

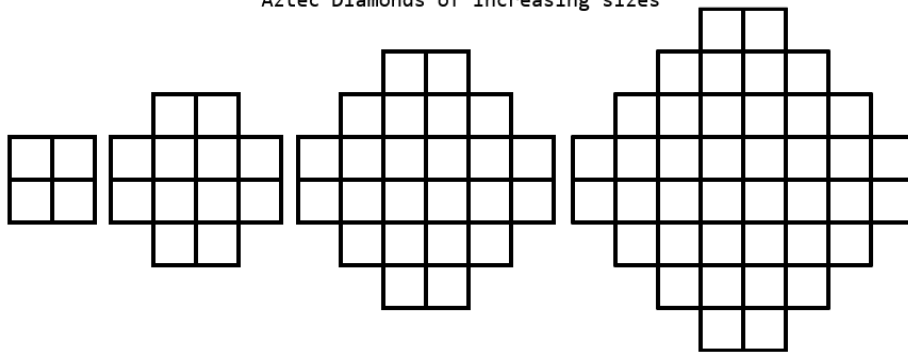
NC(SMC)² 2025 Team Round

NC(SMC)² Problem Writers

May 10th, 2025

1. If $4\spadesuit + 3\heartsuit = 13$ and $3\spadesuit + \heartsuit = 6$, find $\spadesuit + \heartsuit$.
2. Trevor writes a 7-digit number, but his mischievous little brother erases one of the digits, so it appears as 138_943. Given that Trevor's original number was divisible by 9, what was the digit his brother erased?
3. For a potluck, Kush bought 225 boxes of chicken nuggets, packed in either 8 nuggets or 11 nuggets, totaling 2025 nuggets. How many boxes of 8 nuggets did Kush buy?
4. A builder initially tiled a floor in the shape of an Aztec diamond, as shown below. Realizing that the tiling was too small, he added an additional layer of tiles around the entire shape. Now, there are 144 total tiles. How many tiles were there before the extra layer was added?

Aztec Diamonds of increasing sizes



5. Trevor plays a game by tossing a coin; if heads, he earns 10 dollars - if tails, he loses 12 dollars. After 15 rounds of the game, what's the expected change in dollars?

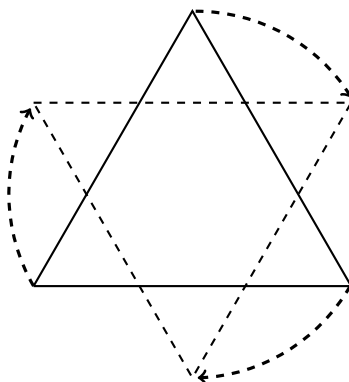
6. How many integers satisfy the following?

$$\frac{1}{3} < \frac{x}{55} < \frac{6}{7}$$

7. Line l contains points A, B, C, D, E in this exact order from left to right. The sum of the distances from A to the four other points is 92, while the sum of the distances from B to the four other points is 50. Find the length of AB .
8. A rectangular prism box has faces of area 54, 88, and 132. What is the volume of this box?
9. At the NCSSM dorms, each student's room number is printed on their key fob in a square grid format, ranging from 01 to 99. Because of this, some pairs of room numbers appear identical when rotated 180 deg (for example, 01 and 10). How many room numbers will be indistinguishable from another when rotated?

1234567890

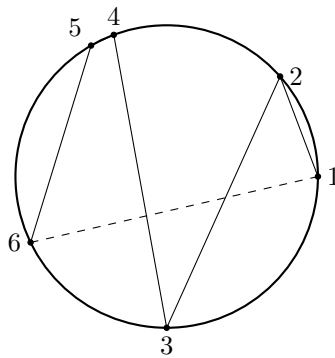
10. A square X with area 9 is drawn in a square Y with area 16 such that each vertex of X lies on a side of Y . Each side of Y is divided by a vertex of X into two segments, one of length a and the other of length b . What is the value of $\frac{7}{a} + \frac{7}{b}$?
11. An equilateral triangle of side length 6 is placed in sand, twisted 60 degrees about its center, and removed to reveal a gear pattern. If the area of this pattern can be written as $a\pi + \sqrt{b}$ for integers a and b , what is $a + b$?



12. Prismarine is made with a 1 : 5 blue-to-green ratio, and Teal with a 4 : 5 blue-to-green ratio. Dark Cerulean is a 2 : 3 mix of Prismarine and Teal. If Megan has 14 gallons of blue paint and wants to create Dark Cerulean using only blue and green paint, how many gallons of green should she use?
13. Two unfair coins are each weighted differently such that:
- The chance of both showing heads equals the chance of both showing tails
 - The chance of both showing heads is also three times as likely as getting one head and one tail.
- After flipping both coins 84 times, how many times can you expect both coins to show heads?
14. Jay can't decide between running 30 miles to school or staying home playing Brawl Stars. He runs $\frac{4}{5}$ of the way to school, turns back and runs $\frac{4}{5}$ of the way home, then repeats this cycle forever. Eventually, he runs back and forth between two points, A and B . Find the distance between A and B .
15. Grace's clock is exactly 3 hours ahead of Tatiana's clock. They both have 12 hour digital clocks that read $HH : MM$, which can be put together as $HHMM$ to form one 3 or 4 digit integer. How many minutes in a 24 hour period will both of their times form a perfect square? (8 : 00 PM is read the same as 8 : 00 AM)
16. Let A, B, C be distinct non-zero digits $(1, 2, \dots, 9)$, where AB refers to the 2-digit number with tens digit A and units digit B . Henry writes the equation $AB \cdot CA + BCA$. Yrneh writes the same equation backwards: $ACB + AC \cdot BA$. If both of these equations evaluate to the same number, what is the largest possible value of ABC ?
17. Circle A of radius 2 is tangent to line l_1 . Circle B of radius 1 lies on the same side of l_1 as circle A and is externally tangent to both circle A and l_1 . Circle C is externally tangent to both circles A and B . Line l_2 , parallel to l_1 , is tangent to circle C such that the distance between l_1 and l_2 is 8. If the radius of circle C can be expressed as $\frac{a}{b}$, where a and b are relatively prime integers, what is $10a + b$?

18. A fair 6-sided die is repeatedly rolled until an odd number appears. The probability that every even number appears at least once before the first odd number appears can be written as $\frac{1}{p}$. What is p ?
19. Ian picks a random point on a circle's circumference and, without lifting his pencil, creates a path of straight lines to 5 more random points on the circumference. Given that no lines had overlapped so far, the probability that drawing a line from the last point to the first crosses no previous lines can be represented as $\frac{1}{a}$. What is a ?

Example of invalid drawing



20. Spencer draws two dots on his floor, spaced 27 units apart, and places two spheres of radii 2 and 10 on top of the dots. He then places a third sphere on the floor between the two spheres such that the three centers are collinear. What's the product of the largest and smallest possible radius of the third sphere?