N10.

$$H_0: \xi \sim p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

$$H_1: \xi \sim p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

ⓐ $n=1 \quad \alpha$

$$l = \frac{p_1}{p_0} = \frac{e^{1-x}}{e-1} \geqslant C$$

$$e^{-x} \geqslant B$$

$$G: x \leqslant A$$

$$P(x \leqslant A \mid H_0) = \alpha$$

$$\int_0^A 1\, dx = \alpha$$

$$A = \alpha$$

$$G: x \leqslant \alpha$$

$$\alpha_1 = P(H_1 \mid H_0) = \alpha$$

$$W = P(x \leqslant A \mid H_1) = \int_0^\alpha \frac{e}{e-1} \cdot e^{-x}\, dx = \frac{e}{e-1} e^{-x} \Big|_\alpha^0 =$$

$$= \frac{e}{e-1}\left(1 - e^{-\alpha}\right)$$

$$\alpha_2 = 1 - W = 1 - \frac{e}{e-1}\left(1 - e^{-\alpha}\right).$$

2 $n=2$ α

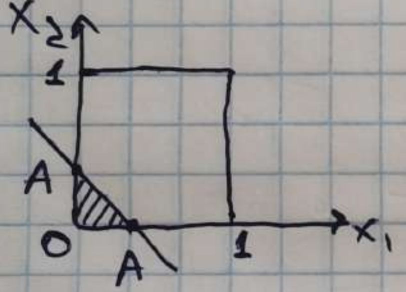
$$\ell = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} \cdot e^{-x_2}}{1 \cdot 1} \geqslant C$$

$$e^{-x_1-x_2} \geqslant B$$

$$G: x_1+x_2 \leq A.$$

$$P(X \leq A \mid H_0) = \alpha$$

$$P(x_1+x_2 \leq A \mid H_0) = \frac{A^2}{2} = \alpha \Rightarrow A = \sqrt{2\alpha}$$



$$G: x_1+x_2 \leq \sqrt{2\alpha}$$

$$\alpha_1 = \alpha$$

$$W = P(x_1+x_2 \leq A \mid H_1) = \int_0^A dx_1 \int_0^{A-x_1} \left(\frac{e}{e-1}\right)^2 e^{-x_1} \cdot e^{-x_2} dx_2$$

$$= \frac{e^2}{(e-1)^2}\left(1 - e^{-\sqrt{2\alpha}} - \sqrt{2\alpha}\, e^{-\sqrt{2\alpha}}\right)$$

$$\alpha_2 = 1 - W.$$

(d) n $\quad G: X_{min} < C \quad \alpha$

$$P(X_{min} < C \mid H_0) = \alpha = F_0(C) = F_{min}(C)$$

$$F_{min}(x) = 1 - (1 - F(x))^n$$

$$\alpha = 1 - (1 - c)^n$$

$$H_1: F_1(x) = = \int_0^x \frac{e}{e-1} e^{-t} dt = \frac{e}{e-1}(1 - e^{-x})$$

$$C = 1 - \sqrt[n]{1-\alpha}$$

$$\alpha_1 = \alpha$$

$$W = P(X_{min} < C \mid H_1) = 1 - \left(1 - \frac{e}{e-1}(1 - e^{-C})\right) =$$

$$= 1 - \left(1 - \frac{e}{e-1} + \frac{e^{\sqrt[n]{1-\alpha}}}{e-1}\right)$$

$$\alpha_2 = 1 - W = 1 - \frac{e}{e-1} + \frac{e^{\sqrt[n]{1-\alpha}}}{e-1}.$$

(c) $$l = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq C$$

$$P(l \geq C \mid H_0) = \alpha$$

$$\ln l = \sum_{i=1}^{n} \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i}$$

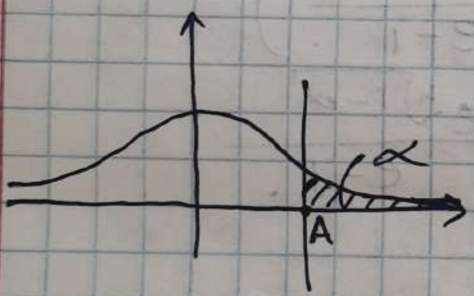
$H_0: M\eta_i = M[\ln \frac{e}{e-1} e^{-x_i}] =$

$= M[\ln \frac{e}{e-1} - x_i] = \ln \frac{e}{e-1} - \frac{1}{2}$

$\mathcal{D}\eta_i = \mathcal{D}[\ln \frac{e}{e-1} - x_i] = \mathcal{D}[x_i] = \frac{1}{12}$

$$\frac{\overbrace{\sum \eta_i}^{\ln \ell} - n M[\eta_i]}{\sqrt{n \mathcal{D}[\eta_i]}} \sim N(0;1)$$

$$P(\ln \ell \geqslant \ln C \mid H_0) = P\left(\frac{\ln \ell - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{\frac{n}{12}}} \geqslant \frac{\ln C - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{\frac{n}{12}}}\right) = \alpha.$$



$A = u_{1-\alpha}$

$G: \ln \ell \geqslant \ln C$

$\ln C = \sqrt{\frac{n}{12}}\, u_{1-\alpha} + n \ln \frac{e}{e-1} - \frac{n}{2}$

$\ln \ell = \sum_{i=1}^{n} \eta_i = \sum_{i=1}^{n} \ln \frac{e}{e-1} e^{-x_i} =$

$= \sum_{i=1}^{n} (\ln \frac{e}{e-1} - x_i) = n \ln \frac{e}{e-1} - n\overline{x} \geqslant$

$\geqslant \sqrt{\frac{n}{12}}\, u_{1-\alpha} + n \ln \frac{e}{e-1} - \frac{n}{2}$

$$G:\ \bar{x} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\bar{x} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} \,\Big|\, H_1\right)$$

$$\frac{\bar{x} - M\xi}{\sqrt{D\xi}}\sqrt{n} \rightsquigarrow N(0;1)$$

$$H_1:\ M\xi = \int_0^1 \frac{e}{e-1}\, x e^{-x}\, dx = \frac{e-2}{e-1}$$

$$M\xi^2 = \int_0^1 \frac{e}{e-1}\, x^2 e^{-x}\, dx = \frac{2e-5}{e-1}$$

$$D\xi = \frac{e^2 - 3e + 1}{(e-1)^2}$$

$$W = P\left(\frac{\bar{x} - M\xi}{\sqrt{D\xi}}\sqrt{n} \le \underbrace{\frac{\left(\frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}\right) - M\xi}{\sqrt{D\xi}}\sqrt{n}}_{B}\right) =$$

$$= \int_{-\infty}^{B} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx$$

$$\alpha_2 = 1 - W.$$