TangentSpace.Know

Uncultured Tramp

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1 Manifolds

1.1 Topological Manifolds

$$\texttt{LocallyEuclidean} :: \prod V : \mathsf{TVS}(K) \,.\, ?\mathsf{TOP}$$
$$M : \texttt{LocallyEuclidean} \iff \forall p \in M \,.\, \exists U \in \mathcal{U}(p) \,.\, U \cong_{\mathsf{TOP}} V$$

$$\texttt{chart} :: \prod M : \texttt{LocallyEuclidean} \,.\, M \to \sum U : \texttt{Open}(M).U \cong_{\mathsf{TOP}} V$$
$$\texttt{chart}(p) = \texttt{LocallyEuclidean}(V)(M)(p)$$

$$\texttt{chartCentredAt} :: \prod M : \texttt{LocallyEuclidean} \to \sum U : \texttt{Open}(M).U \cong_{\mathsf{TOP}} V$$
$$\texttt{chartCentredAt}(p) = (U, \phi - \phi(p))$$
$$\quad \texttt{where}(U, \phi) = \texttt{chart}(p)$$

$$\dim :: \texttt{LocallyEuclidean}(V) \to \texttt{Cardinality}$$
$$\dim M = \dim V$$

$$\texttt{Manifold} :: \texttt{LocallyEuclidean}(V)\&\texttt{Hausdorff}\&\texttt{SecondCountable}$$

1.2 Charts And Atlases

$$\texttt{SChart} :: \prod M : \texttt{Manifold}(V) \,.\, ? \sum U : \texttt{Open}(M) \,.\, C^\infty(U, V)$$
$$(U, x) : \texttt{SChart} \iff x : \texttt{Diffio}\big(U, x[U]\big)$$

$$\texttt{Compatible} :: \prod M : \texttt{Manifold}(V) \,.\, ?\texttt{SChart}^2(M)$$
$$\big((U, x), (W, y)\big) : \texttt{Compatible} \iff \phi \circ \psi^{-1} : \texttt{Diffio}\big(\psi(U \cap W), \phi(U \cap W)\big)$$
$$\psi \circ \phi^{-1} : \texttt{Diffio}\big(\phi(U \cap W), \psi(U \cap W)\big)$$

$$\texttt{Atlas} :: \prod M : \texttt{Manifold}(V) \,.\, \prod A : \texttt{set} \,.\, ?A \to \texttt{Chart}(M)$$
$$(U, x) : \texttt{Atlas} \iff U : \texttt{Cover}(M)$$
$$\forall a.b \in A \,.\, \big((U_a, x_a), (U_b, x_b)\big) : \texttt{Compatible}(M)$$

$$\texttt{Maximal} :: \prod M : \texttt{Manifold}(V) \,.\, ?\texttt{Atlas}(M)$$
$$(\mathfrak{M}, \phi) : \texttt{Maximal} \iff \forall (U, x) : \texttt{Atlas}(M) \,.\, \mathfrak{M} \not\subset U$$

$$\texttt{UniqueMaximalAtlas} :: \forall M : \texttt{Manifold}(V) \,.\, \exists! \mathfrak{M} : \texttt{Maximal}(M)$$

1.3 Smooth Manifolds

$$\texttt{SManifold} ::? \texttt{Manifold}$$
$$M : \texttt{SManifold} \iff \mathcal{T}_M = \texttt{toSet}\ \pi_1\ \texttt{UniqueMaximalAtlas}(M)$$