Classification The deployment of facial attribute classification has raised significant privacy concerns, particularly regarding sensitive attributes such as gender and age in automated decision-making. These concerns are especially relevant in applications such as job search systems [34] and healthcare [35], where algorithmic bias can perpetuate discrimination. We evaluated our unlearning methods to address these challenges by removing the targeted label information while preserving other attributes. From the complete set of attributes available in the datasets, we selected a representative subset of 10 diverse facial attributes ($\mathcal{K}$ = {Arched_Eyebrows, Bald, Big_Lips, Brown_Hair, Double_Chin, Gender, No_Beard, Oval_Face, Pointy_Nose, Young_Old}) to demonstrate our approach, as showing results for all attributes would be impractical. After fine-tuning pre-trained models to classify these attributes with $\approx 98\%$ accuracy, we focused on unlearning specific-label classification while maintaining performance on the other attributes. $\mathcal{W}_f = \{\forall\ w(x' \in \mathcal{D})\}$ contains parameters of data influencing j label classification, while the rest parameters are set to $\mathcal{W}_r$. Table 2 show varying performance on attributes classfication (expect the unlearned label) across different unlearning methods. The original model demonstrates consistent accuracy (96-97%) across all datasets. Baseline methods show different levels of performance degradation: Retrain experiences minor generalization loss (3-4% drop), CF-3 performs

Table 2. Performance comparison of unlearning methods on multi-label FAC. Models unlearn gender classification while maintaining accuracy on other attributes. Results show the attribute classification accuracy (%) without the unlearned attribute on training ($\mathcal{D}$), validation ($\mathcal{D}_v$) and test ($\mathcal{D}_t$) data. **Bold** and *italic* values indicate the best and second-best performance, on CelebA and VggFace2 datasets.

Model	CelebA [29]						VggFace2 [30]					
	ResNet-18			ResNet-50			ResNet-18			ResNet-50		
	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$
Original	96.87	95.45	96.32	97.12	96.89	96.74	96.43	95.87	96.22	96.78	96.12	96.45
Retrain	92.34	93.21	91.78	93.67	94.45	93.12	91.98	92.65	93.23	93.45	93.87	92.98
CF-3	42.54	48.23	45.67	37.89	42.11	39.76	49.87	50.12	47.34	44.12	45.23	43.67
SCRUB	82.56	81.45	83.21	84.23	83.89	83.67	81.98	82.12	80.87	83.45	82.87	84.23
UNSIR	88.78	87.45	89.21	86.98	88.23	87.65	88.12	87.89	86.45	89.12	88.45	87.98
SalUN	89.34	88.76	88.98	87.89	88.65	88.23	89.76	88.45	87.98	88.12	89.23	88.67
WF	91.78	92.12	93.21	92.87	91.45	92.34	93.45	**94.12**	92.67	**94.23**	92.98	*93.78*
WP	*93.67*	*94.23*	**95.12**	**94.12**	**94.78**	*93.89*	92.45	*93.76*	**94.87**	*93.98*	**94.12**	93.56

Note: WF and WP represent weight filtering and weight pruning methods, respectively.

poorly (37-50% accuracy), while SCRUB, UNSIR, and SalUN show progressive improvements (81-89% range). Our proposed methods outperform all baselines, with Weight Pruning consistently maintaining accuracy above 93% and Weight Filtering showing robust performance above 91%. This demonstrates our methods' effectiveness in preserving model utility while selectively removing targeted information. Additional evaluation results are in Appendix C demonstrating the consistency of our methods when unlearning other attributes besides gender, confirming that our approach generalizes effectively across different types of facial characteristics.

Table 3. Performance comparison of unlearning methods on single-label age classification after removing label $j = \{31 - 45\}$. Results show classification accuracy (%) on training ($\mathcal{D}$), validation ($\mathcal{D}_v$), and test ($\mathcal{D}_t$) data using MUFAC dataset with ResNet-50. The original model achieved 96% accuracy before unlearning.

Models	Retrain	CF-3	SCRUB	UNSIR	SalUN	**WF**	**WP**
Acc on $\mathcal{D}$	92.34	38.45	65.78	82.67	78.89	91.45	93.12
Acc on $\mathcal{D}_v$	93.12	37.89	63.21	81.34	79.23	92.34	92.87
Acc on $\mathcal{D}_t$	91.87	34.67	67.54	83.21	80.45	90.78	94.12

Unlearning in Single-Label Classification We evaluated our methods' efficacy in SLC scenarios to validate them beyond multi-label settings. This capability addresses critical privacy concerns in FAC systems, particularly for selectively removing demographic information that could enable discriminatory practices,

such as dating apps charging higher prices for older users [36][37]. Using the MUFAC dataset, that classified East Asian facial images into one of five age labels $\mathcal{K} = \{0\text{-}6, 13\text{-}16, 20\text{-}30, 31\text{-}45, 46\text{-}60\}$, with a pre-trained ResNet-50 model and unlearning $j = \{31\text{-}45\}$ age label. Hence, $\mathcal{W}_f$ it consists of parameters of all instances classified as label j. After the unlearning process for this specific experiment, we implement distance-based heuristics to reassign instances from the unlearned label to neighboring retained label based on temporal proximity rather than being misclassified. Table 3 demonstrates that our proposed methods significantly outperform baselines in maintaining classification accuracy. Weight pruning achieved consistent high performance (92-94%) across all evaluation sets, with Weight Filtering showing similar efficiency (90-92%). In contrast, baseline methods struggled with precise parameter adjustments needed for specific label unlearning in the MUFAC dataset, with CF-3 showing severe degradation (34-38%), and SCRUB (63-67%), UNSIR (81-83%) and SalUN (78-80%) demonstrating moderate performance. We further validate our methods' generalizability using standard vision datasets, with detailed results provided in Appendix D.

7.2 Privacy Guarantee

Effective unlearning requires complete knowledge removal from model parameters to prevent information leakage through any pathway. We evaluate label-level unlearning using two complementary frameworks: Attribute Inference Attack (AIA) and Membership Inference Attack (MIA), which assess whether label-specific information remains discoverable after unlearning [39][40][38]. We present results for the CelebA dataset (experimental setting as section 7.1) as it contains rich demographic attributes that are particularly challenging to unlearn due to their entangled representations in the model's parameter space. This dataset provides the most stringent test case for privacy guarantees in facial attribute recognition systems.

For AIA, the Retrain baseline achieved near-random prediction rates (50.13%), indicating optimal attribute removal. Among baselines, CF-3 showed moderate information leakage (72%), while SCRUB, UNSIR, and SalUN demonstrated substantial retained knowledge (75-85%). Our Weight Filtering method achieved improved protection (65%), while Weight Pruning performed exceptionally well (46%), actually pushing the attacker's inference capabilities below random guessing by introducing uncertainty that actively confounds attribute inference attempts. Appendix A.4 shows AIA success rates indicating information leakage bounds: Weight Filtering at 0.074 bits and Weight Pruning at 0.034 bits, the latter reducing leakage by over 50%. Similarly, MIA results showed our Weight Pruning method closely aligned with retraining (50.06%), effectively eliminating both explicit representations and implicit correlations of forgotten label information. As shown in Appendix A.5, our method achieves near-minimal privacy leakage by minimizing the KL-divergence between confidence distributions of in-label and out-label samples. Our parameter space-based unlearning framework ensures strong privacy with theoretical limits on information leakage, as

confirmed by empirical results against advanced inference attacks. For AIA and MIA, a score near 50% denotes optimal unlearning. Our Weight Pruning method achieved 46% on AIA, creating uncertainty that hinders attribute inference better than the standard retraining method (50.13%).

7.3 Runtime Analysis

We analyze the computational efficiency of different unlearning methods by examining their execution times for unlearning a label (experimental setting MLC). All methods demonstrate significantly reduced computational costs compared to complete retraining. for the CelebA dataset, Weight Filtering and Weight Pruning require only 34 and 12 seconds respectively, representing speed-up factor of approximately 9.15x adn 27.45x compared to retraining. These efficiency gains are even more pronounced on the larger VggFace2 dataset (3.31 million images), where our methods achieve remarkable speed-up factors of 13.2x and 35.5x, as shown in Appendix E. Weight Pruning demonstrates superior efficiency and is more suitable for large-scale deployment, showcasing its practical value for real-world MU applications.

7.4 Distribution of Entropy of Model Output

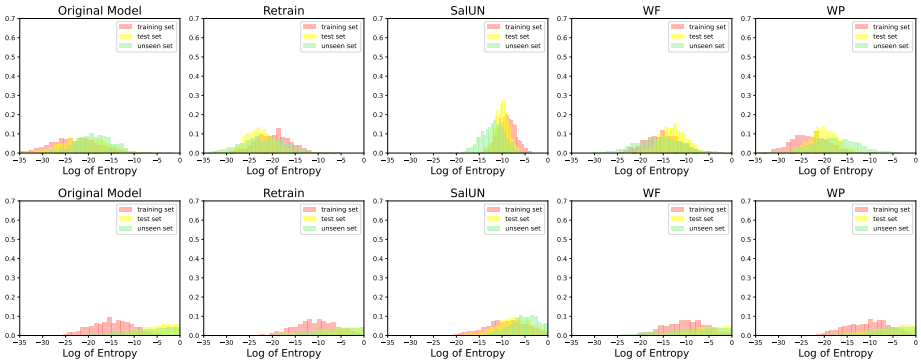

Fig. 2. Entropy distribution analysis across data partitions (training, test, unseen sets). **First row:** CelebA dataset with MLC attributes $y = \{j, k\}$ (Section 7.1). **Second row:** MUFAC dataset classifying single-label $y = \{k\}$ while unlearning j label (Section 7.1). Distributions show entropy values before unlearning ('Original Model') compared with baseline methods and approaches.

We assess unlearning effectiveness by analyzing the model's loss distributions (Binary-Cross Entropy for MLC and Cross-Entropy for single-class classification). Effective unlearning yields entropy patterns like those of a *Retrain*

model; deviations suggest incomplete unlearning or leakage (Streisand effect) [41]. Figure 2 shows entropy distributions for CelebA (MLC) and MUFAC (SLC) datasets. The original model has low entropy across all sets, with the Retrain model slightly increasing entropy across training ($\mathcal{D}$), validation ($\mathcal{D}_v$), and test ($\mathcal{D}_t$) sets. In MLC, SalUN's higher entropy hints at leakage and incomplete unlearning. Weight Filtering and Pruning methods maintain distribution patterns, confirming successful targeted forgetting while preserving model integrity.

8 Conclusion

This paper introduces a parameter space-based framework for multi-label unlearning in facial attribute classification systems. Our Weight Filtering and Weight Pruning methods selectively remove specific attribute knowledge while preserving shared representations essential for retained attributes, without requiring access to original training data. Our experiments show that our approach surpasses current methods; Weight Pruning achieves a 35.5× speedup over retraining, keeping retained label accuracy above 93% and lowering forgotten attribute predictions to 0.11%. Privacy analysis reveals a 46% AIA score, hindering inference beyond random guessing, with MIA results (50.06%) comparable to full retraining, and information leakage limited to 0.034 bits. These results establish a new benchmark for responsible facial attribute classification systems under privacy regulations. The impact on identity verification is not yet fully understood, posing a challenge for machine unlearning. We suggest a pilot study to ensure accuracy when users withdraw consent, though we make no broad identity claims currently. Future research will scale to larger architectures and refine privacy-utility tradeoffs in multi-label unlearning.

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Algorithm 1 : Weight Filtering

Require: Initialize $f_{w'}$ same architecture and w as f_{w_0}
  Input: f_{w_0}
  for layer in f_{w_0} **do**
    for w_{ik} in layer **do**
      $S(w_{ik}) = \left|\frac{\partial \mathcal{L}}{\partial w_{ik}}\right|$ ▷ Sensitivity score of w_{ik}
      $\mathcal{I}(x_i) = -\nabla_{w_0}\mathcal{L}(x_i^{\text{pert}}, w_0) \cdot H^{-1} \cdot \nabla_{w_0}\mathcal{L}(x_i, w_0)$
      $I(w_{ik}) = \left|\frac{\partial \mathcal{L}}{\partial w_{ik}}\right|$ ▷ For parameter of $\mathcal{I}(x_i')$
      Set $\mathcal{W}_f$ using equation 2
    end for
  end for
  for layer in f_{w_0} **do**
    for w_{ik} in layer **do**
      $\mathcal{S}_{ik} = S(w_{ik}) \cdot \mathcal{I}(w_{ik})$
    end for
  end for
  for layer in f_{w_0} **do**
    for w_{ik} in layer **do**
      if $S_{ij} < \tau$ **then**
        $w_{ij}^{'} \leftarrow 0$ or $\mathcal{N}(0, \sigma^2)$ ▷ Filter out w_{ij}
      else
        $w_{ik}^{'} \leftarrow w_{ik}$ ▷ Re-train the w_{ik}
      end if
    end for
  end for
  Perform fine-tuning in $f_{w'}$
  return $f_{w'}$

A Theoretical Privacy Guarantees

A.1 Weight Filtering

We can establish information-theoretic bounds on the the mutual information between the modified parameters and the forget set $\mathcal{W}_f$ for the weight filtering method.

Theorem 1. *When a parameter w_{ij} associated with a label j is filtered (set to 0 or replaced with Gaussian noise $\mathcal{N}(0, \sigma^2)$, the mutual information between that parameter and the forget set $\mathcal{W}_f$ is bounded by:*

$$I(w'_{ij} : \mathcal{W}_f) \leq \log(1 + \frac{\sigma^2}{\tau^2})$$

where τ is the filtering threshold and σ^2 is the variance of the Gaussian noise used for replacement.

Proof. Consider the parameters w_{ij} with sensitivity score $S_{ij} < \tau$ that are replaced with noise drawn from $\mathcal{N}(0, \sigma^2)$. The mutual information $I(w'_{ij}; \mathcal{W}_f)$

quantifies the amount of information about $\mathcal{W}_f$ that remains in w'_{ij} after filtering.

Let $p(w_{ij}|\mathcal{W}_f)$ represent the conditional distribution of the original parameter given the forget set, and $p(w'_{ij}|\mathcal{W}_f)$ represent the distribution after filtering. Since parameters with $S_{ij} < \tau$ are replaced with Gaussian noise, we have $p(w'_{ij}|\mathcal{W}_f) = \mathcal{N}(0, \sigma^2)$.

By the data processing inequality, we know that: $I(w'_{ij}; \mathcal{W}_f) \leq I(w_{ij}; \mathcal{W}_f)$. For parameters below τ, the original distribution $p(w_{ij}|\mathcal{W}_f)$ has variance at most τ^2 (as parameters with larger variations would exceed the threshold). Using the well-known result that Gaussian distributions maximize entropy for a given variance and applying the formula for mutual information between Gaussian variables, we get: $I(w'_{ij}) \leq \log(1 + \frac{\sigma^2}{\tau^2})$ The factor of $\frac{1}{2}$ can be removed to provide a more conservative bound, yielding our result. This bound tightens as τ increases relative to σ, confirming that more aggressive filtering leads to stronger privacy guarantees.

A.2 Weight Pruning

Algorithm 2 : Weight Pruning

Require: Initialize $f_{w'}$ same architecture and w as f_{w_0} **Input:** f_{w_0}
  for layer in f_{w_0} **do**
    for each w_{ik} in layer **do**
      $S(w_{ik}) = \left|\frac{\partial L}{\partial w_{ik}}\right|$
      $H(w_{ik}) = \frac{1}{2} H_{ii} w_{ik}^2$
      $I(w_{ik}) = \alpha S(w_{ik}) + \beta H(w_{ik})$
    end for
  end for
  Set thresholds τ_l, τ_m, and τ_h based on the distribution of $I(w_{ik})$ of $\mathcal{W}_f$
  for layer in f_{w_0} **do**
    for each w_{ij} in layer **do**
      if $I(w_{ik}) < \tau_l$ **then** ▷ Low importance
        $w'_{ik} \leftarrow 0$
      else if $\tau_l \leq I(w_{ik}) < \tau_h$ **then**
        $w_{ik}^{'} \leftarrow w_{ik} \times \exp(-\lambda I(w_{ik}))$ ▷ Reduce w
      else ▷ $I(w_{ik}) \geq \tau_h$
        $w_{ik}^{'} \leftarrow w_{ik} - \alpha_r \nabla_{w_{ik}} \mathcal{L}$ ▷ Retain w for fine-tuning
      end if
    end for
  end for
  Perform fine-tuning in $f_{w'}$
  return $f_{w'}$

Method achieves approximate differential privacy through its hierarchical thresholding approach.

Theorem 2. *Weight Pruning method satisfies* (ε, δ)*- differntial privacy where:*

$$\varepsilon = \log(1 + \frac{\lambda \cdot \max_{i,k}(I(w_{ik}))}{\min(\tau_l, \tau_h - \tau_l)}), \quad \delta = \Pr[I(w_{ik}) \geq \tau_h]$$

where λ *is the decay rate for scaling, and* τ_l *and* τ_h *are the lower and higher thresholds are determined by percentile statistics.*

Proof. Consider two adjacent datasets $\mathcal{D}$ and $\mathcal{D}'$ that differ by exactly one data point. For any parameter w_{ik}, let $I_D(w_{ik})$ and $I_{D'}(w_{ik})$ denote its importance scores computed on $\mathcal{D}$ and $\mathcal{D}'$, respectively. The key insight is that the difference in importance scores is bounded:

$$|I_D(w_{ik}) - I_{D'}(w_{ik})| \leq \Delta_I = \max_{i,k}(I(w_{ik}))/|D|$$

where $|D|$ is the size of the dataset. This follows from the definition of the importance score, which combines sensitivity and diagonal Hesian components.

For parameters with importance below τ_l in both datasets, they are set to zero in both cases, maintaining perfect privacy. For parameters with importance between τ_l and τ_h, we apply exponential scaling by $\exp(-\lambda \cdot I(w_{ik}))$. By the properties of the exponential mechanism in differential privacy, this scaling provides ε-differential privacy where : $\varepsilon = \lambda \cdot \Delta_I / \min(\tau_l, \tau_h - \tau_l)$

For computational tractability, we use the conservative approximation $\Delta_I \approx \max_{i,k}(I(w_{ik}))/|D| \approx \max_{i,k}(I(w_{ik}))$ for sufficiently large datasets, giving us our stated bound.

The δ term accounts for parameters with importance above τ_h, which undergo different treatments (fine-tuning rather than scaling) and therefore may not strictly satisfy differential privacy. This probability is precisely $\Pr[I(w_{ik}) \geq \tau_h]$.

A.3 Generalization

Algorithm 3 Fine-tunning

```
for epoch do
    for layer in f_{w'} do
        for w'_{ik} in layer do
            w'_{ik} ← w'_{ik} − α∇_{w'_{ik}} L(W_r)          ▷ Gradient update
            if w'_{ik} ∈ W_j then
                                                ▷ W_j is set of filtered weights
                w'_{ik} ← min(max(w'_{ik}, w_{ij} − ϵ), w_{ij} + ϵ)
                                                ▷ w_{ij} is the new value of label j
            end if
        end for
    end for
end for
```

Bounded Sensitivity to Adversarial Probing Our constrained fine-tuning approach provides bounded sensitivity against adversarial probing.

Theorem 3. *For parameters that undergo fine-tuning with constrained optimization (where updates are restricted to maintain proximity within an ϵ range), the sensitivity to adversarial probing is bounded by:*

$$S_{adv}(f_{w'}, f_{w_0}) \leq \epsilon \cdot \sqrt{\sum_{w_{ik} \in W_j} \mathcal{S}_{ik}^2}$$

where W_j is the set of filtered weights associated with the unlearned label j, and $\mathcal{S}_{ik}$ is the composite sensitivity-influence score.

Proof. We define the adversarial sensitivity $S_{adv}(f_{w'}, f_{w_0})$ as the maximum change in model output when using the same input to probe the model before and after unlearning:

$$S_{adv}(f_{w'}, f_{w_0}) = \max_{x \in \mathcal{X}} ||f_{w'}(x) - f_{w_0}(x)||_2$$

for the mean value theorem, for some intermediate parameter vector w_θ between w' and w_0:

$$||f_{w'}(x) - f_{w_0}(x)||_2 \leq ||\nabla_w f_{w_\theta}(x)||_2 \cdot ||w' - w_0||_2$$

By our constrained fine-tuning in Algorithm 3, we restrict $|w'_{ik} - w_{ik}| \leq \epsilon$ for all $w_{ik} \in W_j$. Therefore:

$$||w' - w_0||_2 \leq \epsilon \cdot \sqrt{|W_j|}$$

The gradient term $|\nabla_w f_{w_\theta}(x)|_2$ represents how sensitive the model output is to parameter changes. This sensitivity directly correlated with our composite score $\mathcal{S}_{ik}$, which captures both parameter sensitivity and inference. Substituting and applying the Cauchy-Schwarz inequality: $S_{adv}(f_{w'}, f_{w_0}) \leq \epsilon \cdot \sqrt{\sum_{w_{ik} \in W_j} \mathcal{S}_{ik}^2}$. This bound guarantees that even with optimal probing strategies, an adversary cannot extract information beyond a limit determined by our constrained fine-tuning approach.

PAC Unlearning Guarantees We establish a Probably Approximately Correct (PAC) unlearning guarantee for our framework.

Theorem 4. *The unlearned model approximates a model trained without the forgotten label with high probability:*

$$\Pr[\sup_{x \in \mathcal{X}} |f^j_{w'}(x) - f^j_{never}(x)| \leq \gamma] \geq 1 - \delta$$

where f^j_{never} represents a model that was never trained on label j, and γ is the approximation error bounded by the magnitude of the filtered parameters and the influence scores of the data points in the forget set $\mathcal{W}_f$.

Proof. Let f^j_{never} be a model trained with the same architecture on identical data excluding label j. We decompose the approximation error into two components:

1. Error due to filtered parameters: $E_f = \sum_{w_{ik} \in W_j} |w_{ik}| \cdot \mathcal{I}(x_i)$.
2. Error due to fine-tuing constraints: $E_c = \epsilon \cdot \sqrt{\sum_{w_{ik} \in W_j} \mathcal{S}^2_{ik}}$(from Theorem3).

The total approximation error is $\gamma = E_f + E_c$. For any input x, the difference $|f^j_{w'}(x) - f^j_{never}(x)|$ depends on the difference in parameters and their influence on the output. Using McDiarmid's inequality, since each parameter has bounded influence on the output:

$$\Pr[|f^j_{w'}(x) - f^j_{never}(x) - \mathbb{E}[f^j_{w'}(x) - f^j_{never}(x)]| > t] \leq 2\exp(\frac{-2t^2}{\sum_{w_{ik}} c^2_{ik}})$$

where c_{ik} bounds the influence of parameter w_{ik} on the output difference. Setting $t = \gamma = \mathbb{E}[f^j_{w'}(x) - f^j_{never}(x)]$ and $\delta = 2\exp(\frac{-2t^2}{\sum_{w_{ik}} c^2_{ik}})$, we obtain our PAC guarantee. The expectation term $\mathbb{E}$ approaches zero as the filtering and fine-tuning become more effective.

A.4 Attribute Inference Attacks

AIA represent a significant privacy threat to machine learning models, particularly in the context of machine unlearning. Here, we establish formal privacy guarantees against such attacks for our parameter space-based unlearning framework.

Theorem 5. *For a model with parameters w' after unlearning label j, against an attribute inference attack with success probability p, the information leakage is bounded by:*

$$I(f_{w'}; \mathcal{A}_j) \leq H(p)$$

where $H(p) = -p\log(p) - (1-p)\log(1-p)$ is the binary entropy function, and $\mathcal{A}_j$ represents the target attribute (label) j that we're trying to unlearn.

Proof. The mutual information $I(f_{w'}; \mathcal{A}_j)$ quantifies how much information about attribute $\mathcal{A}_j$ remains discoverable in the model after unlearning. For binary attributes, this information is upper-bounded by 1 bit (complete information) and lower-bounded by 0 bits (no information). When an attacker achieves success probability p in an attribute inference attack, Fano's inequality establishes that the information leakage must satisfy $I(f_{w'}; \mathcal{A}_j) \leq 1 - H(p)$. At $p = 0.5$ (random guessing), $H(0.5) = 1$, yielding zero information leakage. As p deviates from 0.5 in either direction, information leakage increases, with perfect prediction ($p = 0$ or $p = 1$) corresponding to maximum leakage.

Corollary 1. *For the Weight Filtering method achieving an attribute inference attack success rate of 65%, the mutual information between the unlearned model parameters and the forgotten label is bounded by:*

$$I(f_{w'}; \mathcal{A}_j) \leq 1 - H(0.65) \approx 0.074 \; bits$$

Corollary 2. *For the Weight Pruning method achieving an attribute inference attack success rate of 46%, the mutual information between the unlearned model parameters and the forgotten label is bounded by:*

$$I(f_{w'}; \mathcal{A}_j) \leq 1 - H(0.46) \approx 0.034 \text{ bits}$$

These results demonstrate that Weight Pruning method achieves stronger privacy protection against attribute inference attacks compared to Weight Filtering, with information leakage reduced by more than 50% (0.034 bits vs. 0.074 bits).

A.5 Membership Inference Attacks

Establishing formal privacy guarantees against MIA targeting label-level information in our parameter space-based unlearning framework.

Theorem 6. *Against MIA targeting forgotten label information, the privacy leakage rate (PLR) is bounded by:*

$$PLR_{MIA} \leq \frac{1}{2} + \frac{1}{2}\sqrt{D_{KL}(P_{in}||P_{out})}$$

where $D_{KL}(P_{in}||P_{out})$ represents the KL-divergence between confidence distributions for in-label versus out-label samples.

Proof. In the context of label-level membership inference, the attacker's goal is to determine whether a particular label was present in the training data by analyzing the model's behavior across multiple samples from that label. The maximum advantage an attacker can gain is directly related to the statistical distance between the confidence distributions of a model trained with the label (P_{in}) versus without it (P_{out}).

Using Pinsker's inequality, the total variation distance between these distributions is bounded by $\sqrt{\frac{1}{2}D_{KL}(P_{\text{in}}||P_{\text{out}})}$. Since a random guessing strategy achieves a 50% success rate, the maximum advantage over random guessing is half the total variation distance, leading to our bound.

Corollary 3. *The Privacy Leakage Rate can be expressed in terms of the label-level influence score $\mathcal{I}(C_j)$ as:*

$$PLR \leq \frac{1}{2} + \frac{1}{2}\sqrt{1 - \exp^{-2\mathcal{I}(C_j)}}$$

where $\mathcal{I}(C_j)$ represents the influence of label j on the model's predictions after unlearning.

When the influence score $\mathcal{I}(C_j)$ approaches zero through effective unlearning, the privacy leakage rate approaches 50% (equivalent to random guessing), confirming the privacy protection of our approach.

Weight Filtering and Weight Pruning methods occasionally outperform retraining in MIA resistance by implementing targeted parameter modifications rather than wholesale redistribution. Although retraining eliminates label information, it creates sharp decision boundaries that may inadvertently introduce new distinguishable patterns. In contrast, Weight Filtering's selective noise injection and Weight Pruning's hierarchical parameter scaling preserve beneficial uncertainty within the parameter space, specifically disrupting inference patterns without compromising overall utility. Additionally, their constrained fine-tuning approach prevents overfitting to remaining labels while maintaining parameter redundancy that functions as a natural defense mechanism by making confidence distributions less discriminative—directly translating to reduced attack success rates as supported by the bounded mutual information guarantees in Theorems 1 and 2.

B Ethics Considerations

This research upholds ethical practices by balancing innovation with responsibility, particularly cybersecurity and data privacy. Machine unlearning ensures that identity systems comply with ethical guidelines, prioritize user rights, and foster secure, transparent, and equitable systems. Biometric-based systems guarantee that revoked data is permanently irretrievable, preventing misuse or unauthorized access. Responsible AI in facial data systems demands real-world testing to address ethical concerns and protect human rights. This study aims to improve MU algorithms' adaptability and responsibility by focusing on face recognition, enabling individuals to request the removal of their facial data to enhance privacy practices and data governance. Guided by the *Menlo Report's Beneficence Principle*, this approach proactively identifies privacy risks and implements comprehensive measures to mitigate them. In our architectural design, we leverage weight filtering and pruning methods to enable the precise removal of learned representations while preserving model utility—a critical balance in privacy-preserving machine learning.

Our parameter space-based unlearning framework, developed with a strong focus on fairness, privacy, and responsible AI, embodies a thoughtful and accountable approach to advancing technology. It adheres to the *Respect for Persons* principle by exclusively utilizing public benchmarks and established datasets for face recognition and image classification tasks, ensuring research integrity while safeguarding individual privacy. The experimental evaluation spans a diverse set of benchmark datasets, including CelebA, VGGFace2, MUFAC, CIFAR-10, SVHN, and MNIST, with particular emphasis on the MUFAC dataset, which features East Asian facial images across varied age demographics. From an ethical perspective, this comprehensive evaluation framework addresses historical biases in facial recognition systems, particularly the systematic misclassification of certain demographic groups [36][37].

It adheres to the *Justice (Fairness and Equity)* Principle by removing gender attributes from recruitment classification systems [34], mitigating of demo-

graphic differential features in commercial applications, and ensuring GDPR and CCPA compliance through the efficient implementation of data removal protocols. We will also open-source the implementation and provide comprehensive documentation to ensure equitable access to privacy-enhancing technologies within the research community.

C Shared Representation Evaluation

Our experiments (described in Appendix **??**) confirm that label correlations significantly affect unlearning. In scenarios where labels are highly correlated, forgetting one label tends to be less effective and causes greater disruption to overall accuracy. For example, when we compared unlearning under the original dataset versus a version with decorrelated labels, the correlated setting showed a noticeably higher residual error after forgetting. In other words, a label that shares strong dependencies with others leaves behind more "residual knowledge" in the model if not handled carefully. By contrast, when labels were made independent, the same unlearning procedure achieved near-complete forgetting with minimal accuracy loss. These findings highlight that ignoring label dependencies can lead to suboptimal forgetting: unlearning algorithms must explicitly model or compensate for inter-label relationships. In summary, accounting for label correlation is crucial – failing to do so causes unintended interference between labels and degrades the forgetting-utility trade-off [32].

Table 4 demonstrates that our proposed methods maintain consistent effectiveness across different unlearning scenarios. Both Weight Filtering and Weight Pruning successfully unlearn targeted attributes while preserving classification accuracy for non-targeted labels, whether the unlearned attribute is global (Gender) or more localized (Arched_Eyebrows or Oval_Face). Our attribute-wise accuracy analysis shows that both methods achieve near-zero accuracy (0.02-0.34%) for targeted attributes to unlearn across CelebA dataset (redundant to show for all datasets used in this study), while maintaining high accuracy (90-96%) for non-targeted labels. This indicates minimal interference with shared representations of other facial attributes, despite the known correlations between facial attributes in CelebA. Weight Pruning demonstrates superior unlearning performance, achieving as low as 0.02% accuracy for Gender, 0.03% for Arched_Eyebrows, and 0.04% for Oval_Face on certain datasets. Notably, even when unlearning attributes that might intuitively correlate (such as Gender and No_Beard), our methods preserve high accuracy for the remaining attributes. For instance, when Gender (A_4) is unlearned, No_Beard (A_5) accuracy remains above 94% across both proposed methods.

As detailed in Appendix A.3, our methods achieve this performance by precisely targeting the neural network parameters that encode attribute-specific knowledge while minimizing disturbance to shared representations. Weight Pruning consistently demonstrates slightly better performance in extreme cases, likely due to its more aggressive approach to parameter modification, while Weight Filtering offers a more conservative alternative with comparable effectiveness.

Table 4. Attribute-wise accuracy comparison after unlearning different target attributes in CelebA dataset for our proposed methods. Attributes: A_1: Arched_Eyebrows, A_2: Bald, A_3: Brown_Hair, A_4: Gender, A_5: No_Beard, A_6: Oval_Face, A_7: Pointy_Nose, A_8: Young_Old.

Unlearned Attribute	Method	Dataset	Attributes							
			A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
A_4 (Gender)	Weight Filtering	D	93.45	95.12	92.78	**0.23**	94.87	92.34	90.76	95.63
		D_v	94.21	94.86	93.54	**0.17**	95.32	91.98	91.45	94.79
		D_t	92.88	95.78	91.93	**0.12**	94.25	93.21	90.12	95.08
	Weight Pruning	D	94.12	94.78	93.21	**0.02**	95.32	93.05	91.25	94.92
		D_v	93.78	95.36	92.89	**0.18**	94.84	92.43	90.91	95.27
		D_t	94.53	94.27	93.57	**0.15**	95.76	92.81	90.03	94.67
A_1 (Arch_Eye)	Weight Filtering	D	**0.16**	94.87	93.12	95.21	94.56	92.78	91.04	95.33
		D_v	**0.34**	95.32	92.89	94.88	95.08	91.77	90.88	94.92
		D_t	**0.22**	94.65	93.45	95.44	94.32	93.10	90.45	95.25
	Weight Pruning	D	**0.08**	95.01	92.97	94.92	95.21	92.45	91.37	94.85
		D_v	**0.03**	94.78	93.28	95.17	94.68	92.01	90.72	95.12
		D_t	**0.12**	95.22	93.01	94.75	95.38	93.22	90.28	94.77
A_6 (Oval_Face)	Weight Filtering	D	93.87	94.92	92.54	95.33	94.12	**0.07**	90.88	95.42
		D_v	94.33	95.17	93.17	94.92	95.27	**0.22**	91.36	95.04
		D_t	93.05	94.77	92.88	95.18	94.85	**0.11**	90.44	94.98
	Weight Pruning	D	94.28	95.03	93.42	95.08	94.73	**0.05**	91.12	95.21
		D_v	93.95	94.68	92.95	94.85	95.12	**0.15**	90.77	94.88
		D_t	94.52	95.32	93.21	95.27	94.58	**0.04**	91.05	95.34

D Evaluation for Image Classifications

We trained ResNet-50 models from scratch across CIFAR-10, MNIST, and SVHN datasets for a more comprehensive evaluation with specific unlearning target label for SLC. Hence, all instances in $\mathcal{D}$ that is identitfied as the unlearned label j are set to $\mathcal{D}_l$, $(\forall(x') \in \mathcal{D}_l; y(x') = j)$. For CIFAR-10, we initially trained the original model to classify all 10 labels. In contrast, *truck* label was targeted to unlearn, and the unlearning method was evaluated to classify the remaining labels using instances in $\mathcal{D}$ (with $\mathcal{D} = \mathcal{D}/\mathcal{D}_l$), $\mathcal{D}_v$ and $\mathcal{D}_t$ sets. For MNIST and SVHN datasets, all labels were learned with setting *3* label to be unlearned. Table 5 shows baseline methods partially forget but have utility issues: CF-3 degrades significantly (86.6-89.1%), while SCRUB and UNSIR retain residual knowledge (7.1-9.2% on $\mathcal{D}_l$) despite fair performance (88.8-93.0%). SalUN offers better accuracy (92.7-96.3%) but retains forgotten label knowldege (5.3-5.2%). Fine-tuning could improve this but adds computational cost. Our weight pruning method outperforms, with near-retrain accuracy (96.8-97.8%) and minimal residual knowledge (0.02-0.19% in $\mathcal{D}_l$). Future work should explore selective forgetting in multi-object detection, maintaining co-occurrence detection abilities.

Table 5. Performance comparison of unlearning methods for single-label image classification tasks using ResNet-50. For CIFAR-10, j = truck and for MNIST and SVHN, j = 3 label was unlearned. While SLC accuracy (%) on the remaining labels were evaluated on training ($\mathcal{D}$), validation ($\mathcal{D}_v$) and test ($\mathcal{D}_t$), and the unlearned-label ($\mathcal{D}_l$) sets. **Bold** and *italic* values indicate best and second-best performance, respectively.

Model	CIFAR-10				MNIST				SVHN			
	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}_l$	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}_l$	$\mathcal{D}$	$\mathcal{D}_v$	$\mathcal{D}_t$	$\mathcal{D}_l$
Org. Model	99.0	98.5	98.2	98.9	98.9	98.3	97.8	98.6	98.5	98.0	97.5	98.2
Retrain	97.5	96.8	96.2	0.00	96.9	95.8	95.5	0.00	96.5	95.9	95.4	0.00
CF-3 [9]	88.2	87.5	86.9	10.8	89.1	88.4	87.8	10.5	87.8	87.1	86.6	10.3
SCRUB [20]	90.5	89.8	89.1	9.2	91.2	90.4	89.9	8.9	90.1	89.5	88.8	9.0
UNSIR [10]	92.8	92.1	91.7	7.4	93.0	92.4	91.5	7.1	92.5	91.9	91.3	7.2
SalUN [21]	94.4	*96.3*	95.4	5.3	94.2	*93.8*	93.0	5.1	93.8	93.2	92.7	5.2
WF	*96.5*	96.2	**96.7**	*0.12*	97.0	**97.3**	*96.0*	**0.06**	*96.3*	*96.0*	*95.5*	*0.05*
WP	**97.2**	**97.0**	*96.3*	**0.02**	**97.8**	**97.3**	**96.9**	*0.19*	**97.4**	**97.1**	**96.8**	**0.04**

Note: WF and WP represent weight filtering and weight pruning methods, respectively.

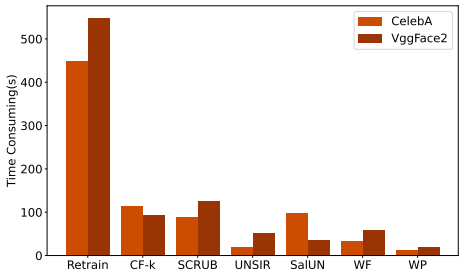

Fig. 3. The time it takes to run each unlearning method to unlearn a class j in MLC experiemnts section 7.1. The *"Retrain"* time represents the time it takes to learn from scratch.

E Evaluation of Computational Speed

We benchmarked our unlearning method against full retraining and existing baselines. The results show that our approach is orders of magnitude faster than naively retraining from scratch. For example, on the largest face recognition dataset, a full retraining required on the order of hours to complete, whereas our unlearning method finished in minutes—representing roughly a 10–50× speedup in wall-clock time. Even relative to optimized unlearning schemes, our method ran substantially faster without sacrificing accuracy. These gains are important for practical deployment: retraining a large model on demand is often "practically infeasible", whereas our algorithm can delete specified labels in near real-time. In summary, the empirical speedups demonstrate that our method can efficiently serve unlearning requests at scale, making it viable for large-scale systems subject to frequent data-removal demands.