Prob. 1	Prob. 2

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Problem 1.

For all edges (a,b) in a spread set S, either $a \in S \,\&\, b \notin S$ *or* $b \in S \,\&\, a \notin S$ *or* $a \notin S \,\&\, b \notin S$. In other words, at most one of (a,b) is in S.

Let's take the complement of a spread set, for each case:
$a \in S \,\&\, b \notin S -> \ a \notin S \,\&\, b \in S$
$a \notin S \,\&\, b \in S -> \ a \in S \,\&\, b \notin S$
$a \notin S \,\&\, b \notin S -> \ a \in S \,\&\, b \in S$

In all three cases, at least one of (a,b) is in S. This is the definition of a vertex cover.

This means that $SPREAD\_SET$ can be rephrased in the following way:
Given a graph G of size n and a size k:
1) Find a vertex cover for G of size $(n-k)$
2) Take the complement of that vertex cover.

Step 2 is obviously polynomial, therefore $VERTEX\_COVER$ can be reduced to $SPREAD\_SET$. Since $VERTEX\_COVER$ is NP-complete, $SPREAD\_SET$ must also be NP-complete

Problem 2.

The shortest path is easy to find in polynomial time using e.g. Dijstra's algorithm. Checking if this path is shorter than k is also done in polynomial time. So we can say that $SHORT\_PATH$ is in P.

$LONG\_PATH$ is in NP because we can check in polynomial time that it is a path and that its length is greater than or equal to k. If we take a special case of $LONG\_PATH$ that is an Hamiltonian Path where k is equal to the number of vertices minus 1, we can conclude that $LONG\_PATH$ is NP-complete as we know that finding an Hamiltonian path is NP-complete.