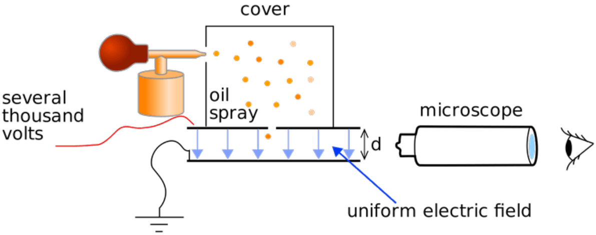
General Physics II

Homework #1

Due 2021/09/29

P1-1. In Milliken's experimental apparatus, oil droplets with different charge q are subject to gravitational force, buoyancy force, and drag force $F_d = 6\pi r\eta v_1$, where r is the droplet radius, η is the viscosity of air and v_1 is the terminal velocity of the droplet. When a uniform electric field E is turned on, the droplet is moving up due to the additional electric force $F_q = qE$ with a terminal speed v_2.



(a) Assume the droplet is spherical. Show that the radius of the droplet is

$$r = \sqrt{\frac{9\eta v_1}{2g(\rho - \rho_{\mathrm{air}})}},$$

where the density of the oil droplet is ρ and the density of air is ρ_{air}. g is the gravitational acceleration.

(b) Calculate the charge of the droplet. Milliken repeated this measurement for a large number of observed droplets and found the charge to be integer multiples of a single number, the fundamental electric charge. Therefore, the experiment confirmed that charge is quantized.

P1-2. Show that the components of $\vec{E}$ due to a dipole are given, at a distant point P in the xz plane, by

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{3pxz}{(x^2 + z^2)^{5/2}}$$

and

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{p(2z^2 - x^2)}{(x^2 + z^2)^{5/2}},$$

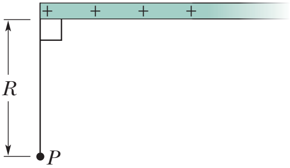
where x and z are coordinates of point P.

P1-3. Suppose N electrons can be placed in either of two configurations. In configuration 1, they are all placed on the circumference of a narrow ring of radius R and are uniformly distributed so that the distance between adjacent electrons is the same everywhere. In configuration 2, $N - 1$ electrons are uniformly distributed on the ring and one electron is placed in the center of the ring.

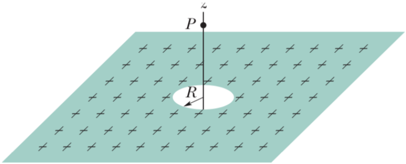
(a) What is the smallest value of N for which the second configuration is less energetic than the first?

(b) For that value of N, consider any one circumference electron—call it e_0. How many other circumference electrons are closer to e_0 than the central electron is?

P1-4. A semi-infinite nonconducting rod has uniform linear charge density λ. Show that the electric field $\vec{E}_p$ at point P makes an angle of 45° with the rod and that this result is independent of the distance R.



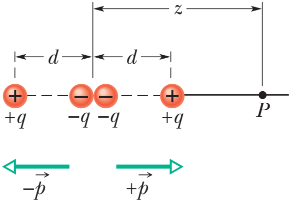
P1-5. A small circular hole of radius $R = 1.80$ cm has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density $\sigma = 4.50\ \text{pC/m}^2$. A z axis, with its origin at the hole's center, is perpendicular to the surface. In unit vector notation, what is the electric field at point P at $z = 2.56$ cm?



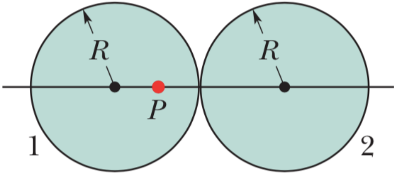
P1-6. A type of electric quadrupole consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Show that the value of E on the axis of the quadrupole for a point P at a distance z from its center (assume $z \gg d$) is given by

$$E = \frac{3Q}{4\pi\epsilon_0 z^4},$$

in which $Q = 2qd^2$ is known as the *quadrupole moment* of the charge distribution.



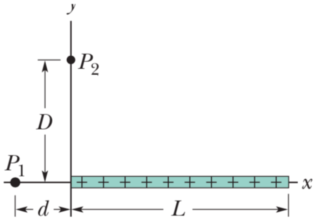
P1-7. In cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance $R/2.00$ from the center of sphere 1. If the net electric field at point P is zero, what is the ratio q_2/q_1 of the total charges?



P1-8. A thin plastic rod of length $L = 10.0$ cm has a nonuniform linear charge density $\lambda = cx$, where $c = 49.9$ pC/m^2.

(a) With $V = 0$ at infinity, find the electric potential at point P_2 on the y axis at $y = D = 3.56$ cm.

(b) Find the electric field component E_y at P_2.



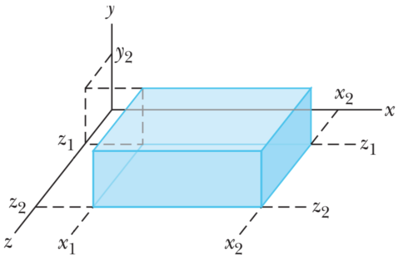
(c) Why cannot the field component E_x at P_2 be found using the result of (a)?

P1-9. An electron is constrained to the central axis of the ring of charge of radius R, with $z \ll R$. Show that the electrostatic force on the electron can cause it to oscillate through the ring center with an angular frequency

$$\omega = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}},$$

where q is the ring's charge and m is the electron's mass.

P1-10. A box-like Gaussian surface encloses a net charge of $+24.0\varepsilon_0$ C and lies in an electric field given by $\vec{E} = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}]$ N/C, with x and z in meters and b a constant. The bottom face is in the xz plane; the top face is in the horizontal plane passing through $y_2 = 1.00$ m. For $x_1 = 1.00$ m, $x_2 = 4.00$ m, $z_1 = 1.00$ m, $z_2 = 3.00$ m, what is b?



P1-A1.* (You can try if you are interested.) Solve the electro-static potential function $V(x, y, z)$ and the electrostatic field $\mathbf{E}(x, y, z)$ in space, with a positive charge $+Q$ located at $(-D, 0, 0)$ and a conducting sphere of radius R (not grounded) at origin ($R<D$).