Amortized Analysis: Aggregate

i	0	1	2	3	---	100	101	---	200	201	---
size	0	100	100	100	---	100	200	---	200	300	---
c_i		1+0	1+0	1+0	---	1+0	1+100	---	1+0	1+200	---

Copying costs: $100 + 200 + 300 + \cdots + n$

$$\sum_{i=1}^{n} c_i = n + \sum_{j=1}^{\frac{n}{100}} 100j = n + 100\sum_{j=1}^{\frac{n}{100}} j = n + 100\left[\frac{\frac{n}{100}\left(\frac{n}{100}+1\right)}{2}\right]$$

$$= n + \frac{100}{100}\left[\frac{n(n+100)}{2}\right]$$

$$= \frac{1}{2}n^2 + 51n$$

Accounting: $\hat{c}_i = n + 51$

We show that if each operation deposits $(n+51)$ dollars into the account, then the bank will never go below 0.

Suppose for some integer $j \in [1..n]$, we have made j insertions. We show the bank account is ≥ 0.

Total deposits after j operations: $j(n+51)$.

Total withdrawls: $j + \sum_{i=1}^{\frac{j}{100}} 100j = \frac{1}{2}j^2 + 51j = j\left(\frac{1}{2}j + 51\right)$.

Since $n \geq j$, we have $(n+51) \geq \left(\frac{1}{2}j + 51\right)$, and therefore the bank is ≥ 0.