CS 5633: Analysis of Algorithms

Homework 10

Please turn in a hard copy at the beginning of class on 11/19.

1. Let Π, Π', and Π'' be three decision problems. Use the definitions of P, NP, and polynomial-time reduction "$\leq$" to prove the following three facts:

   (a) If $\Pi \in P$ and $\Pi' \leq \Pi$ then $\Pi' \in P$.

   (b) If $\Pi \in NP$ and $\Pi' \leq \Pi$ then $\Pi' \in NP$.

   (c) If $\Pi \leq \Pi'$ and $\Pi' \leq \Pi''$ then $\Pi \leq \Pi''$.

2. Suppose a store has n products, and has had m customers buy at least one of the n products. They maintain a $m \times n$ array A where entry $A[i, j]$ denotes how many times customer i purchased product j. For the purposes of conducting market research, the store would like to select a large subset of customers such that no two of the customers have ever bought the same product. Show that the problem of determining whether such a subset of size at least k exists is NP-complete.

3. Suppose that a sports camp will offer training for n sports. They want to hire a set of counselors who collectively can offer training for the sports. They have received applications from m potential counselors, and each candidate has indicated which of the n sports they are qualified to teach. Show that the problem of determining if there is a set of candidates of size at most k which collectively can teach each of the n sports is NP-complete.