

ClosedGraph1 :: $\forall E, F : \text{Banach} . \forall T : \mathcal{M}_{\text{TVS}(\mathbb{R})} : \text{Iso}(\text{SET})(E, F) . T : \text{Iso}(\text{TVS}(\mathbb{R}))(E, F)$
Proof :
 $\text{A } E, F : \text{Banach},$
 $\text{A } T : \mathcal{M}_{\text{TVS}(\mathbb{R})} : \text{Iso}(\text{SET})(E, F),$
 $\text{Iso}(\text{SET})(E, F)(T) \Rightarrow \exists f : \text{Inverse}(\text{SET})(T) \text{ E } (T^{-1}),$
 $\text{A } v, w \in F,$
 $\text{Inverse}(\text{SET})(T)(T^{-1}) \Rightarrow \exists a, b \in E : Ta = v \wedge Tb = w \text{ E},$
 $T^{-1}(v + w) = T^{-1}(Ta + Tb) = T^{-1}T(a + b) = a + b = T^{-1}Ta + T^{-1}Tb = T^{-1}v + T^{-1}w \text{ as (1);}$
 $\text{A } v \in F,$
 $\text{A } r \in \mathbb{R},$
 $\text{Inverse}(\text{SET})(T)(T^{-1}) \Rightarrow \exists a \in E : Ta = v \text{ E},$
 $T^{-1}rv = T^{-1}rTa = T^{-1}Tra = ra = rT^{-1}Ta = rT^{-1}v \text{ as (2); ;}$
 $\text{Linear}(\mathbb{R})(T^{-1}, (1), (2)) \Rightarrow T^{-1} : \mathcal{M}_{\text{NVS}(\mathbb{R})}(F, E) \text{ as (3),}$
 $\text{A } y : \text{Convergent}(F),$
 $Y \triangleq \lim_{n \rightarrow \infty} y_n, X \triangleq T^{-1}Y, x \triangleq T^{-1}y,$
 $0 = \lim_{n \rightarrow \infty} \|Y - y_n\| = \lim_{n \rightarrow \infty} \|TX - Tx_n\| = \lim_{n \rightarrow \infty} \|T(X - x_n)\| = \left\| T \left(\lim_{n \rightarrow \infty} X - x_n \right) \right\| \text{ as (4),}$
 $\text{Norm}(\|\cdot\|)_1(4) \Rightarrow T \left(\lim_{n \rightarrow \infty} X - x_n \right) = 0 \rightarrow \text{Iso}(\text{SET})(E, F)(T) \Rightarrow \lim_{n \rightarrow \infty} x_n = X \text{ as (5),}$
 $\triangleq (Y, y)(5) \Rightarrow \lim_{n \rightarrow \infty} T^{-1}y_n = T^{-1}Y \text{ as (6);}$
 $\text{Continous}(T^{-1}, (6)) \Rightarrow T^{-1} : \mathcal{M}_{\text{TOP}}(F, E) \text{ as (7),}$
 $(3, 7) \Rightarrow T^{-1} : \mathcal{M}_{\text{TVS}(\mathbb{R})}(E, F) \Rightarrow T : \text{Iso}(\text{TVS}(\mathbb{R})); ; \square$

ClosedGraph2 :: $\forall E : \text{Banach} . \forall A : \text{Subspace}(E) . \forall B : \text{Split}(F) .$
 $. \Lambda a \in A . \Lambda b \in B . a + b : \text{Iso}(\text{TVS}(\mathbb{R}))(A \oplus B, E)$
Proof :
 $\text{A } E : \text{Banach},$
 $\text{A } A : \text{ClSubspace}(E),$
 $\text{A } B : \text{Split}(A),$
 $s \triangleq \Lambda a \in A . \Lambda b \in B . a + b,$
 $\text{Split}(A)(B) \Rightarrow A + B = E \Rightarrow s : \text{Surjective as (1),}$
 $\text{Split}(A)(B) \Rightarrow A \cap B = \{0\} \Rightarrow s : \text{Injective as (2),}$
 $(1, 2) \Rightarrow s : \text{Iso}(\text{SET})(E, F) \rightarrow \text{ClosedGraph1}(E : \text{TVS}(\mathbb{R})) \rightarrow s : \text{Iso}(\text{TVS}(\mathbb{R}))(A \oplus B, E); ; \square$