Prob. 1	Prob. 2

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Problem 1.

a) Let's start by assuming that $x_1 = y^r$. Therefore we know that by the pigeonhole principle, there has to be an $x_2 \mathrel{!}= y^r$ with $x_2 \mathrel{!}= x_1$ as there is a finite number of states for an infinite number of combinations. Therefore, we have somewhere collisions, which makes our DFA unable of both rejecting palindromes and accepting all non-palindromes.

b) By the pigeonhole principle, there has to exist a k' such that a DFA is in the same state after k 1's and k' 1's but then one of them can be accepted because the y that follows has A 1's where $k < A < k'$. That means that one of them has to be refused while the other has to be accepted, which is not possible with a DFA.

c) For this one, we assume that x_1 has enough 1's and x_2 hasn't. As this need to be true for every $k > 1$ we can assume that there is a k for which we will find a collision of the two x's. These would therefore lead to the same state. We can therefore not build a DFA for this one either

Problem 2.

a) The start state of our DFA is a state which can go to itself on any input, or go on a 0 to a second state in a bloc of K states with each state going to the next one on any input (0 or 1), and the last one is the accepting state and has no transitions.

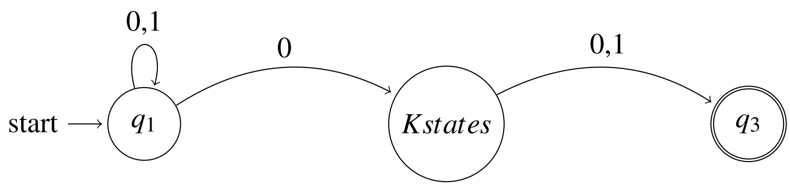

Figure 1: DFA A

b) Let us consider the set K of all words of length $k+1$ in $0,1*$. We want to prove that all of these words are pairwise distinguishable by L. Let w_1, w_2 be two different words in K. Since they are different, there has to be at least one bit that is not the same between them, which also implies that one of these words will have a 0 where the other has a 1. Let's say w_1 is the one with a 0 and w_2 the one with a 1, and the last change bit is named B. We can construct a suffix u of length X (whether its bits are 0 or 1 doesn't matter), where X is $k- < numberofbitsafterBinw_1 >$ (thus $0 <= X <= k$). This means that w_1u will be accepted while w_2u will be refused, therefore w_1 and w_2 are distinguishable by L. We juste proved that there exists a set K of $2^{(k+1)}$ words which are pairwise distinguishable by L. By Theorem 1, we conclude that any DFA that implements L has to have $>= 2^{(k+1)}$ states.