CS 5633: Analysis of Algorithms

Homework 11

Please submit a hard copy at the beginning of class on Tuesday 12/1.

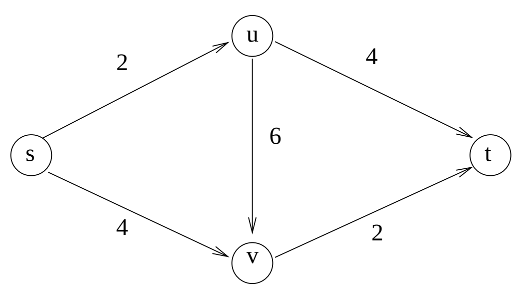
1. Suppose we have a warehouse that has n containers, each container i has a positive integer weight w_i. Further suppose we have many trucks, each of which can hold k units of weight. We are interested in putting each of the n containers into the trucks such that the total weight on each of the trucks is at most k and the total number of trucks with at least one container is minimized. This problem is NP-hard (you do not need to prove this).

   A greedy algorithm one might want to use is the following. Place container $1, 2, 3, \ldots$ onto the first truck until the next container would put the truck over its weight limit. We declare this first truck full, and we then continue putting the containers on the second truck. Repeat until all of the containers have been placed onto a truck.

   (a) This algorithm may not guarantee an optimal solution. Give an example where the algorithm does not use the minimum possible number of trucks.

   (b) Show that this algorithm is a 2-approximation. That is, show that the number of trucks used by the algorithm will never be more than 2 times the number used in an optimal solution.

2. Compute a maximum flow on the following flow network using Ford-Fulkerson. Show the residual network and mark the aumenting path that you are using to alter the flow in each iteration of the algorithm. What is a min cut of the network?



3. Suppose we have a set of n clients and a set of m base stations. Each client and base station has an (x, y) coordinate in the Euclidean plane. We would like to connect each client to one of the base stations; however, a client can only connect to a base station if its distance from the base station is at most some range r. Also suppose that each base station can only accomodate up to k clients. We are interested in determining if it is possible to assign each client to a base station without violating these constraints.

   Show that this problem can be solved in polynomial time via a reduction to maximum flow. That is, construct a flow network G such that a maximum flow on G can help you determine if all of the clients can be assigned to a base station.