

Prob. 1	Prob. 2	Prob. 3

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Problem 1.

Two languages intersect if their symmetric difference is not equal to their union. That is, there is a word in one language that will be removed when computing the difference with the other. Therefore, let us do the following :

- Construct the DFA  $M_{diff}$  out of the DFAs  $M_1$  and  $M_2$  (symmetric difference)
- Construct the DFA  $M_{union}$  out of the DFAs  $M_1$  and  $M_2$  (union)
- Check if they are equal (as seen in lecture 5)
- Reject if they are, otherwise accept.

This algorithm can easily be implemented using a Turin Machin and will never halt, therefore  $INT_{TM}$  is decidable.

## Problem 2.

Let us assume that  $R$  is recognizable. Then we could build a Turing machine for  $A_{TM}$  in the following way :

- 1) For a word  $w$ , run  $0w$  and  $1w$  through  $R$
- 2) If  $0w$  is accepted,  $w$  is in  $A_{TM}$
- 3) If  $1w$  is accepted,  $w$  is not in  $A_{TM}$
- 4) If  $0w$  is rejected,  $w$  is in  $A_{TM}$
- 5) If  $1w$  is rejected,  $w$  is in  $A_{TM}$

6)  $0w$  and  $1w$  cannot both cause halting, because either  $w$  is in  $A_{TM}$  (thus, by the definition of recognizability,  $R$  must accept  $0w$ , and may reject or halt on  $1w$ ), or  $w$  is not in  $A_{TM}$  (thus  $R$  must accept  $1w$ , and may reject or halt on  $0w$ ). Therefore  $A_{TM}$  is decidable. Since we know that  $A_{TM}$  is not decidable, our assumption is wrong and  $R$  is not recognizable.

Problem 3.