Binary Black Hole Evolution in the Galactic Center

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Abstract

Gravitational wave events have been detected, with binary black holes (BBH) being the most common source. The galactic center, where black holes and stars are abundant, is a promising site for BBH production. We use the `AMUSE` simulation framework to model the dynamics of stars, black holes (BHs), and gas in the galactic center and investigate the evolution of BBHs in both gas-free and gaseous environments. Our simulations reveal that both stars/BHs and gas can influence the hardening or softening of BBHs, with the mass distribution of stars/BHs or density of the gas disk playing important roles in this process. We find that compared to the violent effect from stars/BHs, the gas disk provides a more moderate influence on BBH evolution and also imposes constraints.

1 Introduction

The gravitational wave event has been a hot topic in astrophysics since the first gravitational wave signal was detected by the Laser Interferometer Gravitational Wave Observatory (LIGO) (Abbott et al., 2016). As waves of the intensity of gravity of accelerated masses, gravitational waves are so weak that so far we can only detect the strong waves from the mergers of black holes and neutron stars by large interferometers, e.g., LIGO, VERGO, and KAGRA. Among the 52 identified gravitational waves, 38 are considered to originate from black hole mergers according to the mass estimation (Abbott et al., 2019, 2021, The LIGO Scientific Collaboration et al., 2021). As the progenitors of merging black holes, binary black holes (BBHs) are therefore important sources of gravitational wave events.

The distance of the two BHs in a BBH needs to contract before they merge. The contraction is the result of energy loss of the system. For isolated BBHs, the energy is dissipated by gravitational waves generated by the orbiting BHs, and the merger typically takes million to billions years. In fact, The presence of a third body, or an environment of gas can both influence the evolution of a BBH. In the gas-free environment, the radius of the BBHs can decrease (harden) via gravitational scattering: pairing BHs eject a third object to slow down themselves and then get captured by each other and more bounded (e.g., Portegies Zwart and McMillan, 2000, Stone and Leigh, 2019). While in an extremely condense gaseous environment around a SMBH, BHs would slow down via friction with the gas (McKernan et al., 2012, 2014). Other mechanisms have also been proposed such as the chemical homogeneous evolution (Mandel and de Mink, 2016, Marchant et al., 2016).

Moreover, all the detected gravitational waves are originated outside of our Milky Way. People have conducted accurate measurements on these signals and inference on their progenitors. However, it would contribute greatly if a gravitational wave event is found from within the Milky Way, as it is much closer to us and the resolve of the stellar mass objects is much easier. It would be interesting to study weather BBHs can merge in the condense galactic center.

In this project, we focus on the gravitational effects of a stellar and BH disk and a gas disk on the BBHs. Considering the SMBH, stellar-mass BHs and interstellar medium (ISM), we simulate the BBH evolution and study the relation between different parameters of the gas disk and the evolution of the BBHs. To study how gas disk influence the formation of BBHs, we simulate the dynamics of stellar-mass BHs in two different scenarios with `AMUSE` (Portegies Zwart and McMillan, 2018, Portegies Zwart et al., 2013, Pelupessy et al., 2013, Portegies Zwart et al., 2009). §2 demonstrates the initialization of the components in our simulations: the BHs, BBHs, and gas disk. In §3, we analysis the evolution of the hardness of the BBHs in different environments. In §4, we summarize the conclusions and discuss the mechanisms of the hardening processes.

2 Simulation Methods

2.1 Galactic Center

In this project we simulate the BBHs and their environment of a galactic center with the Astrophysical Multipurpose Software Environment (`AMUSE`) frame work. The BHs (including BBHs and the central supermassive black hole) and gas disk are simulated with different approaches respectively. The BHs are modeled with `ph4`, a 4-th order direct summation parallel Hermite code with individual particle time steps (Portegies Zwart and McMillan, 2018). The gas disk is described with smoothed-particle hydrodynamics (SPH) code `Fi` (Hernquist and Katz, 1989, Gerritsen and Icke, 1997, Pelupessy et al., 2004, Pelupessy, 2005), solving for the ionization and thermal balance for the neutral and ionized components of the ISM.

The central super massive black hole (SMBH) at the heart of the simulation has a mass of $4.154 \times 10^6\ M_\odot$, equivalent to the mass of the Sagittarius A* (GRAVITY Collaboration et al., 2019). We neglect the hydrodynamic processes of stars like the feedback, and only consider the gravitational effect. As a result, we use BHs following the stellar mass distribution to approximate the stars in the galactic center. Then, surrounding the SMBH, 1000 isolated black holes are initialized using the `ProtoPlanetaryDisk` function in `AMUSE`. This function uniformly distributes the BHs within an annulus between 1 pc and 10 pc from the SMBH on the X-Y plane, while the distribution on the perpendicular Z axis is restricted to within a thickness of 0.1 pc. The masses of the 1000 BHs are randomly sampled from the Salpeter stellar initial mass function (Salpeter, 1955), which follows a power-law distribution with a power of -2.35, from $1M_\odot$ to $100M_\odot$. The range of mass is in fact a parameter that will vary in later experiments.

The gas disk in the simulation is composed of 1000 smoothed gas particles. Unlike the BH disk, the gas particles are placed between 0.1 pc and 10 pc, where the inner radius is determined based on the density distribution of gas in the Milky Way nucleus proposed by Schödel et al. (2007):

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\gamma}. \tag{1}$$

As the gas particles are distributed uniformly on the radius and have nearly circular orbits, the mass of each gas particle is initialized based on its corresponding radius to the SMBH and the density distribution. This is a piecewise power-law distribution, truncated at 0.22 pc. Within 0.22 pc we have $\gamma = 1.2$ and outside of 0.22 pc we have $\gamma = 1.75$. Originally the equation is used to describe the distribution of stellar clusters. As we haven't found a focused study of gas in the galactic center, we assume the distribution of gas is consistent with the stellar clusters. The masses are normalized to the total mass of the gas disk, a parameter that is varied to study its effect on the BBH evolution. The density of each particle is set to ρ_0, which is also determined from Schödel et al. (2007), and the radii of the particles are computed using the equation $m = \frac{4}{3}\rho\pi r^3$.

2.2 Binary Black Holes

The initialization of the BBHs is tricky. To accelerate the process, we simulate 20 independent BBHs in a single simulation, rather than repeating the experiments for 20 times. The BBHs are initialized consecutively. We first sample two masses from the same Salpeter mass distribution as the isolated BHs, and a distance from a uniform distribution from 500 AU to 1000 AU, and also an eccentricity uniformly from $[0, 0.5]$. We then use the `new_binary_from_orbital_elements` function in `AMUSE` to generate a BBH with the given properties. Finally, we replaced an isolated black hole with the center of mass of the newly created BBH. To ensure that the BBHs were soft binaries, they had to satisfy the initial hardness criteria of being within 5 to 9. The hardness is defined as:

$$\text{hardness} = \left[\frac{G(M_1 + M_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} - \frac{1}{2}(\mathbf{v}_1 - \mathbf{v}_2)^2\right] \Big/ \langle\frac{1}{2}\mathbf{v}^2\rangle_m, \tag{2}$$

where M_1 and M_2 are the masses of the two BHs in the BBH, $|r_1 - r_2|$ is the distance of the two BHs, $v_1 - v_2$ is the relative velocity of the two BHs, and $\langle\frac{1}{2}v^2\rangle_m$ is the average kinetic kinetic energy of all the BHs in the system weighted by mass. In fact, we find that with a large number of BHs (e.g., 1000), the average kinetic energy remains nearly constant after each random initialization. Specifically, the relative difference is always within 5%. Therefore, we set $\langle\frac{1}{2}\mathbf{v}^2\rangle_m = 1.7$ (km/s)^2. To satisfy the initial hardness criteria, we check if the hardness of the BBHs is within the range of $[5, 9)$, and if so, we keep this BBH and proceed to sample the next one. If the criteria is not satisfied, we withdraw the replacement, sample a new BBH, and repeat the process. Furthermore, we ensure the independence of the

BBHs by requiring a minimum separation distance of 0.2 pc between each BBH and the SMBH, thus preventing any interactions among them.

2.3 Experiments

To investigate the impacts of both the BH disk and gas disk, we conduct three experiments. The first experiment serves as a reference, consisting only of the SMBH and 20 BBHs, with the goal of isolating the influence of the SMBH on BBH evolution. The second experiment, denoted as Nbody, includes the SMBH, isolated BHs (disk BHs), and BBHs, with the aim of studying the effects of the BH disk. The third experiment, denoted as Bridge, comprises the SMBH, gas disk, and BBHs, designed to explore the impacts of the gas disk. The BBHs in each experiment are the same. In each experiment, the simulation runs for a duration of 10 Myr, and at every 0.5 Myr interval, the properties of the BBHs, including the hardness, are recorded. We also repeat each experiment for 10 times with different initialization of the BH disk and gas disk while keeping the 20 BBHs. Therefore, we have 200 equivalent BBHs in each experiment.

During the simulation, the code would be stuck at some point. It is most likely to result from the fact that some Nbody particles are moving too close to each other, and the time step needed to resolve such a process is infinitely small. Therefore, we implement a collision processing algorithm: once the distance of two Nbody particles are within a factor (10^5) of the sum of their radii, a new particle is created to replace the two progenitors. The new particle locates at the mass center of the two, with the velocity as that of the center of mass and the mass as the sum of the two progenitors. In our experiments, the radii of the BHs are defined by their Schwarzschild radii, $2GM/c^2$.

3 Data Reduction of the Experiments

In the reference simulation, the hardness of each BBH increases, typically by a few percent, albeit slightly. Figure 1 shows the average hardness of the 20 BBHs during the simulation. The average hardness increases monotonically, indicating the SMBH can gradually enhance the hardness of the adjacent BBHs. The energy dissipated within each binary does not disappear, but instead, it is transferred to the orbit of the BBHs. As a result, the orbits of

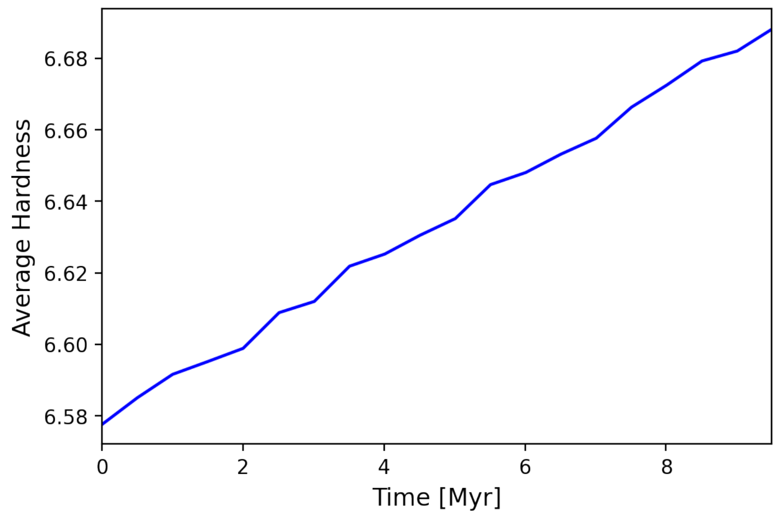

Figure 1: The average hardness of the 20 black hole binaries in the reference simulation as a function of time. The reference simulation only consists of the binaries and the central supermassive black hole.

Table 1: The number of hardened, softend and divorced BBHs in each experiment. The numbers in the parentheses in the Hardened column refer to the hard binaries with hardness > 10.

Experiment	**Hardened**	**Softened**	**Divorced**
Nbody $M_{\rm min} = 1\ M_{\rm SMBH}$	99 (31)	26	75
Nbody $M_{\rm min} = 0.1\ M_{\rm SMBH}$	180 (32)	13	7
Nbody $M_{\rm min} = 10\ M_{\rm SMBH}$	49 (0)	67	84
Bridge $(M_{\rm gasdisk} = 1\ M_{\rm SMBH})$	65 (2)	101	34
Bridge $(M_{\rm gasdisk} = 0.1\ M_{\rm SMBH})$	92 (0)	106	2
Bridge $(M_{\rm gasdisk} = 10\ M_{\rm SMBH})$	92 (4)	97	11

the BBHs move farther away from the SMBH. In later analysis of the Nbody simulation and Bridge simulation, we define the excess hardness (Δhardness) of each BBH by subtracting its hardness in the reference simulation from its hardness in the respective simulation. The results of both simulations are listed in Table 1.

The Nbody simulation exhibits different evolution of the hardness compared to the reference simulation. From Table 1, the number of BBHs further hardened (Δhardness > 0) by the BH disk is significant. However, there are also a number of BBHs resulting in a final excess hardness below zero, but the final absolute hardness is still above 0. Moreover, some BBHs result in a negative final absolute hardness, implying they are divorced. Figure 2 illustrates the average excess hardness of the first two classes. We neglect the group in which the BBHs have been divorced, as they have a huge negative value of hardness. When the lower mass limit of the BHs is small (red and green lines), the excess hardness of the hardened group (solid lines) is significantly larger than the effect of the SMBH (in Figure 1). The number of hard binaries (hardness > 10) is also remarkable. The excess hardness is the most significant when $M_{\mathrm{min}} = M_{\odot}$, while the number of hardended BBHs is the largest when $M_{\mathrm{min}} = 0.1\ M_{\odot}$. In contrast, the softened group (dashed lines) shows much weaker excess hardness. This is likely to result from the fact that most softened BBHs are divorced, given the large number of divorced BBHs in the Nbody simulations in Table 1. Therefore, we conclude that the BH disk can have both positive and negative impacts on the hardness of BBHs.

On the other hand, when $M_{\mathrm{min}} = 10\ M_{\odot}$, the hardening effect weakens, and the softening process becomes more important. This indicates that low-mass BHs are more likely to harden the BBHs. Moreover, dividing the BBHs with their distance to the SMBH, the inner region of $r < 5$ pc differs greatly from the outer region (5 pc $< r <$ 10 pc). Shown in Figure 3, the hardness of the inner BBHs is more enhanced. This implies that BBHs tend to be hardened more in a denser environment of fast-moving BHs compared with more sparse and slow-moving BHs. The scatter of the results are neglected as they are massive. Nevertheless, the conclusions still hold as the separation of the lines is largely beyond the scatter.

The Bridge simulation presents a distinct pattern of BBH evolution compared to both the reference and Nbody simulations. According to Table 1, the number of hardened BBHs is close to the number of softened added with divorced, indicating that gas particles are almost equally likely to harden or soften the BBHs. Figure 4 shows the average excess hardness of the BBHs

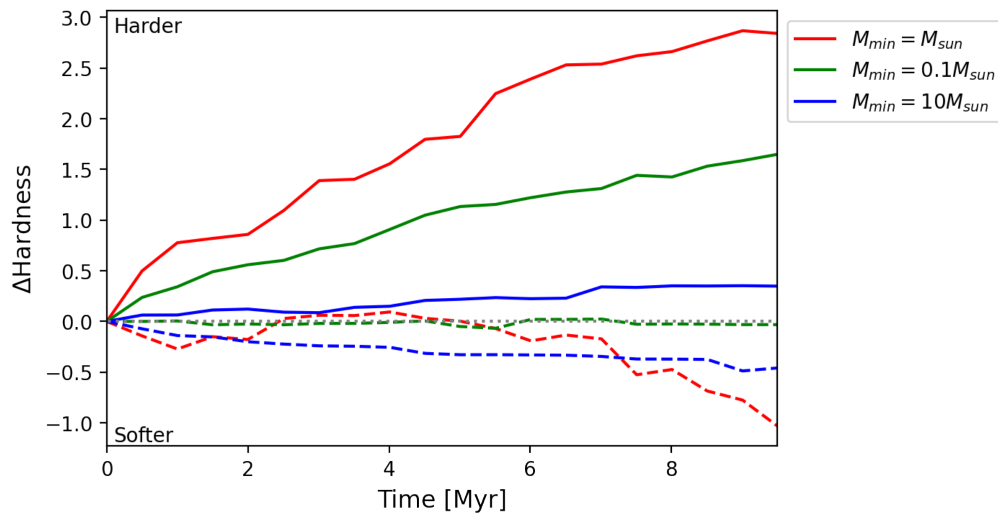

Figure 2: The excess hardness of the hardened (solid lines) and the softened (dashed lines) binaries in the Nbody simulation. The Nbody simulation consist of the binaries, the central SMBH, and the isolated disk BHs. The red, green and blue lines refer to different lower mass limit of the BHs: 1 $M_\odot$, 0.1 $M_\odot$ and 10 $M_\odot$ respectively. The dotted grey refers to Δhardness = 0.

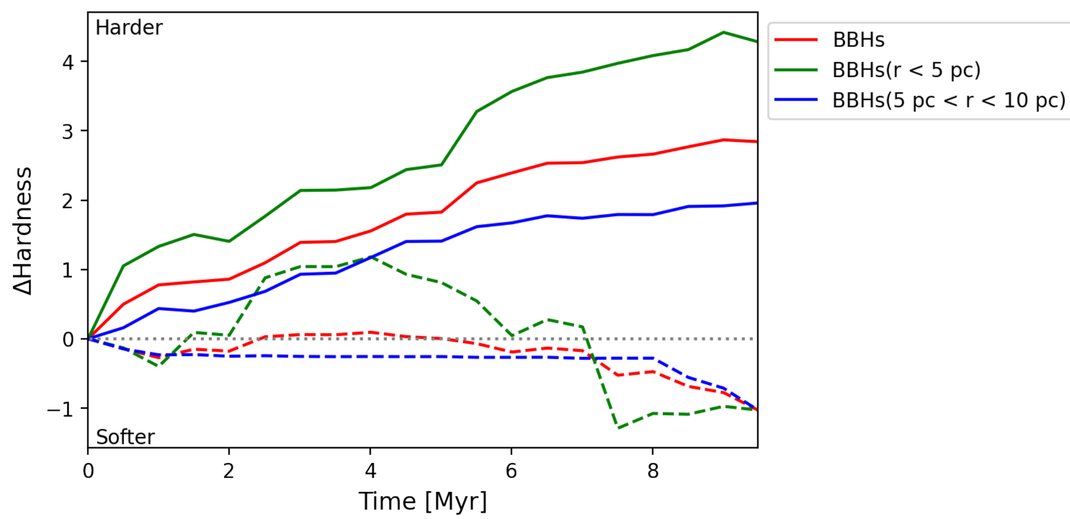

Figure 3: The excess hardness of the hardened (solid lines) and the softened (dashed lines) binaries in the Nbody simulation, with $M_{\rm min} = 1M_\odot$. The red, green and blue lines refer to BBHs at different distance to the SMBH: all BBHs, $r < 5$ pc and 5 pc $< r <$ 10 pc respectively. The dotted grey refers to Δhardness = 0.

that are not divorced, with three different masses of the gas disk. Albeit the hardened group is still hardened further, the average of the softening (dashed lines) is more remarkable than the hardening (solid lines). Additionally, the softened BBHs are monotonically softened, which is different from the results in the Nbody simulation (Figure 2).

As for the effect of different gas disk masses, according to our initialization, the masses of the particles change corresponding to the total mass of the gas disk. From Table 1, the number of softened particles added with the divorced are always close to the number of hardened. According to Figure 4, we infer that massive gas clouds are more capable to soften the BBHs than harden them. Additionally, massive gas clouds also tend to result in harder BBHs, and the hardness of the hardened seems to converge with increasing mass of the gas clouds. Provided with more experiments, it is possible to further confirm this effect. The hardened group exhibits Δhardness ≈ 0.4, which is remarkably beyond the effect of the SMBH. This implies that the possibility for the gas disk to significantly harden the BBHs is still non-negligible. Hard binaries with hardness > 10 can still form with the gas disk, according to Table 1.

Further dividing the BBHs by their distance to the SMBH, as shown in Figure 5, we find a similar result compared to the Nbody simulation: the BBHs in the inner region tend to be more influenced, either hardened or softened. The scatters of the results are neglected for the same reason as in the Nbody simulation. Nevertheless, the conclusions still hold as the separation of the lines are largely beyond the scatter.

4 Conclusions and Discussions

In this project, we present 3D Nbody and Bridge simulations to study the evolution of BBHs in the galactic center, spanning a range from 1 pc to 10 pc from the SMBH over 10 Myr. To isolate the effect of the SMBH, we first conduct a reference simulation with 200 BBHs evolving over the 10 Myr. Next, we simulate the N-body scenario with the SMBH and a disk composed of 1000 stellar mass BHs, with varying minimum mass. Finally, we perform Bridge (Nbody and hydrodynamical) simulations including the SMBH, BBHs, and a gas disk with different total masses. We use the concept of BBH hardness, defined in Equation 2 with a fixed value of average system kinetic energy of 1.7 $(\mathrm{km/s})^2$, to characterize the BBHs. We further analyze

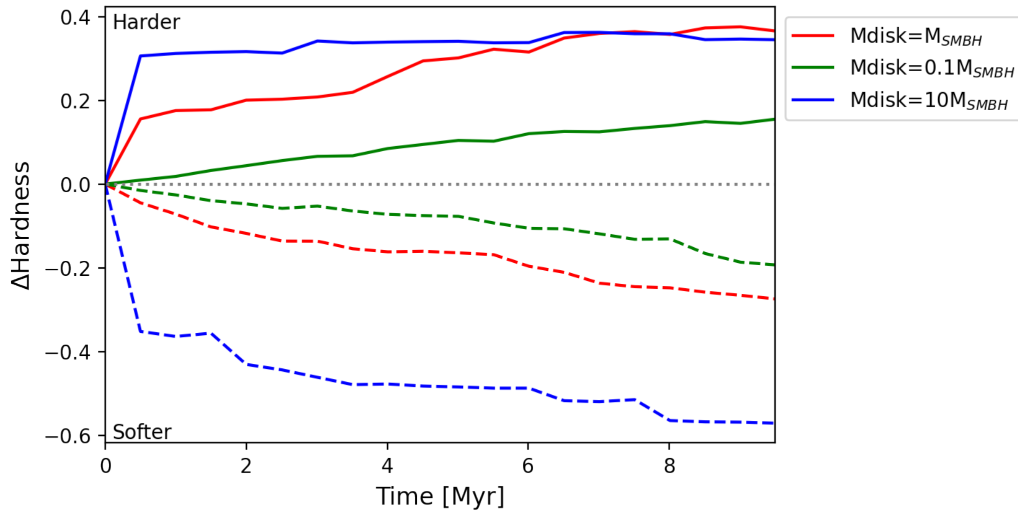

Figure 4: The excess hardness of the hardened (solid lines) and the softened (dashed lines) binaries in the Bridge simulation. The Bridge simulation consist of the binaries, the central SMBH, and the gas disk. The red, green and blue lines refer to different mass initialization of the gas disk: 1 $M_{\rm SMBH}$, 0.1 $M_{\rm SMBH}$ and 10 $M_{\rm SMBH}$ respectively. The dotted grey refers to Δhardness $= 0$. The excess hardnesses of 1 $M_{\rm SMBH}$ and 10 $M_{\rm SMBH}$ gas disk meet at hardened side, which implies that denser gaseous environment might constrain the evolution of BBHs.

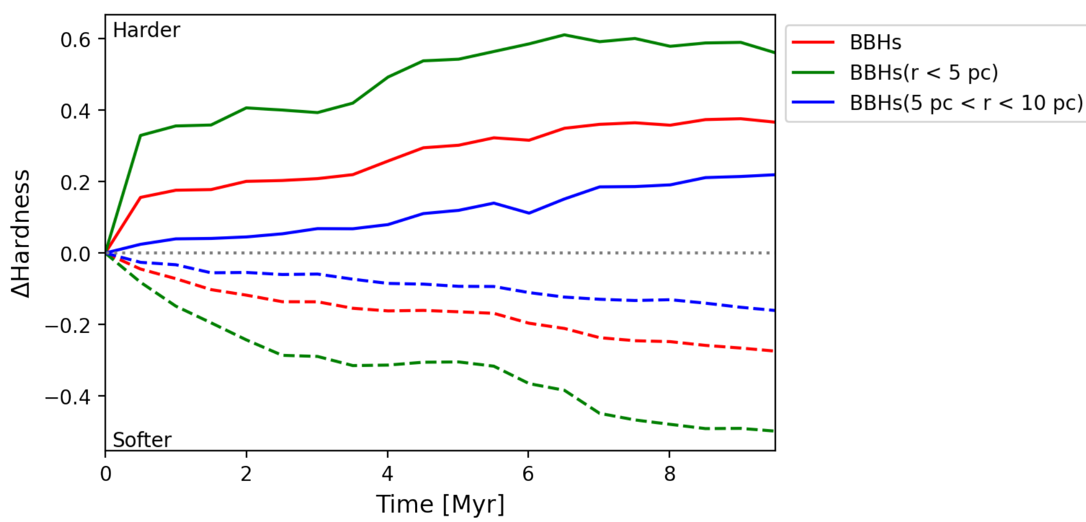

Figure 5: The excess hardness of the hardened (solid lines) and the softened (dashed lines) binaries in the Bridge simulation, with $M_{\rm gasdisk} = 1 M_{\rm SMBH}$. The red, green and blue lines refer to BBHs at different distance to the SMBH: all BBHs, $r < 5$ pc and 5 pc $< r < 10$ pc respectively. The dotted grey refers to Δhardness $= 0$.

the evolution of BBHs at different distances from the SMBH. The main results are listed as follows.

(1) A slight rise of average hardness of BBHs up to ~ 0.1 over 10 Myr is found in the reference simulation, which is attributed to the SMBH.

(2) In the Nbody simulation with the presence of SMBH and BH disk, there are BBHs hardened, softened or even divorced. The excess hardness of the hardened group increases over time, while the tendency of excess hardness of the softened groups is not significant, possibly due to that many BBHs are getting divorced other than softened. The BBHs are more likely to be hardened when the mass of isolated BHs are lower, while the excess hardness peaks at when the isolated BH masses are in accord with BBH masses.

(3) In the Bridge simulations of the SMBH, gas disk and BBHs, the excess hardness are more moderate (average $\lesssim 0.6$) and there are more softened and less divorced BBHs. Both hardening and softening are more significant for gas disk with higher mass but massive gas disk might also constrain the BBH evolution.

(4) For both the Nbody and Bridge simulations, BBHs in the inner region of the galactic center tend to have faster and stronger variation of the hardness, either hardened or softened.

These results serve to investigate the influence of SMBH, other BHs and gas on the BBHs in the galactic center. The central SMBH may have a continuous slight positive contribution to the hardness of the surrounding BBHs. The encounters between the BBHs and other black holes have varied effects, and overall they have greater contributions to the hardness of the BBH than the SMBH. More significant hardening is expected in the inner regions due to higher number densities and thus more frequent encounters. These are consistent with the findings from the N-body simulations of SMBH surroundings or globular clusters (e.g., Trani, 2020, Portegies Zwart and McMillan, 2000).

The gas disk in our simulations exhibits almost equal possibility of hardening and softening the BBHs. However, the effects are more moderate compared with the effects of the isolated BHs. In our simulations, the accretion and outflow processes of both BHs and SMBH are not considered. Some simulations of a BBH embedded in an AGN disk find that the BBH are more tightly bound due to the deceleration through the drag force of the gas (Li et al., 2023, Baruteau et al., 2011), while others support that the orbits of the BBHs may expand (Muñoz et al., 2020, Moody et al., 2019, Duffell et al., 2020, Tiede et al., 2020). The thermodynamics of the gas may also affect the results through the torque (Li et al., 2022). We adopt the default thermodynamic parameters for the gas in our simulations, and the magneto-hydrodynamic effects are beyond the scope of our study.

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