Final - Math 53, Fall 2025

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YOUR NAME IS _

YOUR TA'S NAME IS _

YOUR SECTION NUMBER IS _ _ _ _ _ _ _ _ _ _

- There are ten problems. Each is worth ten points, for a total of 100 points.
- Hunt and choose. Look through the exam, find the easy ones, and do them!
- Do all work on the Exam.
- If you need more space, continue on to the back (and say there's more there).
- An answer with no explanation (except for the true/false questions) will receive no credit.
- Write legibly.
- Stay Calm: Show us what you know.
- **PUT A BOX AROUND YOUR ANSWERS!!!!!**

Question 1

Find the point (or points) on the surface $xyz = 1$ such that the tangent plane is parallel to the surface $2x + 2y + 2z = 0$.

Question 1b

Find the extreme (maximal and minimal) values of the function $2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

Question 2a

Let C be the boundary of the box whose four corners are at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$.

Evaluate

$$\int_C \left[(17y - e^{\sin x})\, dx + (18x + \sqrt{y^4 + 1})\, dy \right]$$

where the curve is integrated in the counterclockwise direction.

Question 2b

Let S be the parabolic cap $z = -(x^2 + y^2) + 9$ above the xy plane.

Let $\vec{F}$ be the vector field $\left(-\frac{y^3}{3}, \frac{x^3}{3}, 0 \right)$.

Let $\vec{G} = \nabla \times \vec{F}$.

Find the flux of $\vec{G}$ through the surface S in the outwards pointing normal direction.

Question 3

Consider the vector field $\vec{F} = (x/3, y/3, z/3)$. Find the flux of $\vec{F}$ through the closed surface formed by the xy plane and the surface $1 = x^2 + y^2 + z^2$, where $z \geq 0$.

Question 4

Find the extreme (maximal and minimal) values of the function $x^2 + y^2 + z^2$ such that $x^4 + y^4 + z^4 = 1$. Be sure to give the input points for these extreme values.

Question 5

What is the worst movie you have ever seen? (Yes, this is the free question)

Question 6

Let $\vec{F} = (x^2 + y^2, xz + yz + xy, y^2 + z^2)$. Let S be the open surface given by $y = (1 - (x^2 + z^2))$, where $y \geq 0$. Find the flux of curl of $\vec{F}$ through this open surface, that is, find:

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

Question 7

Let $x \neq 0$, and f be differentiable. Show that the tangent planes to the surface $z = xf\left(\frac{y}{x}\right)$ all pass through the origin.

Question 8

Let $f(x,y,z) = \left(\frac{x^2}{2} - \frac{x^4}{12} - \frac{y^4}{12} - \frac{z^4}{12}\right)$.

Let S be a closed, non-intersecting surface in $\mathbb{R}^3$ with a clearly defined inside and outside.

Let $A(S)$ be the outward normal flux of ∇f through all of S , that is,

$$A(S) = \iint_S \nabla f \cdot \vec{n}$$

Among all such surfaces, what is the volume of the one that makes $A(S)$ as big as possible?

Question 9

Evaluate the integral:

$$\int_0^1 \int_0^1 e^{\max(x^2, y^2)} \, dy \, dx$$

where $\max(x^2, y^2)$ eans take the bigger of x^2 nd y^2 .

Question 10

Evaluate the integral:

$$\iint_R e^{(x+y)/(x-y)}\, dA$$

where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,-2)$, $(0,-1)$. Hint: Use the transformation:

$$u = x + y \quad v = x - y$$