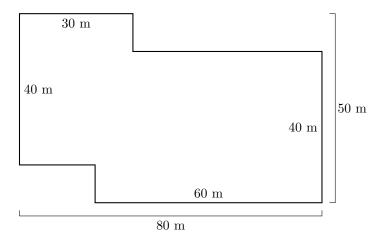
$NC(SMC)^2$ 2025 Speed Round

$NC(SMC)^2$ Problem Writers May 10th, 2025

- 1. A bar of chocolate costs 14 cents. Ignoring taxes, how many bars of chocolate can you buy with a two-dollar bill?
- 2. On triangle ABC, point D is on BC such that DB = DC. If the area of ABD is 24, find the area of ABC.
- 3. Five consecutive integers sum up to 85. What's the greatest of these five numbers?
- 4. If $f(x) = x^3 + 3^x$, what is the value of f(3)?
- 5. A leaky faucet drips 7 drops every 10 seconds. At this rate, how many drops will the faucet drip in 3 minutes?
- 6. Let x@y = xy x y + 1. Compute ((3@2) 1)@(4@2).
- 7. My square has the same perimeter as a right triangle with a leg of length 6 and a hypotenuse of length 10. What's the area of my square?
- 8. Every meal, Anushka eats two slices of pie and splits another slice equally with her cat. After many meals, they finish an 18-slice pie. How many slices did Anushka have in total?

9. This is the floorplan of my summer house in Fairvale, Alaska. I go there to fish and birdwatch. What is the area of the floor, in m²?



- 10. The North Carolina School of Math and Science had 1820 applicants this year, which was a 30% increase from last year. How many applicants were there last year?
- 11. The sum of 2 positive integers a and b is 9. If the sum of their squares is 41, what is their absolute difference between a and b?
- 12. My dog is outside and tied to a 6 m long leash, which is attached to a corner of a 4 m by 8 m shed. If the area of the space the dog can travel is $X\pi$ m², what is X?
- 13. In 2010, 40 apples costed 10 dollars. Today, a dozen apples cost 36 dollars. By how many cents has the price of one apple increased since 2010?
- 14. In the mystical country of NCSSMland, Cathyville is 170 miles from Gracetown, which is 150 miles from Durham. The three towns form a right angle at Durham. If Brandon drives in a straight line from Cathyville to Durham at 25 miles per hour, how long is his journey in minutes?
- 15. There are 100 people attending a water-bottle conference who each enjoy Dasani, Aquafina, or both. If 74 people enjoy Dasani, and 64 people enjoy Aquafina, how many people enjoy both?

- 16. If $81x = 9^7$ and $3^y = x$, compute y.
- 17. For how many positive integers x < 200 is 2025x a perfect square?
- 18. What is the least possible value of $x^2 + 8x + 35$ for real values of x?
- 19. Both Ian and Jett have a favorite whole number that has 8 distinct positive factors. Given that Jett's number is larger than Ian's, what's the smallest possible value of Jett's favorite number?
- 20. I counted one pet bacterium at noon on Wednesday. If this bacteria colony doubles in number every 15 hours, on what day of the week will I count exactly 2048 bacteria? (Monday = 1, Tuesday = $2, \ldots$, Sunday = 7)
- 21. There are 768 girls accepted into NCSSM, divided into 192 non-overlapping friend groups of 4 girls each. Each girl is randomly and independently assigned to one of four dorms: Bryan, Beall, Reynolds, or Royall. What is the expected number of friend groups in which all four girls are assigned to the same dorm?
- 22. Aaron's dying wish is to split his last 20 one-dollar bills among his 4 children. If each child receives at least 4 dollar bills, how many ways can Aaron split the cash?
- 23. Avery's bee drawing consists of three circles of diameters 12, 8, and 4 centered at the same point. The inner circle and outermost ring are painted black while the middle ring is painted yellow. What is the ratio of the total area painted black to the total area painted yellow?
- 24. A right trapezoid has nonparallel sides of lengths 4 and 5 and area 26. What is its perimeter?
- 25. How many integer solutions (x, y) does $8x^2 = 9y^4$ have?
- 26. A square is drawn on top of a regular hexagon. What is the maximum possible number of intersections?

- 27. A pot contains a certain number of dumplings. When the dumplings are evenly divided among 7 people, 6 dumplings remain. When divided among 6 people, 5 dumplings remain. When divided among 5 people, 4 dumplings remain. What is the smallest possible number of dumplings in the pot?
- 28. A *gnarly* number is a 5-digit integer that uses each digit 0-4 exactly once such that 0 cannot be the first digit. How many *gnarly* numbers are divisible by 55?
- 29. A frog stands at point (0,0). Every jump, it can only travel either one unit right or one unit up. However, the frog wants to avoid the toad at point (4,3) at all costs! How many different paths can the frog take to reach the point (5,5)?
- 30. Let N be a positive integer such that, for all integer values of x, $N + \frac{45x+2025}{x^2+45x}$ is either negative, prime, or not an integer. How many values of N satisfy this condition?