

TangentSpace.Know

Uncultured Tramp

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1 Tensors

1.1 Basic Definitions

`TensorType` :: $\prod n \in \mathbb{N} . n \rightarrow \mathbb{N} \times (0|1)$

`vectorize` :: `TensorType`(n) \times `MOD`(R) \rightarrow `List`(`MOD`(R))

`vectorize`(T, V) = $V^T := \bigoplus_{i=1}^n \text{if } \pi_2 T_i == 0 \text{ then } [V]^{\oplus \pi_1 T_i} \text{ else } [V^*]^{\oplus \pi_1 T_i}$

`TensorMap` :: $\prod T : \text{TensorType}(n) . \prod V, W : \text{MOD}(R) . \mathcal{L}(V^T, W)$
 $T(V, W)$

`covariantNumber` :: `TensorType`(n) $\rightarrow \mathbb{N}$

`covariantNumber`(T) = $n(T) := \sum_{i=1}^n \text{if } \pi_2 T_i == 0 \text{ then } 1 \text{ else } 0$

`contravariantNumber` :: `TensorType`(n) $\rightarrow \mathbb{N}$

`contravariantNumber`(T) = $(T)n := \sum_{i=1}^n \text{if } \pi_2 T_i == 0 \text{ then } 0 \text{ else } 1$

`Consolidated` :: `?TensorProduct`(n)

$T : \text{Consolidated} \iff \exists j \in n + 1 . \forall i \in n . \text{if } (-, 1) = T_i \iff i \geq j$

`KronekerDeltaTensor` :: $\prod V : \text{MOD}(R) . \binom{1}{-1}(V, \alpha)$

`KronekerDeltaTensor` = $\delta := (w, v) \mapsto \langle w, v \rangle$