Modules.Know

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$\texttt{data}\quad \texttt{Module} :: \prod R : \texttt{Ring} \;.\; \sum M : \texttt{Abelean} \;.\; \texttt{Homo}\ R\ \ (\mathrm{End}_{\mathsf{Ab}}\ M)$

$\qquad R\text{-}\texttt{Module} := \texttt{Module}(R)$

$\texttt{function}\quad \texttt{moduleMult} :: \prod (M, f) : R\text{-}\texttt{Module} \;.\; R \times M \to M$

$\qquad ra := \texttt{moduleMult}(M, f)(r, a) := f(r)(a)$

$\texttt{function}\quad \texttt{implicit} :: R\text{-}\texttt{Module} \to \texttt{Abelean}$

$\qquad (M, f) := M$

$\texttt{thm}\quad \texttt{basic0} :: \prod M : R\text{-}\texttt{Module} \;.\; \prod m \in M \;.\; 0_R m = 0_M$

$\qquad \texttt{proof}\ M\ m =$

$\qquad\quad (= I)(0_R m) : 0_R m = 0_R m \to (0_R + 0_R)m = 0_R m \to$

$\qquad\quad \to 0_R m + 0_R m = 0_R m \to 0_R m = 0_M \quad \square$

$\texttt{thm}\quad \texttt{basic1} :: \prod M : R\text{-}\texttt{Module} \;.\; \prod m \in M \;.\; -1_R m = -m$

$\qquad \texttt{proof}\ M\ m =$

$\qquad\quad \texttt{basic0}\ M\ m : 0_R m = 0_M \to (1_R - 1_R)m = 0_M \to$

$\qquad\quad \to m + (-1_R)m = 0_M \to -1_R m = -m \quad \square$

$\texttt{thm}\quad \texttt{abGroupZModularity} :: \prod A : \texttt{Abelean} \,.\, \exists!\ f : \texttt{Homo}\ \mathbb{Z}\quad (\mathrm{End}_{\mathsf{Ab}}\ A)\ .$

$\qquad .\ (A, f) : \mathbb{Z}\ \texttt{-Module}$

$\texttt{proof}\ A =$

$\quad \mathbb{Z}\ : \texttt{Init}(\mathsf{Ring}) \to \exists! f : \texttt{Homo}\ \mathbb{Z}\quad (\mathrm{End}_{\mathsf{Ab}}\ A) \multimap f$

$\quad (A, f) : \mathbb{Z}\ \texttt{-Module} \to$

$\quad \to \exists!\ f : \texttt{Homo}\ \mathbb{Z}\quad (\mathrm{End}_{\mathsf{Ab}}\ A)\ .\ (A, f) : \mathbb{Z}\ \texttt{-Module}\quad \square$

$\texttt{predicate}\quad \texttt{Linear}\ :: \prod A, B : R\text{-}\texttt{Module}\ .\ ?\texttt{Group.Homo}\ A\ B$

$\quad f : \texttt{Linear}\ \Leftrightarrow \prod a \in A\ .\ \prod r \in R\ .\ f(ra) = rf(a)$

$\texttt{category}\quad \mathsf{Mod} :: \texttt{Ring} \to \texttt{category}$

$\quad R\text{-}\mathsf{Mod}\ := \mathsf{Mod}(R) := ($

$\qquad Obj := R\text{-}\texttt{Module};$

$\qquad Hom := \Lambda\ A, B : Obj\ .\ \texttt{Linear}\ A\ B;$

$\qquad \cdot := \circ)$

$\texttt{data}\quad \texttt{Algebra} :: \prod R : \texttt{Commutative}\ .\ \sum S : \texttt{Ring}\ .\ \texttt{Homo}\ R\quad \mathrm{center}\, S$

$\quad R\text{-}\texttt{Algebra} := \texttt{Algebra}(R)$

$\texttt{function}\quad \texttt{algebraMult} :: \prod (S, f) : R\text{-}\texttt{Algebra}\ .\ R \times S \to S$

$\quad ra := \texttt{algebraMult}(M, f)(r, a) := f(r)(a)$

$\texttt{function}\quad \texttt{implicit} :: R\text{-}\texttt{Algebra} \to \texttt{Ring}$

$\quad (S, f) := S$

$\texttt{predicate}\quad \texttt{AlgHomo} :: \prod A, B : R\text{-}\texttt{Algebra}\ .\ \texttt{Homo}\ A\ B$

$\quad f : \texttt{AlgHomo} \Leftrightarrow \prod a \in A\ .\ \prod r \in R\ .\ f(ra) = rf(a)$

$\texttt{category}\quad \mathsf{Alg} :: \texttt{Ring} \to \texttt{category}$

$$R\text{-}\mathsf{Alg} := \mathsf{Mod}(R) := ($$

$$Obj := R\text{-}\texttt{Algebra};$$

$$Hom := \Lambda\ A, B : Obj\ .\ \texttt{AlgHomo} A\ B;$$

$$\cdot := \circ)$$

$\texttt{fact}\ (*) : \texttt{zero}\ R\text{-}\mathsf{Mod}$

$\texttt{fact} \prod R : \texttt{Commutative}\ .\ \prod A, B : R\text{-}\texttt{Module}\ .\ Hom_{R\text{-}\mathsf{Mod}}\ A\ B : R\text{-}\texttt{Module}$

$\texttt{predicate}\quad \texttt{Submodule} :: \prod S : R\text{-}\texttt{Module}\ .\ ?\texttt{Subset}\ S$

$$A : \texttt{Submodule} \Leftrightarrow A : R\text{-}\texttt{Module}$$

$\texttt{function}\quad \texttt{quotient} :: \prod M : R\text{-}\texttt{Module}\ .\ \texttt{Submodule}\ S \to R\text{-}\texttt{Module}$

$$\frac{M}{A} := \texttt{quotient}\ S\ A := (\{m + A | m \in M\}, \cdot)$$

$\texttt{fact} \prod M : R\text{-}\texttt{Module}\ .\ \prod S : \texttt{Subset}\ M\ .\ \texttt{iff}\ S : \texttt{Submodule}\ M\ .$

$$.\ \exists N : R\text{-}\texttt{Module}\ .\ \exists f : \texttt{Linear}\ M\ N. \ker f = S$$