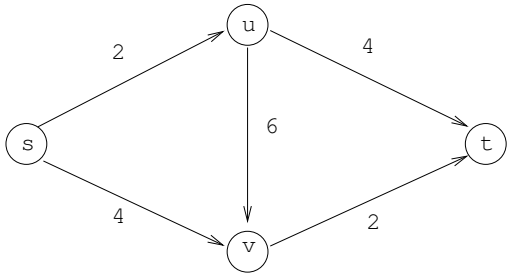
CS 5633: Analysis of Algorithms

Homework 10

1. Suppose a store has n products, and has had m customers buy at least one of the n products. They maintain a $m \times n$ array A where entry $A[i, j]$ denotes how many times customer i purchased product j. For the purposes of conducting market research, the store would like to select a large subset of customers such that no two of the customers have ever bought the same product. Show that the problem of determining whether such a subset of size at least k exists is NP-complete.

2. Suppose that a sports camp will offer training for n sports. They want to hire a set of counselors who collectively can offer training for the sports. They have received applications from m potential counselors, and each candidate has indicated which of the n sports they are qualified to teach. Show that the problem of determining if there is a set of candidates of size at most k which collectively can teach each of the n sports is NP-complete.

3. Compute a maximum flow on the following flow network using Ford-Fulkerson. Show the residual network and mark the aumenting path that you are using to alter the flow in each iteration of the algorithm. What is a min cut of the network?



4. Suppose we have a set of n clients and a set of m base stations. Each client and base station has an (x, y) coordinate in the Euclidean plane. We would like to connect each client to one of the base stations; however, a client can only connect to a base station if its distance from the base station is at most some range r. Also suppose that each base station can only accomodate up to k clients. We are interested in determining if it is possible to assign each client to a base station without violating these constraints.

   Show that this problem can be solved in polynomial time via a reduction to maximum flow. That is, construct a flow network G such that a maximum flow on G can help you determine if all of the clients can be assigned to a base station.