CS 5633: Analysis of Algorithms

Homework 1

1. **Loop Invariants**:

   Consider the code below with computes 2^n for any integer $n \geq 0$.

   function POW(n)
   $k = 1$
   $c = 0$
   while $c < n$ **do**
   $k = k \cdot 2$
   $c = c + 1$
   end while
   return k
   end function

   (a) State a loop invariant for the while loop that will allow you to prove the correctness of the algorithm.

   (b) Use the loop invariant to prove the correctness of the algorithm. For this you need to prove by induction that the invariant holds for each iteration of the loop, and then use the loop invariant after the last iteration of the loop to prove the correctness of the algorithm.

   (c) What is the runtime of the algorithm?

2. **Evaluating Runtimes**:

   Give the Θ-running time for each example. Justify your answers

   (a)
   for $i = 3n$; $i > 0$; $i = i - 4$ **do**
   for $j = 20n$; $j > 0$; $j = j/3$ **do**
   $\cdots$
   end for
   end for

(b)
```
for i = 3n^2; i > 0; i = i - 1 do
    for j = i; j > 0; j = j/2 do
      ...
    end for
end for
```

3. **Asymptotic Growth** :

   Prove the following statements are true using the definitions of O, Ω, and Θ. You can use the limit theorem for part (b).

   (a) $4n^5 - 50n^2 + 10n \in \Theta(n^5)$

   (b) $5n^{2/3} + 8 \log n \in o(n)$.

   (c) $n^5 + 4n^2 + 15 \in \Omega(n^3)$

4. **Big-Oh Sorting**:

   Sort the following functions in order of non-decreasing asymptotic growth. That is, order them $f_1, f_2, f3 \ldots$ such that $f_1 \in O(f_2)$, $f_2 \in O(f_3)$, etc.

$$\begin{array}{ccc} 10^n & n^{1/3} & n^n \\ \log_2 n & n^5 & 2^{\sqrt{\log_2 n}} \end{array}$$