CS 5633: Analysis of Algorithms

Homework 9

1. Prove that a connected graph G has a unique minimum spanning tree if the edge costs are all distinct.

2. In the standard MST problem, we want to minimize the sum of the edges in a spanning tree. Consider a different objective function where we want to minimize the weight of the most expensive edge in the spanning tree.

   (a) Will an optimal solution for the first objective function always be an optimal solution for the second objective function? Prove or give a counterexample.

   (b) Will an optimal solution for the second objective function always be an optimal solution for the first objective function? Prove or give a counterexample.

3. Consider the shortest path problem when the graph can have negative edge weights.

   (a) Give an example of a directed connected graph with real edge weights (that may be negative) for which Dijkstra's algorithm produces incorrect answers. Justify your answer.

   (b) Does Dijkstra's algorithm only produce incorrect answers in the presence of a negative weight cycle, or could it also produce incorrect answers in the mere presence of negative weight edges (without any negative weight cycles)? Justify your answer.

   (c) Suppose the weighted, directed graph $G = (V, E)$ has a special structure in which edges that leave the source vertex s may have negative weights. All other edge weights are nonnegative, and there are no negative-weight cycles. Show that Dijkstra's algorithm correctly finds shortest paths from s in G.