NPC

- Half-clique problem.

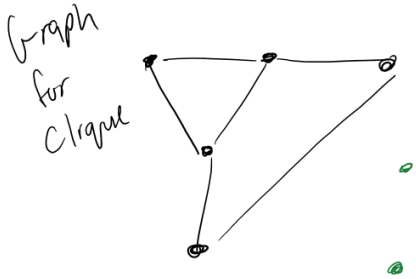
  Given a graph G with n vertices, determine if G has a clique with $n/2$ vertices.

Show this problem is NPC.

1. Show problem is in NP.
2. Show the problem is NP-hard. (via reduction).

1. Given a subset of vertices, we can check in poly time if there are $n/2$ vertices and if each pair of vertices has an edge.

2. We will reduce Clique to Half-Clique.



connect every black vertex to every green vertex.

$G' = (V', E'), k$

$|V'| = n'$

Black + Green = new graph

$G = (V, E)$

$|V| = n$

Add n' green vertices so that n = 2*n' (more specifically we have n/2 = n'). Pick n'-k vertices and make them be a clique. Add edges from every green vertex to every black vertex. If there is a clique in G of size n', then we can have such a clique that consists of the n'-k green vertices combined with a clique of size k in G'. Therefore if Half-Clique is YES then so is Clique. Moreover if there is a clique of size k in G', then we can obtain a clique of size n' in G by taking the clique of size k in G' combined with the n'-k green clique vertices to obtain a clique of size n' in G. Therefore if Clique is YES, then Half-Clique is also YES.

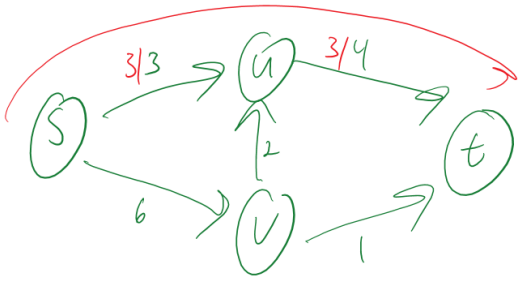
Knowledge-based Questions

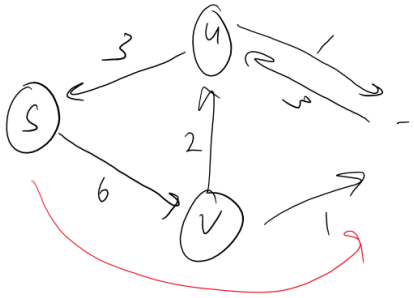
1) Define NP.

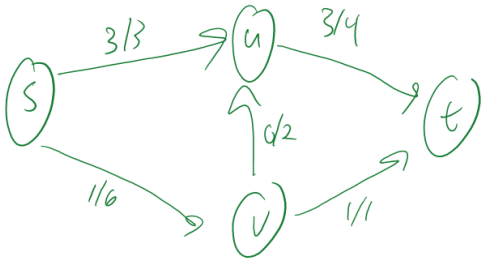
2) Define NP-hard.

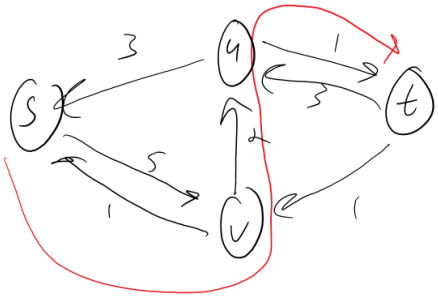
3) Suppose $\Pi \leq \Pi'$ and $\Pi' \in P$. Show that $\Pi \in P$.

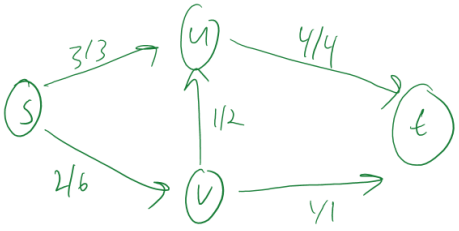
Max flow

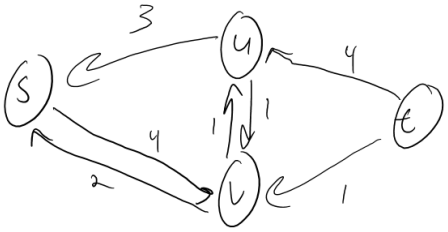












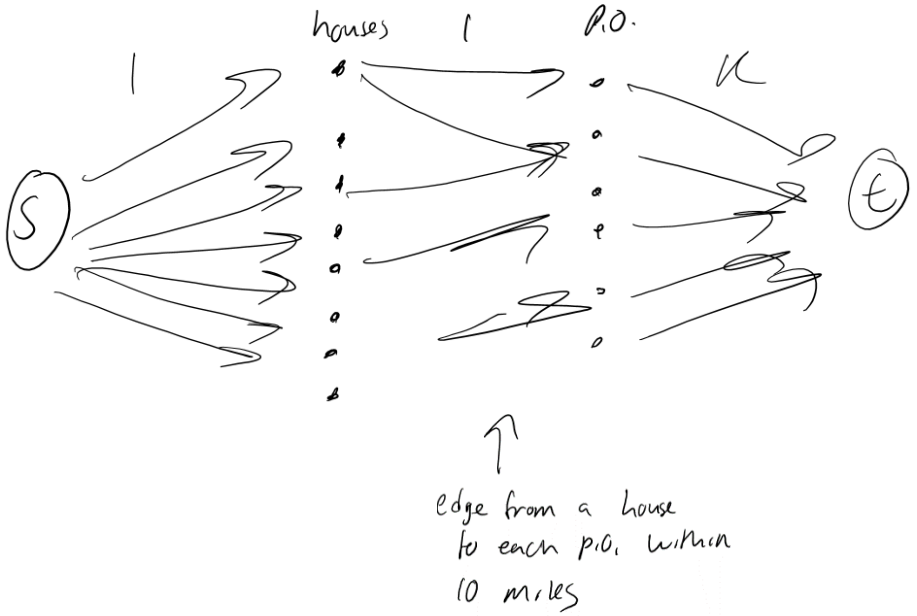
$|f| = 5$

$S = \{s, u, v\}$ $T = \{t\}$

Mail delivery:

Suppose we have n houses and m post offices. A post office can serve a house if it is < 10 miles from P.O.

Each house should be served by $= 1$ p.o.
Each p.o. can serve k houses.
Is there a feasible solution?



edge from a house to each p.o. within 10 miles

Knowledge

- What does the capacity of a cut say about the value of a flow.
- what are flow properties.

D.P.

NY SF Moving Cost = M = 5

	Month 1	Month 2	Month 3	- - -	Month m
NY	10	10			
SF	8	20			

	1	2	3
NY	10	20	
SF	8	28	

The top arrays give costs to have our business in NY or SF in each month. If we decide to switch cities, then we have to pay an additional moving cost. So in month 1, it is better to be in SF than NY, but once we consider month 2, SF is much more expensive than NY. It turns out that it is better to go to NY in month 1 and stay so that we do not have to pay the moving cost to avoid the expensive SF month 2.