

The Proper Light Speed Theory

(The unification of Maxwell's Eqs, gravity, and Quantum Mechanics)

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Abstract

The Michelson–Morley experiment was done by using a half silvered mirror to produce 2 perpendicular beams of light from one light source, and then reflecting each beam back to a common screen to observe the interference fringes. The device was then rotated in an attempt to detect the ether. The results of the experiment are consistent with a constant speed of light in all directions, but they are also consistent with a variable speed of light that is the same in all directions locally within an undetectable variance. In other words, if the speed of light is approx. the same in all directions over the distance that the 2 beams traversed, then any variance in the speed of light could go unnoticed. In this model, light speed dilates in the same capacity that time is thought to dilate in general relativity so if clock speed dilation isn't detectable over the distance in which the experiment is run, then light speed dilation won't be detectable either. In fact, according to this model, the difference in the speed of light at the surface of the earth and 100 meters up is only distinguished at the 6th decimal place (m/s). In this article, the author shows that a variable speed of light is consistent with all of the experimental results of special and general relativity while also resolving some of the current issues in physics. In light of this different structure for the universe, complications in quantum mechanics are also resolved. It is advised to read this with an open mind and then make your conclusions afterwards. An experiment is proposed in which this theory and General Relativity could not both explain the outcome.

Outline

The weak, strong, gravitational, and electromagnetic forces propagate at a rate that is either c (force carriers without mass) or a percentage of c (force carriers with mass). If the speed of light slows down, all of the fundamental forces therefore propagate at a slower speed causing a clock to tick slower. Since your body utilizes these same forces, your biological processes slow down as well so you never notice clock speed dilation locally. When measuring the speed of light, scientists are therefore effectively measuring the speed of light with itself so they will always measure the same value, hence why c is considered constant. Effectively, clock speed changes proportionally with light speed so no matter how much the speed of light changes, the value of c measured remains constant. Why do all of the fundamental forces propagate slower just because light speed slows down? The same thing that causes light to slow down, causes these other force carriers to slow down as well.

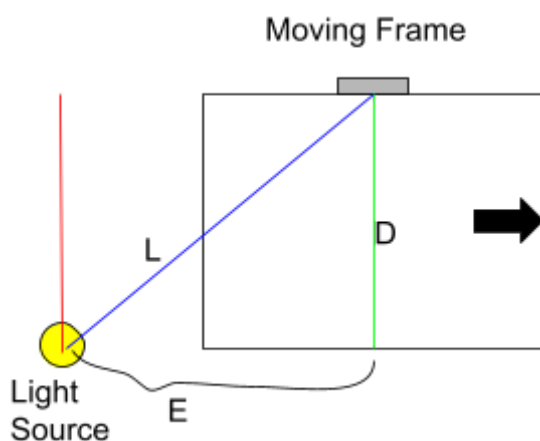


Figure A: The typical setup for deriving the equations of time dilation in special relativity. The blue line represents the path of light from the stationary reference frame to the moving reference frame. At the moment that the moving frames mirror is directly over the light source, a flash occurs producing the red and blue photon paths. The moving reference frame travels just fast enough to ensure the blue line hits the mirror.

In Figure A there is the typical setup for the derivation of time dilation in special relativity. As the moving frame goes from left to right, at the moment that the center of the mirror in the moving frame is directly over the light source, a flash occurs. Two photons are emitted, one in the vertical direction (red) and one up and to the right (blue). In special relativity, the green line is treated as if it were the red line because the speed of light is the same in all reference frames. Thus:

$$L = c\Delta t, D = c\Delta t_0, E = v\Delta t$$

$$\therefore L^2 = D^2 + E^2 \Rightarrow (c\Delta t)^2 = (c\Delta t_0)^2 + (v\Delta t)^2$$

Simplifying yields:

$$\Delta t_0 = \Delta t * \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (A)$$

Leading to the conclusion that time itself is malleable. Now consider the same setup with the following interpretation. The green line in Figure A is simply the vertical component of the blue line representing the vertical component of the photon's path. This means that the observer in the moving reference frame only sees the vertical component of the light and thus in the moving reference frame, the speed of light is slower. The equations are as follows:

$$L = c\Delta t, D = c_0\Delta t, E = v\Delta t$$

$$\therefore L^2 = D^2 + E^2 \Rightarrow (c\Delta t)^2 = (c_0\Delta t)^2 + (v\Delta t)^2$$

Simplifying yields:

$$c_0 = c * \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (B)$$

Where c_0 is the proper speed of light, and c is the speed of light. As the velocity increases, the proper speed of light decreases (just like proper time was thought to decrease) and all of the fundamental forces propagate more slowly causing a moving clock to tick slower. All of the results of special relativity are recovered with a variable speed of light as seen by comparing equations (A) and (B).

So, you have the local speed of light which is how fast a local stationary reference frame perceives the speed of light (this changes radially), and then you have the proper speed of light (rather than proper time) which is how a moving reference frame perceives the speed of light relative to a stationary reference frame in zero-g (this changes with velocity). The faster an object moves, the slower its proper light speed, which means that all of the fundamental forces slow down causing the moving clock to tick slower than the stationary one. The faster a muon travels, the slower its proper light speed and therefore all of the fundamental forces involved in its decay slow down resulting in it existing longer as measured from a stationary reference frame. It follows that matter and energy change the local index of refraction causing gravitational lensing, gravitational redshifting, clock-speed dilation, etc.. The event horizon of a black hole is where the speed of light is zero and thus clocks do not tick there, but to someone falling into the event horizon, their biological processes slow down with the speed of light (the particles in the body use the same forces) and thus they do not notice anything changing from a clock-speed perspective. Effectively all of the results of general relativity are recovered but for a different reason: light speed dilates not time. It is important to note the difference between time and clock-speed: clock-speed is what we measure, and time is as defined below. Since clock-speed is a result of the fundamental forces propagating which is ultimately bounded by c , clocks do not measure time directly but rather measure how far light traveled between the events: they measure how quickly the events took place relative to the speed of light. In physics, what is called time is actually clock-speed.

If x is nothing, then by definition, all components of x are also nothing (otherwise x is something). That is, if $a, b \in x$, then $[a = x] \pm [b = x] = x$ proving something cannot logically be produced from nothing. Since x is nothing, even if all of the laws of physics and logic did not hold, x doesn't exist for it to matter. Two quantities, A and B , that are conserved $| A - B = x$, are therefore never produced from nothing (unless the conserved property is nothingness). This includes Fourier Conjugates. This is shown in more detail in P_2 below.

Since something cannot be produced from nothing, and something currently exists, existence has always existed. A less philosophical way of saying that is that fundamentals of everything that we observe have always existed (otherwise at some point in the past they must be made from nothing). If we therefore define All Existence (AE) as the set of all of the fundamental components that have always existed, then all that can be done is these fundamentals permute: they cannot be created or destroyed. It follows that information can never be lost. Time is the ability for change. Now, it is important to clarify this definition. Think of time like a for loop in programming that is always running in the background. Each loop, if you will, can result in light propagating, clocks ticking, and objects moving. Light can travel 0 meters per loop, or it can travel 299,792,458 m per loop. You cannot have a loop without having the ability for change. It is shown below that such a framework is consistent with experimentation, and simplifies some of the complications currently considered unresolved in physics.

So what is being proposed is that AE has always existed, and thus its existence doesn't violate causality. Time (ability for change) is a property of existence. At some place within AE the properties of our universe that are described by our laws of physics became defined, and the local speed of light became non-zero. The universe then began to organize itself from fundamentals, based on these physical properties, in a manner that is consistent with the evolutionary processes described by science (the current leading theory being that of the Big Bang which ironically includes GR). It follows that the observable universe is a subset of AE, and is therefore not all that exists. Time is not a dimension and does not dilate, but clock-speed dilates for the reasons stated above. It is important to note here the difference between laws of physics and logical principles. Any consistent set of laws of physics that tell fundamentals how to permute could theoretically apply to a universe, but at no point could a universe be inconsistent with logic. This is an important distinction. Just because different laws of physics might be viable, doesn't mean that different rules of logic are.

As predicted by the Big Bang model, after it begins, the universe starts to cool down resulting in the formation of particles, atoms, and the cosmic microwave background radiation (not from nothing). Eventually stars and galaxies form due to the pull of gravity, planets emerge from orbiting dust clouds, and abiogenesis/evolution begin to play out. Notice how each of these steps only occur due to the physical properties of the universe: without these physical properties nothing happens. We can test these physical properties with science, but such tests only help us to understand what the physical properties are, not why they exist in the first place. We therefore need a model that explains why the physical properties of the universe are what they are. Consider the following:

Any universe that emerges is a subset of AE as if this is not the case, said universe would need to be created from nothing. Assume that our universe emerged through natural processes. Since our universe emerged, it follows that with time being a property of existence (as shown below), and existence always existing, statistically other universes have emerged as well within AE. It follows that remnants of said universes would statistically be observable in our cosmological data in the form of galaxies and light headed towards us from all over. Since we do not observe these remnants in our cosmological data, it follows that the universe didn't emerge through natural means. While there are specific cases to consider, like the universe existing as a singularity for an eternity before "erupting", each of these appears to violate known logical principles as shown further down in the article in more detail. Now, if such data were to be found, this could change the conclusion, but so far no indication of such data is known to the author. Since a universe emerging through natural means doesn't fit the cosmological data, it follows

that the universe emerged through unnatural means. Now, please don't hate the author. You are welcome to point to any spot in which there is a mistake, and if you can't do that, then perhaps consider that what is stated is accurate.

Since time is effectively eternal, if it is possible, it is guaranteed that at some point fundamentals naturally come together to form G. Given an infinite period, anything that can happen will happen (with constraints of course), so if it is possible for G to exist, then it is guaranteed that G exists. We now take the infinite period T in which AE exists, and we divide it into 2 infinite periods T_1 and T_2 , where T_1 precedes T_2 . Since T_1 is infinite, if the natural formation of G is possible then statistically it is guaranteed to have occurred in T_1 . Since T_1 precedes T_2 , and T_2 is infinite, the formation of G occurred an infinite time ago. Thus, no matter how far back in time you theoretically look, G has always existed. By this same logic, if it is possible for G to learn everything, become perfect, or develop rules that statistically yield the best outcome, then this was done an infinite time ago. Just to be clear, logically speaking, if G can be created naturally then we could just skip G and assume the universe could be naturally occurring. The problem is that this approach doesn't match the data as explained above, but if such data were to be found then this would be reasonable but unfortunately not conclusive. It follows that G must be the cause of the physical properties of the universe that cause it to organize itself.

Figure B: All Existence (AE) is represented by the 3D box, and the universe U is represented by the red plane contained within. The laws of physics hold inside of U, but not necessarily in $AE \setminus U$.

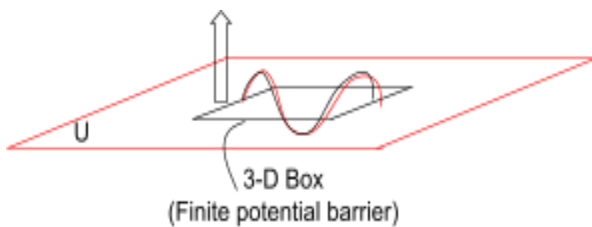
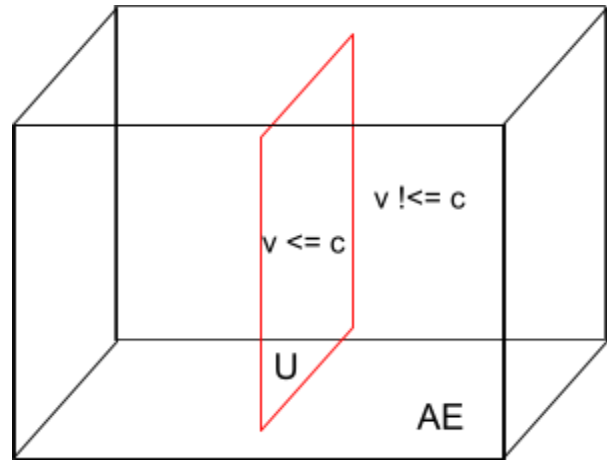


Figure C: The universe U as depicted in Figure B with a semi-plane contained inside representing a 3D box of finite potential barrier. The wave function of a particle contained inside is represented by the black wave, and it extends perpendicular to U in a higher dimension. At some point, a perturbation occurs in the wave resulting in the endpoints extending past the barrier (red wave) resulting in tunneling.

Since time is not a dimension, the structure of the universe is different from that predicted by general relativity, and thus quantum mechanics must also be interpreted differently. Consider that shown in Figure B where AE is represented by a 3D box, and the universe U is represented by a red plane contained inside. All we know is that the laws of physics hold inside of U, but they do not necessarily hold in $AE \setminus U$. In Figure C there is the same 2D representation of U as that in Figure B, with a 2D representation of a 3D box of finite potential barrier contained inside. A particle placed inside of the box has a wave function shown in black. Since the box is 3D, the wave function is 4D, thus its amplitude extends perpendicular to U. When a perturbation occurs, one of the black wave end-points move to the exterior of the box as shown by the red wave. The particle therefore has a probability of

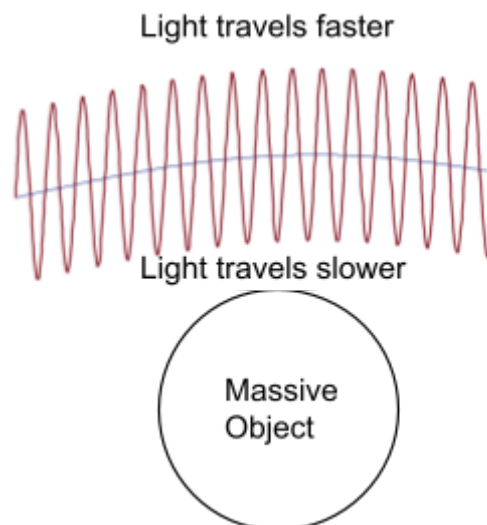
escaping resulting in quantum tunneling. Since the speed v is not bounded by c in $AE \setminus U$, force carriers for entangled particles are not limited to propagation speed c provided that they travel in $AE \setminus U$.

Take the particle depicted in Figure C that is represented by the black wave, and remove the box (particle in free space). Since $v \leq c$ in $AE \setminus U$, the particle is allowed to move rapidly. It therefore changes quantum states rapidly giving the illusion that all of the states exist at the same time. You can therefore take a measurement, and one of those rapidly changing states is observed. So to be clear, the states predicted by quantum mechanics do not exist at the same time, the particle simply changes states faster than the speed of light allows giving the illusion that the states exist simultaneously.

Take the particle depicted in Figure C but without the box (particle in free space again). It travels through U , but also perpendicular to U and thus it doesn't exist in U continuously (it appears to pop in and out of existence in U). Now suppose that such a particle is joined with a second particle that has the exact opposite wave function. The 2 particles wave functions cancel out, and therefore they begin to definitively exist in U . In reality, 2 particles are unlikely to have the exact opposite wave functions so a 2 particle pair will still travel in and out of U . However, the more particles that come together to form heavier objects, the more likely that the superposition of the individual waves becomes zero, and therefore heavier objects exist more definitively in U than lighter ones. So to be clear, if you take n individual particles, their wave functions cause them to travel in and out of U . When those particles are joined together, their wave functions cancel out causing them to exist more definitively in U which is why large objects such as cars and people are always observable. You can then convert those heavier objects back into individual particles due to $E = mc^2$ and those individual particles then move in and out of U again and the object disappears. Those particles can then be forced back into U through a strong magnetic field as predicted by the Schwinger Effect.

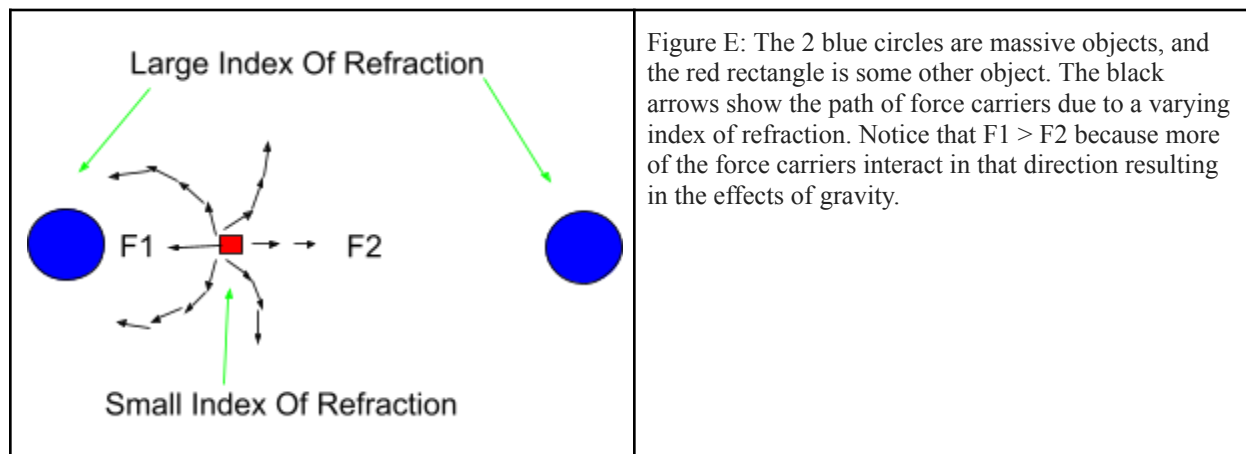
Since a particle travels in and out of U , the information that we have about the particle only pertains to when it is in U , and therefore it can only be described statistically. However, with additional information pertaining to the particles' higher dimensional components, everything is deterministic as explained in more detail below.

Figure D: A photon (red) travels past a massive object causing what is observed as gravitational lensing. The photon travels slower when it is closer to the massive object, then when it is further away causing the path to curve.



This model of quantum mechanics is consistent with wave particle duality, superposition, quantization of fundamentals, entanglement, uncertainty, quantum tunneling, interference patterns, and it never violates causality. It explains why we obtain the experimental results that we do without any paradoxes. So how do we tie this model of

quantum mechanics to the Proper Light Speed theory? Notice that both light and particles travel in waves. If the wave path provides the particle with information causing it to move, then light speed dilation and quantum mechanics can be tied together. In Figure D, a photon travels past a massive object. As the photon travels back and forth along its path, it obtains information about the environment in which it traverses. When the photon's path is closest to the massive object, the photon travels slower than when it is further away due to a larger index of refraction. This causes the light curve resulting in gravitational lensing. The same thing occurs in quantum mechanics, where the wave-particle duality provides the particle with information about the environment and it responds accordingly. It is the wave-particle duality of nature, the fact that particles travel in and out of U, and the varying index of refraction of U that results in all experimental outcomes.



In Figure E, two massive objects represented by the two blue circles, and one smaller object represented by a red rectangle, are interacting. Due to the varying index of refraction, the force carriers for the red object tend to propagate towards the blue circle on the left resulting in a stronger force in that direction which we call gravity.

Putting all of this together: Time is not a dimension, it is a property of existence (It is the ability for change.). The fundamental forces all propagate collectively at a rate that is bounded by what the object perceives the speed of light to be (proper speed of light is dependent on energy density and velocity and is due to a varying index of refraction). Since all particles use these forces, the particles within a clock and within the body all dilate based on proper light speed and this results in all of the experimental outcomes of general relativity such as clock time dilation. Particles have a wave function that causes them to travel in and out of U. The more particles that come together to form an object, the more those wave functions cancel resulting in the object existing more definitively in U and therefore we can observe it. The object can then be converted into energy according to $E = mc^2$ where the particles separate resulting in their wave functions no longer canceling and thus they begin to move in and out of U again. Those particles can then be forced to exist in U by pulling them from the “vacuum” with a strong magnetic field. The wave particle duality of nature results in the ability for particles to interact with the environment. Due to the larger index of refraction near massive objects, the force carriers tend to propagate towards the massive object resulting in a larger force towards it that we call gravity.

With this said, this model is not a mathematical representation of quantum mechanics, it is a restructuring of the universe that the principles of quantum mechanics are applied to. Maxwell's equations for light propagation in a gravitational field are derived using this new structure of the universe, as well as the equations modeling the proper speed of light. Therefore, this structure of the universe is consistent with light propagation, gravity, and quantum mechanics thus unifying these theories into one.

With this said, everything that we observe points to the universe being isomorphic to a simulation, and simulations do not need to have mechanisms. The rest of the article is dedicated to mathematically showing the above conclusions to be consistent with experimental results and observations in general relativity as everything stated herein is dependent on the variable speed of light.

Definitions

System (A) refers to an object with a given state A_n . An **isolated system** is one in which a state change cannot be caused by anything outside of the system (or vice versa), and where nothing leaves or enters.

Existence is the state of being. **Clarification:** If some property of X is measured, then X exists. If X exists, this does not mean that some property of X can be measured.

Universe (U) refers to all of the fundamental components of what is referenced in General Relativity as our spacetime object. If anything exists beyond the universe, that isn't included in the definition. In the case of cyclic models in which a new universe emerges from within the previous, both the old and the new universe are included in the definition.

All Existence (AE) refers to all of the fundamental components of all that exists. If anything exists beyond this universe, AE includes the fundamental components of that as well. AE is by definition isolated since P_2 below holds. Thus, $U \subseteq AE$.

Nothing (\emptyset) is defined as the absence of existence. Mathematically, if $C = \emptyset$, then $\forall C_i \subseteq C, C_i = \emptyset$. **Clarification:** This is important for showing that at no point can we actually produce existence from non-existence, but we can produce something from something else in such a manner as to conserve certain properties.

E is an event $E_n \equiv A_n \rightarrow A_{n+1}$ of system **A** going from one state to the next.

Causality means that cause precedes effect.

Time is the ability for change [source needed]. You either have an ability for change, or you don't: there aren't varying degrees of time. The fundamental forces all propagate at a speed that is collectively bounded by c . When those forces propagate (a change), the clock ticks, and events are measured against the clock. So the property of time gives rise to our clock-speed measurement, but the clock-speed measurement is based on the local speed of light. The local speed of light dilates based on velocity and energy density giving rise to clock-speed dilation. When referencing herein that time is eternal, what is meant is that time is a property of existence and existence has always existed. Therefore the property of time has always existed. Without the ability for change, the universe cannot begin. It is important to clarify that time as used in physics is clock-speed. Time itself is just an ability for change.

where each component of y is $x \stackrel{\text{def}}{=} 0 \mid k(0) = \left| \frac{y}{0} \right|(0) = z$, where $\left| \frac{y}{0} \right| \stackrel{\text{def}}{=} \infty$. It follows that

$$z = \left| \frac{y}{0} \right|(0) = \left| \frac{y}{0} \right|\left(\frac{0}{2}\right) = \frac{1}{2} \left[\left| \frac{y}{0} \right|(0) \right] = \frac{z}{2} \Rightarrow z = \frac{z}{2} \text{ which is only true for } z = 0, \infty. \text{ Let}$$

$z = \infty = \left| \frac{y}{0} \right|$ so that $\left| \frac{y}{0} \right| = \left| \frac{y}{0} \right|(0) \Rightarrow 1 = (0)$ resulting in a contradiction. Let $z = 0$ so that $0 = \left| \frac{y}{0} \right|(0) \Rightarrow 1 = \infty$ also resulting in a contradiction. Thus, all values of z result in a contradiction proving that when you attempt to produce something from nothing, you get a contradiction. This is an important case to consider, as some try to use division by zero as a means of attempting to produce something from nothing.

Consider producing y from $x \mid y + (-y) = x$. Since y doesn't exist to produce $(-y)$, and $(-y)$ doesn't exist to produce y , y and $(-y)$ must be produced from x independently. Thus the above proof holds for such cases.

P_3 : Time has always passed, AE has always existed, this doesn't violate causality, and each state of an isolated system is finite in time.

Proof: Let \mathbf{A} represent an isolated system in the state A_{n+1} where A is the set of all states of \mathbf{A} in order of occurrence; A_n and $A_{n+1} \in A$; $n \in \mathbb{Z}$; and $A_{n+1} \neq \emptyset$. Let $\check{T}(A_i)$ be the length of time in which \mathbf{A} is in state A_i .

1) Prove that if $A_c \in A$, then $A_c \neq \emptyset$:

Since $A_{n+1} \neq \emptyset$, and \mathbf{A} is isolated, then by P_2 , $A_c \neq \emptyset$.

2) Prove that $A_{n-1} \in A$:

a) Suppose that $\check{T}(A_n) = \infty$. Since $|\{A_n, A_{n+1}\}| = [2 < \infty]$, by P_1 , state A_{n+1} isn't possible, contradicting the premise that $A_{n+1} \in A$. Since \mathbf{A} is isolated and P_2 holds, by contradiction, $\check{T}(A_n) < \infty$, thus $A_{n-1} \in A$.

b) Suppose that $\check{T}(A_n) < \infty$. Since P_2 holds, $\exists A_{n-1} \in A$.

3) Prove that the $|A| = \infty$:

Since A_n and A_{n+1} being elements of A proves that $A_{n-1} \in A$, A_{n-1} and A_n being elements of A proves that $A_{n-2} \in A$. It follows that $\exists A_{k+1} \in A \forall k \leq n$, where $k \in \mathbb{Z} \Rightarrow |A| = \infty$.

Since AE is isolated because P_2 holds, and it has at least 2 states that are not \emptyset , we can let $\mathbf{A} = \text{AE}$.

1. Since $|A| = \infty$, $|\text{AE}| = \infty$, thus time has always passed. Time is a property of existence.
2. Since $\exists A_{k-1}$ (cause) $\forall A_k$ (effect), every effect has a cause. Thus AE always existing doesn't violate causality.

The Proper Light Speed Theory

(The Proper Light Speed theory is based on the framework of GR, and it is only intended to show that the above claims are consistent with experimentation. If GR were found to be wrong, it would not change the fact that the above statements are consistent with experimental results to date. Any differences would need to then be compared.)

In this model, $U \subseteq AE$, and time is a change of state of AE. As time passes, light travels at some rate relative to time. Since all measurement devices effectively operate on light, we therefore measure events against the speed of light, not directly against time. Essentially time passes, light travels causing clocks to tick, and then we measure events against the clock. Since time is a property of existence, it follows that time can't go to 0, and thus the speed of light is what dilates, not time. Since our ability to measure time is dependent on the speed of light, when the speed of light slows down it can appear as if it is time that is dilating. It follows that as what is referred to as the proper time τ decreases, the speed of light is what actually decreases $\Rightarrow c_0 dt = c d\tau$, where c_0 is the proper speed of

light. Since $c d\tau = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}$, it therefore follows that $c_0 = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}$ where $g_{\mu\nu}$ is the metric tensor. The equations of general relativity are used without time dilation to show that it is the speed of light that changes, and this change in the speed of light results in what is observed as time dilation, and length contraction, without either actually occurring. This theory shows that light experiences time in the same capacity as everything else, hence why light can move.

Postulate 1: Time runs at a constant rate throughout AE, but our ability to measure time is tied to the local speed of light.

Postulate 2: To someone observing the effects of what we call time dilation, one cannot tell if it is the speed of light, or time that is changing.

Starting with the Schwartzchild metric having only radial components, we get $c_0 = \sqrt{\frac{r-r_s}{r} c^2 - \frac{r}{r-r_s} v^2}$, where the Schwartzchild radius $r_s = \frac{2GM}{c^2}$. Plotting this as in Figure 1, we see that c_0 changes as a function of r , r_s , and v . Therefore:

$$c_0(r, r_s, v) = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} \quad (1)$$

Thus, the proper speed of light is tied to velocity. Setting $r_s = 0$, and $v = c$, we see that $c_0 = 0$ meaning that light doesn't observe other light catching up to it: This holds true for light regardless of what the local speed of light C is. From Figure 1, we see that the magnitude of the local speed of light is the first term in the metric which for the Schwartzchild metric yields:

$$C = \sqrt{\frac{r-r_s}{r}} c \quad (2)$$

It follows that since c was measured on earth, the actual speed of light in zero-g would need to be modified by the inverse of equation (2): We shall ignore this.

Since $|c_0| \leq \sqrt{\frac{r-r_s}{r}}c$, and $\lim_{r^+ \rightarrow r_s} \sqrt{\frac{r-r_s}{r}} = 0$, it follows that $c_0(r_s, r_s, v) = 0$ which $\Rightarrow v = 0$ for both

light and mass at the event horizon of a black hole, and that the event horizon is never actually reached.

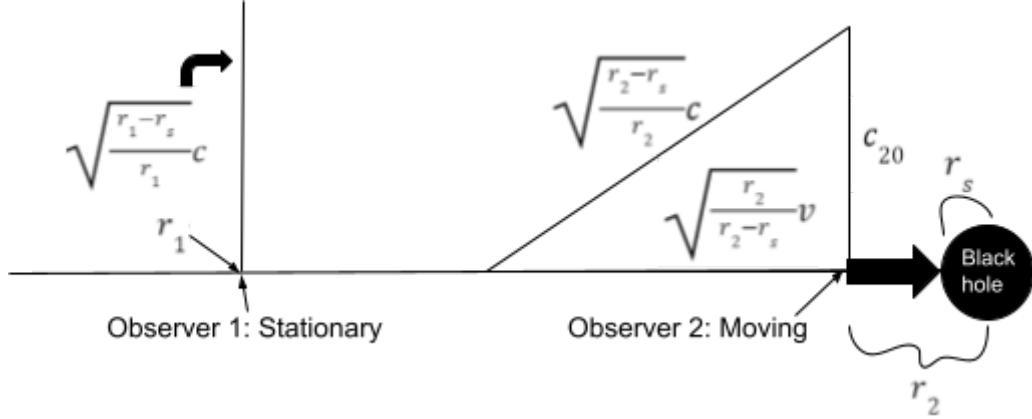


Figure 1: Shows the relationship between the components of the Schwartzchild metric.

From Figure 1, suppose that observer 2 starts at $r_2 = \infty$ (far left), and moves towards Observer 1. The proper velocity is:

$$v_0 = \sqrt{\frac{r_2}{r_2 - r_s}} v_2 \quad (3)$$

So initially, the proper velocity is $v_0 = \sqrt{\frac{\infty}{\infty - r_s}} v = v$. This means that when Observer 2 passes Observer 1,

$v_0 = \sqrt{\frac{r_1}{r_1 - r_s}} v_1 = \sqrt{\frac{r_2}{r_2 - r_s}} v_2$, where v_1 is the velocity of Observer 2 according to Observer 1. Since v_2 is the velocity of Observer 2 as viewed from a stationary reference frame at r_2 , for Observer 1,

$$v_1 = \sqrt{\frac{r_2(r_1 - r_s)}{r_1(r_2 - r_s)}} v_2 \quad (4)$$

Since this is also constant, as $r_2 \rightarrow r_s$, $v_2 \rightarrow 0$. It follows that Observer 1 sees Observer 2's velocity go to 0, and Observer 2 sees their velocity remaining the same. This is only possible because all velocities are measured against their local speeds of light (see the definition of Time above). This tells us that:

$$v_0 = \frac{c}{c_0} \frac{dr}{dt} \quad (5)$$

where c_0 is the proper speed of light at r , and $\frac{dr}{dt}$ is how quickly r changes with time (not relative to light) as viewed from $r = \infty$. For example, $v_0 = \frac{c}{c} \frac{dr}{dt}$ at $r = \infty$; and since $c_0(r_s, r_s, v) = 0$, $\frac{dr}{dt} = 0$ at the event horizon. For light, $c_0 = 0$, so $\frac{dr}{dt} = 0$ as perceived at $r = \infty$ (you can't see light once it is gone). It's important to note that length contraction doesn't physically occur. If we wanted to say that there is some clock speed τ in the universe relating velocity to the

local speed of light such that $v_0 = \frac{dr}{d\tau}$, then $\frac{dr}{d\tau} = \frac{c}{c_0} \frac{dr}{dt} \Rightarrow c_0 dt = c d\tau$ which is what we concluded above. In fact, all we really care about is the relationships between clock speeds so it is easiest to use τ in calculations, but in reality it is the speed of light that dilates, not time. This leads us to:

Postulate 3: The proper time τ ties velocity to the local speed of light (not to time). Therefore, we replace t with τ in all of our non-relativistic equations, and we replace c with C . To observe what the function looks like locally we leave it in terms of τ , and to see what it would look like from far away we convert τ into t using the metric. To a distant observer τ goes to 0 at the event horizon so all motion stops: To an observer headed towards a black hole, nothing changes. The laws of physics are the same everywhere but they play out according to the local speed of light. Thus, if there are any inconsistencies, this needs to be addressed in the equations of our local laws, not necessarily in the metric (or this theory).

Example (Free Particle over a small distance and slow velocity): The time solution is of the form

$\Psi(\tau) = Ae^{-i\omega\tau}$. Thus $\Psi(t) = Ae^{-i\omega\sqrt{\frac{r-r_s}{r}}t}$, which means that to an observer in zero-g, the time component of the particle's wave is stretched out due to the slower speed of light in the field compared to out of the field.

These differences might seem minute, but this is important because time is not actually a dimension: Time is a property of existence, and therefore the structure of this “lightspace” doesn’t include time. This means that wormholes do not exist, time travel into the past is impossible, length contraction doesn’t physically occur, causality always holds in relation to t , yet black holes still exist.

Deriving Maxwell’s Equations of light in a gravitational field: From equation (1), for a photon traveling in a plane containing the COM of some object of mass M we get ($c_0 = 0$ for light):

$$\frac{r-r_s}{r}c^2 - \frac{r}{r-r_s} \left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\theta}{dt}\right)^2 = 0 \quad (6)$$

From Figure 2, we see that the components of equation (6) require a velocity vector of:

$$\vec{v} = \left\langle \frac{dx}{d\tau} = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \cos(\theta) - r \frac{d\theta}{dt} \sin(\theta), \frac{dy}{d\tau} = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \sin(\theta) + r \frac{d\theta}{dt} \cos(\theta) \right\rangle \quad (7)$$

Where $x = r\cos(\theta)$, $y = r\sin(\theta)$, and $r = \pm \sqrt{x^2 + y^2}$. Thus:

$$\frac{dy}{dx} = \frac{\sqrt{\frac{r}{r-r_s}} \sin(\theta) + r \frac{d\theta}{dr} \cos(\theta)}{\sqrt{\frac{r}{r-r_s}} \cos(\theta) - r \frac{d\theta}{dr} \sin(\theta)} \quad (8)$$

Notice that τ is used instead of t as required by postulate 3. Dividing the x-component in equation (7) by dx , squaring both sides, and multiplying by $\partial^2 E_x$ yields:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \cos(\theta) - r \frac{d\theta}{dt} \sin(\theta) \right]^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial \tau^2} \quad (9)$$

Repeating the same process for the y-component we get:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \sin(\theta) + r \frac{d\theta}{dt} \cos(\theta) \right]^2 \frac{\partial^2 E_y}{\partial y^2} = \frac{\partial^2 E_y}{\partial \tau^2} \quad (10)$$

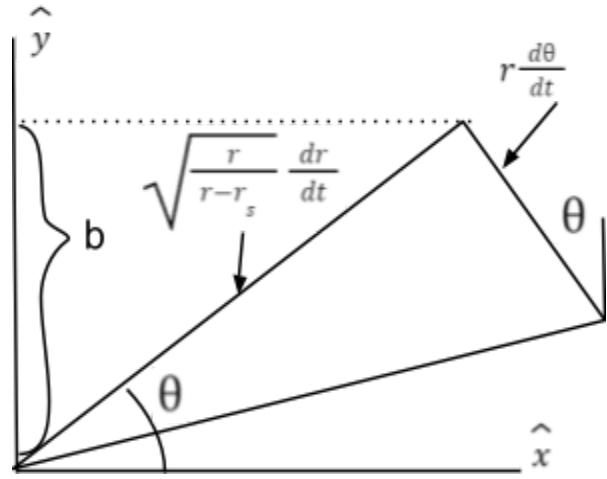
Equations (9) and (10) are Maxwell's Equations for light in a gravitational field in the \hat{x} and \hat{y} directions respectively, where the light is traveling through some plane going through the COM. Notice that when $r_s = \theta = 0$ equation (9) yields:

$$\left[\frac{dr}{dt} = c\right]^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial \tau^2} \quad (\text{Maxwell's Eq for light in zero-g, } \hat{x} \text{ - direction})$$

Likewise, when $r_s = 0$, and $\theta = \frac{\pi}{2}$, equation (10) yields:

$$\left[\frac{dr}{dt} = c\right]^2 \frac{\partial^2 E_y}{\partial y^2} = \frac{\partial^2 E_y}{\partial \tau^2} \quad (\text{Maxwell's Eq for light in zero-g, } \hat{y} \text{ - direction})$$

Figure 2: This figure shows how the components of equation (6) fit geometrically. For a mass positioned at (0,0), a photon released in the x-direction from (x=0, y=b) will curve downwards.



Clarification: Since the proper time of spacetime is also 0 for light, these equations can be derived using the spacetime metric as well.

General Relativity Experimental Results Comparison

Clock-speed dilation: From equation (3), $v_0 = \sqrt{\frac{r}{r-r_s}} v$. Thus $\frac{dr}{d\tau} \sqrt{1 - \frac{r_s}{r}} = \frac{dr}{dt}$, and therefore $d\tau = \sqrt{1 - \frac{r_s}{r}} dt$ which is of course the exact Schwartzchild solution for the time dilation of a non-rotating object in space.

Black holes: Since $c_0(r, r_s, v) = 0$ for light, equation (1) yields $\frac{dr}{dt} = \pm \frac{r-r_s}{r} c$ (assuming no radial components). $\frac{dr}{dt} = -\frac{r-2}{r} c$ is plotted in Figure 3, where we see that inside the event horizon the velocity is positive, and outside the event horizon the velocity is negative. Therefore, all of the light of a black hole moves to the event horizon, and mass follows. Thus, there isn't a singularity inside of a black hole. If you consider the acceleration of light $\frac{d^2 r}{dt^2} = \frac{r-r_s}{r^3} c^2$, you see that inside the event horizon, the acceleration changes direction. These results are consistent with Susskind's proof that the amount of information inside of a black hole is proportional to the surface area of the event horizon as all of the information is actually on the event horizon separated by what is assumed to

be the minimal distance allowed by quantum mechanics. Notice that the acceleration for light is 0, not ∞ , at the event horizon.

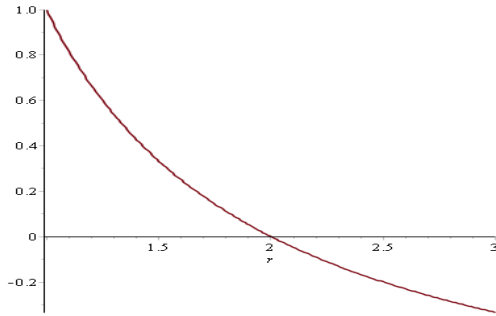


Figure 3: This is a plot of the velocity of light which shows that light always moves towards the event horizon (shown as $r = 2$), not a singularity.

Gravitational Redshift: Using equation (9) in only the \hat{x} - direction yields:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \right]^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2} \quad (11)$$

Since there aren't any rotational velocities, equation (6) tells us that $\frac{r-r_s}{r} c^2 = \frac{r}{r-r_s} \left(\frac{dr}{dt} \right)^2$. Thus, equation (11) becomes:

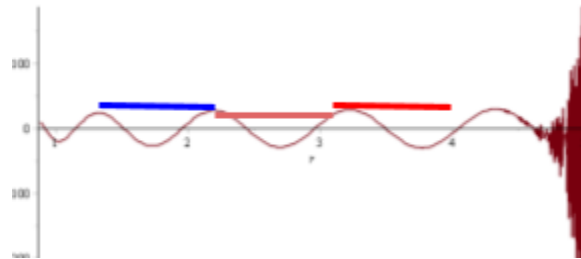
$$\frac{r-r_s}{r} c^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2} \quad (12)$$

Setting $E = R(r)T(\tau)$ we get:

$$\frac{d^2 R(r)}{dr^2} = - \left[k^2 \frac{r}{r-r_s} \right] R(r) \quad (13)$$

The solutions for equation (13) are Whittaker functions shown in figure 4 for arbitrary values simply to show the shape. As you can see from the red and blue stripes, gravitational redshifting occurs as the photon gets further and further from the event horizon.

Figure 4: The blue and red stripes are the same length, thus, this figure shows a gravitational redshift as the light gets further from the event horizon on the left.



From equation (13):

$$k \sqrt{\frac{r}{r-r_s}} = \frac{2\pi}{\lambda} \quad (14)$$

Thus:

$$\lambda = \frac{2\pi}{k} \sqrt{\frac{r-r_s}{r}} = \lambda_\infty \sqrt{\frac{r-r_s}{r}} \quad (15)$$

Where equation (15) is the exact relationship between λ and λ_∞ as predicted by GR.

Gravitational Lensing: From equation (11), $C = \sqrt{\frac{r-r_s}{r}} c = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt}$. This suggests that the index of refraction is either $n_1 = \frac{r}{r-r_s}$, or $n_2 = \sqrt{\frac{r}{r-r_s}}$, depending on the frame of reference. When deriving the gravitational lens equation for GR using a Fermat Surface, $n = \sqrt{\frac{r+r_s}{r-r_s}}$ (Bacon). Setting n_1 equal to n we get: $\sqrt{\frac{r+r_s}{r-r_s}} = \alpha_1 \frac{r}{r-r_s} \Rightarrow \alpha_1 = \frac{\sqrt{r^2 - r_s^2}}{r}$ which is ~ 1 for the exterior of anything that isn't a black hole. Setting n_2 equal to n we get $\sqrt{\frac{r+r_s}{r-r_s}} = \alpha_2 \sqrt{\frac{r}{r-r_s}} \Rightarrow \alpha_2 = \sqrt{\frac{r+r_s}{r}}$ which is also ~ 1 for the same. Using the calculus of variations to minimize the functional $\int_b^R N \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2} dr$, where $N \in \{n, n_1, n_2\}$ and θ is as shown in Figure 2, we can derive $\theta(r)$. However, the solutions are integrals, so it is easiest to compare $\frac{d\theta}{dr}$ as shown in Figure 5 where we use the mass of the sun. It is important to note that the left side doesn't matter since those points are inside of the sun. In fact, $r_{sun} \cong 6.96 * 10^8$ meters, so the entire plot shown in Figure 5 is still inside of the sun. This means that the predictions of this theory and those of GR are identical in any region that can be tested in our solar system.

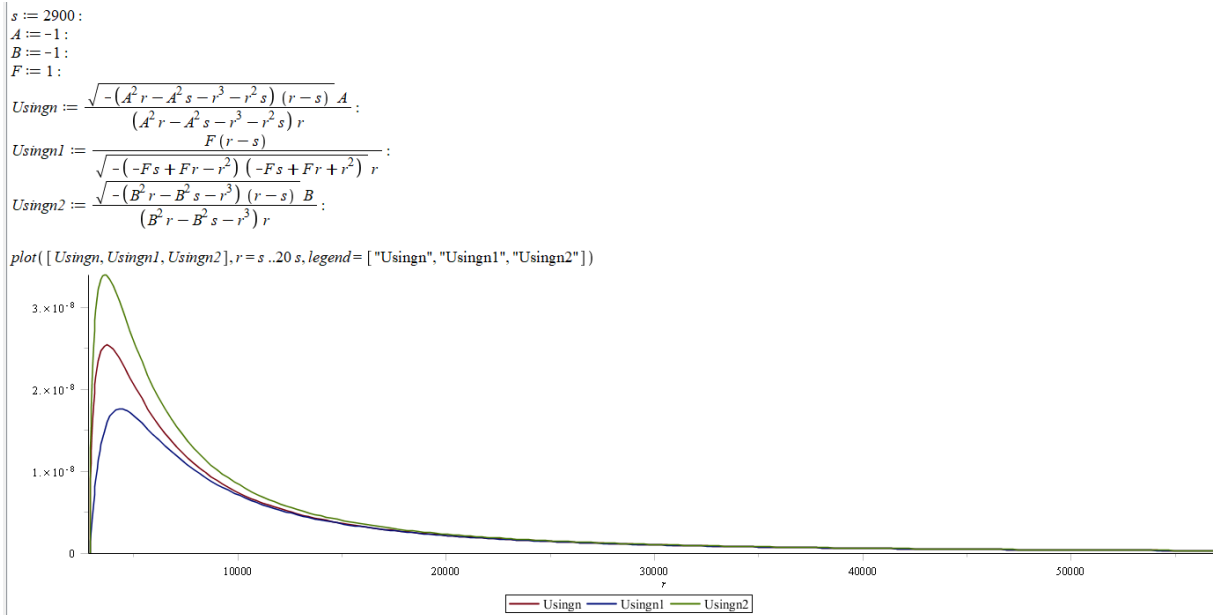


Figure 5: (Units in meters) These are plots of $\frac{d\theta(r)}{dr}$ where $s = r_s = r_{s, sun}$. They are only meant to show the relationship between the solutions as the constants were arbitrarily selected. Note that every point on this plot is still inside of the sun, so in the region where this is testable, the solutions are identical.

Gravitational Waves: It appears that there exists some fabric of lightspace that expands and contracts based on energy density, and this expanding and contracting changes its local index of refraction in the same capacity as time was thought to do so. It thus makes sense that such expanding and contracting would propagate through space as a wave.

The Universe's Expansion: Has not been looked into.

Muon Decay: From Figure 1, $c_0^2 = \frac{r-r_s}{r} c^2 - \frac{r}{r-r_s} \left(\frac{dr}{dt}\right)^2$. In zero-g this yields: $c_0^2 = c^2 - v^2$, and

restructuring this gives us: $\frac{c_0}{c} = \sqrt{1 - \left(\frac{v}{c}\right)^2}$, so any perceived effects of special relativity are immediately recovered without time and length contraction, and gravity is already built-in. So the lorentz factor appearing in the relativistic decay equation can be interpreted further as meaning that the half life of a particle is a function of how the particle perceives light. That is:

$$T_{1/2}(c_0) = \tau_{1/2} \frac{c}{c_0} \quad (21) \quad (\text{Approximation})$$

Thus, the faster the particle moves, the slower the particle perceives light, and thus everything that occurs within the particle is slowed down without the need for time or length contraction.

Length Contraction: Imagine that Muons are produced at a height H above the surface of the earth, and that the percentage of Muons to hit the surface before decaying is consistent with the predictions of special relativity (which it is). It should be clear that H didn't physically shrink in either reference frame in any capacity. Thus, while special relativity does yield the correct answer, how it got to the correct answer is not author approved. The logical conclusion here is that H stays the same, and the particle's decay rate depends on how the particle perceives light as shown in equation (21). This makes sense because all of the forces involved communicate at a rate that is governed by the speed of light.

Quantum Mechanics Experimental Results Comparison

Entanglement: Since entangled particles share a wave function, the force carriers simply need to propagate in AE\U so that they are not limited by c.

Quantum Eraser: The entangled photons share a wave function, and the force carriers are not limited by c as explained above. One photon can hit the screen, and the information is carried in AE\U at faster than light speed to the second particle. The second particle then hits its screen, and sends information back to the first particle changing outcomes at faster than the speed of light.

Superposition: Particles are only bound by the speed limit c when they exist in U. When traveling in AE\U they change states very rapidly, and when they pass back through U they are in a different state. This rapid change in quantum states gives the illusion that all of the states exist simultaneously.

Double Slit: The wave function of the current state passes through the 2 slits causing them to head in the direction of one of the interference patterns. The particle follows and hits the screen.

Additional Definitions and Claims

Deterministic refers to an event $E_n \equiv A_n \rightarrow (C_{a1}A_{a1} \vee C_{a2}A_{a2} \dots \vee C_{am}A_{am})$ with fixed probabilities $(C_{a1}, C_{a2}, \dots, C_{am})$ respectively. The fact that there is a fixed weight or probability, tells us that there is an underlying rule that ensures those weights are obeyed. In the classical case $am = n+1$, $A_n \rightarrow A_{am}$, and $C_{am} = 1$. In the quantum case, A_{a1} through A_{am} represent the possible states, and C_a through C_{am} are the probabilities of those states. Thus quantum mechanics is deterministic.

Free Will refers to an event $E_n \equiv A_n \rightarrow (C_{a1}A_{a1} \vee C_{a2}A_{a2} \dots \vee C_{am}A_{am})$ where the respective probabilities $(C_{a1}, C_{a2}, \dots, C_{am})$ are not fixed.

Suppose that you have a dart board with different states or sections $(A_{a1} \vee A_{a2} \dots \vee A_{am})$, each with their respective probability $(C_{a1}, C_{a2}, \dots, C_{am})$ of getting hit. As long as the board is far enough away that one's skill in throwing darts is not valuable, the probabilities of each state can be calculated based entirely on the areas of each state in relation to the others (ignoring misses). This is only true because everything about the dart is deterministic. If the dart had free will, there would be no way to assign such probabilities.

Claim 1: There must exist $G = \{\dots, A_{n-1,j}, A_{n,j}\} \subseteq AE$ where $A_{n,j} \subseteq A_n$ | G has free will, and G organized U.

In this section a case is made for why the author believes that each standard model pertaining to the beginning of the universe isn't possible:

Conformal Cyclic Cosmology:

Let u be a m -dimensional volume in which the laws of quantum mechanics, and general relativity hold, and let $u \subseteq U$. Since space is expanding \exists some distance D in which 2 events are non-existent to each other due to the finite speed limit c . We thus define a point P in U in which event $E_{n,P \pm \epsilon} \equiv U_{n,P \pm \epsilon} \rightarrow U_{n+1,P \pm \epsilon}$ representing the universe's beginning (or this cycle of it) occurs, and then define u as being the volume enclosed by distance D around P in m -space. Since space expands uniformly, there is nothing unique about point P , thus every point in u has the same probability for a similar event $E_{a,i}$ ($a \geq n$) to occur. We thus establish all of the points in u as a grid, where each point is separated by a planck length l_p : We then define t_{min} (from P_1) to be the Planck time | every t_{min} an event $E_{a,i}$ could occur at each point in u (outside of the light radius of P). We now calculate the probability that $E_{n,P \pm \epsilon}$ is the only such event that occurs in u over time D/c , for $m = 3$.

Let the radius of a sphere be a multiple (k) of l_p . We divide the area of the sphere by the area of an equilateral triangle of side length l_p , to get the \sim number of triangles that grid the sphere. Thus, the approx. number of triangles $\blacktriangle(k)$ is:

$$\blacktriangle(k) \cong \frac{4Pi(k \cdot l_p)^2}{(\frac{l_p^2 \sin(Pi/3)}{2})} = \frac{16Pi(k)^2}{\sqrt{3}}$$

The number of triangles on the surface of the sphere relates to the number of points $\ast(k)$ by the approx. relation $\ast(k) \cong \frac{1}{2} \blacktriangle(k)$. Thus:

$$\ast(k) = \frac{8Pi(k)^2}{\sqrt{3}}$$

It follows, that at the moment of $E_{n,P \pm \epsilon}$, there existed $\frac{8Pi}{\sqrt{3}} \sum_{k=1}^q (k)^2$ opportunities for $E_{a,i}$ to occur elsewhere within u , where $q = \lfloor D/l_p \rfloor$. By the time that light from P reached the next planck length to communicate that $E_{n,P \pm \epsilon}$ occurred, another $\frac{8Pi}{\sqrt{3}} \sum_{k=2}^q (k)^2$ opportunities passed, followed by $\frac{8Pi}{\sqrt{3}} \sum_{k=3}^q (k)^2$ the following $t_{min} \dots$ Thus, the number of opportunities for $E_{a,i}$ to occur in D is:

$$\begin{aligned} \frac{8Pi}{\sqrt{3}} \sum_{j=1}^q \sum_{k=j}^q (k)^2 &= \frac{8Pi}{\sqrt{3}} \sum_{j=1}^q \left(\frac{q(q+1)(2q+1)}{6} - \frac{(j-1)j(2j-1)}{6} \right) \\ &= \frac{4Pi}{3\sqrt{3}} \left(q^2(q+1)(2q+1) - \sum_{j=1}^q j(2j^2-3j+1) \right) \\ &= \frac{4Pi}{3\sqrt{3}} \left(q^2(q+1)(2q+1) - \left[2\left(\frac{q(q+1)}{2}\right)^2 - 3\frac{q(q+1)(2q+1)}{6} + \frac{q(q+1)}{2} \right] \right) \\ &= \frac{2q^2Pi}{\sqrt{3}} (q^2 + 2q + 1) \end{aligned}$$

If we now think of each point in u as a die with η distinct states in which only 1 results in $E_{a,i}$, then the probability that $E_{a,i}$ doesn't ever occur in u over time D/c is $\left(\frac{\eta-1}{\eta}\right)^{\frac{2q^2Pi}{\sqrt{3}}(q^2+2q+1)}$. If we set this to greater than or equal to what is typically considered to be "impossible" we get $\left(\frac{\eta-1}{\eta}\right)^{\frac{2q^2Pi}{\sqrt{3}}(q^2+2q+1)} \geq 10^{-50}$, which $\Rightarrow \eta \geq \frac{1}{1-10^{\frac{-25\sqrt{3}}{q^2Pi(q+1)^2}}}$. This means that the vacuum of space must have at least η distinct states (not energy

levels) with one being able to cause $E_{n,P \pm \epsilon}$. Using just the values for when D is one light-second we get $\eta > 10^{170}$. There isn't anything in the vacuum that is known to even remotely come close to this value. Thus $E_{a,i}$ should have occurred many times over, and those events occurring should be apparent in the cosmological data: We should observe galaxies and radiation heading towards us from all directions.

In addition to this, if $E_{a,i}$ and $E_{n,P \pm \epsilon}$ both occur, which according to P_1 this must happen at some point, neither event is aware of the other, so the scale factor in the Conformal Cyclic Cosmology model could not be coordinated leading to inconsistencies.

It follows that the CCC model of cosmology, or any theory attempting to "smoothly" cycle through universes, suffers from these issues.

Other Cyclic Models:

These models are either known to violate the laws of entropy, or they are known to require a beginning a finite number of cycles into the past.

Big Bang Model:

From $P_4 - 3$, the universe cannot exist in a singularity state for an infinite period. So, the BB could occur, but a non-singularity state had to precede the singularity in which our universe emerged: This can't happen without violating the laws of entropy unless an outside force caused it ([non-singularity state \rightarrow singularity state (for all of U)] is a decrease in total entropy.).

Multiverse:

This model has the same issues as the CCC in that we should observe interactions of other universes with our own, and those interactions should be apparent through the CMB which they aren't. Additionally, unless the universes get recycled, they must be made from nothing (even if AE is infinite) violating P_2 .

Conclusion: The universe can't be cyclic (infinitely), it can only begin with a BB if the BB were started from an outside force, and the multiverse isn't supported by the cosmological data. The only other known option is that the process that organized U is controlled. Therefore there must exist $G = \{\dots, A_{n-1,j}, A_{n,j}, A_{n+1,j}, \dots\} \subset AE$ where $A_{n,j} \subset A_n$ | G has free will, and G organized U.

Claim 2: G has the properties attributed to God in the bible, assuming that such qualities are obtainable.

Proof: Let T represent all time, and divide T into 2 infinite periods T_1 and T_2 , where T_1 precedes T_2 . Since T_1 is infinite, by P_1 , if it is possible for G to become perfect, obtain all knowledge, develop moral codes that statistically yield the best results, and organize universes, then G has done so in T_1 . Since T_2 is also infinite, G obtained these qualities an infinite time ago. Thus, no matter how far back you go in time, G has these qualities, hence why "God is the same yesterday, today, and forever". While this is not a proof that the bible is valid, it is a proof that the bible got this correct.

Proposed Experiment

From equation (2), the speed of light remains constant in a stationary reference frame when traveling in the tangential direction, but varies when traveling in the radial direction. Thus a time measurement between photons traveling the same distance vertically and horizontally could be compared. However, this is not currently feasible due to the high precision necessary unless the photons could travel substantial distances. However, the calculations are as follows for consideration:

$$D = [c * \sqrt{\frac{R-r_s}{R}}] * t_h = c * [\frac{1}{D} \int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr] * t_v \quad (22)$$

Where D is the one-way path distance traveled (D must be small enough that the change in radius can be ignored), c is the speed of light in zero-g, R is the radius of the earth, t_h is the one-way time of travel for the photon in the horizontal direction, and t_v is the one-way time of travel for the photon in the vertical direction. Therefore:

$$t_h = \left[\frac{1}{D} \int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr \right] * \sqrt{\frac{R}{R-r_s}} * t_v \quad (23)$$

OR

$$\Delta t = t_h - t_v = \frac{D}{c} \left[\sqrt{\frac{R}{R-r_s}} - \frac{D}{\int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr} \right] \quad (24)$$

For $D = 1000$ meters, $\Delta t \approx 1.8 * 10^{-19}$ s giving an idea as to why the speed of light appears constant. To make this test feasible, a means of measuring the time of travel for 100 seconds or better is needed but of course this exceeds the bounds for D so the equation for t_h would need to be modified.

Conclusion

In this paper we showed that time is a property of existence. It was then shown that the same experimental results of GR can be achieved by allowing time to pass at the same rate while also allowing the local speed of light to vary. This approach shows that what we perceive as time, is tied to our local speed of light, and thus the appearance of time dilation occurs when measuring time where the local speed of light is different than our own. This approach yields the same results as predicted by GR for gravitational lensing (approx.) and redshifting, and it removes all paradoxes known to the author at the time of writing: time travel into the past is impossible; there are no singularities of a black hole; all laws of physics hold everywhere (assuming they hold locally) when they are written in terms of C and τ ; time passed before the universe began thus allowing it to make sense for the universe to begin in the first place; and you never have to create something from nothing. This theory points to a universe that was organized from fundamentals that have always existed, not created from nothing.

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Above all else, the KJV was used as this theory was structurally organized according to the bible.

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