0.1 500-word summary

This dissertation is a literature study that investigates the validity of different linear models in application. Validity in this context refers to how well simplified lower dimensional models compare to more realistic higher dimensional models. This dissertation will investigate the validity of the Timoshenko beam theory and the Reissner-Mindlin plate theory. The higher dimensional models are a two- and three-dimensional beam and a three-dimensional plate. In the first chapter, the models are provided.

Next, a discussion of the existence and uniqueness of solutions for a general vibration problem. The article discussed provide four assumptions, and prove there exists an infinitesimal generator of a C_0-semigroup of contractions, that rewrites the problem into an initial value problem, with a unique solution. The authors then prove this is also a solution of the general vibration problem. An application of this theory is demonstrated using a cantilever Timoshenko beam.

Thereafter a discussion of two theoretical results for the Finite Element Method. The first result is from an article where the convergence of the Galerkin Approximation is proved. A Galerkin Approximation of the general vibration problem is derived. Using the Finite Element Method, this Galerkin Approximation is rewritten into an initial value problem, that has a unique solution. The authors show that there exists error estimates that approximate the error between the general vibration problem and the initial value problem. The next result is from a textbook on the convergence of the eigenvalues and eigenvectors of a one-dimensional vibration problem. This section rewrites the textbook's results with updated notation, aiming for greater clarity and understanding.

The forth chapter discusses results for the Timoshenko beam theory. First an article is examined that provides a method to calculate the exact eigenvalues and eigenvectors. The method is then applied to a cantilever and a free-free beam model. Next is a discussion of an empirical study, that compares the natural frequencies of a physical free-free beam to the theoretical results of the Timoshenko beam theory. Finally an extension on a technical report of a cantilever Timoshenko beam with a tip-body and elastic interfaces. This model serves as an example of the application of the Timoshenko theory in a complex model and adds to the article by expanding the numerical results.

In the fifth chapter, the Finite Element Method is applied to the two and three-dimensional models of this dissertation. The goal of this chapter is to obtain a numerical formula that can be used to calculate the eigenvalues and

eigenvectors of the models in preparation for the final chapter where the models are compared.

The last chapter compares the eigenvalues and eigenvectors of the models. The approach is based on an article where a cantilever Timoshenko beam model is compared to a cantilever two-dimensional model. The article is discussed and results replicated. The article is then extended to a comparison of a two-dimensional model to a three-dimensional model. Finally, a cantilever Reissner-Mindlin plate model is compared to a cantilever three-dimensional plate model.

493 Words

0.2 Summary

Chapter 1

This dissertation is a literature study that investigates the validity of different linear models. The term validity in this dissertation means how well a model compares to a more realistic model for real world applications. The first chapter of the dissertation introduces the models. The main model is the Timoshenko beam theory. The other models are a two-dimensional elastic model, a three-dimensional elastic model and a Reissner-Mindlin plate model. The goal of this disseration to validate the use of the Timoshenko beam model and the Reissner-Mindlin plate model in applications. These models are simplified one-dimensional beam and two-dimensional plate models. But more realistic models exists, such as a two-dimensional and three-dimensional beam model and a three-dimensional plate model. Using modal analysis, it is shown that the solutions of the models can be represented as a linear combination of the eigenvalues and eigenfunctions. Therefore it is only required to compare the eigenvalues and eigenfunctions of the models to determine the difference of the solutions.

Chapter 2

In this section, the existence and solutions for a general vibration problem is investigated. The main article that is discussed is [**VV02**]. In this article, the authors define a general vibration problem. The authors state four assumptions that need to be satified. Under these four assumptions, it can be shown that for the general vibration problem there exists a linear operator A, which is an infinitesimal generator of a C_0-semigroup of contractions with a domain

in the problem space. This operator allows the general vibration problem to be rewritten as a system of first order differential equations $x' = Ax + f$. This differential equations are rewritten into an initial value problem which can be proven using semi-group theory to have a unique solution. The authors then provide the necessary proofs to show that a solution of the initial value problem is also a solution of the general vibration problem. An example of the application of this theory is provided using a cantilever Timoshenko beam.

Chapter 3

In this chapter, some theory for the Finite Element Method is discussed. This chapter contains two parts, that covers two different results. The first result is the convergence of the Galerkin approximation of a general vibration problem. This result is presented in the article [**BV13**]. The second result concerns the convergence of the eigenvalues and eigenfunctions of a one-dimensional vibration problem when applying the Finite Element Method. This result is presented in the textbook [**SF97**]. In the first section, the authors continue with the general vibration problem that is used in chapter 2. The general vibration problem has a solution as shown in chapter 2. The Galerkin Approximation is derived from the general vibration probelm and is rewritten into a system of ordinary differential using the Finite Element Method. This oridinary differential equation can be proven to have a unique solution. The main results of [**BV13**] shows that this solution of the Galerkin Approximation converges to the solution of the general vibration problem. The approach of the authors is to calculate the error estimates. The error is split into two parts, a semi-discrete and a fully discrete problem. Two assumptions are added to the assumptions from chapter 2. Using these assumptions, the authors obtain error estimates for the semi-discrete and fully discrete problems.

The second part of the chapter considers work done in a textbook [**SF97**]. The specific work discussed, covers the convergence of the eigenvalues and eigenfunctions of a one-dimensional vibration problem when applying the Finite Element Method. The authors consider a general eigenvalue problem. Then using the Rayleigh-quotient, from the Rayleigh-Ritz method, as well as an approximation theorem from [**ORXX**], the main result is proven. The specific work done in this section is updating the notation of the textbook, as well as expanding some results so that the results are easier to understand.

Chapter 4

This chapter is a focus on the main model theory of this disseration, the Timoshenko beam theory. The first section is a discussion of modal analysis applied to the Timoshenko beam theory, and specifically a discussion of the article [**VV06**]. In this article, the authors present a method to calculate the exact eigenvalues and eigenfunctions of a Timoshenko beam. This method considers the boundary conditions, which differs from most other research. Starting with a general eigenvalue problem for a Timoshenko beam model, the authors derive a general solution for this problem. The authors then explain the method by hands of an example by applying the method to a cantilever beam model. The next sections of this chapter also then looks at examples of applying the method, first to a cantilever beam model, and then to a free-free beam model. The focus of these applications are to show that the method works and that the numerical results can be obtained easily. Similar numerical results will be required in chapter 6. The next section is a short discussion on a suspended beam model, afterwards an interesting article [**SP06**] is discussed. In this article, the authors investigate the validity of the Timoshenko beam theory, by comparing the eigenvalues (natural frequencies) for a emperically mesasured beam, to a Timoshenko beam and a three-dimensional beam using Finite Element Analysis. The specific model used is a free-free beam. The emperical results are obtained by vibrating a suspended steel beam and measuring the natural frequencies. These results are then compared to the theoretical results. Finally the chapter finished by looking at an interesting model of a cantilever beam with a tip-body with elastic interfaces. These results combine modal analysis and looks at the behavior of the model. This section serves as an example of how useful the Timoshenko beam theory can be in a complex model.

Chapter 5

In this chapter, the Finite Element Method is applied to cantilever two-dimensional,and three-dimensional cantilever beams and a cantilever Reissner-Mindlin plate. The aim of this section is to obtain a numerical formula to calculate the eigenvalues and eigenfunctions of the models. The results of these formulas will be used in chapter 6. The Finite Element Method need not be applied to the Timoshenko beam theory, as chapter 4 provides an alternative method. For all the models, the Finite Element Method is applied using bi-cubic or tri-cubic basis functions to improve the rate of convergence and reduce the processing required to obtain accurate results.

Chapter 6

In this chapter, the main comparisons of the models are conducted. The first section is a discussion of the article [**LVV09**]. Each section in this chapter follows the same structure as this article. In this article, the authors investigate the validity of a cantilever Timoshenko beam models, by comparing it to a two-dimensional cantilever Timoshenko beam model. To make this comparison, the authors compare the eigenvalues and eigenfunctions of the models. But since the two-dimensional model is more complex, there are 'non-beam type' eigenvalues. These are eigenvalue that are not in the Timoshenko beam theory, and does not relate to beam type problems. To be able to find the beam type eigenvalues that can be compared, the authors compare the mode shapes. This allows the eigenvalues to be matched up. The results of the article show that the Timoshenko beam model compares very well to the two-dimensional beam. The comparison improves when the application is for a long and slender beam. The results of the article are verified and more results are added to extend the article and also to be using the the following sections. But in real-world applications, a beam is a three-dimensional model. Therefore it would be better to use a three-dimensional model to investigate the validity of the Timoshenko beam theory. The authors of [**LVV09**] mentions this, and suggests using the two-dimensional model as an intermediate step to avoid complexities. So in the second section of this chapter, the validity of the two-dimensional model is investigated using the three-dimensional model as a reference. The results show that the two-dimensional model also compares very well to the three-dimensional model. Similar to the one-dimensional case, the results improve for a long and slender beam, and also if the width of the three-dimensional beam is not larger than the height of the beam. When increasing the width past the size of the height, the two-dimensional model is less accurate. This shape does however bring into question the use of a beam model, and something like a plate model might be better suited. In the last section, the validiy of the Reissner-Mindlin plate model is investigated. Similar to how the Timoshenko beam theory improves on the Euler-Bernoulli beam theory, the Reissner-Mindlin plate theory improves on the classical plate theory. The numerical results do show that this Reissner-Mindlin plate model compares well to a three-dimensional plate model, also when the dimensions of the models are realistic for application.