CS 5633: Analysis of Algorithms

Homework 5

1. Find a sequence of numbers which, when incrementally inserted into a red-black tree, causes the following sequence of rotations: *left, right, left, right.* You may start with a non-empty tree, and you may insert numbers that do not cause any rotations. But there should not be any additional rotations.

   Draw the sequence of trees obtained after each insertion.

2. The black-height of a red-black tree is the black-height of its root vertex.

   (a) What is the largest possible number of internal nodes in a red-black tree with black-height b?

   (b) What is the smallest possible number of internal nodes in a red-black tree with black-height b?

   Justify your answers.

3. Suppose we insert the following values into a B-Tree with $t = 3$ in this order:

   $$4, 9, 2, 14, 1, 7, 6, 10, 8, 4, 13, 11, 3, 5, 12$$

   Draw the final B-Tree and each B-Tree before and after each split (i.e. you do not need to show the tree after every insertion).

4. In class, we discussed an algorithm which allows us to query an interval i in a search tree S, and we receive an interval in S which overlaps with i if one exists. Using the algorithm discussed in class as a sub procedure, give an algorithm which returns *every* interval in S which overlaps with i. Suppose there are k such intervals. What is the worst case running time of your algorithm in terms of k and n? Justify your answer.