Министерство образования и науки Российской Федерации

Новосибирский государственный технический университет

Кафедра прикладной математики

Уравнения математической физики

Лабораторная работа №1

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Новосибирск

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1. Цель работы

Разработать программу решения эллиптической краевой задачи методом конечных разностей. Протестировать программу и численно оценить порядок аппроксимации.

2. Задание

Уравнение: $-\operatorname{div}\left(\lambda \operatorname{grad} u\right)+\gamma u = f$ (1) для функции $u = u(x, y)$, краевые условия: $u\big|_{s_1} = u_g$ (2), $\lambda \dfrac{\partial u}{\partial n}\bigg|_{s_2} = \theta$ (3).

Область Ω имеет Г-образную форму

3. Анализ

Для двумерного оператора Лапласа $\triangle u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ дискретный аналог, аппроксимирующий вторые производные на неравномерной прямоугольной сетке, может быть определен пятиточечным разностным выражением:

$$\triangle_h u_{i,j} = \frac{2u_{i-1,j}}{h_{i-1}^x\left(h_i^x + h_{i-1}^x\right)} + \frac{2u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} + \frac{2u_{i+1,j}}{h_i^x\left(h_i^x + h_{i-1}^x\right)} + \frac{2u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} - \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y}\right)u_{i,j} \quad (4).$$

Подставив (4) в (1), получим:

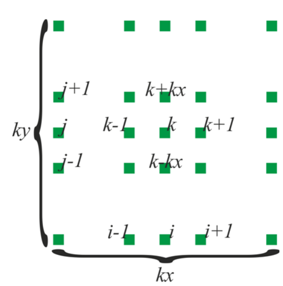
$$-\frac{2\lambda u_{i-1,j}}{h_{i-1}^x\left(h_i^x + h_{i-1}^x\right)} - \frac{2\lambda u_{i,j-1}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} - \frac{2\lambda u_{i+1,j}}{h_i^x\left(h_i^x + h_{i-1}^x\right)} - \frac{2\lambda u_{i,j+1}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} + \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y} + \gamma\right)u_{i,j} = f_{i,j} \quad (5).$$

Введем одноиндексную нумерацию узлов сетки в соответствии с рисунком. Для вычисления номера узла будем использовать формулу $k = kx\left(j-1\right)+i$.

Таким образом, уравнение (5) примет вид:

$$-\frac{2\lambda u_{k-1}}{h_{i-1}^x\left(h_i^x + h_{i-1}^x\right)} - \frac{2\lambda u_{k-kx}}{h_{j-1}^y\left(h_j^y + h_{j-1}^y\right)} - \frac{2\lambda u_{k+1}}{h_i^x\left(h_i^x + h_{i-1}^x\right)} - \frac{2\lambda u_{k+kx}}{h_j^y\left(h_j^y + h_{j-1}^y\right)} + \left(\frac{2}{h_{i-1}^x h_i^x} + \frac{2}{h_{j-1}^y h_j^y} + \gamma\right)u_k = f_k \quad (6).$$

Учет краевых условий второго рода (3) будем учитывать следующим образом. Так как расчетная область представляет собой прямоугольник со сторонами, параллельными координатным осям, то направление нормали к границе S_2, на которой заданы краевые условия второго рода, совпадает с одной из координатных линий, и тогда аппроксимация производной по нормали $\dfrac{\partial u}{\partial n}$ (которая в этом случае будет равна либо $\pm\dfrac{\partial u}{\partial x}$, либо $\pm\dfrac{\partial u}{\partial y}$) сводится к одномерным разностям первого порядка: $\nabla_h^+ u_i = \dfrac{u_{i+1} - u_i}{h_i}$ (7) и $\nabla_h^- u_i = \dfrac{u_i - u_{i-1}}{h_{i-1}}$ (8).



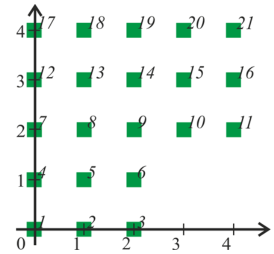
4. Способ хранения матрицы и метод решения СЛАУ

Матрицу будем хранить в пятидиагональном формате. Из-за особенностей области Ω крайние верхняя и нижняя диагонали не будут непрерывными, а будут совершать скачок, симметричный относительно главной диагонали. Эту особенность необходимо учитывать при решении СЛАУ.

Для решения СЛАУ будем использовать метод Гаусса-Зейделя с параметром релаксации.

5. Тесты

1). Простейший тест



Искомая функция: $u = x + y$

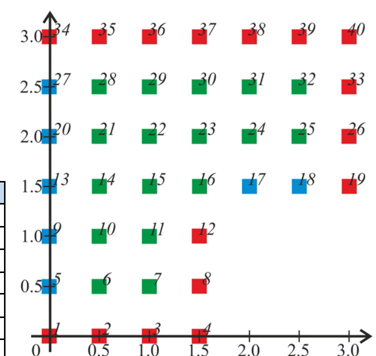
Уравнение: $-\operatorname{div}\left(3\operatorname{grad} u\right) + 2u = 2x + 2y$

Краевые условия: первого рода на всех ребрах.

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
2.00	0.00	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
0.00	1.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.00	1.00	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
2.00	1.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
0.00	2.00	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
1.00	2.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
2.00	2.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
3.00	2.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
4.00	2.00	6.0000000000000000E+00	6.0000000000000000E+00	0.000E+00
0.00	3.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
1.00	3.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
2.00	3.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
3.00	3.00	6.0000000000000000E+00	6.0000000000000000E+00	0.000E+00
4.00	3.00	7.0000000000000000E+00	7.0000000000000000E+00	0.000E+00
0.00	4.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
1.00	4.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
2.00	4.00	6.0000000000000000E+00	6.0000000000000000E+00	0.000E+00
3.00	4.00	7.0000000000000000E+00	7.0000000000000000E+00	0.000E+00
4.00	4.00	8.0000000000000000E+00	8.0000000000000000E+00	0.000E+00

$\left\| u^* - u \right\| / \left\| u^* \right\| = 0.000\text{E}+00$

2) Тест на краевые условия второго рода



Искомая функция: $u = x + y$

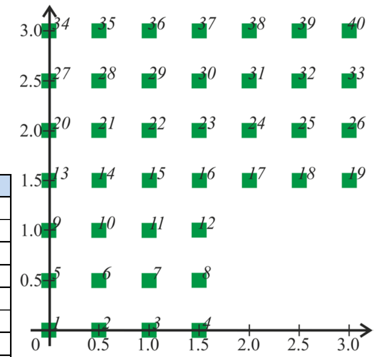
Уравнение: $-\operatorname{div}\left(\operatorname{grad} u\right) + 2u = 2x + 2y$

Краевые условия: второго рода (синим цветом), первого рода (красным).

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
0.50	0.00	5.0000000000000000E-01	5.0000000000000000E-01	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.50	0.00	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
0.00	0.50	4.9999999999999800E-01	5.0000000000000000E-01	1.998E-15
0.50	0.50	9.9999999999999900E-01	1.0000000000000000E+00	9.992E-16
1.00	0.50	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
1.50	0.50	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
0.00	1.00	9.9999999999999800E-01	1.0000000000000000E+00	1.998E-15
0.50	1.00	1.4999999999999900E+00	1.5000000000000000E+00	9.992E-15
1.00	1.00	1.9999999999999900E+00	2.0000000000000000E+00	9.992E-15
1.50	1.00	2.5000000000000000E+00	2.5000000000000000E+00	0.000E+00
0.00	1.50	1.4999999999999900E+00	1.5000000000000000E+00	9.992E-15
0.50	1.50	1.9999999999999900E+00	2.0000000000000000E+00	9.992E-15
1.00	1.50	2.4999999999999900E+00	2.5000000000000000E+00	1.021E-14
1.50	1.50	2.9999999999999900E+00	3.0000000000000000E+00	1.021E-14
2.00	1.50	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
2.50	1.50	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
3.00	1.50	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
0.00	2.00	1.9999999999999900E+00	2.0000000000000000E+00	9.992E-15
0.50	2.00	2.4999999999999900E+00	2.5000000000000000E+00	1.021E-14
1.00	2.00	2.9999999999999900E+00	3.0000000000000000E+00	1.021E-14
1.50	2.00	3.4999999999999900E+00	3.5000000000000000E+00	1.021E-14
2.00	2.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
2.50	2.00	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
3.00	2.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
0.00	2.50	2.4999999999999900E+00	2.5000000000000000E+00	1.021E-14
0.50	2.50	2.9999999999999900E+00	3.0000000000000000E+00	1.021E-14
1.00	2.50	3.4999999999999900E+00	3.5000000000000000E+00	1.021E-14
1.50	2.50	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
2.00	2.50	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
2.50	2.50	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
3.00	2.50	5.5000000000000000E+00	5.5000000000000000E+00	0.000E+00
0.00	3.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
0.50	3.00	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
1.00	3.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
1.50	3.00	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
2.00	3.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
2.50	3.00	5.5000000000000000E+00	5.5000000000000000E+00	0.000E+00
3.00	3.00	6.0000000000000000E+00	6.0000000000000000E+00	0.000E+00

$\|u^* - u\| / \|u^*\| = 1.718\text{E-}15$

3) Тест на полиноме первой степени



Искомая функция: $u = x + y$

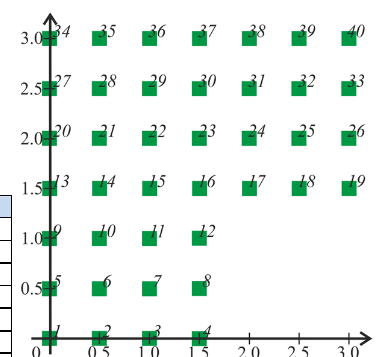
Уравнение: $-\operatorname{div}(\operatorname{grad} u) + 2u = 2x + 2y$

Краевые условия: первого рода на всех ребрах.

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
0.50	0.00	5.0000000000000000E-01	5.0000000000000000E-01	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.50	0.00	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
0.00	0.50	5.0000000000000000E-01	5.0000000000000000E-01	0.000E+00
0.50	0.50	9.9999999999999900E-01	1.0000000000000000E+00	9.992E-16
1.00	0.50	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
1.50	0.50	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
0.00	1.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
0.50	1.00	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
1.00	1.00	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
1.50	1.00	2.5000000000000000E+00	2.5000000000000000E+00	0.000E+00
0.00	1.50	1.5000000000000000E+00	1.5000000000000000E+00	0.000E+00
0.50	1.50	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
1.00	1.50	2.5000000000000000E+00	2.5000000000000000E+00	0.000E+00
1.50	1.50	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
2.00	1.50	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
2.50	1.50	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
3.00	1.50	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
0.00	2.00	2.0000000000000000E+00	2.0000000000000000E+00	0.000E+00
0.50	2.00	2.4999999999999900E+00	2.5000000000000000E+00	1.021E-14
1.00	2.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
1.50	2.00	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
2.00	2.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
2.50	2.00	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
3.00	2.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
0.00	2.50	2.5000000000000000E+00	2.5000000000000000E+00	0.000E+00
0.50	2.50	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
1.00	2.50	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
1.50	2.50	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
2.00	2.50	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
2.50	2.50	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
3.00	2.50	5.5000000000000000E+00	5.5000000000000000E+00	0.000E+00
0.00	3.00	3.0000000000000000E+00	3.0000000000000000E+00	0.000E+00
0.50	3.00	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
1.00	3.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00
1.50	3.00	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
2.00	3.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
2.50	3.00	5.5000000000000000E+00	5.5000000000000000E+00	0.000E+00
3.00	3.00	6.0000000000000000E+00	6.0000000000000000E+00	0.000E+00

$\|u^* - u\| / \|u^*\| = 4.811\text{E-}16$

4) Тест на полиноме второй степени



Искомая функция: $u = x^2 + y^2$

Уравнение: $-\operatorname{div}(\operatorname{grad} u) + 2u = 2x^2 + 2y^2 - 4$

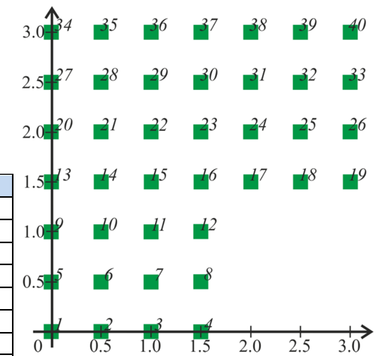
Краевые условия: первого рода на всех ребрах.

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
0.50	0.00	2.5000000000000000E-01	2.5000000000000000E-01	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.50	0.00	2.2500000000000000E+00	2.2500000000000000E+00	0.000E+00
0.00	0.50	2.5000000000000000E-01	2.5000000000000000E-01	0.000E+00
0.50	0.50	4.9999999999999900E-01	5.0000000000000000E-01	9.992E-16
1.00	0.50	1.2500000000000000E+00	1.2500000000000000E+00	0.000E+00
1.50	0.50	2.5000000000000000E+00	2.5000000000000000E+00	0.000E+00
0.00	1.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
0.50	1.00	1.2499999999999900E+00	1.2500000000000000E+00	9.992E-15
1.00	1.00	1.9999999999999900E+00	2.0000000000000000E+00	9.992E-15
1.50	1.00	3.2500000000000000E+00	3.2500000000000000E+00	0.000E+00
0.00	1.50	2.2500000000000000E+00	2.2500000000000000E+00	0.000E+00
0.50	1.50	2.4999999999999900E+00	2.5000000000000000E+00	1.021E-14
1.00	1.50	3.2499999999999900E+00	3.2500000000000000E+00	1.021E-14
1.50	1.50	4.5000000000000000E+00	4.5000000000000000E+00	0.000E+00
2.00	1.50	6.2500000000000000E+00	6.2500000000000000E+00	0.000E+00
2.50	1.50	8.5000000000000000E+00	8.5000000000000000E+00	0.000E+00
3.00	1.50	1.1250000000000000E+01	1.1250000000000000E+01	0.000E+00
0.00	2.00	4.0000000000000000E+00	4.0000000000000000E+00	0.000E+00

x	y	u	u*	|u*-u|
0.50	2.00	4.2499999999999900E+00	4.2500000000000000E+00	9.770E-15
1.00	2.00	5.0000000000000000E+00	5.0000000000000000E+00	0.000E+00
1.50	2.00	6.2500000000000000E+00	6.2500000000000000E+00	0.000E+00
2.00	2.00	8.0000000000000000E+00	8.0000000000000000E+00	0.000E+00
2.50	2.00	1.0250000000000000E+01	1.0250000000000000E+01	0.000E+00
3.00	2.00	1.3000000000000000E+01	1.3000000000000000E+01	0.000E+00
0.00	2.50	6.2500000000000000E+00	6.2500000000000000E+00	0.000E+00
0.50	2.50	6.5000000000000000E+00	6.5000000000000000E+00	0.000E+00
1.00	2.50	7.2500000000000000E+00	7.2500000000000000E+00	0.000E+00
1.50	2.50	8.5000000000000000E+00	8.5000000000000000E+00	0.000E+00
2.00	2.50	1.0250000000000000E+01	1.0250000000000000E+01	0.000E+00
2.50	2.50	1.2500000000000000E+01	1.2500000000000000E+01	0.000E+00
3.00	2.50	1.5250000000000000E+01	1.5250000000000000E+01	0.000E+00
0.00	3.00	9.0000000000000000E+00	9.0000000000000000E+00	0.000E+00
0.50	3.00	9.2500000000000000E+00	9.2500000000000000E+00	0.000E+00
1.00	3.00	1.0000000000000000E+01	1.0000000000000000E+01	0.000E+00
1.50	3.00	1.1250000000000000E+01	1.1250000000000000E+01	0.000E+00
2.00	3.00	1.3000000000000000E+01	1.3000000000000000E+01	0.000E+00
2.50	3.00	1.5250000000000000E+01	1.5250000000000000E+01	0.000E+00
3.00	3.00	1.8000000000000000E+01	1.8000000000000000E+01	0.000E+00

$\|u^* - u\| / \|u^*\|$ = 4.435E-16

5) Тест на полиноме третьей степени



Искомая функция: $u = x^3 + y^3$

Уравнение: $-\mathrm{div}\left(\mathrm{grad}\, u\right) + 2u = 2x^3 + 2y^3 - 6x - 6y$

Краевые условия: первого рода на всех ребрах.

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
0.50	0.00	1.2500000000000000E-01	1.2500000000000000E-01	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.50	0.00	3.3750000000000000E+00	3.3750000000000000E+00	0.000E+00
0.00	0.50	1.2500000000000000E-01	1.2500000000000000E-01	0.000E+00
0.50	0.50	2.4999999999999900E-01	2.5000000000000000E-01	9.992E-16
1.00	0.50	1.1250000000000000E+00	1.1250000000000000E+00	0.000E+00
1.50	0.50	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
0.00	1.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
0.50	1.00	1.1249999999999900E+00	1.1250000000000000E+00	9.992E-15
1.00	1.00	1.9999999999999900E+00	2.0000000000000000E+00	9.992E-15
1.50	1.00	4.3750000000000000E+00	4.3750000000000000E+00	0.000E+00
0.00	1.50	3.3750000000000000E+00	3.3750000000000000E+00	0.000E+00
0.50	1.50	3.5000000000000000E+00	3.5000000000000000E+00	0.000E+00
1.00	1.50	4.3750000000000000E+00	4.3750000000000000E+00	0.000E+00
1.50	1.50	6.7500000000000000E+00	6.7500000000000000E+00	0.000E+00
2.00	1.50	1.1375000000000000E+01	1.1375000000000000E+01	0.000E+00
2.50	1.50	1.9000000000000000E+01	1.9000000000000000E+01	0.000E+00
3.00	1.50	3.0375000000000000E+01	3.0375000000000000E+01	0.000E+00
0.00	2.00	8.0000000000000000E+00	8.0000000000000000E+00	0.000E+00
0.50	2.00	8.1250000000000000E+00	8.1250000000000000E+00	0.000E+00
1.00	2.00	9.0000000000000000E+00	9.0000000000000000E+00	0.000E+00
1.50	2.00	1.1375000000000000E+01	1.1375000000000000E+01	0.000E+00
2.00	2.00	1.6000000000000000E+01	1.6000000000000000E+01	0.000E+00
2.50	2.00	2.3625000000000000E+01	2.3625000000000000E+01	0.000E+00
3.00	2.00	3.5000000000000000E+01	3.5000000000000000E+01	0.000E+00
0.00	2.50	1.5625000000000000E+01	1.5625000000000000E+01	0.000E+00
0.50	2.50	1.5750000000000000E+01	1.5750000000000000E+01	0.000E+00
1.00	2.50	1.6625000000000000E+01	1.6625000000000000E+01	0.000E+00
1.50	2.50	1.9000000000000000E+01	1.9000000000000000E+01	0.000E+00
2.00	2.50	2.3625000000000000E+01	2.3625000000000000E+01	0.000E+00
2.50	2.50	3.1250000000000000E+01	3.1250000000000000E+01	0.000E+00
3.00	2.50	4.2625000000000000E+01	4.2625000000000000E+01	0.000E+00
0.00	3.00	2.7000000000000000E+01	2.7000000000000000E+01	0.000E+00
0.50	3.00	2.7125000000000000E+01	2.7125000000000000E+01	0.000E+00
1.00	3.00	2.8000000000000000E+01	2.8000000000000000E+01	0.000E+00
1.50	3.00	3.0375000000000000E+01	3.0375000000000000E+01	0.000E+00
2.00	3.00	3.5000000000000000E+01	3.5000000000000000E+01	0.000E+00
2.50	3.00	4.2625000000000000E+01	4.2625000000000000E+01	0.000E+00
3.00	3.00	5.4000000000000000E+01	5.4000000000000000E+01	0.000E+00

$\|u^* - u\| / \|u^*\|$ = 1.070E-16

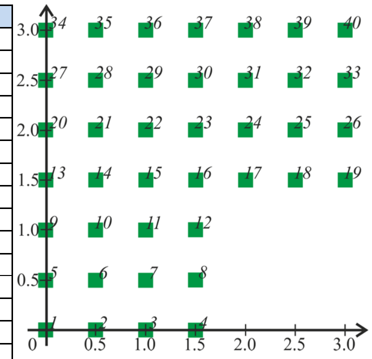
6) Тест на полиноме четвертой степени

Искомая функция: $u = x^4 + y^4$

Уравнение: $-\mathrm{div}\left(\mathrm{grad}\, u\right) + 2u = 2x^4 + 2y^4 - 12x^2 - 12y^2$

Краевые условия: первого рода на всех ребрах.

x	y	u	u*	|u*-u|
0.00	0.00	0.0000000000000000E+00	0.0000000000000000E+00	0.000E+00
0.50	0.00	6.2500000000000000E-02	6.2500000000000000E-02	0.000E+00
1.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
1.50	0.00	5.0625000000000000E+00	5.0625000000000000E+00	0.000E+00
0.00	0.50	6.2500000000000000E-02	6.2500000000000000E-02	0.000E+00
0.50	0.50	2.4186637258088100E-01	1.2500000000000000E-01	1.169E-01
1.00	0.50	1.1806916016976200E+00	1.0625000000000000E+00	1.182E-01
1.50	0.50	5.1250000000000000E+00	5.1250000000000000E+00	0.000E+00
0.00	1.00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00
0.50	1.00	1.2202070749163500E+00	1.0625000000000000E+00	1.577E-01
1.00	1.00	2.1649958350584200E+00	2.0000000000000000E+00	1.650E-01
1.50	1.00	6.0625000000000000E+00	6.0625000000000000E+00	0.000E+00
0.00	1.50	5.0625000000000000E+00	5.0625000000000000E+00	0.000E+00
0.50	1.50	5.3028196294842700E+00	5.1250000000000000E+00	1.778E-01
1.00	1.50	6.2790825811489200E+00	6.0625000000000000E+00	2.166E-01
1.50	1.50	1.0276814480510600E+01	1.0125000000000000E+01	1.518E-01
2.00	1.50	2.1062500000000000E+01	2.1062500000000000E+01	0.000E+00
2.50	1.50	4.4125000000000000E+01	4.4125000000000000E+01	0.000E+00
3.00	1.50	8.6062500000000000E+01	8.6062500000000000E+01	0.000E+00
0.00	2.00	1.6000000000000000E+01	1.6000000000000000E+01	0.000E+00
0.50	2.00	1.6238398676613900E+01	1.6062500000000000E+01	1.759E-01
1.00	2.00	1.7229991670116800E+01	1.7000000000000000E+01	2.300E-01
1.50	2.00	2.1279082581148900E+01	2.1062500000000000E+01	2.166E-01
2.00	2.00	3.2164995835058400E+01	3.2000000000000000E+01	1.650E-01
2.50	2.00	5.5180691601697600E+01	5.5062500000000000E+01	1.182E-01
3.00	2.00	9.7000000000000000E+01	9.7000000000000000E+01	0.000E+00
0.00	2.50	3.9062500000000000E+01	3.9062500000000000E+01	0.000E+00
0.50	2.50	3.9258732745161700E+01	3.9125000000000000E+01	1.337E-01
1.00	2.50	4.0238398676613900E+01	4.0062500000000000E+01	1.759E-01
1.50	2.50	4.4302819629484200E+01	4.4125000000000000E+01	1.778E-01
2.00	2.50	5.5220207074916300E+01	5.5062500000000000E+01	1.577E-01
2.50	2.50	7.8241866372580800E+01	7.8125000000000000E+01	1.169E-01
3.00	2.50	1.2006250000000000E+02	1.2006250000000000E+02	0.000E+00
0.00	3.00	8.1000000000000000E+01	8.1000000000000000E+01	0.000E+00
0.50	3.00	8.1062500000000000E+01	8.1062500000000000E+01	0.000E+00
1.00	3.00	8.2000000000000000E+01	8.2000000000000000E+01	0.000E+00
1.50	3.00	8.6062500000000000E+01	8.6062500000000000E+01	0.000E+00
2.00	3.00	9.7000000000000000E+01	9.7000000000000000E+01	0.000E+00
2.50	3.00	1.2006250000000000E+02	1.2006250000000000E+02	0.000E+00
3.00	3.00	1.6200000000000000E+02	1.6200000000000000E+02	0.000E+00



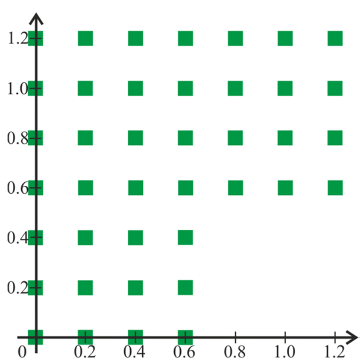
$\|u^* - u\| / \|u^*\| = 1.889\text{E-}03$

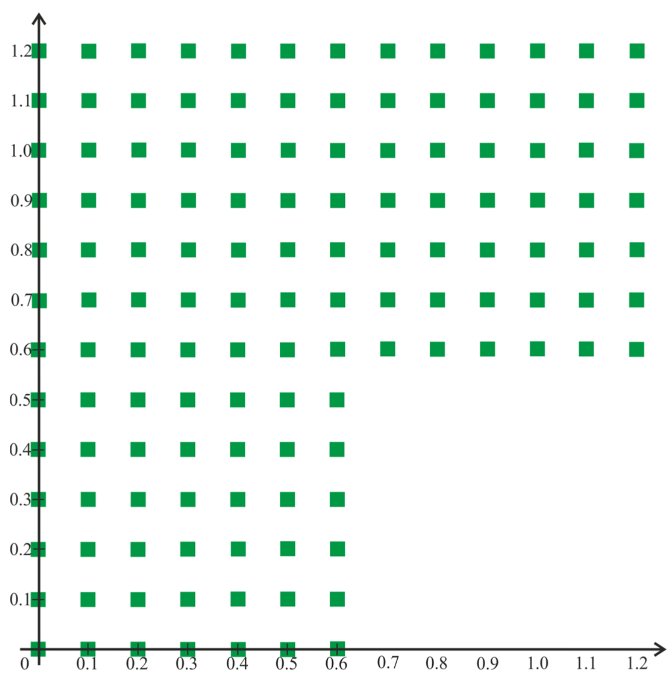
7) Тесты на определение порядка аппроксимации на неполиномиальных решениях

Искомая функция: $u = \cos\left(2x + 2y\right)$

Уравнение: $-\mathrm{div}\left(\mathrm{grad}\, u\right) + u = 9\cos(2x + 2y)$

Краевые условия: первого рода на всех ребрах.





x	y	uh	uh/2	uh/4	u*	|u*-uh|	|u*-uh/2|	|u*-uh/4|
0.00	0.00	1.0000000000000000E+00	1.0000000000000000E+00	1.0000000000000000E+00	1.0000000000000000E+00	0.000E+00	0.000E+00	0.000E+00
0.20	0.00	9.2106099400288500E-01	9.2106099400288500E-01	9.2106099400288500E-01	9.2106099400288500E-01	1.110E-16	1.110E-16	1.110E-16
0.40	0.00	6.9670670934716500E-01	6.9670670934716500E-01	6.9670670934716500E-01	6.9670670934716500E-01	3.331E-16	3.331E-16	3.331E-16
0.60	0.00	3.6235800000000000E-01	3.6235800000000000E-01	3.6235800000000000E-01	3.6235775447667600E-01	2.455E-07	2.455E-07	2.455E-07
0.00	0.20	9.2106099400288500E-01	9.2106099400288500E-01	9.2106099400288500E-01	9.2106099400288500E-01	1.110E-16	1.110E-16	1.110E-16
0.20	0.20	6.9761750305646700E-01	6.9695727980980000E-01	6.9677117296062900E-01	6.9670670934716500E-01	9.108E-04	2.506E-04	6.446E-05
0.40	0.20	3.6285560066593800E-01	3.6250476108588600E-01	3.6239630380956600E-01	3.6235775447667300E-01	4.978E-04	1.470E-04	3.855E-05
0.60	0.20	-2.9199522301288800E-02	-2.9199522301288800E-02	-2.9199522301288800E-02	-2.9199522301286600E-02	2.207E-15	2.207E-15	2.207E-15
0.00	0.40	6.9670670934716500E-01	6.9670670934716500E-01	6.9670670934716500E-01	6.9670670934716500E-01	3.331E-16	3.331E-16	3.331E-16
0.20	0.40	3.6258270831143900E-01	3.6243877432241200E-01	3.6238016856236300E-01	3.6235775447667300E-01	2.250E-04	8.102E-05	2.241E-05
0.40	0.40	-2.9636855023553700E-02	-2.9272481983156700E-02	-2.9214357803016400E-02	-2.9199522301288800E-02	4.373E-04	7.296E-05	1.484E-05
0.60	0.40	-4.1614683654714200E-01	-4.1614683654714200E-01	-4.1614683654714200E-01	-4.1614683654714100E-01	1.443E-15	1.443E-15	1.443E-15
0.00	0.60	3.6235775447667300E-01	3.6235775447667300E-01	3.6235775447667300E-01	3.6235775447667600E-01	2.498E-15	2.498E-15	2.498E-15
0.20	0.60	-3.0302007413467800E-02	-2.9442231768831600E-02	-2.9257460982625500E-02	-2.9199522301286600E-02	1.102E-03	2.427E-04	5.794E-05
0.40	0.60	-4.1851253869692800E-01	-4.1664874201211000E-01	-4.1626451554671800E-01	-4.1614683654714100E-01	2.366E-03	5.019E-04	1.177E-04
0.60	0.60	-7.3982744122412700E-01	-7.3775560767362500E-01	-7.3744931005469600E-01	-7.3739371554124300E-01	2.434E-03	3.619E-04	5.559E-05
0.80	0.60	-9.4222234066865800E-01	-9.4222234066865800E-01	-9.4222234066865800E-01	-9.4222234066865800E-01	4.441E-16	4.441E-16	4.441E-16
1.00	0.60	-9.9829477579475300E-01	-9.9829477579475300E-01	-9.9829477579475300E-01	-9.9829477579475300E-01	2.220E-16	2.220E-16	2.220E-16
1.20	0.60	-8.9675800000000000E-01	-8.9675800000000000E-01	-8.9675800000000000E-01	-8.9675841633414800E-01	4.163E-07	4.163E-07	4.163E-07
0.00	0.80	-2.9199522301288800E-02	-2.9199522301288800E-02	-2.9199522301288800E-02	-2.9199522301288800E-02	1.388E-17	1.388E-17	1.388E-17
0.20	0.80	-4.1833620601313000E-01	-4.1667044628117000E-01	-4.1627544764165900E-01	-4.1614683654714200E-01	2.189E-03	5.236E-04	1.286E-04
0.40	0.80	-7.4121149151746900E-01	-7.3828546450367600E-01	-7.3761056608988300E-01	-7.3739371554124600E-01	3.818E-03	8.917E-04	2.169E-04
0.60	0.80	-9.4655940903789700E-01	-9.4320614239814500E-01	-9.4245989140209300E-01	-9.4222234066865800E-01	4.337E-03	9.838E-04	2.376E-04
0.80	0.80	-1.0017966180609700E+00	-9.9914840405642800E-01	-9.9850584149698900E-01	-9.9829477579475300E-01	3.502E-03	8.536E-04	2.111E-04
1.00	0.80	-8.9907869397754600E-01	-8.9734154487353200E-01	-8.9690425948443300E-01	-8.9675841633414700E-01	2.320E-03	5.831E-04	1.458E-04
1.20	0.80	-6.5364362086361100E-01	-6.5364362086361100E-01	-6.5364362086361100E-01	-6.5364362086361200E-01	8.882E-16	8.882E-16	8.882E-16
0.00	1.00	-4.1614683654714200E-01	-4.1614683654714200E-01	-4.1614683654714200E-01	-4.1614683654714200E-01	3.886E-16	3.886E-16	3.886E-16
0.20	1.00	-7.3955238990384900E-01	-7.3792853594724800E-01	-7.3752675952916700E-01	-7.3739371554124500E-01	2.159E-03	5.348E-04	1.330E-04
0.40	1.00	-9.4562453438777300E-01	-9.4304932585640300E-01	-9.4242679829891000E-01	-9.4222234066865800E-01	3.402E-03	8.270E-04	2.045E-04
0.60	1.00	-1.0020644190698100E+00	-9.9920516437436600E-01	-9.9851981688980500E-01	-9.9829477579475300E-01	3.770E-03	9.104E-04	2.250E-04
0.80	1.00	-9.0001177399612600E-01	-8.9756205731549400E-01	-8.9695822461673900E-01	-8.9675841633414700E-01	3.253E-03	8.036E-04	1.998E-04
1.00	1.00	-6.5570987906965600E-01	-6.5416094319321600E-01	-6.5377281568897500E-01	-6.5364362086361200E-01	2.066E-03	5.173E-04	1.292E-04
1.20	1.00	-3.0733286997841900E-01	-3.0733286997841900E-01	-3.0733286997841900E-01	-3.0733286997841900E-01	3.331E-16	3.331E-16	3.331E-16
0.00	1.20	-7.3739399999999900E-01	-7.3739399999999900E-01	-7.3739399999999900E-01	-7.3739371554124500E-01	2.845E-07	2.845E-07	2.845E-07
0.20	1.20	-9.4222234066865800E-01	-9.4222234066865800E-01	-9.4222234066865800E-01	-9.4222234066865800E-01	1.110E-16	1.110E-16	1.110E-16
0.40	1.20	-9.9829477579475300E-01	-9.9829477579475300E-01	-9.9829477579475300E-01	-9.9829477579475300E-01	1.110E-16	1.110E-16	1.110E-16
0.60	1.20	-8.9675841633414700E-01	-8.9675841633414700E-01	-8.9675841633414700E-01	-8.9675841633414800E-01	9.992E-16	9.992E-16	9.992E-16
0.80	1.20	-6.5364362086361100E-01	-6.5364362086361100E-01	-6.5364362086361100E-01	-6.5364362086361200E-01	8.882E-16	8.882E-16	8.882E-16
1.00	1.20	-3.0733286997841900E-01	-3.0733286997841900E-01	-3.0733286997841900E-01	-3.0733286997841900E-01	3.331E-16	3.331E-16	3.331E-16
1.20	1.20	8.7498999999999900E-02	8.7498999999999900E-02	8.7498999999999900E-02	8.7498983439446400E-02	1.656E-08	1.656E-08	1.656E-08

$\|u^*-u_h\| / \|u^*\| = 2.413\text{E-}03$, $\|u^*-u_{h/2}\| / \|u^*\| = 5.668\text{E-}04$, $\|u^*-u_{h/4}\| / \|u^*\| = 1.385\text{E-}04$

$$\log_2 \frac{\|u^*-u_h\|}{\|u^*-u_{h/2}\|} \approx 2.09 , \qquad \log_2 \frac{\|u^*-u_{h/2}\|}{\|u^*-u_{h/4}\|} \approx 2.03$$

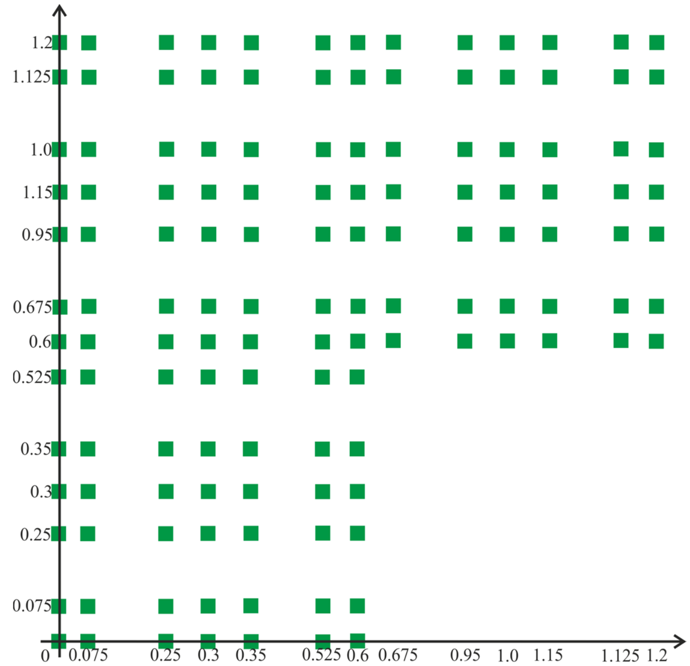
8) Тест на неравномерной сетке

Искомая функция: $u = \cos\left(2x+2y\right)$

Уравнение: $-\operatorname{div}\left(\operatorname{grad} u\right)+u = 9\cos(2x+2y)$

Краевые условия: первого рода на всех ребрах.

$\left\|u^* - u\right\| / \left\|u^*\right\| = 3.462\text{E-}02$



6. Вывод

Исследованный метод хорошо справляется с решением уравнений с полиномиальными решениями степени до 3 включительно. При полиномах более высокого порядка, а также при неполиномиальных решениях результаты получаются несколько хуже. Однако, при достаточной малости шага, метод пригоден и для них.

Исследования на порядок аппроксимации подтвердили теоретические предположения – на равномерных сетках с краевыми условиями первого рода метод имеет второй порядок аппроксимации. Использование неравномерных сеток или краевых условий второго рода может привести к понижению порядка вплоть до первого.

7. Код программы

```
module module_fdm
    implicit none

    type :: area_g
        double precision,allocatable :: mesh_x(:),mesh_y(:)
        integer :: num_x,num_y,num_g_x,num_g_y,num_all
        double precision :: lambda_,gamma_,bound_corner(5)
    contains
        procedure :: input_g
    end type

    type :: slae
        double precision,allocatable :: di(:),du1(:),du2(:),dl1(:),dl2(:),df(:),x(:)
        integer :: shift1,shift2,dshift,n,maxiter=10000
        double precision :: eps=1d-15,def_x=1d0,omega=1.0d0
    contains
        procedure :: calc_gauss_seidel
        procedure,private :: norm
    end type

    type :: fdm
        type(area_g) :: area_g
        type(slae) :: slae
    contains
        procedure :: f
        procedure :: gettypeofb
        procedure :: getvalueofb
        procedure :: getslae
        procedure :: dealloc
        procedure :: write_
    end type

contains

    function f(this,x,y)
        implicit none
        type(fdm) :: this
        double precision :: f,x,y
        !# Test 1,2,3
        !f=2d0*x+2d0*y
        !# Test 4
        !f=2d0*x**2+2d0*y**2-4d0
        !# Test 5
        !f=2d0*x**3+2d0*y**3-6d0*x-6d0*y
        !# Test 6
```

```
    !f=2d0*x**4+2d0*y**4-12d0*x**2-12d0*y**2
    !# Test 7
    f=9d0*cos(2d0*x+2d0*y)
end function

! ----------
! |   6    |
! |        | 5
! | 2   ----
! |     |   4
! |  1  | 3
! -------
function gettypeofb(this,numb)
    implicit none
    type(fdm) :: this
    integer :: numb,gettypeofb
    !# Test 1,3,4,5,6,7
    if(numb.eq.1.or.numb.eq.2.or.numb.eq.3.or.numb.eq.4.or.numb.eq.5.or.numb.eq.6) then
        gettypeofb=1
    end if
    !# Test 2
    !if(numb.eq.1.or.numb.eq.3.or.numb.eq.5.or.numb.eq.6) then
    !    gettypeofb=1
    !else
    !    gettypeofb=2
    !end if
end function

function getvalueofb(this,numb,x,y)
    implicit none
    type(fdm) :: this
    integer :: numb
    double precision :: getvalueofb,x,y
    !# Test 1,3
    !getvalueofb=x+y
    !# Test 2
    !if(numb.eq.1.or.numb.eq.3.or.numb.eq.5.or.numb.eq.6) then
    !    getvalueofb=x+y
    !else if(numb.eq.2) then
    !    getvalueofb=1d0
    !else
    !    getvalueofb=1d0
    !end if
    !# Test 4
    !if(numb.eq.1.or.numb.eq.3.or.numb.eq.5.or.numb.eq.6) then
    !    getvalueofb=x**2+y**2
    !else if(numb.eq.2) then
    !    getvalueofb=2d0*x
    !else
    !    getvalueofb=2d0*y
    !end if
    !# Test 4
    !getvalueofb=x**2+y**2
    !# Test 5
    !getvalueofb=x**3+y**3
    !# Test 6
    !getvalueofb=x**4+y**4
    !# Test 7
    getvalueofb=cos(2d0*x+2d0*y)
end function

subroutine input_g(this)
    implicit none
    type(area_g) :: this
    integer :: i
    open(10,file='../area.txt',status='old')
    read(10,*) this%num_x,this%num_y
    allocate(this%mesh_x(this%num_x))
    allocate(this%mesh_y(this%num_y))
    read(10,*) (this%mesh_x(i), i=1,this%num_x)
    read(10,*) (this%mesh_y(i), i=1,this%num_y)
    read(10,*) this%num_g_x,this%num_g_y
    read(10,*) this%lambda_,this%gamma_
    close(10)
    this%num_all=this%num_g_x*(this%num_g_y-1)+(this%num_y-this%num_g_y+1)*this%num_x
    open(20,file='../bcorner.txt',status='old')
    read(20,*) (this%bound_corner(i), i=1,5)
    close(20)
end subroutine

subroutine getslae(this)
    implicit none
    type(fdm) :: this
    integer :: i,j,t
    double precision :: hx1,hx2,hy1,hy2
    this%slae%n=this%area_g%num_all
    allocate(this%slae%di(this%slae%n))
    allocate(this%slae%du1(this%slae%n))
    allocate(this%slae%du2(this%slae%n))
    allocate(this%slae%dl1(this%slae%n))
    allocate(this%slae%dl2(this%slae%n))
    allocate(this%slae%df(this%slae%n))
    this%slae%di=0d0
    this%slae%du1=0d0
    this%slae%du2=0d0
    this%slae%dl1=0d0
```

```
this%slae%dl2=0d0
this%slae%df=0d0
this%slae%shift1=this%area_g%num_g_x
this%slae%shift2=this%area_g%num_x

! нижняя плоскость "Г"
this%slae%di(1)=1d0
this%slae%df(1)=this%area_g%bound_corner(1)
hy1=dabs(this%area_g%mesh_y(1)-this%area_g%mesh_y(2))
do i=2,this%area_g%num_g_x-1
    this%slae%df(i)=this%getvalueofb(1,this%area_g%mesh_x(i),this%area_g%mesh_y(1))
    if(this%gettypeofb(1).eq.2) then
        this%slae%di(i)=-this%area_g%lambda_/hy1
        this%slae%du2(i)=this%area_g%lambda_/hy1
    else
        this%slae%di(i)=1d0
    end if
end do
t=this%area_g%num_g_x
this%slae%di(t)=1d0
this%slae%df(t)=this%area_g%bound_corner(2)
t=t+1

! ножка "Г"
do i=2,this%area_g%num_g_y-1
    hy1=dabs(this%area_g%mesh_y(i)-this%area_g%mesh_y(i-1))
    hy2=dabs(this%area_g%mesh_y(i+1)-this%area_g%mesh_y(i))
    this%slae%df(t)=this%getvalueofb(2,this%area_g%mesh_x(1),this%area_g%mesh_y(i))
    if(this%gettypeofb(2).eq.2) then
        hx1=dabs(this%area_g%mesh_x(1)-this%area_g%mesh_x(2))
        this%slae%di(t)=-this%area_g%lambda_/hx1
        this%slae%du1(t)=this%area_g%lambda_/hx1
    else
        this%slae%di(t)=1d0
    end if
    t=t+1

    do j=2,this%area_g%num_g_x-1
        hx1=dabs(this%area_g%mesh_x(j)-this%area_g%mesh_x(j-1))
        hx2=dabs(this%area_g%mesh_x(j+1)-this%area_g%mesh_x(j))
        this%slae%df(t)=this%f(this%area_g%mesh_x(j),this%area_g%mesh_y(i))
        this%slae%dl1(t-1)=-2d0*this%area_g%lambda_/(hx1*(hx2+hx1))
        this%slae%dl2(t-this%slae%shift1)=-2d0*this%area_g%lambda_/(hy1*(hy2+hy1))
        this%slae%du1(t)=-2d0*this%area_g%lambda_/(hx2*(hx2+hx1))
        this%slae%du2(t)=-2d0*this%area_g%lambda_/(hy2*(hy2+hy1))
        this%slae%di(t)=(2d0/(hx1*hx2)+2d0/(hy1*hy2))*this%area_g%lambda_+this%area_g%gamma_
        t=t+1
    end do

    this%slae%df(t)=this%getvalueofb(3,this%area_g%mesh_x(this%area_g%num_g_x),this%area_g%mesh_y(i))
    if(this%gettypeofb(3).eq.2) then
        hx1=dabs(this%area_g%mesh_x(this%area_g%num_g_x)-this%area_g%mesh_x(this%area_g%num_g_x-1))
        this%slae%di(t)=this%area_g%lambda_/hx1
        this%slae%dl1(t-1)=-this%area_g%lambda_/hx1
    else
        this%slae%di(t)=1d0
    end if
    t=t+1
end do

this%slae%dshift=t

! Между ножкой и шляпкой
hy1=dabs(this%area_g%mesh_y(this%area_g%num_g_y)-this%area_g%mesh_y(this%area_g%num_g_y-1))
hy2=dabs(this%area_g%mesh_y(this%area_g%num_g_y+1)-this%area_g%mesh_y(this%area_g%num_g_y))
this%slae%df(t)=this%getvalueofb(2,this%area_g%mesh_x(1),this%area_g%mesh_y(this%area_g%num_g_y))
if(this%gettypeofb(2).eq.2) then
    hx1=dabs(this%area_g%mesh_x(1)-this%area_g%mesh_x(2))
    this%slae%di(t)=-this%area_g%lambda_/hx1
    this%slae%du1(t)=this%area_g%lambda_/hx1
else
    this%slae%di(t)=1d0
end if
t=t+1

do j=2,this%area_g%num_g_x
    hx1=dabs(this%area_g%mesh_x(j)-this%area_g%mesh_x(j-1))
    hx2=dabs(this%area_g%mesh_x(j+1)-this%area_g%mesh_x(j))
    this%slae%df(t)=this%f(this%area_g%mesh_x(j),this%area_g%mesh_y(this%area_g%num_g_y))
    this%slae%dl1(t-1)=-2d0*this%area_g%lambda_/(hx1*(hx2+hx1))
    this%slae%dl2(t-this%slae%shift1)=-2d0*this%area_g%lambda_/(hy1*(hy2+hy1))
    this%slae%du1(t)=-2d0*this%area_g%lambda_/(hx2*(hx2+hx1))
    this%slae%du2(t)=-2d0*this%area_g%lambda_/(hy2*(hy2+hy1))
    this%slae%di(t)=(2d0/(hx1*hx2)+2d0/(hy1*hy2))*this%area_g%lambda_+this%area_g%gamma_
    t=t+1
end do

do j=this%area_g%num_g_x+1,this%area_g%num_x-1
    this%slae%df(t)=this%getvalueofb(4,this%area_g%mesh_x(j),this%area_g%mesh_y(this%area_g%num_g_x))
    if(this%gettypeofb(4).eq.2) then
        this%slae%di(t)=-this%area_g%lambda_/hy2
        this%slae%du2(t)=this%area_g%lambda_/hy2
    else
        this%slae%di(t)=1d0
    end if
    t=t+1
```

```fortran
    end do

    this%slae%df(t)=this%area_g%bound_corner(3)
    this%slae%di(t)=1d0
    t=t+1

    ! шляпка "Г"
    do i=this%area_g%num_g_y+1,this%area_g%num_y-1
        hy1=dabs(this%area_g%mesh_y(i)-this%area_g%mesh_y(i-1))
        hy2=dabs(this%area_g%mesh_y(i+1)-this%area_g%mesh_y(i))
        this%slae%df(t)=this%getvalueofb(2,this%area_g%mesh_x(1),this%area_g%mesh_y(i))
        if(this%gettypeofb(2).eq.2) then
            hx1=dabs(this%area_g%mesh_x(1)-this%area_g%mesh_x(2))
            this%slae%di(t)=-this%area_g%lambda_/hx1
            this%slae%du1(t)=this%area_g%lambda_/hx1
        else
            this%slae%di(t)=1d0
        end if
        t=t+1

        do j=2,this%area_g%num_x-1
            hx1=dabs(this%area_g%mesh_x(j)-this%area_g%mesh_x(j-1))
            hx2=dabs(this%area_g%mesh_x(j+1)-this%area_g%mesh_x(j))
            this%slae%df(t)=this%f(this%area_g%mesh_x(j),this%area_g%mesh_y(i))
            this%slae%dl1(t-1)=-2d0*this%area_g%lambda_/(hx1*(hx2+hx1))
            this%slae%dl2(t-this%slae%shift1)=-2d0*this%area_g%lambda_/(hy1*(hy2+hy1))
            this%slae%du1(t)=-2d0*this%area_g%lambda_/(hx2*(hx2+hx1))
            this%slae%du2(t)=-2d0*this%area_g%lambda_/(hy2*(hy2+hy1))
            this%slae%di(t)=(2d0/(hx1*hx2)+2d0/(hy1*hy2))*this%area_g%lambda_+this%area_g%gamma_
            t=t+1
        end do

        this%slae%df(t)=this%getvalueofb(5,this%area_g%mesh_x(this%area_g%num_x),this%area_g%mesh_y(i))
        if(this%gettypeofb(5).eq.2) then
            hx1=dabs(this%area_g%mesh_x(this%area_g%num_x)-this%area_g%mesh_x(this%area_g%num_x-1))
            this%slae%di(t)=this%area_g%lambda_/hx1
            this%slae%dl1(t-1)=-this%area_g%lambda_/hx1
        else
            this%slae%di(t)=1d0
        end if
        t=t+1
    end do

    ! верхняя граница шляпки "Г"
    this%slae%df(t)=this%area_g%bound_corner(4)
    this%slae%di(t)=1d0
    t=t+1
    hy1=dabs(this%area_g%mesh_y(this%area_g%num_y)-this%area_g%mesh_y(this%area_g%num_y-1))
    do i=2,this%area_g%num_x-1
        this%slae%df(t)=this%getvalueofb(6,this%area_g%mesh_x(i),this%area_g%mesh_y(this%area_g%num_y))
        if(this%gettypeofb(6).eq.2) then
            this%slae%di(t)=this%area_g%lambda_/hy1
            this%slae%dl2(t-this%slae%shift1)=-this%area_g%lambda_/hy1
        else
            this%slae%di(t)=1d0
        end if
        t=t+1
    end do
    this%slae%di(t)=1d0
    this%slae%df(t)=this%area_g%bound_corner(5)
end subroutine

function norm(this,x)
    implicit none
    type(slae) :: this
    double precision :: x(*),norm
    integer :: i
    norm=0d0
    do i=1,this%n
        norm=norm+x(i)**2
    end do
end function

subroutine calc_gauss_seidel(this)
    implicit none
    type(slae) :: this
    integer :: i,iter
    double precision :: sum_,residual,res_fax,res_f
    allocate(this%x(this%n))
    this%x=this%def_x
    res_f=this%norm(this%df)
    do iter=1,this%maxiter
        res_fax=0d0
        do i=1,this%n
            sum_=this%di(i)*this%x(i)
            if(i.le.this%n-1) then
                sum_=sum_+this%du1(i)*this%x(i+1)
            end if
            if(i.ge.2) then
                sum_=sum_+this%dl1(i-1)*this%x(i-1)
            end if
            if(i.le.this%dshift) then
                sum_=sum_+this%du2(i)*this%x(this%shift1+i)
            else if(i.le.this%n-this%shift2) then
                sum_=sum_+this%du2(i)*this%x(this%shift2+i)
            end if
```

```
                if(i.ge.this%shift1+this%dshift+1) then
                    sum_=sum_+this%dl2(i-this%shift1)*this%x(i-this%shift2)
                else if(i.ge.1+this%shift1) then
                    sum_=sum_+this%dl2(i-this%shift1)*this%x(i-this%shift1)
                end if
                res_fax=res_fax+(this%df(i)-sum_)**2
                this%x(i)=this%x(i)+this%omega/this%di(i)*(this%df(i)-sum_)
            end do
            residual=dsqrt(res_fax/res_f)
            if(mod(iter,10).eq.0) write(*,fmt='( a5 i5 a7 e9.2)') 'Iter=',iter,' Resid=',residual
            if(residual.le.this%eps) goto 100
        end do
100     continue
        write(*,fmt='( a5 i5 a7 e9.2)') 'Iter=',iter,' Resid=',residual
    end subroutine

    subroutine dealloc(this)
        implicit none
        type(fdm) :: this
        deallocate(this%area_g%mesh_x)
        deallocate(this%area_g%mesh_y)
        deallocate(this%slae%di)
        deallocate(this%slae%du1)
        deallocate(this%slae%du2)
        deallocate(this%slae%dl1)
        deallocate(this%slae%dl2)
        deallocate(this%slae%df)
        deallocate(this%slae%x)
    end subroutine

    subroutine write_(this)
        implicit none
        type(fdm) :: this
        integer :: i,j,k=1
        open(30,file='../output.txt',status='unknown')
        do i=1,this%area_g%num_g_y-1
            do j=1,this%area_g%num_g_x
                write(30,fmt='( 3e27.16 )') this%area_g%mesh_x(j),this%area_g%mesh_y(i),this%slae%x(k)
                k=k+1
            end do
        end do
        do i=this%area_g%num_g_y,this%area_g%num_y
            do j=1,this%area_g%num_x
                write(30,fmt='( 3e27.16 )') this%area_g%mesh_x(j),this%area_g%mesh_y(i),this%slae%x(k)
                k=k+1
            end do
        end do
        close(30)
    end subroutine

end module

program prog_main
    use module_fdm
    implicit none
    type(fdm) :: a
    integer :: i,j,l

    call a%area_g%input_g()
    call a%getslae()
    call a%slae%calc_gauss_seidel()
    call a%write_()

    l=a%slae%n-a%area_g%num_x+1
    do i=a%area_g%num_y,a%area_g%num_g_y,-1
        do j=1,a%area_g%num_x
            write(*,fmt='( f8.3 $)') a%slae%x(l)
            l=l+1
        end do
        print*,''
        l=l-2*a%area_g%num_x
    end do
    l=l+a%slae%shift1-1
    do i=a%area_g%num_g_y-1,1,-1
        do j=1,a%area_g%num_g_x
            write(*,fmt='( f8.3 $)') a%slae%x(l)
            l=l+1
        end do
        print*,''
        l=l-2*a%area_g%num_g_x
    end do

    call a%dealloc()
end program
```