TangentSpace.Know

Uncultured Tramp

June 12, 2016

1 Classical Differential Geometry

1.1 Basic Definitions

$$\texttt{Hypersurface} :: \prod M : \texttt{SManifold} \;.\; ?\texttt{Submanifold}(M)$$
$$H : \texttt{Hypersurface} \iff \operatorname{codim} H = 1$$

$$\texttt{Parallel} :: ?TM \times TM$$
$$((p,v),(q,w)) : \texttt{Parallel} \iff (p,v) \parallel (q,w) \iff v = w$$

$$\texttt{FieldAlongCurve} :: \texttt{Curve}(M) \to \texttt{Field}(M)$$
$$Y : \texttt{FieldAlongCurve}(c) \iff \forall t \in \mathbb{R} \;.\; \pi\, Y(c(t)) = \dot{c}(t)$$

$$\texttt{DerivativeOfCurve} :: \texttt{Curve}(\mathbb{R}^n) \to \texttt{Curve}(T\mathbb{R}^n)$$
$$\texttt{DerivativeOfCurve}(c) = c' = \sum_{i=1}^{n} \dot{c}(t)^i \hat{e}_i(c(t))$$

$$\texttt{HigherDerivativeCurve} :: \texttt{Curve}(\mathbb{R}^n) \to \mathbb{N} \to \texttt{Curve}(T\mathbb{R}^n)$$
$$\texttt{HigherDerivativeCurve}(c,k) = c^{(k)} = \sum_{i=1}^{n} \frac{\mathrm{d}^{k-1}\dot{c}(t)^i}{\mathrm{d}t^{k-1}} \hat{e}_i(c(t))$$

$$\texttt{crossProduct} :: C^\infty((\mathbb{R}^n)^{n-1}, \mathbb{R}^n)$$
$$\texttt{crossProduct}(v) = \times(v) := \texttt{asBasis}(\mathbb{R}^n)(\det[v] \otimes x)(x)$$

1.2 Curves

$\texttt{RegularCurve} :: ?\texttt{Curve}(\mathbb{R}^n)$
$\gamma : \texttt{RegularCurve} \iff \forall t \in \text{Dom}\,\gamma\ .\ \|\dot{\gamma}(t)\| \neq 0$

$\texttt{unitTangentField} :: \texttt{RegularCurve}(\mathbb{R}^n) \to \texttt{Curve}(\mathbb{R}^n)$
$\texttt{unitTangentField}(\gamma) = \mathrm{T}_\gamma = t \mapsto \dfrac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}$

$\texttt{Length} :: \texttt{Path}(\mathbb{R}^n) \to \mathbb{R}_+$
$\texttt{Length}(\gamma) = L(\gamma) = \displaystyle\int_0^1 \|\dot{\gamma}(t)\| dt$

$\texttt{arclength} :: \prod \gamma : \texttt{Curve}(\mathbb{R}^n)\ .\ \text{Dom}\,\gamma \to \text{Dom}\,\gamma \to \mathbb{R}$
$\texttt{Length}(\gamma) = h(t_0)(t) = \displaystyle\int_{t_0}^t \|\dot{\gamma}(t)\| dt$

$\texttt{unitSpeedCurve} :: \texttt{RegularCurve}(\mathbb{R}^n) \to \texttt{RegularCurve}(\mathbb{R}^n)$
$\texttt{unitSpeedCurve}(\gamma) = c_\gamma := t \mapsto \gamma \circ (h_\gamma(0))^{-1}(t)$

$\texttt{curvatureVector} :: \texttt{RegularCurve}(\mathbb{R}^n) \to \texttt{Curve}(\mathbb{R}^n)$
$\texttt{curvatureVector}(\gamma) = \kappa_\gamma := t \mapsto \dot{\mathrm{T}}_\gamma(t)$

$\texttt{curvatureFunction} :: \texttt{RegularCurve}(\mathbb{R}^n) \to C^\infty[0,1]$
$\texttt{curvatureVector}(\gamma) = K_\gamma := t \mapsto \|\kappa_\gamma(t)\|$

$\texttt{RealCurve} :: ?\texttt{RegularCurve}$
$\gamma :: \texttt{RealCurve} \iff K_\gamma > 0$