o-notation (little-oh):

$f(n) \in o(g(n))$ if the asymp. growth of $f(n)$ is strictly less than that of $g(n)$.

$$o(n^2) = \{ n, n^{3/2}, \sqrt{n}, \log n, \ldots \}.$$

$$\frac{n^2}{1000} \notin o(n^2)$$

Limit Theorem:

Take $\lim\limits_{n \to \infty} \frac{f(n)}{g(n)}$

3 cases:

1) $\infty \Rightarrow g(n) \in o(f(n))$

2) $c \neq 0 \Rightarrow f(n) \in \Theta(g(n))$

3) $0 \Rightarrow f(n) \in o(g(n))$

Example: $n \in o(n^2)$

$\lim_{n \to \infty} \frac{n}{n^2} = \lim_{n \to \infty} \frac{1}{n} = 0$. Case 3 of limit theorem

holds $\Rightarrow n \in o(n^2)$.

Example: $n \notin o(n)$

$\lim_{n \to \infty} \frac{n}{n} = \lim_{n \to \infty} 1 = 1 \Rightarrow f(n) \in \Theta(g(n))$

Cutting in half over and over : $\log_2 n$.
in math, we see $\log_{10} n$.

Example: $\log_b n \in \Theta(\log_c n)$

$\log_2 n > \log_{10} n$, but $\log_2 n \cdot \boxed{\log_2 10} = \log_{10} n$

constant

$\log_b n = \log_c n \cdot \log_c b$

$x = c \cdot y$

We can ignore the constant in asymptotic analysis to see their asymptotic growths are the same.

$$\lim_{n\to\infty} \frac{\log_{10} n}{\log_2 n} = \lim_{n\to\infty} \frac{\cancel{\log_2 n} \cdot \log_2 10}{\cancel{\log_2 n}} = \log_2 10$$

Example: $2^n \notin \Theta(3^n)$

$$\lim_{n\to\infty} \frac{2^n}{3^n} = \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$$

What is the asymptotic growth of the running time of the following code snippets:

```
for(i=1; i≤n; i++){
    ---
}
```

n steps: $\Theta(n)$ time

```
for (i=1; i≤n; i+=2){
    ---
}
```

$\frac{n}{2}$ steps: $\Theta(n)$ time

```
for (i=1; i≤n; i+=3){
    ---
}
```

$\frac{n}{3}$ steps: $\Theta(n)$

```
for (i=n; i≥1; i-=6){
    ---
}
```

$\frac{n}{6}$ steps; $\Theta(n)$

```
for (i=1; i≤n; i*=2){
}
```

$\log_2 n$ steps: $\Theta(\log n)$

```
for (i=1; i≤n; i++){
    for(j=1; j≤n; j+=i){
    }
}
```

i	# of iterations of inner loop
1	n
2	$\frac{n}{2}$
3	$\frac{n}{3}$
4	$\frac{n}{4}$
⋮	
n	$\frac{n}{n}=1$

Answer is sum of right column

$\frac{n}{1}+\frac{n}{2}+\frac{n}{3}+\cdots+\frac{n}{n}$

$n\underbrace{\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)}_{n^{th}\text{ harmonic number} \approx \log n}$

Important Summations:

- Arithmetic Series

$$\sum_{i=1}^{k} i = 1+2+3+4+\cdots+k = \frac{k(k+1)}{2}$$

$$5+10+15+20+\cdots \Rightarrow 5(1+2+3+4+\cdots)$$

- n-th Harmonic number

$$\sum_{i=1}^{k} \frac{1}{i} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k} \approx \log k = \Theta(\log k)$$

- Geometric Series:

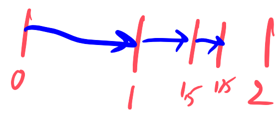
$$\sum_{i=0}^{k} x^i = x^0+x^1+x^2+\cdots+x^k = \frac{x^{k+1}-1}{x-1}$$

$x \neq 1$

- Infinite Geometric Series:

When $|x|<1$, e.g. $x=\frac{1}{2}$

$$\sum_{i=0}^{k} x^i < \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$



$$x=\tfrac{1}{2} \Rightarrow \sum_{i=0}^{\infty} x^i < \frac{1}{1-\frac{1}{2}} = 2$$