

The Proper Light Speed Theory

(The unification of Maxwell's Eqs, gravity, and Quantum Mechanics)

Author: Russell R. Smith

Email: SmithCoDev@gmail.com

Orcid: <https://orcid.org/0009-0001-5302-9646>

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Abstract

General relativity is perhaps one of the most successful theories in all of physics. While it accurately predicts gravitational redshifting, gravitational lensing, black holes, and more, it fails to be compatible with quantum mechanics due to how each theory deals with scale and energy differences; space and time; singularities; and entanglement. Therefore, GTR and QM are not compatible at the most fundamental of levels. This article proposes a different structure than spacetime in which to apply the principles of QM where the aforementioned issues are resolved and experimental results in both GTR and QM are still accurately predicted. This model explains the results in QM, GTR, and the Michelson Morley experiment in terms of a variable speed of light by recognizing the difference between time and clock speed. It is recommended that one read the Overview with an open mind before deciding on the validity of this theory as it accurately models the experimental results of gravitational lensing, redshift, clock speed dilation, black holes, and muon decay; and it explains why the experimental results of QM do not conflict with those of GTR.

Overview

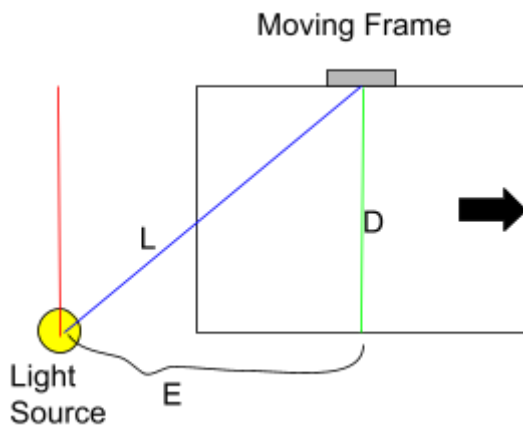


Figure A: This is the typical setup for deriving the equations of time dilation in special relativity. The blue line represents the path of light from the stationary reference frame to the moving reference frame. At the moment that the moving frames mirror is directly over the light source, a flash occurs producing the red and blue photon paths. The moving reference frame travels just fast enough to ensure the blue line hits the mirror.

In Figure A there is the typical setup for the derivation of time dilation in special relativity. As the moving frame goes from left to right, at the moment that the center of the mirror in the moving frame is directly over the light source, a flash occurs. Two photons are emitted, one in the vertical direction (red) and one up and to the right (blue). In special relativity, the green line is treated as if it were the red line because the speed of light is the same in all reference frames. Thus:

$$L = c\Delta t, D = c\Delta\tau, E = v\Delta t$$

$$\therefore L^2 = D^2 + E^2 \Rightarrow (c\Delta t)^2 = (c\Delta\tau)^2 + (v\Delta t)^2$$

Simplifying yields:

$$\Delta\tau = \Delta t * \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (A)$$

Leading to the conclusion that time itself is malleable. Now consider the same setup with the following interpretation. The green line in Figure A is simply the vertical component of the blue line representing the vertical component of the photon's path. This means that the observer in the moving reference frame only sees the vertical

component of the light and thus in the moving reference frame, the speed of light is slower. The equations are as follows:

$$L = C\Delta t, D = c_0\Delta t, E = v\Delta t$$

$$\therefore L^2 = D^2 + E^2 \Rightarrow (C\Delta t)^2 = (c_0\Delta t)^2 + (v\Delta t)^2$$

Simplifying yields:

$$c_0 = C * \sqrt{1 - \left(\frac{v}{C}\right)^2} \quad (B)$$

Where c_0 is the proper speed of light, which is the speed of light in a moving reference frame, and C is the local speed of light in a stationary reference frame. When incorporating the presence of mass, as derived below, the proper speed of light is:

$$c_0 = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} \quad (C)$$

Therefore the proper speed of light dilates exactly as time dilates in GTR. Now consider that all force carriers in the observed RF R_0 propagate collectively at a rate that is bounded by c_0 such that when c_0 decreases, all force carriers within the R_0 RF, including those within a clock and involved in any biological processes, propagate more slowly resulting in clock speed dilation that is not noticed in the R_0 RF. It follows that the clock speed in the R_0 RF $\tau = \tau(c_0)$ is a function of the proper speed of light, and it turns out that the clock speed τ is exactly the proper time of GTR. In other words, the proper time of GTR is simply clock speed dilation caused by light speed dilation. This implies that if we write equations of physics in terms of both the proper time τ , and the proper speed of light c_0 , rather than t and c , then the physics as viewed by an observer in the R_0 RF stays the same everywhere, but it “plays out” at a rate that is entirely determined by the proper speed of light.

Briefly consider that if x is nothing, then by definition, all components of x are also nothing. That is, if $a, b \in x$, then $[a = x] \pm [b = x] = x$ proving something cannot logically be produced from nothing. Even if x isn’t required to follow any rules, or if there were a rule where x had to produce something, x doesn’t exist for it to matter. This is explained in more detail in P_2 for further consideration. It follows that since existence exists today, the fundamentals for such existence have always existed, and therefore they cannot be created or destroyed, they can only permute. Information is therefore never lost.

With that said, let’s consider what time is. Time is simply the ability for change. It is not a dimension, it is a property of existence (“things” have the ability to change). Time allows the fundamentals that have always existed to permute. Since time is a property of existence, the force carriers propagate inside of the clock causing the clock to tick, and we measure events against the ticking of the clock. Since the force carriers cause the clock to tick, we effectively measure events by how far those force carriers propagated inside the clock between the events. This is confusing because it means that a clock doesn’t measure time, it measures how far the force carriers traveled between events due to them having the property of time. When referencing the “passing of time” or “time being eternal”, such terminology references the fact that existence has always existed and time is a property of existence, and thus time is also eternal. One can think of time as a loop in programming: nothing has to change in each loop, but the ability for change is always there. In some areas of this hypothetical program, the proper speed of light is x pixels/loop, and in others it is y pixels/loop, but everywhere the physics stays the same.

With that said, let's clarify some definitions:

c_0 : The speed of light in the observed RF R_0 . This changes with energy density and speed.

c : The speed of light in a RF R_{zero} with zero-g (and no velocity).

t : This is the clock speed in the R_{zero} RF.

τ : This is the clock speed in the moving RF R_0 .

Notice that $c_0 = c$ and $\tau = t$ when $v = m = 0$.

Since clock speed dilates with light speed, it follows that:

$$c_0 dt = c d\tau \quad (D)$$

That is, compared to the R_{zero} RF, light is moving in the R_0 RF at speed c_0 (left side of equation (D)), and in the R_0 RF light is moving at speed c and the clock speed there is τ (right side of equation (D)). Now this can be confusing because the speed of light cannot actually be measured in the R_0 RF from the R_{zero} RF as when the light gets to the R_{zero} RF it is then traveling at c again. So this is not comparing an observation, this is comparing what is actually happening in both RFs. The right side of equation (D) tells us why the speed of light is always measured as constant even though it dilates: The clock speed dilates proportionally with light speed dilation so effectively the speed of light is always measured against itself resulting in a constant value of c .

Since the laws of physics must be the same everywhere, writing equations in terms of τ and c_0 would ensure that both the physics of the event and the clock used to measure it in the same RF would dilate proportionally. This would ensure that a local measurement of events would remain the same everywhere. When light from those events travel elsewhere, the speed of the light in the observer's RF is proportional to the observer's local clock speed so the event could be observed differently. For example, suppose that a rock concert occurred near a black hole. The speed of light near the event would be very slow relative to someone viewing it in freespace, so the event plays out as normal to those at the event, but the event appears "drug out" to the other observer. This is shown by the clock

speed solution for a free particle near the event horizon where $\Psi(\tau) = Ae^{-i\omega\tau}$, and where $\Psi(t) = Ae^{-i\omega\sqrt{\frac{r-r_s}{r}}*t}$ (r is the radius in which the event took place, not where it is being observed from), which means that to an observer in zero-g, the clock speed component of the particle's wave is stretched out due to the slower speed of light in the field compared to out of the field. Let's consider the wave equation for light $\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$. The effects of clock speed dilation have no relevance on this equation, but if we write it as something like $\nabla^2 E = \frac{1}{c_0^2} \frac{\partial^2 E}{\partial \tau^2}$ (not exact) all of the physics can remain the same in every reference frame that light is propagating. The exact equation is derived below, and it accurately models gravitational lensing and redshifting while reducing to $\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$ in zero-g (stationary RF). It is important to note that c_0 is not from the photons perspective (where $c_0 = 0$), it is from the observer's RF (C). It is also important to mention that the equations of physics that we currently have would need; c_0 dependent terms that cancel out yielding the results obtained through experiments (see equation (21) as an example); a re-evaluation of the constant values such as c and h (See equation (2) as an example). So this theory doesn't

necessarily tell us what the equations of physics are so that they hold everywhere, it tells us that such equations need to be written in terms of proper time and proper light speed.

Since you cannot produce something from nothing, we define all existence (AE) as the set of fundamentals that have always existed. Since AE has always existed, it doesn't violate causality. At some place within AE the properties of our universe that are described by our laws of physics became defined, and the local speed of light became non-zero. The universe then began to organize itself from fundamentals, based on these physical properties, in a manner that is consistent with the evolutionary processes described by science. It follows that the observable universe is a subset of AE, and is therefore not all that exists. As predicted by the Big Bang model, after it begins, the universe starts to cool down resulting in the formation of particles, atoms, and the cosmic microwave background radiation (not from nothing). Eventually stars and galaxies form due to the pull of gravity, planets emerge from orbiting dust clouds, and abiogenesis/evolution begin to play out. Notice how each of these steps only occur due to the physical properties of the universe: without these physical properties nothing happens. We can test these physical properties with science, but such tests only help us to understand what the physical properties are, not why they exist in the first place. We therefore need a model that explains why the physical properties of the universe are what they are. It is important to note here the difference between laws of physics and logical principles. Any consistent set of laws of physics that tell fundamentals how to permute could theoretically apply to a universe, but at no point could a universe be inconsistent with logic. This is an important distinction. Just because different laws of physics might be viable, doesn't mean that different rules of logic are. Consider the following:

Any universe that emerges is a subset of AE as if this is not the case, said universe would need to be created from nothing. Assume that our universe emerged through natural processes. Since our universe emerged, it follows that with time being eternal other universes have emerged as well within AE. It follows that remnants of said universes would statistically be observable in our cosmological data in the form of galaxies and light headed towards us from all over. Since we do not observe these remnants in our cosmological data, it follows that the universe didn't emerge through natural means. While there are specific cases to consider, like the universe existing as a singularity for an eternity before "erupting", each of these violate known logical principles as shown further down in the article in more detail. Now, if such data were to be found, this could change the conclusion, but so far no indication of such data is known to the author. Since a universe emerging through natural means doesn't fit the cosmological data, it follows that the universe emerged through unnatural means. Now, please don't hate the author. You are welcome to point to any spot in which there is a mistake, and if you can't do that, then perhaps consider that what is stated is accurate.

Since time is a property that has always existed, if it is possible, it is guaranteed that at some point fundamentals naturally come together to form G. Given an infinite period, anything that can happen will happen (with constraints of course), so if it is possible for G to exist, then it is guaranteed that G exists. We now take the infinite period T in which AE exists, and we divide it into 2 infinite periods T_1 and T_2 , where T_1 precedes T_2 . Since T_1 is infinite, if the natural formation of G is possible then statistically it is guaranteed to have occurred in T_1 . Since T_1 precedes T_2 , and T_2 is infinite, the formation of G occurred an infinite time ago. Thus, no matter how far back in time you theoretically look, G has always existed. By this same logic, if it is possible for G to learn everything, become perfect, or develop rules that statistically yield the best outcome, then this was done an infinite time ago. Just to be clear, logically speaking, if G can be created naturally then we could just skip G and assume the universe could be naturally occurring. The problem is that this approach doesn't match the data as explained above, but if such data were to be found then this would be reasonable but unfortunately not conclusive. It follows that G must be the cause of the physical properties of the universe that cause it to organize itself.

The remaining part of the overview is not a mathematical derivation, it is a means of interpreting the results of quantum mechanics in terms of the framework mentioned above so that light propagation, QM, and the experimental results of GTR are entirely compatible.

Figure B: All Existence (AE) is represented by the 3D box, and the universe U is represented by the red plane contained within. The laws of physics hold inside of U, but not necessarily in $AE \setminus U$.

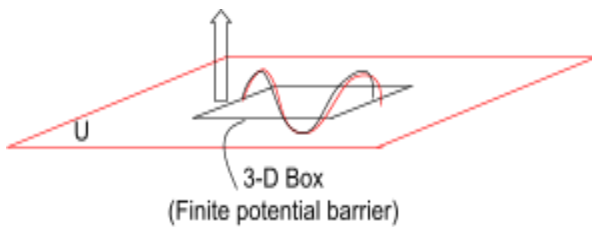
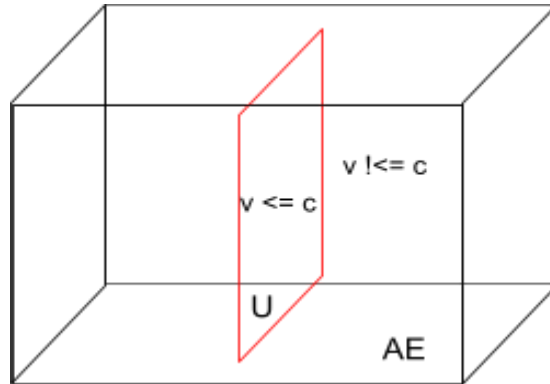


Figure C: The universe U as depicted in Figure B with a semi-plane contained inside representing a 3D box of finite potential barrier. The wave function of a particle contained inside is represented by the black wave, and it extends perpendicular to U in a higher dimension. At some point, a perturbation occurs in the wave resulting in the endpoints extending past the barrier (red wave) causing quantum tunneling.

Since time is not a dimension, the structure of the universe is different from that predicted by general relativity, and thus quantum mechanics must also be interpreted differently. Consider that shown in Figure B where AE is represented by a 3D box, and the universe U is represented by a red plane contained inside. All we know is that the laws of physics hold inside of U, but they do not necessarily hold in $AE \setminus U$. In Figure C there is the same 2D representation of U as that in Figure B, with a 2D representation of a 3D box of finite potential barrier contained inside. A particle placed inside of the box has a wave function shown in black. Since the box is 3D, the wave function amplitude is 4D (ignore time), thus its amplitude extends perpendicular to U. When a perturbation occurs, one of the black wave end-points move to the exterior of the box as shown by the red wave. The particle therefore has a probability of escaping resulting in quantum tunneling. Since the speed v is not bounded by c in $AE \setminus U$, force carriers for entangled particles are not limited to propagation speed c provided that they travel in $AE \setminus U$.

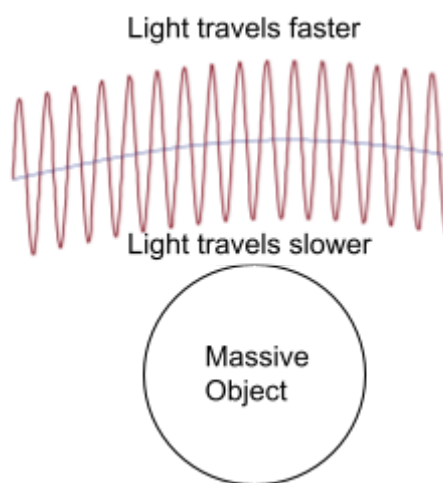
Take the particle depicted in Figure C that is represented by the black wave. Assume that $v \gg c$ in $AE \setminus U$, so that the particle is allowed to move rapidly. It therefore changes quantum states rapidly giving the illusion that all of the states exist at the same time. You can therefore take a measurement, and one of those rapidly changing states is observed. So to be clear, the states predicted by quantum mechanics do not exist at the same time, the particle simply changes states faster than the speed of light allows giving the illusion that the states exist in superposition.

Take the particle depicted in Figure C but without the box (particle in free space again). It travels through U, but also perpendicular to U and thus it doesn't exist in U continuously (it appears to pop in and out of existence in U). Now suppose that such a particle is joined with a second particle that has the exact opposite wave function. The 2 particle's wave functions cancel out, and therefore they begin to definitively exist in U. In reality, 2 particles are unlikely to have the exact opposite wave functions so a 2 particle pair will still travel in and out of U as one might conclude the uncertainty principle requires. However, the more particles that come together to form heavier objects,

the more likely that the superposition of the individual waves becomes zero, and therefore heavy objects exist more definitively in U than quantum ones. So to be clear, if you take n number of individual particles, their wave functions cause them to travel in and out of U. When those particles are joined together to form larger objects, their wave functions cancel out causing them to exist more definitively in U which is why large objects such as cars and people are always observable. You can then convert those heavier objects back into individual particles due to $E = mc^2$ and those individual particles then move in and out of U again and the object disappears. Those particles can then be forced back into U through a strong magnetic field as predicted by the Schwinger Effect.

Since a particle travels in and out of U, the information that we have about the particle only pertains to when it is in U, and therefore it can only be described statistically. However, with additional information pertaining to the particles' higher dimensional components (in AE\U), everything is deterministic. It is the lack of information regarding the higher dimension that results in the conclusion that QM is random.

Figure D: Since the speed of light in a stationary reference frame decreases with radius from a massive object, the index of refraction increases with a decrease in radius. Since photons curve towards higher index of refraction a photon (red) travels past a massive object causing what is observed as gravitational lensing. The photon travels slower when it is closer to the massive object, then when it is further away causing the path to curve.



This structure is consistent with wave particle duality, superposition, quantization of fundamentals, entanglement, uncertainty, quantum tunneling, interference patterns, and it never violates causality. It explains why we obtain the experimental results that we do without any paradoxes.

In figure D, a photon travels past a massive object so the side of the wave that is closest to the massive object travels slower than the opposite side resulting in gravitational lensing. The path that light takes in a gravitational field is derived herein and it accurately models the experimental results of GR for gravitational lensing and gravitational redshifting. One could say that it is because light travels in a wave, that it curves around massive objects. In this same manner, the wave-particle duality of all quantum particles is what causes the experimental results that are obtained through science.

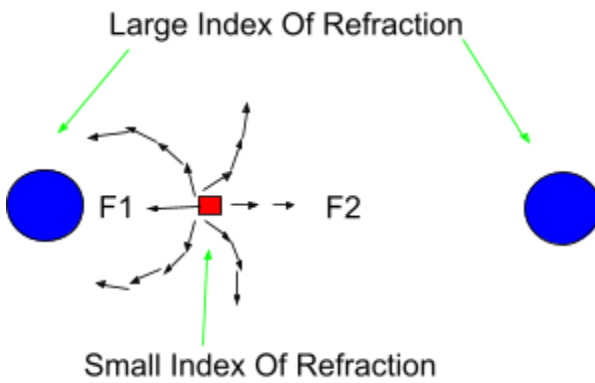


Figure E: The 2 blue circles are massive objects, and the red rectangle is some other object. The black arrows show the path of force carriers due to a varying index of refraction. Notice that $F1 > F2$ because more of the force carriers interact in that direction resulting in the effects of gravity.

In Figure E, two massive objects represented by two blue circles, and one smaller object represented by a red rectangle, are interacting. Since the speed of light decreases with a decrease in radius, this can be thought of as the index of refraction increasing with a decrease in radius. Since light tends to curve in the direction of the larger index of refraction, the path of the force carriers curve towards the massive object creating a non-uniform field which results in $F1 > F2$ (gravity).

This structure for the universe (and AE) is consistent with quantum mechanics, light propagation, and experimental results of GTR. With that said, everything that we observe points to the universe being isomorphic to a simulation meaning that every aspect of our reality can be modeled by a computer due to the universe's "mathematical nature". If the universe is a simulation, any programmer who designed it would know to write all of the physical laws in terms of one parameter so that everything changes collectively. In this case that parameter is light speed, and the programmer is G. The rest of the article is dedicated to mathematically proving the premises on which this theory is based, and showing them to be consistent with experimental results and observations in GTR as everything stated herein is dependent on the variable speed of light.

Definitions

System (A) refers to an object with a given state A_n . An **isolated system** is one in which a state change cannot be caused by anything outside of the system (or vice versa), and where nothing leaves or enters.

Universe (U) refers to all of the fundamental components of what is referenced in General Relativity as our spacetime object. If anything exists beyond the universe, that isn't included in the definition. In the case of cyclic models in which a new universe emerges from within the previous, both the old and the new universe are included in the definition.

All Existence (AE) refers to all of the fundamental components of all that exists. If anything exists beyond this universe, AE includes the fundamental components of that as well. AE is by definition isolated since P_2 below holds. Thus, $U \subseteq AE$.

Nothing (\emptyset) is defined as the absence of existence. Mathematically, if $C = \emptyset$, then $\forall C_i \subseteq C, C_i = \emptyset$. **Clarification:**

This is important for showing that at no point can we actually produce existence from non-existence, but we can produce something from something else in such a manner as to conserve certain properties.

E is an event $E \equiv A_n \rightarrow A_{n+1}$ of system **A** going from one state to the next.

Causality means that cause precedes effect.

Time is the ability for change. You either have an ability for change, or you don't: there aren't varying degrees of time. When referencing that time is eternal, what is meant is that time is a property of existence and existence has always existed. Therefore the property of time has always existed. The same logic applies when referencing the passing of time. Without the ability for change, the universe cannot begin. It is important to clarify that time as used in physics is the same as clock speed.

Premises

P_1 : Given an infinite period for an isolated system **A** with a finite set of distinct possible states, any event **E** that is possible will happen. (variation of the Poincare Recurrence Theorem).

Proof: Let $B = \{B_1, \dots, B_m, B_{m+1}\}$ be the set of distinct states of **A** (which is in state B_j), and let $E_j \equiv B_j \rightarrow B_{j+1}$ be an event with a probability $P_1(E_j) = \varepsilon_j$ of occurring, where $0 < \varepsilon_j \leq 1 \ \forall j \in [1, m]$. It follows that $P_1(\neg E_j) = 1 - \varepsilon_j$, and $P_k(\neg E_j) = (1 - \varepsilon_j)^k$ where k is the number of opportunities. Since $|B| < \infty$, $1 < m < \infty$, and thus we define an infinite period $T = \{T_1, T_2, \dots, T_m\} \mid [T_j = \frac{T}{m} = \infty \text{ and } T_i \cap T_j = \emptyset] \ \forall i \in [1, m] \text{ where } i \neq j$. We also define some minimal unit of time $\infty > t_{min} > 0$ in which a state change can occur $\mid k = \lfloor \frac{t}{t_{min}} \rfloor$. Since $\lim_{t \rightarrow T_j} P_{\lfloor \frac{t}{t_{min}} \rfloor}(\neg E_j) = 0 \ \forall j$, all of the states of **A** have a 0 probability of not occurring in T . Since T is arbitrary, this holds for any infinite period. If such a t_{min} doesn't exist, then $\varepsilon = 0$.

Clarification: Suppose that *A* and *B* are 2 mutually exclusive events each with a non-zero probability of occurring \mid once either *A* or *B* occurs, the probability of the other event occurring becomes 0.

Let t_a and $t_b \in [0, t)$ be the respective time periods in which events *A* and *B* remain possible. We let *A* represent the event that occurs, and since *A* and *B* are mutually exclusive, they cannot occur at the same time. Thus, $0 \leq t_b < t_a \leq t$. Thus event *B* not occurring doesn't violate P_1 even as $t \rightarrow \infty$ since $t_b < t$.

P_2 : Something cannot come from nothing.

Clarification: Suppose that $x = \emptyset$. It follows that since $x \text{ DNE}$, x is not restricted to follow any physical laws or logical principles. However, $x \text{ DNE}$ to utilize such properties so even though x has no restrictions, $x \text{ DNE}$ for it to matter. Therefore, even if some law required that x produce $y \neq \emptyset$, x doesn't exist to follow said rule. Therefore nothing cannot produce something. It is therefore not a logical issue, but an existence one.

Since A_n and A_{n+1} being elements of A proves that $A_{n-1} \in A$, A_{n-1} and A_n being elements of A proves that $A_{n-2} \in A$. It follows that $\exists A_{k+1} \in A \forall k \leq n$, where $k \in \mathbb{Z} \Rightarrow |A| = \infty$.

Since AE is isolated because P_2 holds, and it has at least 2 states that are not \emptyset , we can let $A = AE$.

1. Since $|A| = \infty$, $|AE| = \infty$, thus time has always passed since time is a property of existence.
2. Since $\exists A_{k-1}$ (cause) $\forall A_k$ (effect), every effect has a cause. Thus AE always existing doesn't violate causality.
3. From 2) $\check{T}(A_n) < \infty \forall A_n \in A$.

The Proper Light Speed Theory

In this model, $U \subseteq AE$, and time is the ability for change. As time passes (see definition of time), light travels at some rate. Since all measurement devices effectively operate on light, we therefore measure events against the speed of light, not directly against time. Essentially time passes, light travels causing clocks to tick, and then we measure events against the clock. Since time is a property of existence, it follows that time can't go to 0, and thus the speed of light is what dilates, not time. Since our ability to measure time is dependent on the speed of light, when the speed of light slows down it can appear as if it is time that is dilating. It follows that as what is referred to as the proper time τ decreases, the speed of light is what actually decreases $\Rightarrow c_0 dt = c d\tau$, where c_0 is the proper speed of

light. Since $c d\tau = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$, it therefore follows that $c_0 = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}$ where $g_{\mu\nu}$ is the metric tensor. The equations of general relativity are used without time dilation to show that it is the speed of light that changes, and this change in the speed of light results in what is observed as time dilation, and length contraction, without either actually occurring. This theory shows that light experiences time in the same capacity as everything else.

Starting with the Schwartzchild metric having only radial components, we get $c_0 = \sqrt{\frac{r-r_s}{r} c^2 - \frac{r}{r-r_s} v^2}$,

where the Schwartzchild radius $r_s = \frac{2GM}{c^2}$. Plotting this as in Figure 1, we see that c_0 changes as a function of r , r_s , and v . Therefore:

$$c_0(r, r_s, v) = \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} \quad (1)$$

Thus, the proper speed of light is tied to velocity. Setting $r_s = 0$, and $v = c$, we see that $c_0 = 0$ meaning that light doesn't observe other light catching up to it: This holds true for light regardless of what the local speed of light C is. From Figure 1, we see that the magnitude of the local speed of light is the first term in the metric which for the Schwartzchild metric yields:

$$C = \sqrt{\frac{r-r_s}{r}} c \quad (2)$$

It follows that since c was measured on earth, the actual speed of light in zero-g would need to be modified by the inverse of equation (2): We shall ignore this.

Since $|c_0| \leq \sqrt{\frac{r-r_s}{r}} c$, and $\lim_{r^+ \rightarrow r_s} \sqrt{\frac{r-r_s}{r}} = 0$, it follows that $c_0(r_s, r_s, v) = 0$ which $\Rightarrow v = 0$ for both

light and mass at the event horizon of a black hole, and that the event horizon is never actually reached.

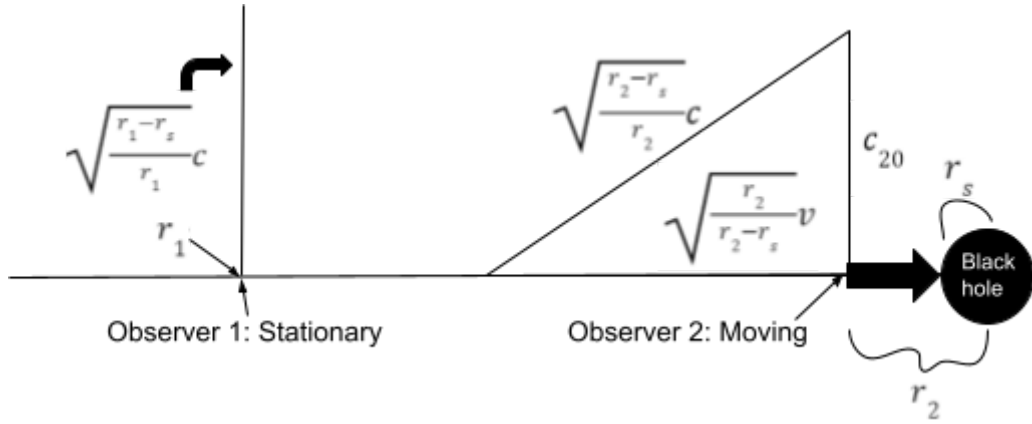


Figure 1: Shows the relationship between the components of the Schwartzchild metric.

From Figure 1, suppose that observer 2 starts at $r_2 = \infty$ (far left), and moves towards Observer 1. The proper velocity is:

$$v_0 = \sqrt{\frac{r_2}{r_2 - r_s}} v_2 \quad (3)$$

So initially, the proper velocity is $v_0 = \sqrt{\frac{\infty}{\infty - r_s}} v = v$. This means that when Observer 2 passes Observer 1,

$v_0 = \sqrt{\frac{r_1}{r_1 - r_s}} v_1 = \sqrt{\frac{r_2}{r_2 - r_s}} v_2$, where v_1 is the velocity of Observer 2 according to Observer 1. Since v_2 is the velocity of Observer 2 as viewed from a stationary reference frame at r_2 , for Observer 1,

$$v_1 = \sqrt{\frac{r_2(r_1 - r_s)}{r_1(r_2 - r_s)}} v_2 \quad (4)$$

Since this is also constant, as $r_2 \rightarrow r_s$, $v_2 \rightarrow 0$. It follows that Observer 1 sees Observer 2's velocity go to 0, and Observer 2 sees their velocity remaining the same. This is only possible because all velocities are measured against their local speeds of light (see the definition of Time above). This tells us that:

$$v_0 = \frac{c}{c_0} \frac{dr}{dt} \quad (5)$$

where c_0 is the proper speed of light at r , and $\frac{dr}{dt}$ is how quickly r changes with time (not relative to light) as viewed from $r = \infty$. For example, $v_0 = \frac{c}{c} \frac{dr}{dt}$ at $r = \infty$; and since $c_0(r_s, r_s, v) = 0$, $\frac{dr}{dt} = 0$ at the event horizon. For light, $c_0 = 0$, so $\frac{dr}{dt} = 0$ as perceived at $r = \infty$ (you can't see light once it is gone). It's important to note that length contraction doesn't physically occur. If we wanted to say that there is some clock speed τ in the universe relating velocity to the local speed of light such that $v_0 = \frac{dr}{d\tau}$, then $\frac{dr}{d\tau} = \frac{c}{c_0} \frac{dr}{dt} \Rightarrow c_0 dt = c d\tau$ which is what we concluded above. In fact, all we really care about is the relationships between clock speeds so it is easiest to use τ in calculations, but in reality it is the speed of light that dilates, not time. This leads us to:

Postulate 1: The proper time τ ties velocity to the local speed of light (not to time).

These differences might seem minute, but this is important because time is not actually a dimension: Time is a property of existence, and therefore the structure of this "lightspace" doesn't include time. This means that wormholes do not exist, time travel into the past is impossible, length contraction doesn't physically occur, causality always holds in relation to t , yet black holes still exist.

Deriving Maxwell's Equations of light in a gravitational field: From equation (1), for a photon traveling in a plane containing the COM of some object of mass M we get ($c_0 = 0$ for light):

$$\frac{r-r_s}{r} c^2 - \frac{r}{r-r_s} \left(\frac{dr}{dt}\right)^2 - r^2 \left(\frac{d\theta}{dt}\right)^2 = 0 \quad (6)$$

From Figure 2, we see that the components of equation (6) require a velocity vector of:

$$\vec{v} = \left\langle \frac{dx}{d\tau} = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \cos(\theta) - r \frac{d\theta}{dt} \sin(\theta), \frac{dy}{d\tau} = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \sin(\theta) + r \frac{d\theta}{dt} \cos(\theta) \right\rangle \quad (7)$$

Where $x = r \cos(\theta)$, $y = r \sin(\theta)$, and $r = \pm \sqrt{x^2 + y^2}$. Thus:

$$\frac{dy}{dx} = \frac{\sqrt{\frac{r}{r-r_s}} \sin(\theta) + r \frac{d\theta}{dr} \cos(\theta)}{\sqrt{\frac{r}{r-r_s}} \cos(\theta) - r \frac{d\theta}{dr} \sin(\theta)} \quad (8)$$

Notice that τ is used instead of t as required by postulate 1. Dividing the x-component in equation (7) by dx , squaring both sides, and multiplying by $\partial^2 E_x$ yields:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \cos(\theta) - r \frac{d\theta}{dt} \sin(\theta) \right]^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial \tau^2} \quad (9)$$

Repeating the same process for the y-component we get:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \sin(\theta) + r \frac{d\theta}{dt} \cos(\theta) \right]^2 \frac{\partial^2 E_y}{\partial y^2} = \frac{\partial^2 E_y}{\partial \tau^2} \quad (10)$$

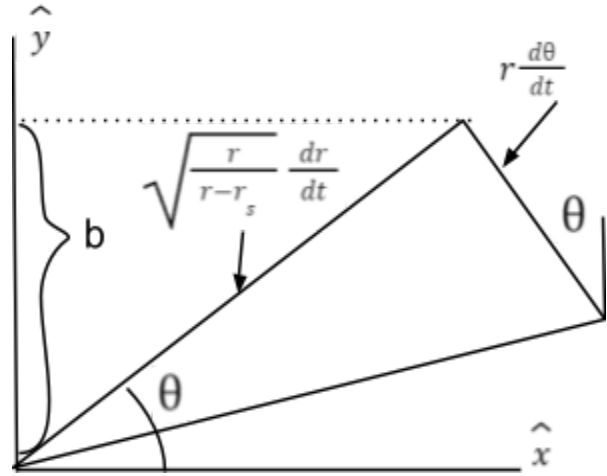
Equations (9) and (10) are Maxwell's Equations for light in a gravitational field in the \hat{x} and \hat{y} directions respectively, where the light is traveling through some plane going through the COM. Notice that when $r_s = \theta = 0$ equation (9) yields:

$$\left[\frac{dr}{dt} = c \right]^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial \tau^2} \quad (\text{Maxwell's Eq for light in zero-g, } \hat{x} \text{ - direction})$$

Likewise, when $r_s = 0$, and $\theta = \frac{\pi}{2}$, equation (10) yields:

$$\left[\frac{dr}{dt} = c\right]^2 \frac{\partial^2 E_y}{\partial y^2} = \frac{\partial^2 E_y}{\partial \tau^2} \quad (\text{Maxwell's Eq for light in zero-g, } \hat{y} \text{ - direction})$$

Figure 2: This figure shows how the components of equation (6) fit geometrically. For a mass positioned at (0,0), a photon released in the x-direction from (x=0, y=b) will curve downwards.



General Relativity Experimental Results Comparison

Clock-speed dilation: From equation (3), $v_0 = \sqrt{\frac{r}{r-r_s}} v$. Thus $\frac{dr}{d\tau} \sqrt{1 - \frac{r_s}{r}} = \frac{dr}{dt}$, and therefore $d\tau = \sqrt{1 - \frac{r_s}{r}} dt$ which is of course the exact Schwartzchild solution for the time dilation of a non-rotating object in space.

Black holes: Since $c_0(r, r_s, v) = 0$ for light, equation (1) yields $\frac{dr}{dt} = \pm \frac{r-r_s}{r} c$ (assuming no radial components). $\frac{dr}{dt} = -\frac{r-2}{r} c$ is plotted in Figure 3, where we see that inside the event horizon the velocity is positive, and outside the event horizon the velocity is negative. Therefore, all of the light of a black hole moves to the event horizon, and mass follows. Thus, there isn't a singularity inside of a black hole. If you consider the acceleration of light $\frac{d^2 r}{dt^2} = \frac{r-r_s}{r^3} c^2$, you see that inside the event horizon, the acceleration changes direction. These results are consistent with Susskind's proof that the amount of information inside of a black hole is proportional to the surface area of the event horizon as all of the information is actually on the event horizon separated by what is assumed to be the minimal distance allowed by quantum mechanics. Notice that the acceleration for light is 0, not ∞ , at the event horizon.

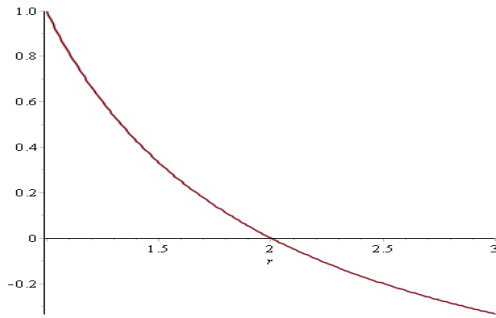


Figure 3: This is a plot of the velocity of light which shows that light always moves towards the event horizon (shown as $r = 2$), not a singularity.

Gravitational Redshift: Using equation (9) in only the \hat{x} - direction yields:

$$\left[\sqrt{\frac{r}{r-r_s}} \frac{dr}{dt} \right]^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2} \quad (11)$$

Since there aren't any rotational velocities, equation (6) tells us that $\frac{r-r_s}{r} c^2 = \frac{r}{r-r_s} \left(\frac{dr}{dt} \right)^2$. Thus, equation (11) becomes:

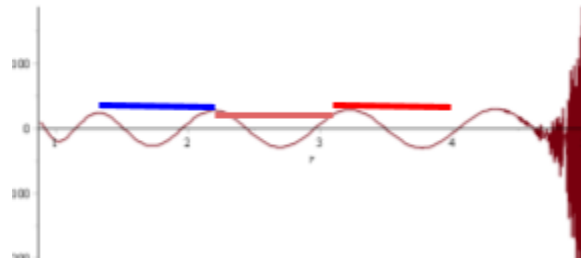
$$\frac{r-r_s}{r} c^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2} \quad (12)$$

Setting $E = R(r)\mathbf{T}(\tau)$ we get:

$$\frac{d^2 R(r)}{dr^2} = - \left[k^2 \frac{r}{r-r_s} \right] R(r) \quad (13)$$

The solutions for equation (13) are Whittaker functions shown in figure 4 for arbitrary values simply to show the shape. As you can see from the red and blue stripes, gravitational redshifting occurs as the photon gets further and further from the event horizon.

Figure 4: The blue and red stripes are the same length, thus, this figure shows a gravitational redshift as the light gets further from the event horizon on the left.



From equation (13):

$$k \sqrt{\frac{r}{r-r_s}} = \frac{2\pi}{\lambda} \quad (14)$$

Thus:

$$\lambda = \frac{2\pi}{k} \sqrt{\frac{r-r_s}{r}} = \lambda_\infty \sqrt{\frac{r-r_s}{r}} \quad (15)$$

Where equation (15) is the exact relationship between λ and λ_∞ as predicted by GR.

Gravitational Lensing: From equation (11), $C = \sqrt{\frac{r-r_s}{r}} c = \sqrt{\frac{r}{r-r_s}} \frac{dr}{dt}$. This suggests that the index of refraction is either $n_1 = \frac{r}{r-r_s}$, or $n_2 = \sqrt{\frac{r}{r-r_s}}$, depending on the frame of reference. When deriving the gravitational lens equation for GR using a Fermat Surface, $n = \sqrt{\frac{r+r_s}{r-r_s}}$ (Bacon). Setting n_1 equal to n we get:

$$\sqrt{\frac{r+r_s}{r-r_s}} = \alpha_1 \frac{r}{r-r_s} \Rightarrow \alpha_1 = \frac{\sqrt{r^2 - r_s^2}}{r} \text{ which is } \sim 1 \text{ for the exterior of anything that isn't a black hole.}$$

Setting n_2 equal to n we get $\sqrt{\frac{r+r_s}{r-r_s}} = \alpha_2 \sqrt{\frac{r}{r-r_s}} \Rightarrow \alpha_2 = \sqrt{\frac{r+r_s}{r}}$ which is also ~ 1 for the same. Using the calculus of variations to minimize the functional $\int_b^R N \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2} dr$, where $N \in \{n, n_1, n_2\}$ and θ is as shown in Figure 2, we can derive $\theta(r)$. However, the solutions are integrals, so it is easiest to compare $\frac{d\theta}{dr}$ as shown in Figure 5 where we use the mass of the sun. It is important to note that the left side doesn't matter since those points are inside of the sun. In fact, $r_{sun} \cong 6.96 * 10^8$ meters, so the entire plot shown in Figure 5 is still inside of the sun. This means that the predictions of this theory and those of GR are identical in any region that can be tested in our solar system.

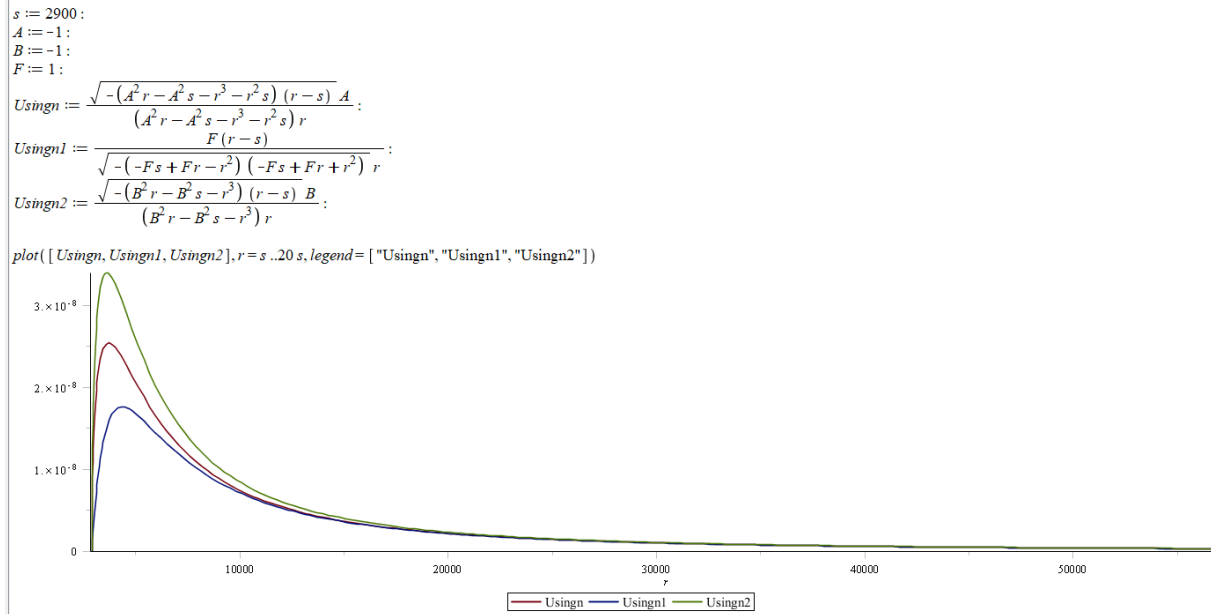


Figure 5: (Units in meters) These are plots of $\frac{d\theta(r)}{dr}$ where $s = r_s = r_{s, sun}$. They are only meant to show the relationship between the solutions as the constants were arbitrarily selected. Note that every point on this plot is still inside of the sun, so in the region where this is testable, the solutions are identical.

Gravitational Waves: It appears that there exists some fabric of lightspace that expands and contracts based on energy density, and this expanding and contracting changes its local index of refraction in the same

capacity as time was thought to do so. It thus makes sense that such expanding and contracting would propagate through space as a wave.

The Universe's Expansion: Has not been looked into.

Muon Decay: From Figure 1, $c_0^2 = \frac{r-r_s}{r} c^2 - \frac{r}{r-r_s} \left(\frac{dr}{dt}\right)^2$. In zero-g this yields: $c_0^2 = c^2 - v^2$, and

restructuring this gives us: $\frac{c_0}{c} = \sqrt{1 - \left(\frac{v}{c}\right)^2}$, so any perceived effects of special relativity are immediately recovered without time and length contraction, and gravity is already built-in. So the lorentz factor appearing in the relativistic decay equation can be interpreted further as meaning that the half life of a particle is a function of how the particle perceives light. That is:

$$T_{1/2}(c_0) = \tau_{1/2} \frac{c}{c_0} \quad (21) \quad (\text{Approximation})$$

Thus, the faster the particle moves, the slower the particle perceives light, and thus everything that occurs within the particle is slowed down without the need for time or length contraction.

Length Contraction: Imagine that Muons are produced at a height H above the surface of the earth, and that the percentage of Muons to hit the surface before decaying is consistent with the predictions of special relativity (which it is). It should be clear that H didn't physically shrink in either reference frame in any capacity. Thus, while special relativity does yield the correct answer, how it got to the correct answer is not author approved. The logical conclusion here is that H stays the same, and the particle's decay rate depends on how the particle perceives light as shown in equation (21). This makes sense because all of the forces involved communicate at a rate that is governed by the speed of light.

Quantum Mechanics Experimental Results Comparison

Entanglement: Since entangled particles share a wave function, the force carriers simply need to propagate in AE\U so that they are not limited by c.

Quantum Eraser: The entangled photons share a wave function, and the force carriers are not limited by c as explained above. One photon can hit the screen, and the information is carried in AE\U at faster than light speed to the second particle. The second particle then hits its screen, and sends information back to the first particle changing outcomes at faster than the speed of light.

Superposition: Particles are only bound by the speed limit c when they exist in U. When traveling in AE\U they change states very rapidly, and when they pass back through U they are in a different state. This rapid change in quantum states gives the illusion that all of the states exist simultaneously.

Double Slit: The wave function of the current state passes through the 2 slits causing them to head in the direction of one of the interference patterns. The particle follows and hits the screen.

Additional Definitions and Claims

Deterministic refers to an event $E_n \equiv A_n \rightarrow (C_{a1}A_{a1} \vee C_{a2}A_{a2} \dots \vee C_{am}A_{am})$ with fixed probabilities $(C_{a1}, C_{a2}, \dots, C_{am})$ respectively. The fact that there is a fixed weight or probability, tells us that there is an underlying rule that ensures those weights are obeyed. In the classical case $am = n+1$, $A_n \rightarrow A_{am}$, and $C_{am} = 1$. In the quantum case, A_{a1} through A_{am} represent the possible states, and C_a through C_{am} are the probabilities of those states. Thus quantum mechanics is deterministic.

Free Will refers to an event $E_n \equiv A_n \rightarrow (C_{a1}A_{a1} \vee C_{a2}A_{a2} \dots \vee C_{am}A_{am})$ where the respective probabilities $(C_{a1}, C_{a2}, \dots, C_{am})$ are not fixed.

Suppose that you have a dart board with different states or sections $(A_{a1} \vee A_{a2} \dots \vee A_{am})$, each with their respective probability $(C_{a1}, C_{a2}, \dots, C_{am})$ of getting hit. As long as the board is far enough away that one's skill in throwing darts is not valuable, the probabilities of each state can be calculated based entirely on the areas of each state in relation to the others (ignoring misses). This is only true because everything about the dart is deterministic. If the dart had free will, there would be no way to assign such probabilities.

Claim 1: There must exist $G = \{\dots, A_{n-1,j}, A_{n,j}\} \subset AE$ where $A_{n,j} \subset A_n \mid G$ has free will, and G organized U .

In this section a case is made for why the author believes that each standard model pertaining to the beginning of the universe isn't possible:

Conformal Cyclic Cosmology:

Let u be a m -dimensional volume in which the laws of quantum mechanics, and general relativity hold, and let $u \subseteq U$. Since space is expanding \exists some distance D in which 2 events are non-existent to each other due to the finite speed limit c . We thus define a point P in U in which event $E_{n,P \pm \epsilon} \equiv U_{n,P \pm \epsilon} \rightarrow U_{n+1,P \pm \epsilon}$ representing the universe's beginning (or this cycle of it) occurs, and then define u as being the volume enclosed by distance D around P in m -space. Since space expands uniformly, there is nothing unique about point P , thus every point in u has the same probability for a similar event $E_{a,i}$ ($a \geq n$) to occur. We thus establish all of the points in u as a grid, where each point is separated by a planck length l_p : We then define t_{min} (from P_1) to be the Planck time | every t_{min} an event $E_{a,i}$ could occur at each point in u (outside of the light radius of P). We now calculate the probability that $E_{n,P \pm \epsilon}$ is the only such event that occurs in u over time D/c , for $m = 3$.

Let the radius of a sphere be a multiple (k) of l_p . We divide the area of the sphere by the area of an equilateral triangle of side length l_p , to get the \sim number of triangles that grid the sphere. Thus, the approx. number of triangles $\blacktriangle(k)$ is:

$$\blacktriangle(k) \cong \frac{4Pi(k * l_p)^2}{\left(\frac{l_p^2 \sin(Pi/3)}{2}\right)} = \frac{16Pi(k)^2}{\sqrt{3}}$$

The number of triangles on the surface of the sphere relates to the number of points $*(k)$ by the approx. relation $*(k) \cong \frac{1}{2} \blacktriangle(k)$. Thus:

$$*(k) = \frac{8Pi(k)^2}{\sqrt{3}}$$

It follows, that at the moment of $E_{n,P \pm \epsilon}$, there existed $\frac{8Pi}{\sqrt{3}} \sum_{k=1}^q (k)^2$ opportunities for $E_{a,i}$ to occur elsewhere within u , where $q = \lfloor D/l_p \rfloor$. By the time that light from P reached the next planck length to communicate that $E_{n,P \pm \epsilon}$ occurred, another $\frac{8Pi}{\sqrt{3}} \sum_{k=2}^q (k)^2$ opportunities passed, followed by $\frac{8Pi}{\sqrt{3}} \sum_{k=3}^q (k)^2$ the following $t_{min} \dots$ Thus, the number of opportunities for $E_{a,i}$ to occur in D is:

$$\begin{aligned} \frac{8Pi}{\sqrt{3}} \sum_{j=1}^q \sum_{k=j}^q (k)^2 &= \frac{8Pi}{\sqrt{3}} \sum_{j=1}^q \left(\frac{q(q+1)(2q+1)}{6} - \frac{(j-1)j(2j-1)}{6} \right) \\ &= \frac{4Pi}{3\sqrt{3}} \left(q^2(q+1)(2q+1) - \sum_{j=1}^q j(2j^2-3j+1) \right) \\ &= \frac{4Pi}{3\sqrt{3}} \left(q^2(q+1)(2q+1) - \left[2\left(\frac{q(q+1)}{2}\right)^2 - 3\frac{q(q+1)(2q+1)}{6} + \frac{q(q+1)}{2} \right] \right) \\ &= \frac{2q^2Pi}{\sqrt{3}} (q^2+2q+1) \end{aligned}$$

If we now think of each point in u as a die with η distinct states in which only 1 results in $E_{a,i}$, then the probability that $E_{a,i}$ doesn't ever occur in u over time D/c is $\left(\frac{\eta-1}{\eta}\right)^{\frac{2q^2Pi}{\sqrt{3}}(q^2+2q+1)}$. If we set this to greater than or equal to what is typically considered to be "impossible" we get $\left(\frac{\eta-1}{\eta}\right)^{\frac{2q^2Pi}{\sqrt{3}}(q^2+2q+1)} \geq 10^{-50}$, which $\Rightarrow \eta \geq \frac{1}{1-10^{-\frac{25\sqrt{3}}{q^2Pi(q+1)^2}}}$. This means that the vacuum of space must have at least η distinct states (not energy levels) with one being able to cause $E_{n,P \pm \epsilon}$. Using just the values for when D is one light-second we get $\eta > 10^{170}$. There isn't anything in the vacuum that is known to even remotely come close to this value. Thus $E_{a,i}$ should have occurred many times over, and those events occurring should be apparent in the cosmological data: We should observe galaxies and radiation heading towards us from all directions.

In addition to this, if $E_{a,i}$ and $E_{n,P \pm \epsilon}$ both occur, which according to P_1 this must happen at some point, neither event is aware of the other, so the scale factor in the Conformal Cyclic Cosmology model could not be coordinated leading to inconsistencies.

It follows that the CCC model of cosmology, or any theory attempting to "smoothly" cycle through universes, suffers from these issues.

Other Cyclic Models:

These models are either known to violate the laws of entropy, or they are known to require a beginning a finite number of cycles into the past.

Big Bang Model:

From $P_4 - 3$, the universe cannot exist in a singularity state for an infinite period. So, the BB could occur, but a non-singularity state had to precede the singularity in which our universe emerged: This can't happen without violating the laws of entropy unless an outside force caused it ([non-singularity state \rightarrow singularity state (for all of U)] is a decrease in total entropy.).

Multiverse:

This model has the same issues as the CCC in that we should observe interactions of other universes with our own, and those interactions should be apparent through the CMB which they aren't. Additionally, unless the universes get recycled, they must be made from nothing (even if AE is infinite) violating P_2 .

Conclusion: The universe can't be cyclic (infinitely), it can only begin with a BB if the BB were started from an outside force, and the multiverse isn't supported by the cosmological data. The only other known option is that the process that organized U is controlled. Therefore there must exist $G = \{..., A_{n-1,j}, A_{n,j}, A_{n+1,j}, \dots\} \subset AE$ where $A_{n,j} \subset A_n$ | G has free will, and G organized U.

Claim 2: G has the properties attributed to God in the bible, assuming that such qualities are obtainable.

Proof: Let T represent all time, and divide T into 2 infinite periods T_1 and T_2 , where T_1 precedes T_2 . Since T_1 is infinite, by P_1 , if it is possible for G to become perfect, obtain all knowledge, develop moral codes that statistically yield the best results, and organize universes, then G has done so in T_1 . Since T_2 is also infinite, G obtained these qualities an infinite time ago. Thus, no matter how far back you go in time, G has these qualities, hence why "God is the same yesterday, today, and forever". While this is not a proof that the bible is valid, it is a proof that the bible got this correct.

Proposed Experiment

From equation (2), the speed of light remains constant in a stationary reference frame when traveling in the tangential direction, but varies when traveling in the radial direction. Thus a time measurement between photons traveling the same distance vertically and horizontally could be compared. However, this is not currently feasible due to the high precision necessary unless the photons could travel substantial distances. However, the calculations are as follows for consideration:

$$D = [c * \sqrt{\frac{R-r_s}{R}}] * t_h = c * [\frac{1}{D} \int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr] * t_v \quad (22)$$

Where D is the one-way path distance traveled (D must be small enough that the change in radius can be ignored), c is the speed of light in zero-g, R is the radius of the earth, t_h is the one-way time of travel for the photon in the horizontal direction, and t_v is the one-way time of travel for the photon in the vertical direction. Therefore:

$$t_h = \left[\frac{1}{D} \int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr \right] * \sqrt{\frac{R}{R-r_s}} * t_v \quad (23)$$

OR

$$\Delta t = t_h - t_v = \frac{D}{c} \left[\sqrt{\frac{R}{R-r_s}} - \frac{D}{\int_R^{R+D} \sqrt{\frac{r-r_s}{r}} dr} \right] \quad (24)$$

For $D = 1000$ meters, $\Delta t \approx 1.8 * 10^{-19}$ s giving an idea as to why the speed of light appears constant. To make this test feasible, a means of measuring the time of travel for 100 seconds or better is needed but of course this exceeds the bounds for D so the equation for t_h would need to be modified. The question then would be: Would an experiment in which electricity were run vertically and horizontally be acceptable or would the claim against the results be that the electrons length contracted?

Conclusion

In this paper we showed that time is a property of existence. It was then shown that the same experimental results of GR can be achieved by allowing time to pass at the same rate while also allowing the local speed of light to vary. This approach shows that what we perceive as time, is tied to our local speed of light, and thus the appearance of time dilation occurs when measuring time where the local speed of light is different than our own. This approach yields the same results as predicted by GR for gravitational lensing (approx.) and redshifting, and it removes all paradoxes known to the author at the time of writing: time travel into the past is impossible; there are no singularities of a black hole; all laws of physics hold everywhere (assuming they hold locally) when they are written in terms of C and τ ; time passed before the universe began thus allowing it to make sense for the universe to begin in the first place; and you never have to create something from nothing. This theory points to a universe that was organized from fundamentals that have always existed, not created from nothing.

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