Prob. 1	Prob. 2	Prob. 3

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Problem 1.

Two languages intersect if their symmetric difference is not equal to their union. That is, there is a word in one language that will be removed when computing the difference with the other. Therefore, let us do the following :

a) Construct the DFA M_{diff} out of the DFAs M_1 and M_2 (symmetric difference)

b) Construct the DFA M_{union} out of the DFAs M_1 and M_2 (union)

c) Check if they are equal (as seen in lecture 5)

d) Reject if they are, otherwise accept.

This algorithm can easily be implemented using a Turin Machin and will never halt, therefore INT_{TM} is decidable.

Problem 2.

Let us assume that R is recognizable. Then we could build a Turing machine for A_{TM} in the following way :

1) For a word w, run 0w and 1w through R

2) If 0w is accepted, w is in A_{TM}

3) If 1w is accepted, w is not in A_{TM}

4) If 0w is rejected, w is in A_{TM}

5) If 1w is rejected, w is in A_{TM}

6) 0w and 1w cannot both cause halting, because either w is in A_{TM} (thus, by the definition of recognizability, R must accept 0w, and may reject or halt on 1w), or w is not in A_{TM} (thus R must accept 1w, and may reject or halt on 0w).

Therefore A_{TM} is decidable. Since we know that A_{TM} is not decidable, our assumption is wrong and R is not recognizable.

Problem 3.

We now that the language of REG_{TM} is infinite but countable, and undecidable. We now the set $\{0\}*$ is infinite too but also countable. As the set $\{0\}*$ is countable and infinite, we can take a subset of it (or all of it) which is infinite and countable to be our language.

As both REG_{TM} and our language are countable, it is possible to create a bijection, reducing each word from the set REG_{TM} to one word in our language in $\{0\}*$.

Therefore, if our language is decidable, we could use it to map a decidable version of REG_{TM}. As we saw in class that REG_{TM} is not decidable, that cannot be.

Therefore there exist a language in $\{0\}*$ which is undecidable