1)

$$[F:\mathbb{Q}] = \phi(9) = 6$$

as $F$ is cyclotomic extension, which means that

$$\mathrm{Gal}(F/\mathbb{Q}) = [\mathbb{Z}/9\mathbb{Z}]^*$$

.

2) It is possible to write minimal polynomial $p \in \mathbb{Q}(\alpha)[X]$ as :

$$p(X) = (X - e^{2\pi i/9})(X - e^{-2\pi i/9}) = X^2 - 2\cos(2\pi/9)X + 1$$

as conjugation is a homomorphism of $\mathbb{Q}$ - algebras and all non rational reals in $F$ are result of conjugation of elements, which are themselves are $\mathbb{Q}$-linear combinations of basis elements generated by powers of $\zeta$, which already contain conjugates. That is

$$\mathbb{Q}(\alpha) = F \cap \mathbb{R}$$

3) $X^9 - 5$ is the minimal polynomial for $\gamma$ so $[L:\mathbb{Q}] = 9$. The subfield $K$ must have degree which divides 9 but is not 1 or 9. So degree of $K$ is 3. $\mathbb{Q}(\gamma^3)$ has degree 3 and is a subfield of $L$ so it must, indeed, be equal to $K$ (consider minimal polynomial $X^3 - \gamma^3$ of $\gamma$ over $K$.)

4) In case of nontrivial intersection $F \cap L = K$ as it must be nontrivial subextension of $L$. It is also know that extension $F$ is Galois as cyclotomic extension, however $K$ does not split any polynomials, so it is not normal and hence not Galois. And As every subextension of $F$ must be Galois (abelean Galois group) it is clear that $F \cap L = \mathbb{Q}$.

This means that $[M:\mathbb{Q}] = [L:\mathbb{Q}][F:\mathbb{Q}] = 54$ as $M = \mathbb{Q}(\zeta, \gamma)$ .

5) $H$ is a subgroup inherited from $L$, the subgroup generated by action $\sqrt[9]{5} \mapsto \sqrt[9]{25}$ . Another subgroup must be inherited from $F$ . This soubgroup is generated by action $\zeta \mapsto \zeta^2$ . so $H \cong \mathbb{Z}_9^+$ and $S \cong \mathbb{F}_6^+$ . This means that $|G| > 9$, so the galois group is not commutative.

6) All subextensions of degree 2 of $M$ must be also subextensions of $F$. The only such subextension I know is $\mathbb{Q}(\alpha)$

7) There are two subextensions of $M$ of degree 3 but only one of them is Galois. This subextension must belong to $F$. So, it is $\mathbb{Q}(\xi)$ where $\xi$ is the third primitive root of unity.