CHAPTER 14

Indexing

Practice Exercises

14.1 Indices speed query processing, but it is usually a bad idea to create indices on every attribute, and every combination of attributes, that are potential search keys. Explain why.

Answer:
Reasons for not keeping indices on every attribute include:

- Every index requires additional CPU time and disk I/O overhead during inserts and deletions.
- Indices on non-primary keys might have to be changed on updates, although an index on the primary key might not (this is because updates typically do not modify the primary-key attributes).
- Each extra index requires additional storage space.
- For queries which involve conditions on several search keys, efficiency might not be bad even if only some of the keys have indices on them. Therefore, database performance is improved less by adding indices when many indices already exist.

14.2 Is it possible in general to have two clustering indices on the same relation for different search keys? Explain your answer.

Answer:
In general, it is not possible to have two primary indices on the same relation for different keys because the tuples in a relation would have to be stored in different order to have the same values stored together. We could accomplish this by storing the relation twice and duplicating all values, but for a centralized system, this is not efficient.

14.3 Construct a B$^+$-tree for the following set of key values:

(2, 3, 5, 7, 11, 17, 19, 23, 29, 31)

Assume that the tree is initially empty and values are added in ascending order. Construct B$^+$-trees for the cases where the number of pointers that will fit in one node is as follows:

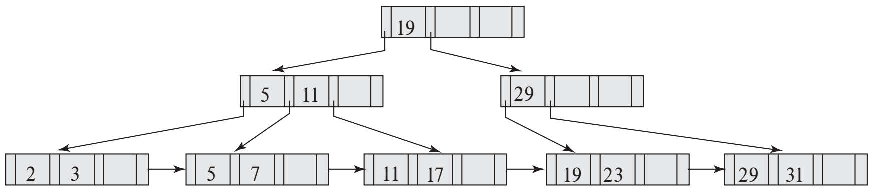
a. Four

b. Six

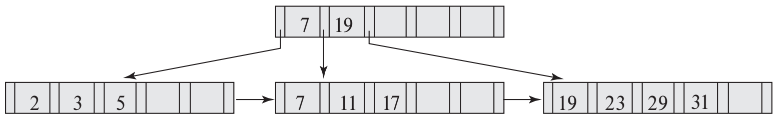
c. Eight

Answer:
The following were generated by inserting values into the B$^+$-tree in ascending order. A node (other than the root) was never allowed to have fewer than $\lceil n/2 \rceil$ values/pointers.

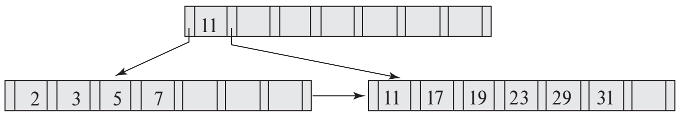
a.



b.



c.



14.4 For each B$^+$-tree of Exercise 14.3, show the form of the tree after each of the following series of operations:

a. Insert 9.

b. Insert 10.

c. Insert 8.

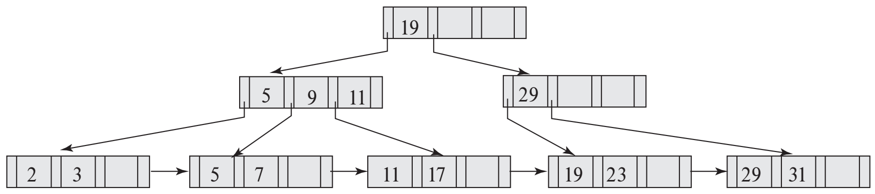
d. Delete 23.

e. Delete 19.

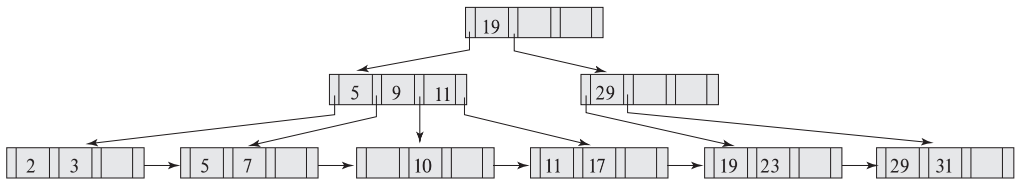
Answer:

- With structure Exercise 14.3.a:

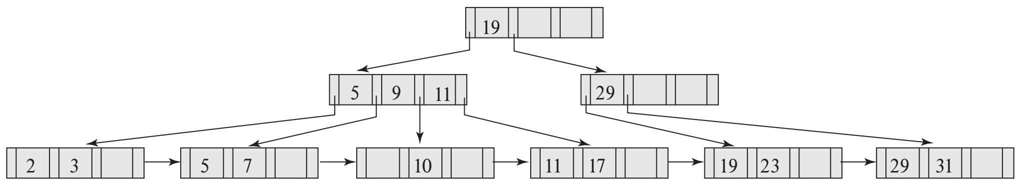
  Insert 9:



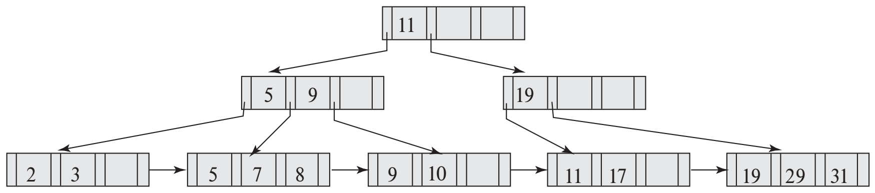
Insert 10:



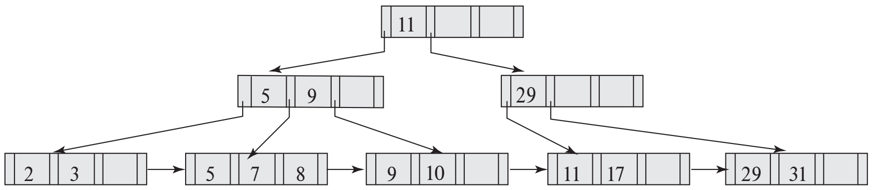
Insert 8:



Delete 23:

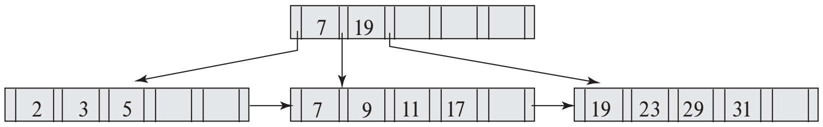


Delete 19:

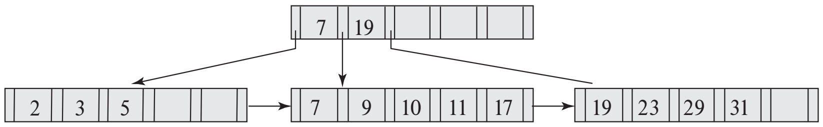


- With structure Exercise 14.3.b:

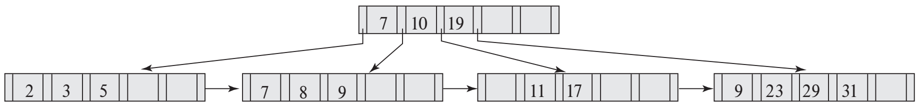
Insert 9:



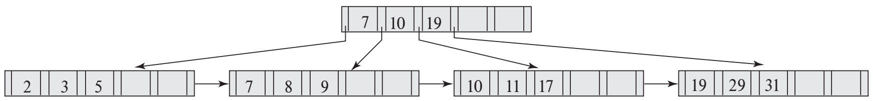
Insert 10:



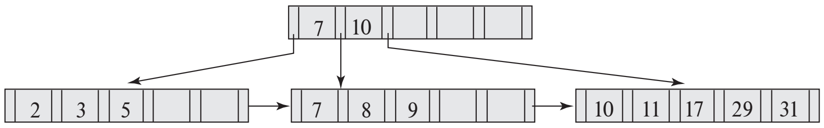
Insert 8:



Delete 23:

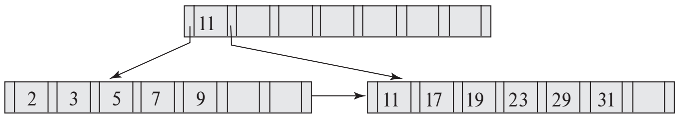


Delete 19:

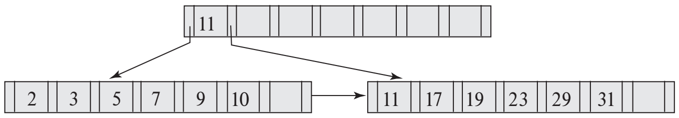


- With structure Exercise 14.3.c:

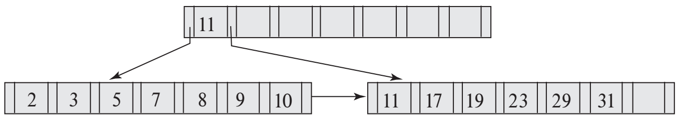
  Insert 9:



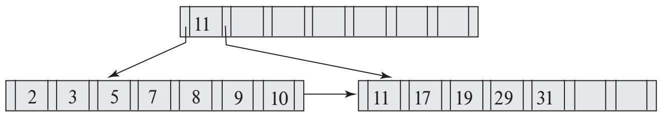
  Insert 10:



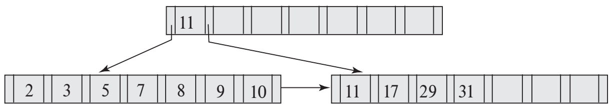
  Insert 8:



  Delete 23:



  Delete 19:



14.5 Consider the modified redistribution scheme for B$^+$-trees described on page 651. What is the expected height of the tree as a function of n?

Answer:

If there are K search-key values and $m - 1$ siblings are involved in the redistribution, the expected height of the tree is: $log_{\lfloor (m-1)n/m \rfloor}(K)$

14.6 Give pseudocode for a B$^+$-tree function findRangeIterator(), which is like the function findRange(), except that it returns an iterator object, as described in Section 14.3.2. Also give pseudocode for the iterator class, including the variables in the iterator object, and the next() method.

Answer:

FILL IN

14.7 What would the occupancy of each leaf node of a B^+-tree be if index entries were inserted in sorted order? Explain why.

Answer:

If the index entries are inserted in ascending order, the new entries get directed to the last leaf node. When this leaf node gets filled, it is split into two. Of the two nodes generated by the split, the left node is left untouched and the insertions take place on the right node. This makes the occupancy of the leaf nodes about 50 percent except for the last leaf.

If keys that are inserted are sorted in descending order, the above situation would still occur, but symmetrically, with the right node of a split never getting touched again, and occupancy would again be 50 percent for all nodes other than the first leaf.

14.8 Suppose you have a relation r with n_r tuples on which a secondary B^+-tree is to be constructed.

a. Give a formula for the cost of building the B^+-tree index by inserting one record at a time. Assume each block will hold an average of f entries and that all levels of the tree above the leaf are in memory.

b. Assuming a random disk access takes 10 milliseconds, what is the cost of index construction on a relation with 10 million records?

c. Write pseudocode for bottom-up construction of a B^+-tree, which was outlined in Section 14.4.4. You can assume that a function to efficiently sort a large file is available.

Answer:

a. The cost to locate the page number of the required leaf page for an insertion is negligible since the non-leaf nodes are in memory. On the leaf level it takes one random disk access to read and one random disk access to update it along with the cost to write one page. Insertions which lead to splitting of leaf nodes require an additional page write. Hence to build a B^+-tree with n_r entries it takes a maximum of $2 * n_r$ random disk accesses and $n_r + 2 * (n_r/f)$ page writes. The second part of the cost comes from the fact that in the worst case each leaf is half filled, so the number of splits that occur is twice n_r/f.

The above formula ignores the cost of writing non-leaf nodes, since we assume they are in memory, but in reality they would also be written eventually. This cost is closely approximated by $2 * (n_r/f)/f$, which is the number of internal nodes just above the leaf; we can add further terms to account for higher levels of nodes, but these are much smaller than the number of leaves and can be ignored.

b. Substituting the values in the above formula and neglecting the cost for page writes, it takes about $10,000,000 * 20$ milliseconds, or 56 hours, since each insertion costs 20 milliseconds.

c.

```
function insert_in_leaf(value $K$, pointer $P$)
    if(tree is empty) create an empty leaf node $L$, which is also the root
    else Find the last leaf node in the leaf nodes chain $L$
    if ($L$ has less than $n - 1$ key values)
        then insert ($K$,$P$) at the first available location in $L$
    else begin
        Create leaf node $L1$
        Set $L.P_n = L1$;
        Set $K1$ = last value from page $L$
        insert_in_parent(1, $L$, $K1$, $L1$)
        insert ($K$,$P$) at the first location in $L1$
    end


function insert_in_parent(level $l$, pointer $P$, value $K$, pointer $P1$)
    if (level $l$ is empty) then begin
        Create an empty non-leaf node $N$, which is also the root
        insert($P$, $K$, $P1$) at the starting of the node $N$
        return
    else begin
        Find the right most node $N$ at level $l$
        if ($N$ has less than $n$ pointers)
            then insert($K$, $P1$) at the first available location in $N$
        else begin
            Create a new non-leaf page $N1$
            insert ($P1$) at the starting of the node $N$
            insert_in_parent($l + 1$, pointer $N$, value $K$, pointer $N1$)
        end
    end
```

The insert_in_leaf function is called for each of the value, pointer pairs in ascending order. Similar function can also be built for descending order. The search for the last leaf or non-leaf node at any level can be avoided by storing the current last page details in an array.

The last node in each level might be less than half filled. To make this index structure meet the requirements of a B$^+$-tree, we can redistribute the keys of the last two pages at each level. Since the last but one node is always full, redistribution makes sure that both of them are at least half filled.

14.9 The leaf nodes of a B$^+$-tree file organization may lose sequentiality after a sequence of inserts.

a. Explain why sequentiality may be lost.

b. To minimize the number of seeks in a sequential scan, many databases allocate leaf pages in extents of n blocks, for some reasonably large n. When the first leaf of a B$^+$-tree is allocated, only one block of an n-block unit is used, and the remaining pages are free. If a page splits, and its n-block unit has a free page, that space is used for the new page. If the n-block unit is full, another n-block unit is allocated, and the first $n/2$ leaf pages are placed in one n-block unit and the remaining one in the second n-block unit. For simplicity, assume that there are no delete operations.

   i. What is the worst-case occupancy of allocated space, assuming no delete operations, after the first n-block unit is full?

   ii. Is it possible that leaf nodes allocated to an n-node block unit are not consecutive, that is, is it possible that two leaf nodes are allocated to one n-node block, but another leaf node in between the two is allocated to a different n-node block?

   iii. Under the reasonable assumption that buffer space is sufficient to store an n-page block, how many seeks would be required for a leaf-level scan of the B$^+$-tree, in the worst case? Compare this number with the worst case if leaf pages are allocated a block at a time.

   iv. The technique of redistributing values to siblings to improve space utilization is likely to be more efficient when used with the preceding allocation scheme for leaf blocks. Explain why.

Answer:

a. In a B$^+$-tree index or file organization, leaf nodes that are adjacent to each other in the tree may be located at different places on disk. When a file organization is newly created on a set of records, it is possible to allocate blocks that are mostly contiguous on disk to leafs nodes that are contiguous in the tree. As insertions and deletions occur on the tree, sequentiality is increasingly lost, and sequential access has to wait for disk seeks increasingly often.

b. i. In the worst case, each n-block unit and each node of the B$^+$-tree is half filled. This gives the worst-case occupancy as 25 percent.

   ii. No. While splitting the n-block unit, the first $n/2$ leaf pages are placed in one n-block unit and the remaining pages in the second n-block unit. That is, every n-block split maintains the order. Hence, the nodes in the n-block units are consecutive.

iii. In the regular B$^+$-tree construction, the leaf pages might not be sequential and hence in the worst-case, it takes one seek per leaf page. Using the block at a time method, for each n-node block, we will have at least $n/2$ leaf nodes in it. Each n-node block can be read using one seek. Hence the worst-case seeks come down by a factor of $n/2$.

iv. Allowing redistribution among the nodes of the same block does not require additional seeks, whereas in regular B$^+$-trees we require as many seeks as the number of leaf pages involved in the redistribution. This makes redistribution for leaf blocks efficient with this scheme. Also, the worst-case occupancy comes back to nearly 50 percent. (Splitting of leaf nodes is preferred when the participating leaf nodes are nearly full. Hence nearly 50 percent instead of exact 50 percent)

14.10 Suppose you are given a database schema and some queries that are executed frequently. How would you use the above information to decide what indices to create?

Answer:
Indices on any attributes on which there are selection conditions; if there are only a few distinct values for that attribute, a bitmap index may be created, otherwise a normal B$^+$-tree index.

B$^+$-tree indices on primary-key and foreign-key attributes.

Also indices on attributes that are involved in join conditions in the queries.

14.11 In write-optimized trees such as the LSM tree or the stepped-merge index, entries in one level are merged into the next level only when the level is full. Suggest how this policy can be changed to improve read performance during periods when there are many reads but no updates.

Answer:
If there have been no updates in a while, but there are a lot of index look ups on an index, then entries at one level, say i, can be merged into the next level, even if the level is not full. The benefit is that reads would then not have to look up indices at level i, reducing the cost of reads.

14.12 What trade offs do buffer trees pose as compared to LSM trees?

Answer:
The idea of buffer trees can be used with any tree-structured index to reduce the cost of inserts and updates, including spatial indices. In contrast, LSM trees can only be used with linearly ordered data that are amenable to merging. On the other hand, buffer trees require more random I/O to perform insert operations as compared to (all variants of) LSM trees.

Write-optimized indices can significantly reduce the cost of inserts, and to a lesser extent, of updates, as compared to B$^+$-trees. On the other hand, the index lookup cost can be significantly higher for write-optimized indices as compared to B$^+$-trees.

14.13 Consider the *instructor* relation shown in Figure 14.1.

a. Construct a bitmap index on the attribute *salary*, dividing *salary* values into four ranges: below 50,000, 50,000 to below 60,000, 60,000 to below 70,000, and 70,000 and above.

b. Consider a query that requests all instructors in the Finance department with salary of 80,000 or more. Outline the steps in answering the query, and show the final and intermediate bitmaps constructed to answer the query.

Answer:
We reproduce the instructor relation below.

ID	*name*	*dept_name*	*salary*
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

a. Bitmap for *salary*, with S_1, S_2, S_3 and S_4 representing the given intervals in the same order

S_1	0	0	1	0	0	0	0	0	0	0	0	0
S_2	0	0	0	0	0	0	0	0	0	0	0	0
S_3	1	0	0	0	1	0	0	1	0	0	0	0
S_4	0	1	0	1	0	1	1	0	1	1	1	1

b. The question is a bit trivial if there is no bitmap on the *dept_name* attribute. The bitmap for the *dept_name* attribute is:

Comp. Sci	1	0	0	0	0	0	1	0	0	0	1	0
Finance	0	1	0	0	0	0	0	0	1	0	0	0
Music	0	0	1	0	0	0	0	0	0	0	0	0
Physics	0	0	0	1	0	1	0	0	0	0	0	0
History	0	0	0	0	1	0	0	1	0	0	0	0
Biology	0	0	0	0	0	0	0	0	0	1	0	0
Elec. Eng.	0	0	0	0	0	0	0	0	0	0	0	1

To find all instructors in the Finance department with salary of 80000 or more, we first find the intersection of the Finance department bitmap and S_4 bitmap of *salary* and then scan on these records for salary of 80000 or more.
Intersection of Finance department bitmap and S_4 bitmap of *salary*.

S_4	0	1	0	1	0	1	1	0	1	1	1	1
Finance	0	1	0	0	0	0	0	0	1	0	0	0
$S_4 \cap$ Finance	0	1	0	0	0	0	0	0	1	0	0	0

Scan on these records with salary 80000 or more gives Wu and Singh as the instructors who satisfy the given query.

14.14 Suppose you have a relation containing the x, y coordinates and names of restaurants. Suppose also that the only queries that will be asked are of the following form: The query specifies a point and asks if there is a restaurant exactly at that point. Which type of index would be preferable, R-tree or B-tree? Why?

Answer:
FILL IN

14.15 Suppose you have a spatial database that supports region queries with circular regions, but not nearest-neighbor queries. Describe an algorithm to find the nearest neighbor by making use of multiple region queries.

Answer:
Start with regions with very small radius, and retry with a larger radius if a particular region does not contain any result. For example, each time the radius could be increased by a factor of (say) 1.5. The benefit is that since we do not use a very large radius compared to the minimum radius required, there will (hopefully!) not be too many points in the circular range query result.