Analysis in Eucleadean space

Uncultured Tramp

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Contents

1 Lebesgue Measure

1.1 Hypervolume

$$\texttt{hypervolume} :: \prod_{d=0}^{\infty} \texttt{OpenInterval}(\mathbb{R}^d) \to \mathbb{R}_+$$

$$\texttt{hypervolume}\,(a, b) = \mathrm{Vol}(a, b) = \lambda^d(a, b) := \prod_{n=1}^{d}(b_n - a_n)$$

`HypervolumeIn1DIsLength` $:: \lambda^1 = \lambda$

`Proof` $=$

$\lambda^1(a, b) = b - a = \lambda(a, b)$.

$\square$

`HypervolumeSubbaditivity` $::$

$: \forall d \in \mathbb{N} \,.\, \forall(A, B) : \texttt{OpenInterval}(\mathbb{R}^d) \,.\, \forall I : \texttt{Countable} \,.\, \forall(a, b) : I \to \texttt{OpenInterval}(\mathbb{R}^d) \,.$

$.\, \forall \aleph : (A, B) \subset \bigcup_{i \in I}(a_{n,n}) \,.\, \lambda^d(A, B) \leq \sum_{i \in I} \lambda^d(A, B)(a_i, b_i)$

`Proof` $=$