# The Proper Light Speed (PLS)

(A unification of Gravity, Quantum Mechanics, and Light propagation V2)

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#### **Abstract**

This paper challenges the conventional notion of time in General Relativity (GTR) by proposing a novel framework that reconciles experimental results with potential solutions to philosophical questions about the universe's origin. It is argued that the Riemannian metric of GTR can be reinterpreted as a 3-dimensional metric, reducing spacetime to space alone. Time then becomes not a separate dimension, but a convenient parameter reflecting the relationship between light, distance, and the "proper time". This framework also proposes a variable speed of light  $(c_0)$ , where the observed constancy arises from reference-frame-dependent variations.

While all observers measure c,  $c_0$  itself adapts to gravitational and other influences. Furthermore, the variable speed of light bridges the gap between the discrete nature of quantum mechanics and the smooth spacetime of GTR. The implications are significant, suggesting a revised interpretation of time dilation and other GTR phenomena while providing a potential pathway for reconciling scientific and philosophical perspectives on the universe's origins. Further investigation is needed to explore the full theoretical and experimental implications of this revised framework.

## Introduction

In general relativity, time and space are tied together using the Riemannian metric. In coordinate space, the Riemannian metric reduces to the Minkowski metric of special relativity allowing a seamless transition from one theory to the other. This mathematical framework ensures that the laws of physics are invariant in all RF's. While this model is very accurate, the concept of time as described in it doesn't appear to be compatible with solutions to philosophical questions like: why and how did the universe begin in the first place? While these questions typically extend beyond the scope of science, science has to be compatible with potential solutions to them, and GTR is not as explained herein. This article is focused on deriving a framework that is isomorphic to that of spacetime (therefore it also obeys the Lorentz transformation) that produces the same experimental outcomes of GTR while adhering to potential solutions to the aforementioned questions. This framework is then shown to be compatible with both light propagation and quantum mechanics.

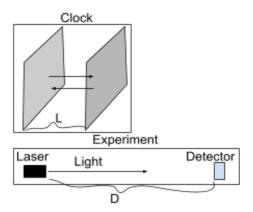


Figure 0: (top) A light clock in which photons pass back and forth between 2 mirrors. (bottom) An experiment in which a laser emits a photon, the photon travels to the detector, and the time of travel is measured using the light clock.

Consider the following thought experiment: From Figure 0, count the number of passes P that a photon makes reflecting between the two mirrors before the detector registers light. The answer is always  $P = \frac{D}{I}$ . Rearranging leads to ([] signifies units):

$$[D] = [L][P]$$

$$= \frac{dist}{pass} pass$$

$$= [c][t]$$

$$= \frac{dist}{time} time$$

Resulting in [time] = passes. It follows that time is not a physical dimension in any capacity, it is simply a convenient parameter modeling how far light travels between two events. Notice that even if the speed of light had a magnitude of a googolplex\*c, or varied as a sign wave, the number of passes will always be  $P = \frac{D}{L}$  since both the light in the clock, and the light in the experiment change proportionally: This is why the speed of light is always measured as constant.

With this said, the reality is that all clocks are light clocks (there isn't any other option since [time] = passes), where different fundamental forces are mediated by force carriers that propagate at a rate that is proportional to the local speed of light (e.g. The W boson propagates at approx. 0.6c). Consider the following examples to ensure this to be consistent with experimental results of STR:

- A) A muon travels at 0.99c through the atmosphere and, according to STR, time dilates by approx. a factor of 7 causing it to exist in coordinate space for 7x as long. The analog to this in terms of the PLS is that the muon perceives the speed of light to be c, but in relation to coordinate space, the speed of light perceived by the muon is 1/7th that of coordinate space. Photons, and the W and Z bosons therefore mediate at 1/7th their coordinate space rate resulting in a prolonged decay in coordinate space.
- B) When a wire isn't conducting electricity, it is electrically neutral. When a voltage is applied, the electrons begin to move resulting in their force carriers slowing down causing an imbalance in the measured charge density.

C) Suppose that observer A is stationary and observer B passes them. The speed of light in B's reference frame causes their clock to tick slower. However, the light from A also slows down as it approaches B's RF, and therefore observer B sees observer A's clock dilate. Likewise, the same occurs for observer A. Both observers measure the speed of light to be c.

Time, in the context of physics, is just a convenient parameter in which the relationship between light, time, and distance has not properly been addressed. This reduces the 4-dimensional metric of GTR to a 3-dimensional metric eliminating the concept of spacetime. However, Einstein did do the heavy lifting for this theory as should become apparent further down.

Since the speed of light varies, and each reference frame always measures the speed of light to be the same value, the proper time  $\tau$  is a function of the proper speed of light  $c_0$ . Additionally, this results in there being a difference between what is really happening, and what is observed to be happening at any point in space other than coordinate space: This difference ties the discreteness of quantum mechanics to the smoothness of GTR making PLS and QM compatible.

While the premise of science is that it is testable, the majority of what is real, or what occurs, doesn't fit such a definition, yet remains true. As an example, while information isn't lost, the ability for science to show that I went for a walk at 2:00am this morning is approx. zero even though it is true. In this regard, most truths are outside of the realm of science. This implies that the truths of science are a subset of all truths, and if we want to know all truths, then we have to be willing to stretch definitions as long as such definitions reduce down to scientific ones where applicable.

# **Definitions (1)**

**Clock Speed** is the local distance that light travels between two events and it is oddly labeled in units of seconds, minutes, or hours. It is the time as defined in classical physics.

 $R_{zero}$ : This is the reference frame having zero-g and no velocity (coordinate space).

 $R_0$ : This is the moving reference frame located anywhere.

 $R_s$ : This is a stationary reference frame located anywhere.  $R_s = R_0$  when velocity is zero;

c: This is the speed of light in the  $R_{zero}$  RF. This is also the speed of light that is measured locally in any RF as all RF's measure the speed of light to be the same.

 $c_0$ : This is the proper speed of light in the  $R_0$  RF. It is the real speed of light in the  $R_0$  RF.

t: This is the clock speed in the  $R_{zero}$  RF. This is also the locally measured clock speed in the  $R_0$  RF.

 $\tau$ : This is the real clock speed in the  $R_0$  RF.

## **Theoretical Framework: Key Concepts**

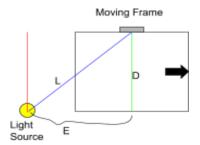


Figure 1: This is the typical setup for deriving the equations of time dilation in special relativity. The blue line represents the path of light from the stationary reference frame to the moving reference frame. At the moment that the moving frames mirror is directly over the light source, a flash occurs producing the red and blue photon paths. The moving reference frame travels just fast enough to ensure the blue line hits the mirror.

In Figure 1 there is the setup for the typical derivation of time dilation in special relativity. As the moving frame goes from left to right, at the moment that the center of the mirror in the moving frame is directly over the light source, a flash occurs. Two photons are emitted, one in the vertical direction (red) and one up and to the right (blue). In special relativity, the green line is treated as if it were the red line because the speed of light is the same in all reference frames. Thus:

$$L = c dt, D = c d\tau, E = v dt$$

$$\therefore L^{2} = D^{2} + E^{2} \Rightarrow (c dt)^{2} = (c d\tau)^{2} + (v dt)^{2}$$

Simplifying yields:

$$d\tau = dt * \sqrt{1 - \left(\frac{v}{c}\right)^2}$$
 (1)

Leading to the conclusion that time itself is malleable. Now consider the same setup with the following interpretation. The green line in Figure 1 is simply the vertical component of the blue line representing the vertical component of the photon's path. This means that the observer in the moving reference frame only sees the vertical component of the light and thus in the moving reference frame, the speed of light is slower. The equations are as follows:

$$L = c dt, D = c_0 dt, E = v dt$$

$$\therefore L^2 = D^2 + E^2 \Rightarrow (c dt)^2 = (c_0 dt)^2 + (v dt)^2$$

Simplifying yields:

$$c_0 = c * \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (STR \text{ analog, 2})$$

From equations (1) and (2),  $\frac{d\tau}{dt} = \frac{c_0}{c}$  resulting in:

$$c_0 dt = \mathrm{cd} \boldsymbol{\tau} \quad (3)$$

Since  $\operatorname{cd} \tau = \sqrt{g_{\mu\nu} dx^{\mu} dx^{\nu}}$  in GTR, from equation (3),  $c_0 dt = \sqrt{g_{\mu\nu} dx^{\mu} dx^{\nu}}$ , and therefore:

$$c_0 = \sqrt{g_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt}}$$
 (GTR analog, 4)

where  $g_{\mu\nu}$  is the metric tensor. A clock moving at the local speed of light has a clock speed of zero. Therefore:

$$[c_0 = 0] = \sqrt{g_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt}} \quad \text{(Photon, 5)}$$

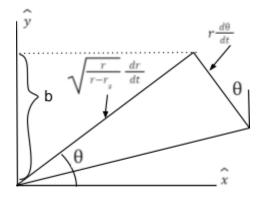
Using the Schwartzchild metric, with  $\varphi = 0$ , yields:

$$c_0 = \sqrt{\frac{r - r_s}{r}c^2 - \frac{r}{r - r_s}v^2 - r^2(\frac{d\theta}{dt})^2}$$
 (Schwarzschild  $\varphi = 0, 6$ )

Therefore the local speed of light in the  $R_s$  RF is  $(c_0 = C)$ :

$$C = \sqrt{\frac{r - r_s}{r}} c \quad \text{(Schwartzehild - stationary, 7)}$$

Figure 2: This figure shows how the components of equation (6) fit geometrically. For a mass positioned at (0,0), a photon released in the x-direction from (x=0, y=b) will curve downwards.



When photons travel in a plane through the COM, equation (6) results in:

$$\frac{r-r_s}{r}c^2 - \frac{r}{r-r_s}\left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\theta}{dt}\right)^2 = 0 \quad (8)$$

From Figure 2, we see that the components of equation (8) can be summed in each direction as:

$$C = \left| < \frac{dx}{d\tau} = \sqrt{\frac{r}{r - r_s}} \frac{dr}{dt} cos(\theta) - r \frac{d\theta}{dt} sin(\theta), \frac{dy}{d\tau} = \sqrt{\frac{r}{r - r_s}} \frac{dr}{dt} sin(\theta) + r \frac{d\theta}{dt} cos(\theta) > \right|$$
 (9)

Notice that  $\tau$  is used instead of t in  $\frac{dx}{d\tau}$  and  $\frac{dy}{d\tau}$ : Locally c, and t are measured, but what is really happening is modeled using  $c_0$  and  $\tau$ . Dividing the x-component in equation (9) by dx, squaring both sides, and multiplying by  $\partial^2 E_{\tau}$  yields:

$$\left[\sqrt{\frac{r}{r-r_s}}\frac{dr}{dt}cos(\theta) - r\frac{d\theta}{dt}sin(\theta)\right]^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial \tau^2} \quad (10)$$

Repeating the same process for the y-component yields:

$$\left[\sqrt{\frac{r}{r-r_{s}}}\frac{dr}{dt}sin(\theta) + r\frac{d\theta}{dt}cos(\theta)\right]^{2}\frac{\partial^{2}E_{y}}{\partial y^{2}} = \frac{\partial^{2}E_{y}}{\partial \tau^{2}}$$
(11)

Equations (10) and (11) model light propagation in a gravitational field where the light is traveling through some plane going through the COM. Notice that when  $r_s = \theta = 0$  equation (10) yields:

$$c^2 \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_x}{\partial t^2}$$
 (Wave eq for light in zero-g,  $\hat{x}$  - direction)

Likewise, when  $r_s = 0$ , and  $\theta = \frac{\pi}{2}$ , equation (11) yields:

$$c^2 \frac{\partial^2 E_y}{\partial y^2} = \frac{\partial^2 E_y}{\partial t^2}$$
 (Wave eq for light in zero-g,  $\hat{y}$  - direction)

Notice that in zero-g,  $c_0 = c$  and  $\tau = t$ . Using equation (11) with  $\theta = \frac{\pi}{2}$ , and y = r yields:

$$\left[\sqrt{\frac{r}{r-r_s}}\frac{dr}{dt}\right]^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2} \quad (12)$$

Since there aren't any rotational velocities, equation (8) tells us that  $\frac{r-r_s}{r}c^2 = \frac{r}{r-r_c}\left(\frac{dr}{dt}\right)^2$ .

Thus, equation (12) becomes:

$$\frac{r-r_s}{r}c^2 \frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E}{\partial \tau^2}$$
 (13)

Setting  $E = k(r)T(\tau)$ , the differential equation for k(r) is:

$$\frac{d^2 k(r)}{dr^2} = -\left[k^2 \frac{r}{r - r_s}\right] k(r) \quad (14)$$

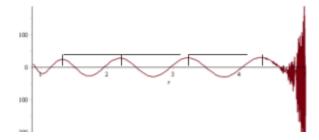


Figure 3: The black strips are the same length, thus, this figure shows a gravitational redshift as the light gets further from the event horizon on the left. The noise on the right-side is believed to be attributed to the complexity of the Whittaker functions, and is not believed to be accurate.

The solutions for equation (14) are Whittaker functions shown in figure 3 for arbitrary values. As you can see from the red and blue stripes, gravitational redshifting occurs as the photon gets

further and further from the event horizon (EH). From equation (14),  $k\sqrt{\frac{r}{r-r_s}} = \frac{2\pi}{\lambda}$ . Thus:

$$\lambda = \frac{2\pi}{k} \sqrt{\frac{r - r_s}{r}} = \lambda_{\infty} \sqrt{\frac{r - r_s}{r}} \quad (15)$$

Where equation (15) is the exact relationship between  $\lambda$  and  $\lambda_{\infty}$  in GTR (I told you Einstein did the heavy lifting here). Notice that equation (13) can be written as:

$$\left[c_{0}^{2}\right]^{2} \frac{\partial^{2} E}{\partial r^{2}} = \frac{\partial^{2} E}{\partial \tau^{2}} \quad (16)$$

It is important to note that  $c_0$  is not from the photons perspective ( $c_0 = 0$ ), it is from the  $R_s$  RF where the photon passes. This is important because when equations of physics are written in terms of  $c_0$  and  $\tau$ , the physics holds everywhere because it models what is really happening, not what is measured (in coordinate space what is real and what is measured are the same). In the  $R_s$  RF,  $C = f\lambda$ , and using equations (7) and (15), this results in:

$$f_{real} = \frac{\sqrt{\frac{r-r_s}{r}}c}{\lambda_{\infty}\sqrt{\frac{r-r_s}{r}}}$$
$$= \frac{c}{\lambda_{\infty}}$$
$$= const$$

However, every RF measures the local speed of light to be c due to clock speed dilation, and therefore what is measured is  $c = f\lambda$ . Using equations (7) and (15) again, this results in:

$$f_{meas} = \frac{c}{\lambda_{\infty} \sqrt{\frac{r - r_{s}}{r}}}$$
$$= \frac{c}{\lambda_{\infty}} \sqrt{\frac{r}{r - r_{s}}}$$
$$= f_{i} \sqrt{\frac{r}{r - r_{s}}}$$

It follows that what is really happening in the  $R_s$  RF is described by  $C = f_{real} \lambda$ , and what is measured from the  $R_s$  RF is described by  $c = f_{meas} \lambda$ . The real energy of the photon is therefore

 $E_{real} = \frac{c}{\lambda_{\infty}} h$  which does not change, and the energy that is measured is  $E_{meas} = [\frac{c}{\lambda_{\infty}} h] \sqrt{\frac{r}{r-r_s}}$ .  $E_{real}$  is constant because even though the wavelength decreases with r, the actual speed of light C slows down as well and those differences exactly cancel. It follows that:

$$E_{meas} = E_{real} \sqrt{\frac{r}{r-r_s}}$$
 (photon viewed from the  $R_s$  RF, 17)

What this means is that the energy  $(E_{real})$  that is really involved in the  $R_0$  RF is quantized by h, but the energy that is measured  $(E_{meas})$  in the  $R_0$  RF is not since  $h\sqrt{\frac{r}{r-r_s}}$  is continuous for all  $r>r_s$ . So quantum mechanics says that everything is quantized, and the PLS theory agrees but shows that what is measured, not what is real, is continuous due to light speed dilation. This is the key to uniting QM with experimental results of GTR: the difference between what is real and what is measured.

PROPERTY VALUES OF A <b>PHOTON</b> AS r DECREASES (AS VIEWED FROM THE $R_{_{\mathcal{S}}}$ RF)						
	Speed of light	Clock Speed	λ	f	Energy	
Real (R <sub>s</sub> RF)	Decrease $C = \sqrt{\frac{r - r_s}{r}} c$	Decrease $d\mathbf{\tau} = \sqrt{\frac{r - r_s}{r}} dt$	Decrease $\lambda = \lambda_{\infty}$ $\sqrt{\frac{r-r_{s}}{r}}$	Constant $f_{real} = \frac{c}{\lambda_{\infty}}$	Constant $E_{real} = \left[\frac{c}{\lambda_{\infty}}h\right]$	
Measur ed (R <sub>s</sub> RF)	Constant c	Constant Change t	Decrease $\lambda = \lambda_{\infty}$ $\sqrt{\frac{r-r_{s}}{r}}$	Increase $f_{meas} = \frac{c}{\lambda_{\infty}} \sqrt{\frac{c}{r}}$	Increase $E_{meas} = \left[\frac{c}{\lambda_{\infty}}h\right]\sqrt{\frac{r}{r}}$	

Table 1: Shows the relationship between the given quantities as r decreases, when viewed from the  $R_s$  RF. Notice that both the speed of light and the proper time scale the same.

In table 1 above, a presentation of how the real and measured quantities change with a decrease in r-value. Notice that the real energy remains constant, while the measured energy increases. The real energy is quantized; and the measured energy is quantized for a given r-value but it is continuous with r. As a particle approaches an EH:

- 1) the measured energy in that RF approaches infinity;
- 2) the measured clock speed in that RF is t;
- 3) the real energy in that RF remains constant;
- 4) and the real clock speed goes to zero.

Equations (7) and (17) can then be used to write  $E_{meas} = \frac{c}{C} E_{real}$  which can be rewritten for any RF as:

$$E_{meas} = \frac{c}{c_0} E_{real}$$
 (photon traveling in and viewed from the  $R_0$  RF, 18)

In regards to particles with mass, it makes sense that velocities scale with  $c_0$  and  $\tau$ , otherwise the physics couldn't be the same in all reference frames. Therefore:

$$v_{real} = v_{meas} \frac{c_0}{c} \quad (19)$$

This is a great point in the article to consider mass as an emergent property. In the  $R_0$  RF the interaction between the higgs boson and the higgs field would be reduced in relation to coordinate space, but it would be measured locally to be the same since the speed of light is always measured locally to be c. Does this result in a reduced mass? Consider  $\frac{mv_{meas}^2}{r} = \frac{GMm}{r^2}$  or equivalently  $v_{meas}^2 = \frac{GM}{r} \implies v_{real}^2 = \frac{GM}{r} (\frac{c_0}{c})^2$  suggesting that either the mass or the gravitational constant G change with  $(\frac{c_0}{c})^2$ . Now consider  $\frac{mv_{meas}^2}{r} = \frac{\mu q_1 q_2}{2\pi r^2} \implies \frac{2}{2\pi r^2}$ 

 $v_{real}^2 = \frac{\mu q_1 q_2}{2\pi mr} \left(\frac{c_0}{c}\right)^2$ . If the mass decreases with  $\left(\frac{c_0}{c}\right)^2$ , the real coulomb force doesn't and this is wrong since it is mediated by the photon. It follows that:

$$m = m \quad (20)$$

$$q_{real} = q_{meas} \frac{c_0}{c} \quad \text{OR} \quad \mu_{real} = \mu_{meas} \left(\frac{c_0}{c}\right)^2 \quad (21)$$

$$G_{real} = G_{meas} \left(\frac{c_0}{c}\right)^2 \quad (22)$$

This means that:

- 1) The rest mass is the same in all reference frames.
- 2) If  $q_{real}$  is quantized, then  $q_{meas}$  isn't (and vis vera) OR  $\mu$  isn't constant.
- 3) If  $G_{real}$  is constant (assumed to be), then  $G_{meas}$  increases with a decrease in radius. Since  $v_{meas}^{2} = \frac{G_{meas}M}{r}$ , and  $G_{meas} = [const](\frac{c}{c_{0}})^{2}$ , the velocity  $v_{meas}$  of a star in orbit would need to increase by a factor of  $\frac{c}{c_{0}}$  on top of what was originally calculated in order for the star to stay in orbit. **Does this account for dark matter?**

Therefore in a stationary RF near the EH:

$$F_{real} = \frac{GmM}{r^2} \frac{(r-r_s)}{r}$$
 (23)  
$$F_{meas} = \frac{GmM}{r^2}$$
 (24)

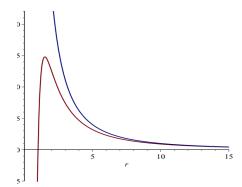


Figure 4: In this figure,  $F_{real}$  is depicted in red, and  $F_{meas}$  in blue. The two forces are nearly identical until just before the EH, where  $F_{meas}$  continues to increase, while  $F_{real}$  drops to 0. Inside the EH,  $F_{real}$  is negative indicating that a particle found inside is forced outwards.

From figure 4,  $F_{real}$  (red) and  $F_{meas}$  remain very similar up until right before the EH.  $F_{meas}$  gets very large, and  $F_{real}$  drops to 0. A particle headed towards the EH feels a tremendous force that tapers off and then drops to 0.  $F_{real}$  is negative inside of the EH implying that a particle trapped inside during the black hole's formation is pushed outwards to the EH and therefore a black hole doesn't have any singularities.

For a particle with mass, starting with  $E = \sqrt{p^2c^2 + m^2c^4}$ :

$$E_{meas} = \sqrt{m^2 v_{meas}^2 c^2 + m^2 c^4}$$
$$= cm \sqrt{v_{meas}^2 + c^2} \quad (25)$$

$$\begin{split} E_{real} &= \sqrt{m^2 v_{real}^2 c_0^2 + m^2 c_0^4} \\ &= \sqrt{m^2 (v_{meas} \frac{c_0}{c})^2 c_0^2 + m^2 c_0^4} \\ &= m c_0 \sqrt{(v_{meas} \frac{c_0}{c})^2 + c_0^2} \\ &= m c_0^2 \sqrt{(v_{meas} \frac{1}{c})^2 + 1} \\ &= m \frac{c_0^2}{c} \sqrt{v_{meas}^2 + c_0^2} \quad (26) \end{split}$$

Therefore:

 $E_{meas} = (c^2/c_0^2)E_{real}$  (particle with mass, in and viewed from the  $R_0$  RF, 27) The Schrodinger Equation represents an observable (measured), and the Hamiltonian is the energy term. Therefore:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(..., t, c) \right] \Psi \quad \text{(measured: } \Psi(t, c), 28)$$

$$i\hbar \frac{\partial Y}{\partial \tau} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(..., \tau, c_0) \right] Y(\frac{c_0}{c})^2 \quad \text{(real: } Y(\tau, c_0), 29)$$

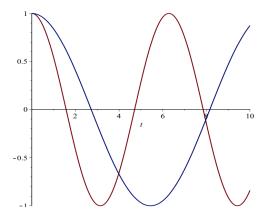
Equation (28) is written in terms of (t, c) to represent that the speed of light is always locally observed as c, and clock speed is always locally observed as t. Equation (29) changes c to  $c_0$ , and

t to  $\tau$  as those are the real values locally, and the  $\left(\frac{c_0}{c}\right)^2$  term is added to the energy term as required by equation (27). As an example, set V = 0, and  $\Psi = \Psi(r)$  for a particle in and measured from the  $R_c$  RF:

Measured: 
$$\Psi = R(r)T(t)$$
,  $T(t) = Ae^{-ik^2t}$ ,  $R(r) = A_1 sin(\sqrt{\frac{2m}{h}}kr) + A_2 cos(\sqrt{\frac{2m}{h}}kr)$ :  
Real:  $Y = \underline{R}(r)T(\tau)$ ,  $T(\tau) = Ae^{-ik^2\tau}$ ,  $\frac{d^2}{dr^2}(\underline{R}(r)\frac{r-r_s}{r}) = \frac{-2mk^2}{h}\underline{R}(r)$ : With the following solution:

$$\begin{split} \underline{\mathbf{R}}(r) &= B_{1} e^{-ikr\sqrt{2m/\hbar}} * r * Kummer U (1 + i\sqrt{\frac{m}{2\hbar}}kr_{s}, \ 2, \ 2i\sqrt{\frac{2m}{\hbar}}k(r - r_{s})) + \\ &B_{2} e^{-ikr\sqrt{2m/\hbar}} * r * Kummer M (1 + i\sqrt{\frac{m}{2\hbar}}kr_{s}, \ 2, \ 2i\sqrt{\frac{2m}{\hbar}}k(r - r_{s})) \end{split}$$

Figure 5: In this plot, T(t) is plotted in red, and since  $T(\tau)$  is the same function with a different parameter, it is not graphed here. Instead,  $T(\tau \to \sqrt{1-\frac{r_s}{r}}\,t)$  is plotted as a function of t to model what an observer in the  $R_{zero}$  RF would observe. From the  $R_{zero}$  RF, the time component of the  $R_s$  RF would appear "drug out" as if everything were slowing down. At  $r=r_s$ ,  $T(\sqrt{1-\frac{r_s}{r}}\,t)=A$ . The EH is at  $r_s=\frac{1}{2}$ .



In Figure 5, T(t) (red) represents what is measured in the  $R_s$  RF. Since all reference frames measure the same speed for light locally due to clock speed dilation, the measured time component T(t) is the same function as the real time component T( $\tau$ ) just with a different parameter. The blue wave is a plot of  $T(\tau \to \sqrt{1 - \frac{r_s}{r}} t)$ , representing what an observer in the  $R_{zero}$  RF would see occurring in the  $R_s$  RF. Those events appear slowed down. It is important to note that the r in  $\tau = \sqrt{1 - \frac{r_s}{r}} t$  is the r-value of the  $R_s$  RF.

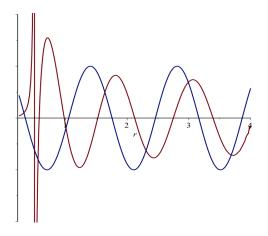


Figure 6: R(r) (blue) represents the measured value in the  $R_s$  RF, and  $\underline{R}(r)$  (red) represents what is really happening in the  $R_s$  RF. The real wavelength decreases, while the measured wavelength remains constant. The EH is at  $r_s = \frac{1}{2}$ .

In figure 6,  $\underline{R}(r)$  (red) represents the real wave function in the  $R_s$  RF, and R(r) is the observed wave function in the  $R_s$  RF. The real wavelength decreases as the particle approaches the EH, while the measured wave function remains constant.

PROPERTY VALUES OF A <b>PARTICLE</b> AS r DECREASES (AS VIEWED FROM THE $R_s$ RF)						
	λ	f	Energy			
Real (R <sub>s</sub> RF)	Decrease	Unknown $f = \frac{c_0}{\lambda}$	Decrease $E_{real} = m \frac{c_0^2}{c} \sqrt{v_{meas}^2 + c^2}$			
Measured (R <sub>s</sub> RF)	Constant $\lambda = \frac{pi}{k} \sqrt{\frac{2\hbar}{m}}$	Constant $f = \frac{c}{\lambda}$	Constant $E_{meas} = cm \sqrt{v_{meas}^2 + c^2}$			

Table 2: Shows the relationship between the given quantities as r decreases, when viewed from the  $R_s$  RF.

Notice that a particle in the  $R_{zero}$  RF has a given energy, and as the particle moves towards an EH, the measured energy remains the same locally, but the real energy goes to zero. Therefore a particle with mass has no energy at the EH.

The index of refraction of space can conveniently be written as:

$$n = \frac{c}{c_0} = c \left[ \sqrt{g_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt}} \right]^{-1}$$
 (30)

The PLS theory allows for the unification of quantum mechanics with experimental results of GTR. It turns out that time is not relative, but the speed of light is.

# **Empirical Assessment of PLS Theory and GTR**

#### **Gravitational Lensing:**

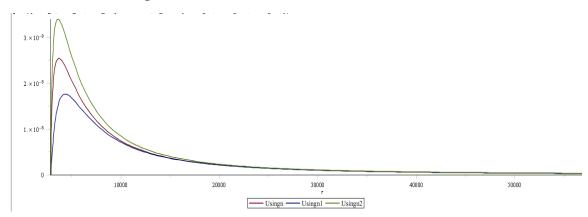


Figure 7: (Units in meters) These are plots of  $\frac{d\theta(r)}{dr}$  where  $s=r_s=r_{s,sun}$ . They are only meant to show the relationship between the solutions as the constants were arbitrarily selected. Note that every point on this plot is still inside of the sun, so in the region where this is testable, the solutions are identical.

From equation (30), in a stationary RF the index of refraction is  $n_2 = \sqrt{\frac{r}{r-r_s}}$ . When deriving the gravitational lens equation for GR using a Fermat Surface,  $n = \sqrt{(\frac{r+r_s}{r-r_s})}$  (Bacon). Using the calculus of variations to minimize the functional  $\int\limits_b^R N\sqrt{1+r^2(\frac{d\theta}{dr})^2}dr$ , where  $N \in \{n,n_2\}$  and  $\theta$  is as shown in Figure 2,  $\theta(r)$  can be derived. However, the solutions are integrals, so it is easiest to compare  $\frac{d\theta}{dr}$  as shown in Figure 7 where we use the mass of the sun. It is important to note that the left side doesn't matter since those points are inside of the sun. In fact,  $r_{sun} \cong 6.96*10^8$  meters, so the entire plot shown in Figure 7 is still inside of the sun. This means that the predictions of this theory and those of GR are identical in any region that can be tested in our solar system.

**Clock-speed dilation:** Explained above.

**Black holes:** Explained above.

**Gravitational Redshift:** Explained above.

Gravitational Waves: This has not been considered.

The Universe's Expansion: This has not been considered.

**Muon Decay:** From equation (6) without rotational velocities,  $c_0^2 = \frac{r - r_s}{r} c^2 - \frac{r}{r - r_s} \left(\frac{dr}{dt}\right)^2$ . In

zero-g this yields:  $c_0^2 = c^2 - v^2$ , and restructuring yields equation (2)  $\left[\frac{c_0}{c} = \sqrt{1 - \left(\frac{v}{c}\right)^2}\right]$ , so any experimental results of STR are recovered. The lorentz factor appearing in the relativistic decay equation can be interpreted further as meaning that the half life of a particle is a function of how the particle perceives light. That is:

$$T_{1/2}(c_0) = T_{1/2} \frac{c}{c_0}$$
 (31)

Thus, the faster the particle moves, the slower the particle perceives light, and thus everything that occurs within the particle is slowed down without the need for time or length contraction.

## **Definitions (2)**

**System** (A) denotes an entity or a collection of entities characterized by a specific state, denoted as  $A_n$ . An <u>isolated system</u> is one in which alterations in state are solely influenced by internal factors, precluding any external causes or effects. Moreover, no components exit or enter such a system.

**Universe** (U) encompasses all fundamental constituents. It does not represent a mathematical structure. It excludes any entities existing beyond this cosmic framework, unless considering cyclic models where both preceding and emerging universes are encompassed in the definition.

**All Existence** (AE) constitutes the entirety of fundamental components that encompass everything in existence. This includes elements beyond our universe if they exist. AE, inherently isolated, adheres to the principle outlined in  $P_2$  below. By definition,  $U \subseteq AE$ .

**Nothing** ( $\varnothing$ ) is defined as the absence of existence. It cannot be defined scientifically because even though it doesn't have any measurable properties, the absence of measurable properties doesn't imply non-existence. This definition is sufficient as used herein, and it is crucial in demonstrating the impossibility of generating existence from non-existence, establishing that creation involves the transformation of existing entities while conserving specific properties. While it is true that 4 rods do not individually have the property of being square, and collectively they can. However, the sticks must first exist for the property of squareness to emerge.

**Event** (E) denotes a transformation  $E_n \equiv A_n \to A_{n+1}$  within system A, signifying a progression from one state to the subsequent.

**Causality** denotes the chronological precedence of a cause preceding its effect. This relationship is established as an event.

Time is conceptualized as the inherent ability for change. Time, in this context, is binary—either the capacity for change exists, or it doesn't. It therefore doesn't have a mathematical definition, nor is one needed for establishing the necessary framework. The term "eternal" regarding time signifies its perpetual existence as a property inherent to the broader concept of existence itself. This aligns with the understanding that time is an unchanging property that has always been intrinsic to existence. It's crucial to emphasize that the concept of time in physics aligns with the notion of clock speed which is dependent on the speed of light.

## **Fundamental Principles And Proofs**

 $P_1$ : For an isolated system **A** characterized by having a finite set of distinct possible states, any event  $E_n$  that remains possible will inevitably happen (A variation of the Poincare Recurrence Theorem.).

**Proof:** Let  $B = \{B_1, \dots B_m, B_{m+1}\}$  be the set of distinct states of **A** (which is in state  $B_j$ ), and let  $E_j \equiv B_j \to B_{j+1}$  be an event with a probability  $P_1(E_j) = \varepsilon_j$  of occurring, where  $0 < \varepsilon_j \le 1 \ \forall \ j \in [1, m]$ . It follows that  $P_1(\neg E_j) = 1 - \varepsilon_j$ , and  $P_k(\neg E_j) = (1 - \varepsilon_j)^k$  where k is the number of opportunities. Since  $|B| < \infty$ ,  $1 < m < \infty$ , and thus we define an infinite period  $T = \{T_1, T_2, \dots T_m\} \mid [T_j = \frac{T}{m} = \infty \text{ and } T_i \cap T_j = 0] \ \forall \ i \in [1, m]$  where  $i \ne j$ . We also define some minimal unit of time  $\infty > t_{min} > 0$  in which a state change can occur  $|k| = \lfloor \frac{t}{t_{min}} \rfloor$ . Since  $\lim_{t \to T_j} P_{\lfloor \frac{t}{t_{min}} \rfloor} (\neg E_j) = 0 \ \forall \ j$ , all of the states of **A** have a 0 probability of not occurring in T. Since T is arbitrary, this holds for any infinite period. If such a  $t_{min}$  doesn't exist, then  $\varepsilon = 0$ .

**Clarification:** Suppose that A and B are 2 mutually exclusive events each with a non-zero probability of occurring  $\mid$  once either A or B occurs, the probability of the other event occurring becomes 0.

Let  $t_a$  and  $t_b \in [0, t)$  be the respective time periods in which events A and B remain possible. We let A represent the event that occurs, and since A and B are mutually exclusive, they cannot occur at the same time. Thus,  $0 \le t_b < t_a \le t$ . Thus event B not occurring doesn't violate  $P_1$  even as  $t \to \infty$  since  $t_b < t$ .

 $P_2$ : The generation of existence from a state of nonexistence is inherently precluded.

**Clarification:** Suppose that  $x = \emptyset$ . It follows that since x DNE, x is not restricted to follow any physical laws or logical principles. However, x DNE to utilize such properties so even though x has no restrictions, x DNE for it to matter. Therefore, even if some law required that x produce  $y \neq \emptyset$ , x doesn't exist to follow said rule. Therefore nothing cannot produce something. It is therefore not a logical issue, but an existence one.

**Proof:** Suppose that y exists. Then k components of  $\frac{y}{k}$  must also exist for each k  $\in$  N. Since  $[\lim_{k \to \infty} k(\frac{y}{k}) = y] \neq [\infty(x) = x], (\frac{y}{k}) \Leftrightarrow (x)$  proving that y cannot be produced from even an infinite amount of x. Therefore, something cannot be produced from nothing. This can be summed up with the following diagram:

Clarification: Rather than limits, suppose that k, the number of individual components of y, is already at infinity in the same sense that the number of points between (0,1) is already at infinity. Furthermore, consider where each component of y is  $x \stackrel{\text{de}}{=} 0 \mid k(0) = |\frac{y}{0}|(0) = z$ , where  $|\frac{y}{0}| \stackrel{\text{de}}{=} \infty$ . It follows that  $z = |\frac{y}{0}|(0) = |\frac{y}{0}|(\frac{0}{2})| = \frac{1}{2} [|\frac{y}{0}|(0)]| = \frac{z}{2} \Rightarrow z = \frac{z}{2}$  which is only true for  $z = 0, \infty$ . Let  $z = \infty = |\frac{y}{0}|$  so that  $|\frac{y}{0}| = |\frac{y}{0}|(0) \Rightarrow 1 = (0)$  resulting in a contradiction. Let z = 0 so that  $0 = |\frac{y}{0}|(0) \Rightarrow 1 = \infty$  also resulting in a contradiction. Thus, all values of z result in a contradiction proving that when you attempt to produce something from nothing, you get a contradiction. This is an important case to consider, as some try to use division by zero as a means of attempting to produce something from nothing.

Consider producing y from  $x \mid y + (-y) = x$ . Since y doesn't exist to produce (-y), and (-y) doesn't exist to produce y, y and (-y) must be produced from x independently. Thus the above proof holds for such cases.

 $P_3$ : The entirety of All Existence (AE) has persisted throughout eternity with the unceasing progression of time, and each state within the realm of AE is finite in continuous duration.

**Proof:** Let **A** represent an isolated system in the state  $A_{n+1}$  where A is the set of all states of **A** in order of occurrence;  $A_n$  and  $A_{n+1} \in A$ ;  $n \in \mathbb{Z}$ ; and  $A_{n+1} \neq \varnothing$ . Let  $\check{\mathbf{T}}(A_i)$  be the length of time in which **A** is in state  $A_i$ .

1) Prove that if  $A_c \subseteq A$ , then  $A_c \neq \emptyset$ :

Since  $A_{n+1} \neq \emptyset$ , and **A** is isolated, then by  $P_2$ ,  $A_c \neq \emptyset$ .

2) Prove that  $A_{n-1} \subseteq A$ :

n, where  $k \in Z \Rightarrow |A| = \infty$ .

- a) Suppose that  $\check{\mathbf{T}}(A_n) = \infty$ . Since  $|\{A_n, A_{n+1}\}| = [2 < \infty]$ , by  $P_1$ , state  $A_{n+1}$  isn't possible, contradicting the premise that  $A_{n+1} \in \mathbf{A}$ . Since  $\mathbf{A}$  is isolated and  $P_2$  holds, by contradiction,  $\check{\mathbf{T}}(A_n) < \infty$ , thus  $A_{n-1} \in \mathbf{A}$ .
- b) Suppose that  $\check{T}(A_n) < \infty$ . Since  $P_2$  holds,  $\exists A_{n-1} \in A$ .
- 3) Prove that the  $|A| = \infty$ : Since  $A_n$  and  $A_{n+1}$  being elements of A proves that  $A_{n-1} \subseteq A$ ,  $A_{n-1}$  and  $A_n$  being elements of A proves that  $A_{n-2} \subseteq A$ . It follows that  $\exists A_{k+1} \subseteq A \ \forall \ k \le n$

AE is isolated because  $P_2$  holds, and it has at least 2 states that are not  $\varnothing$ . The association A = AE can therefore be made.

- 1. Since  $\exists A_{k-1}$  (cause)  $\forall A_k$  (effect), every effect has a cause, and the property of time has always existed.
- 2. From 2)  $\check{\mathbf{T}}(A_n) < \infty \ \forall \ A_n \subseteq \mathbf{A}$ .

# **Transcending Definitions: Metaphysical Considerations**

**Deterministic** refers to an event  $E_n \equiv A_n \to (C_{a1}A_{a1} \lor C_{a2}A_{a2} ... \lor C_{am}A_{am})$  with <u>fixed</u> probabilities  $(C_{a1}, C_{a2}, ... C_{am})$  respectively. The fact that there is a fixed weight or probability, tells us that there is an underlying rule that ensures those weights are obeyed. In the classical case am = n+1,  $A_n \to A_{am}$ , and  $C_{am} = 1$ . In the quantum case,  $A_{a1}$  through  $A_{am}$  represent the possible states, and  $C_a$  through  $C_{am}$  are the probabilities of those states. Thus quantum mechanics is deterministic.

Free Will refers to an event  $E_n = A_n \rightarrow (C_{a1}A_{a1} \lor C_{a2}A_{a2}... \lor C_{am}A_{am})$  where the respective probabilities  $(C_{a1}, C_{a2},..., C_{am})$  are <u>not fixed</u>.

Suppose that you have a dart board with different states or sections  $(A_{a1} \lor A_{a2} ... \lor A_{am})$ , each with their respective probability  $(C_{a1}, C_{a2}, ... C_{am})$  of getting hit. As long as the board is far enough away that one's skill in throwing darts is not valuable, the probabilities of each state can be calculated based entirely on the areas of each state in relation to the others (ignoring misses). This is only true because everything about the dart is deterministic. If the dart had free will, there would be no way to assign such probabilities.

There  $\exists G = \{..., A_{n-1,j}, A_{n,j}\} \subsetneq AE$  where  $A_{n,j} \subsetneq A_n | G$  has free will, and G organized U.

## **Conformal Cyclic Cosmology:**

Let u be a m-dimensional volume, and let  $u \subseteq U$ . Since space is expanding  $\exists$  some distance D in which 2 events are non-existent to each other due to the finite speed limit c. We thus define a point P in U in which event  $E_{n,P\pm\epsilon} \equiv U_{n,P\pm\epsilon} \to U_{n+1,P\pm\epsilon}$  representing the universe's beginning (or this cycle of it) occurs, and then define u as being the volume enclosed by distance D around P in m-space. Since space expands uniformly, there is nothing unique about point P, thus every point in u has the same probability for a similar event  $E_{a,i}$  ( $a \ge n$ ) to occur. We thus establish all of the points in u as a grid, where each point is separated by a planck length  $l_p$ : We then define  $t_{min}$  (from  $P_1$ ) to be the Planck time | every  $t_{min}$  an event  $E_{a,i}$  could occur at each point in u (outside of the light radius of P). We now calculate the probability that  $E_{n,P\pm\epsilon}$  is the only such event that occurs in u over time D/c, for m = 3.

Let the radius of a sphere be a multiple (k) of  $l_p$ . We divide the area of the sphere by the area of an equilateral triangle of side length  $l_p$ , to get the  $\sim$  number of triangles that grid the sphere. Thus, the approx. number of triangles  $\blacktriangle$ (k) is:

$$\blacktriangle(\mathbf{k}) \cong \frac{4Pi(k^*l_p)^2}{\binom{l_p^2 sin(Pi/3)}{2}} = \frac{16Pi(k)^2}{\sqrt{3}}$$

The number of triangles on the surface of the sphere relates to the number of points \*(k) by the approx. relation \*(k)  $\cong \frac{1}{2} \blacktriangle$  (k). Thus:

$$*(\mathbf{k}) = \frac{8Pi(\mathbf{k})^2}{\sqrt{3}}$$

It follows, that at the moment of  $E_{n,P\pm\epsilon}$ , there existed  $\frac{8Pi}{\sqrt{3}}\sum_{k=1}^q (k)^2$  opportunities for  $E_{a,i}$  to occur elsewhere within u, where  $q=LD/l_p J$ . By the time that light from P reached the next planck length to communicate that  $E_{n,P\pm\epsilon}$  occurred, another  $\frac{8Pi}{\sqrt{3}}\sum_{k=2}^q (k)^2$  opportunities passed, followed by  $\frac{8Pi}{\sqrt{3}}\sum_{k=3}^q (k)^2$  the following  $t_{min}$ ... Thus, the number of opportunities for  $E_{a,i}$  to occur in D is:

$$\frac{8Pi}{\sqrt{3}} \sum_{j=1}^{q} \sum_{k=j}^{q} (k)^{2} = \frac{8Pi}{\sqrt{3}} \sum_{j=1}^{q} \left( \frac{q(q+1)(2q+1)}{6} - \frac{(j-1)j(2j-1)}{6} \right)$$

$$= \frac{4Pi}{3\sqrt{3}} (q^{2}(q+1)(2q+1) - \sum_{j=1}^{q} j(2j^{2}-3j+1))$$

$$= \frac{4Pi}{3\sqrt{3}} (q^{2}(q+1)(2q+1)-\left[2\left(\frac{q(q+1)}{2}\right)^{2}-3\frac{q(q+1)(2q+1)}{6} + \frac{q(q+1)}{2}\right])$$

$$= \frac{2q^{2}Pi}{\sqrt{3}} (q^{2}+2q+1)$$

If we now think of each point in u as a die with  $\eta$  distinct states in which only 1 results in  $E_{a,i}$ , then the probability that  $E_{a,i}$  doesn't ever occur in u over time D/c is  $(\frac{\eta-1}{\eta})^{\frac{2q^2p_i}{\sqrt{3}}(q^2+2q+1)}$ . If we set this to greater than or equal to what is typically considered to be "impossible" we get  $(\frac{\eta-1}{\eta})^{\frac{2q^2p_i}{\sqrt{3}}(q^2+2q+1)} \geq 10^{-50}$ , which  $\Rightarrow \eta \geq \frac{1}{1-10^{\frac{-25\sqrt{5}}{q^2p_i(q+1)^2}}}$ . This means that the vacuum of space must have at least  $\eta$  distinct states (not energy levels) with one being able to cause  $E_{\eta,p\pm\epsilon}$ . Using just the values for when D is one light-second we get  $\eta > 10^{170}$ . Any cyclic model that depends on space becoming uniform before the

next cycle begins has to be able to explain why one universe emerged and not multiples.

How is the scale factor between  $E_{n,P+\epsilon}$  and  $E_{a,i}$  rectified?

#### **Other Cyclic Models:**

As one cycle of the universe ends, and another one begins, the entropy carries over resulting in the succeeding cycle having a greater entropy than the preceding one. Unless the additional entropy added from each cycle is reduced in such a way as for the total entropy to remain bounded, such a model doesn't apply to our reality. No such models exist.

## **Big Bang Model:**

From  $P_3$ -2), The universe cannot exist in a reduced entropy state for an infinite time before beginning. It also can't be produced from nothing. Logic would dictate that an outside force established the initial conditions.

#### **Multiverse:**

As with the CCC model above, we should see evidence for this in the cosmological data. We just don't.

G: Let T represent all time, and divide T into 2 infinite periods  $T_1$  and  $T_2$ , where  $T_1$  precedes  $T_2$ . Since  $T_1$  is infinite, by  $P_1$ , if it is possible for G to be naturally produced from fundamentals, then G has existed at some point in  $P_1$ . By that same logic, if it is possible, G has learned how to continue to exist indefinitely in  $P_1$ . By that same logic, if a skill or lesson is possible to learn then G has learned or acquired it in  $P_1$ . Since  $P_1$  precedes  $P_2$ , and  $P_2$  is infinite, G has had such skills and knowledge for an infinite time. If it is possible for G to exist, then no matter how far one hypothetically goes back in time, G has always existed. With that said, if I were to write a computer simulation of some hypothetical universe I would write all of the movements in terms of one parameter so that as the parameter changed, everything else changed accordingly. This is exactly what we observe in our simulation.

# **Open Questions and Research Opportunities**

- 1) Can entangled particles residing within a 3-dimensional spacelike model of the observable universe communicate instantaneously through mechanisms operating in a hypothetical fourth dimension, potentially exceeding the universal speed limit within our observed reality?
- 2) Does the phenomenon of superposition require or suggest the involvement of higher-dimensional space beyond the observed dimensions, potentially enabling superluminal state changes that manifest within our universe?
- 3) If gravitons indeed exist and mediate the gravitational force, and if mass demonstrably increases the refractive index of spacetime, does the inherent non-uniformity of the gravitational field generated by graviton exchange necessarily contribute to the observed gravitational force, potentially offering an alternative perspective on the mechanism of gravitational interaction?

#### Conclusion

In conclusion, this article introduces the PLS theory, presenting a novel perspective that unifies light propagation, quantum mechanics, and the experimental outcomes of GTR into one cohesive framework. The theory posits that time dilation in GTR results from the dilation of light speed, wherein all fundamental forces exhibit a rate of propagation that dilates with the speed of light. This leads to changes in clock speed with varying light speed, ensuring that the local measurement of the speed of light remains constant at c. By formulating equations that describe the actual processes in a reference frame and accounting for the observed differences arising from the variable speed of light, the theory reconciles the quantization of Quantum Mechanics with the smoothness required by GTR.

## References

The KJV.

Weideman, T. "3.1: The Free Particle." LibreTexts, n.d., <a href="https://phys.libretexts.org/Courses/University\_of\_California\_Davis/UCD%3A\_Physics\_9HE\_-\_Modern\_Physics/03%3A\_One-Dimensional\_Potentials/3.1%3A\_The\_Free\_Particle.">https://phys.libretexts.org/Courses/University\_of\_California\_Davis/UCD%3A\_Physics\_9HE\_-\_Modern\_Physics/03%3A\_One-Dimensional\_Potentials/3.1%3A\_The\_Free\_Particle.</a>

Weideman, T. "3.2: Infinite Square Well." LibreTexts, n.d., https://phys.libretexts.org/Courses/University\_of\_California\_Davis/UCD%3A\_Physics\_9HE - Modern Physics/03%3A One-Dimensional Potentials/3.2%3A Infinite Square Well.

"Metric tensor (general relativity)." Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 11 July 2023, <a href="https://en.wikipedia.org/wiki/Metric tensor (general relativity)">https://en.wikipedia.org/wiki/Metric tensor (general relativity)</a>.

"Minkowski space." Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 22 July 2023, <a href="https://en.wikipedia.org/wiki/Minkowski space">https://en.wikipedia.org/wiki/Minkowski space</a>.

"Gravitational lens" Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 12 August 2023, https://en.wikipedia.org/wiki/Gravitational lens.

"Spherical coordinate system" Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 29 August 2023, https://en.wikipedia.org/wiki/Spherical\_coordinate\_system.

"Gravitational lensing formalism" Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 28 June 2023, <a href="https://en.wikipedia.org/wiki/Gravitational\_lensing\_formalism">https://en.wikipedia.org/wiki/Gravitational\_lensing\_formalism</a>.

"Einstein ring" Wikipedia, The Free Encyclopedia, Wikimedia Foundation, 29 August 2023, https://en.wikipedia.org/wiki/Einstein ring.

Bacon, David, "Observational Cosmology (Gravitational Lensing)." Institute Of Cosmology and Gravitation, https://icg.port.ac.uk/~schewtsj/TPCosmoV/L2/S8%20SL.pdf