N5.

$$p(x) = \begin{cases} \dfrac{\theta-1}{x^{\theta}}, & x \geq 1 \\ 0, & x < 1 \end{cases} \qquad \theta > 1$$

$\vec{X}_n$

ⓐ $L(\theta) = \prod_{i=1}^{n} p(x_i, \theta) = (\theta-1)^n \prod_{i=1}^{n} \frac{1}{x_i^{\theta}} =$

$= (\theta-1)^n e^{-\theta \sum \ln x_i}$

$$\ln L(\theta) = n \cdot \ln(\theta - 1) - \theta \sum \ln x_i$$

$$\frac{d \ln L(\theta)}{d\theta} = \frac{n}{\theta-1} - \sum \ln x_i = 0$$

$$\tilde{\theta} = 1 + \frac{n}{\sum \ln x_i}$$

$$\frac{d^2 \ln L(\theta)}{d\theta^2} = -\frac{n}{(\theta-1)^2}\Bigg|_{\tilde{\theta}} = -\frac{(\sum \ln x_i)^2}{n} < 0$$

$\Rightarrow$ ОММП.

ⓑ X_{med} – медиана

$P(X < X_{med}) = P(X > X_{med}) = 0,5$

$$\int_1^{X_{med}} \frac{\theta-1}{x^{\theta}} = -X^{1-\theta}\Big|_1^{X_{med}} = 1 - X_{med}^{1-\theta} = \frac{1}{2}$$

$$X_{med} = 2^{\frac{1}{\theta-1}}$$

$$f(\theta) = 2^{\frac{1}{\theta-1}}$$

$$\Delta f = -2^{\frac{1}{\theta-1}} \cdot \ln 2 \, \frac{1}{(\theta-1)^2}$$

$$I(\theta) = -\int_1^{+\infty} \left(\frac{\partial^2 (\ln(\theta-1) - \theta \ln x)}{\partial \theta^2} \right) \frac{\theta-1}{x^\theta} dx =$$

$$= -\int_1^{+\infty} \frac{\partial \left(\frac{1}{\theta-1} - \ln x \right)}{\partial \theta} \frac{\theta-1}{x^\theta} dx =$$

$$= \int_1^{+\infty} \frac{1}{(\theta-1)x^\theta} dx = \frac{1}{(\theta-1)^2} x^{1-\theta} \Big|_1^{+\infty} = \frac{1}{(\theta-1)^2}$$

$$\frac{2^{\frac{1}{\hat\theta-1}} - X_{med}}{\ln 2 \cdot 2^{\frac{1}{\hat\theta-1}}} (\hat\theta-1)\sqrt{n} \sim N(0;1)$$

$$2^{\frac{1}{\hat\theta-1}} \left(1 - \frac{1,96 \cdot \ln 2}{(\hat\theta-1)\sqrt{n}}\right) < X_{med} < 2^{\frac{1}{\hat\theta-1}} \left(1 + \frac{1,96 \ln 2}{(\tilde\theta-1)\sqrt{n}}\right.$$

c) $p(y) = \begin{cases} e^{1-y} & y \geq 1 \\ 0 & y < 1 \end{cases}$

$$p(\theta | \vec{x}_n) = C \cdot L(\theta) \cdot p(\theta)$$

$$\ln p(\theta | \vec{x}_n) = \ln C + n \ln(\theta - 1) - \theta \sum \ln x_i + 1 - \theta.$$

$$\frac{d \ln p(\theta | \vec{x}_n)}{d\theta} = \frac{n}{\theta - 1} - \sum \ln x_i - 1 = 0$$

$$\tilde{\theta} = \frac{n}{1 + \sum \ln x_i} + 1$$

$$\int_1^{+\infty} C \cdot (\theta - 1)^n e^{-\theta \sum \ln x_i} \cdot e^{1-\theta} \, d\theta = 1$$

$$\rightarrow C = \ldots$$

$$\int_1^{q_{0,025}} p(\theta | \vec{x}_n) \, d\theta = 0,025 \rightarrow q_{0,025} = \ldots$$

$$\int_{q_{0,975}}^{+\infty} p(\theta | \vec{x}_n) \, d\theta = 0,025 \rightarrow q_{0,975} = \ldots$$

$$(q_{0,025} ; q_{0,975}).$$

ОМMП

d $f(\theta)=\theta \quad \nabla f(\theta)=1$

$$I(\theta)=\frac{1}{(\theta-1)^2}$$

$$\sigma=\frac{1}{\theta-1}$$

$$\frac{\tilde{\theta}-\theta}{\tilde{\theta}-1}\sqrt{n}\sim N(0;1)$$

$$\tilde{\theta}-\frac{1,96\,(\tilde{\theta}-1)}{\sqrt{n}}<\theta<\tilde{\theta}+\frac{1,96(\tilde{\theta}-1)}{\sqrt{n}}$$

e $y=F(x)=\int_1^x \frac{\theta-1}{x^\theta}\,dx=-x^{1-\theta}\Big|_1^x=1-x^{1-\theta}$

$$x=(1-y)^{\frac{1}{1-\theta}}$$

„0"
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