Final Project Assignment

June 3, 2020

1 Problem Solving & Statistics Final Project Assignment

1.1 Sejong University, 3-2, Computer Science

1.1.1 Professor : Shake Md Riazul Islam

1.2 Title

1.2.1 Student Title

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1.2.2 Project Title

Demonstrate the Central Limit Theorem (CLT) in Python.

1.2.3 Objective

- Objective: Reflecting the knowledge of sampling distribution using Python programming.
- Report Submission Deadline: June 19, 2020 (11.30 PM, Friday).

1.3 Topics

A gray-scale image is a two-dimensional array of numbers, each of which represents the corresponding pixel intensity. You can obtain this array of numbers (i.e. image read) using various python packages. Consider the supplied “lena_gray.gif” gray-scale image as the population. Based on the population, you need to implement the following tasks:

- Task1
- Task2
- Task3
- Task4

1.3.1 Ready to solve the problem

- import libraries
- load images
- tiny EDA

[293]:
```python
# import libs
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
```

[294]:
```python
# load image
im_nparray = plt.imread("lena_gray.gif")

# Checking
print(type(im_nparray))
print(im_nparray.shape)

# remove 1,2,3 channel because it has no meanings
im_nparray = im_nparray[:,:,0]
print(im_nparray.shape)
plt.figure(figsize = [8,8])
plt.imshow(im_nparray, gray)
```

```
<class numpy.ndarray>
(512, 512, 4)
(512, 512)
```

[294]:
```
<matplotlib.image.AxesImage at 0x20554105828>
```

[295]:
```
flattened_im_nparray = im_nparray.reshape(-1)
print(flattened_im_nparray.shape)
```

```
(262144,)
```

1.4 Task1

Request : Find out the population size , population mean (), population variance (^2), population range, minimum number, maximum number, population mode, and population median.

We already know that : - The image of 512 by 512 (1 channel) lena will contain a total of 512x512 intensity values. - The maximum value for each intensity value is 2^8-1 and the minimum is 0.

[296]:
```
print(flattened_im_nparray.shape)
population_size_n = flattened_im_nparray.shape[0]
```

```
(262144,)
```

[297]:
```
population_mean_m = flattened_im_nparray.mean()
population_variance_v = flattened_im_nparray.var()
population_std_sigma = flattened_im_nparray.std()

print(population mean :, population_mean_m)
print(population variance :, population_variance_v)
print(population std :, population_std_sigma, (likewise {})\n.
 ↪format(population_variance_v**(1/2)))

population_minval = flattened_im_nparray.min()
population_median = np.median(flattened_im_nparray)
population_maxval = flattened_im_nparray.max()
population_range = (population_minval, population_maxval) # use later

print(population min :, population_minval)
print(population median :, population_median)
print(population max :, population_maxval)
print(population range :, population_minval,~, population_maxval)
```

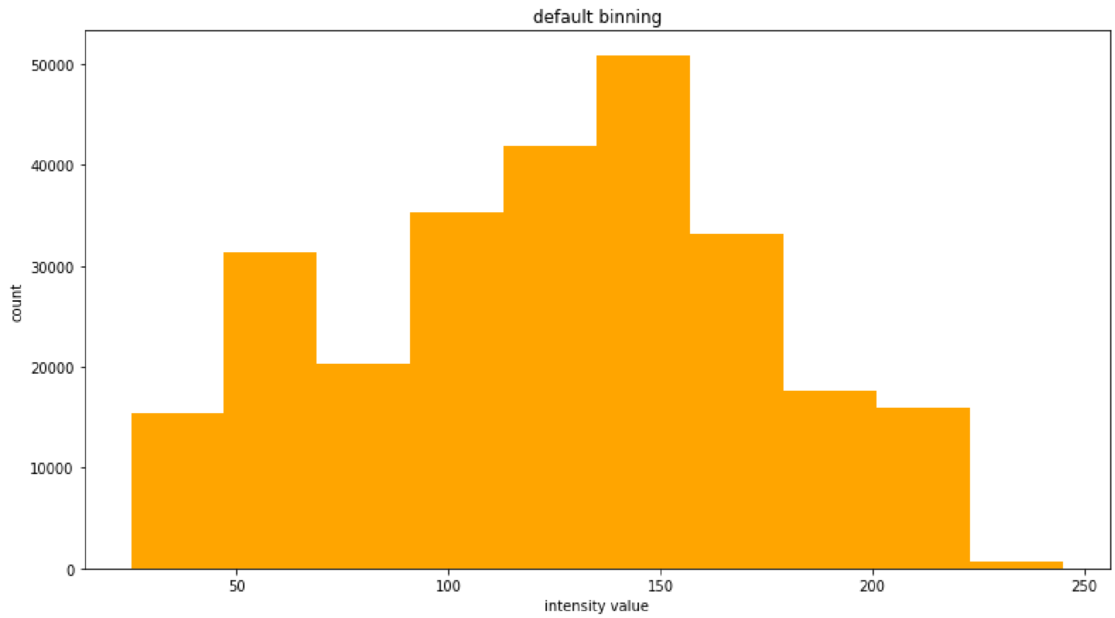
```
population mean : 124.05046081542969
population variance : 2289.9760151074734
population std : 47.853693850187504 (likewise 47.853693850187504)

population min : 25
population median : 129.0
population max : 245
population range : 25 ~ 245
```

1.5 Task2

Request : Find out the histogram of the population. Comment on the population distribution.

[298]:
```
# lena entire image intensity values histogram

plt.figure(figsize = [13,7])

plt.title(default binning)
plt.xlabel(intensity value)
plt.ylabel(count)
plt.hist(np.sort(flattened_im_nparray), color = orange)
```

[298]:
```
(array([15363., 31302., 20304., 35230., 41809., 50786., 33252., 17583.,
         15887.,   628.]),
 array([ 25.,  47.,  69.,  91., 113., 135., 157., 179., 201., 223., 245.]),
 <a list of 10 Patch objects>)
```



It’s not a bell-shaped distribution

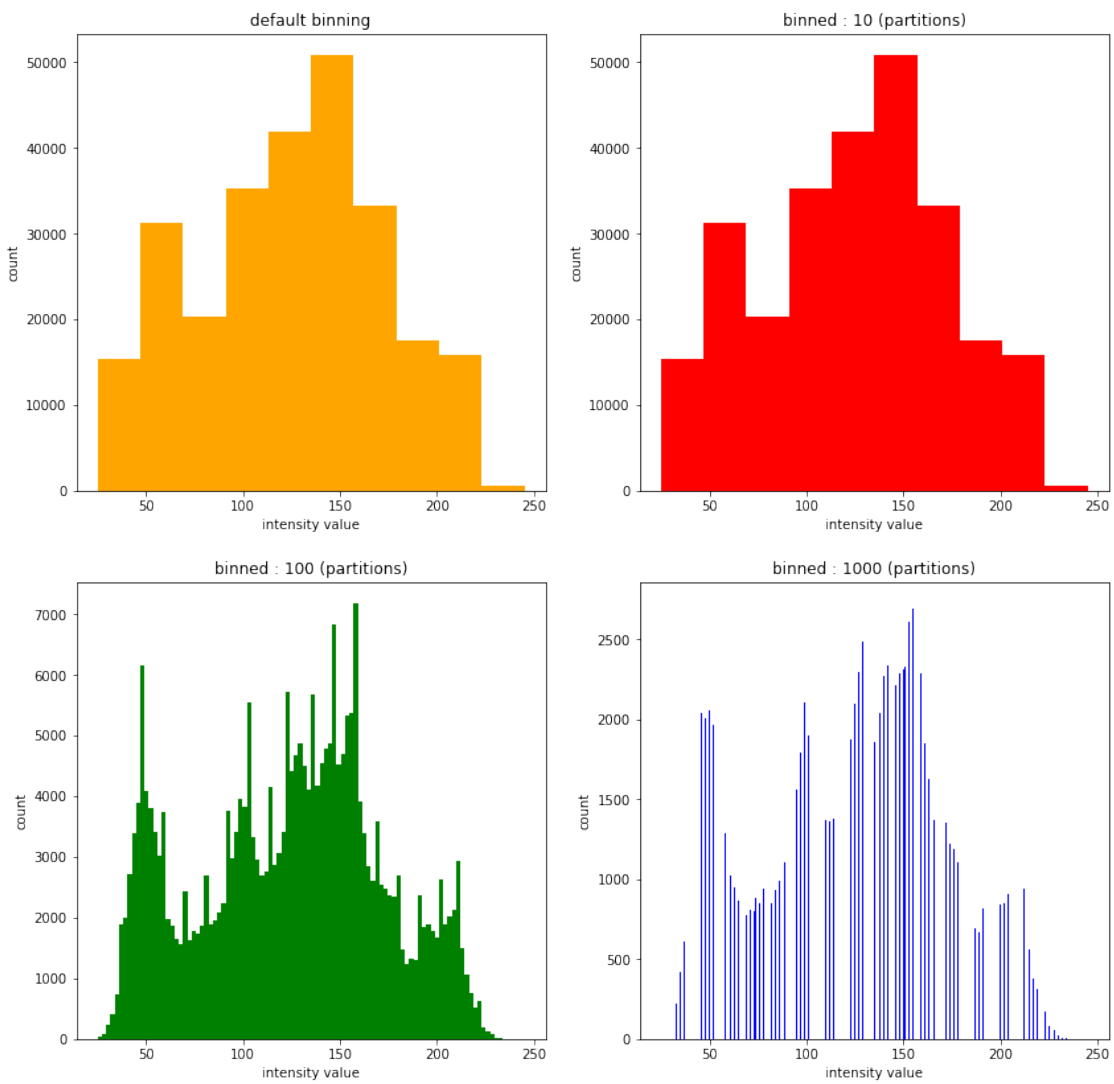
1.6 Task3

Request : Investigate the histogram by changing the number of bins to 10, 100, and 1,000. Provide your observations

[299]:
```
plt.figure(figsize = [14,14])

plt.subplot(2,2,1)
plt.title(default binning)
plt.xlabel(intensity value)
plt.ylabel(count)
plt.hist(np.sort(flattened_im_nparray), color = orange)

plt.subplot(2,2,2)
plt.title(binned : 10 (partitions))
plt.xlabel(intensity value)
plt.ylabel(count)
plt.hist(np.sort(flattened_im_nparray), color = red, bins = 10)


plt.subplot(2,2,3)
plt.title(binned : 100 (partitions))
plt.xlabel(intensity value)
plt.ylabel(count)
plt.hist(np.sort(flattened_im_nparray), color = green, bins = 100)
```

```
plt.subplot(2,2,4)
plt.title(binned : 1000 (partitions))
plt.xlabel(intensity value)
plt.ylabel(count)
plt.hist(np.sort(flattened_im_nparray), color = blue, bins = 1000)

print(end!)
```

```
end!
```



Let's see these graph avobe. As you look into more and more fine sections, you can see that some shape of probability distribution changes. It certainly shows that population-distribution is not a normal distribution.

1.7 Task4

Demonstrate the central limit theorem (i.e., the distribution of the sampling mean will approach towards the normal distribution with the mean and variance 2/ as the sample size increases). Recommended sample sizes are 5, 10, 20, 30, 50, 100. In addition to any content that you think appropriate for this demonstration, you will include various graphical representations such as the respective histogram for each sample size.

1.7.1 What is Central Limit Problem?

In probability theory the central limit theorem (CLT) establishes that in some situations : - when independent random variables are added (X1, X2, X3, … Xn) from X - their (X1, X2, X3, … Xn) properly normalized sum tends toward a normal distribution (informally a bell curve) - even if the original variables themselves are not normally distributed.

1.7.2 Why Central Limit Problem is Important?

CLT is significant because : - The results hold regardless of what shape the original population distribution was which makes it important for statistical inference. - The more data that's gathered, the more accurate the statistical inferences become, meaning more certainty in estimates.

[300]:
```
unlimit = 10
```

[301]:
```
# np.random.choice selects random number from list
# define function
def Xbar(sampling_iterations_with_sample_size_from_population, sample_size) :
    X_bar = []
    for i in range(sampling_iterations_with_sample_size_from_population):
        X1 = np.random.choice(flattened_im_nparray, sample_size)
        X1_mean = X1.mean()
        X_bar.append(X1_mean)
    return X_bar
```

[302]:
```
bucket = {}

# sampling : 1 time
# sample sizes : 5
bucket[1] = Xbar(1, 5)

# sampling : 5 times
# sample sizes : 5
bucket[5] = Xbar(5, 5)

# sampling : 50 times
# sample sizes : 5
bucket[20] = Xbar(50, 5)

# sampling : 250 times
# sample sizes : 5
bucket[250] = Xbar(250, 5)
```

```
# sampling : 1000 times
# sample sizes : 5
bucket[1000] = Xbar(1000, 5)

# sampling : 5000 times
# sample sizes : 5
bucket[5000] = Xbar(5000, 5)

# sampling : 25000 times
# sample sizes : 5
bucket[25000] = Xbar(25000, 5)

# sampling : 100000 times
# sample sizes : 5
bucket[100000] = Xbar(100000, 5)

for i in bucket:
    print(len(bucket[i]))
```

```
1
5
50
250
1000
5000
25000
100000
```

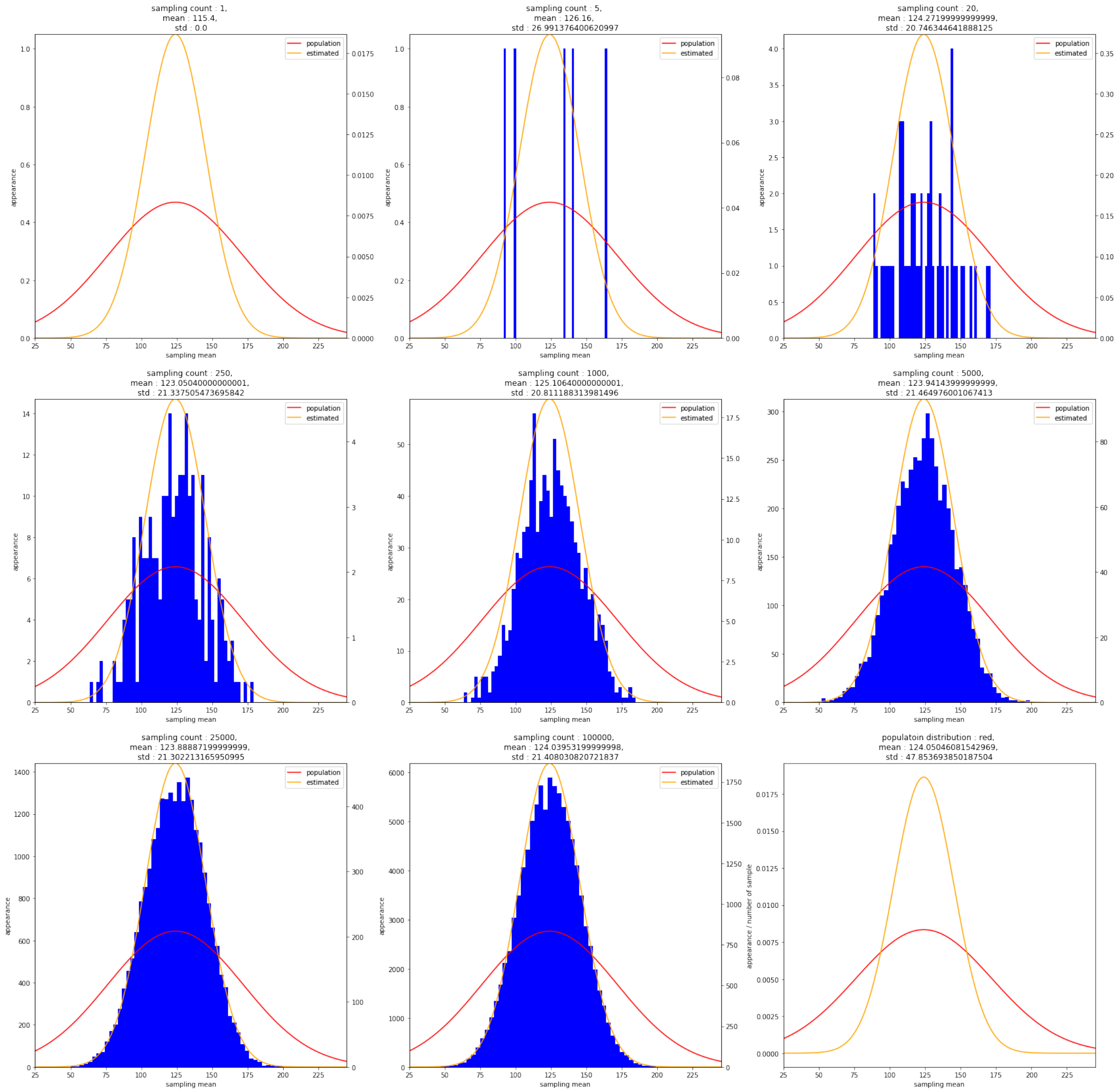
[303]:

```
plt.figure(figsize = [29,29])
domain = np.linspace(0,255,256)

cnt = 1
for i in bucket :
    ax1 = plt.subplot((len(bucket)//3)+1, 3,cnt)
    plt.title(sampling count : {}, \nmean : {}, \nstd : {}.format(i, np.
 ↪array(bucket[i]).mean(),np.array(bucket[i]).std()))
    plt.xlabel(sampling mean)
    plt.ylabel(appearance)
    plt.xlim(population_range)
    plt.hist(np.sort(bucket[i]), color = blue, bins = 50)
    ax2 = ax1.twinx()

    ax2.plot(domain, np.asarray(norm.pdf(domain, population_mean_m,␣
 ↪population_std_sigma))*i,
                  color = red, label="population")
```

```
    ax2.plot(domain, np.asarray(norm.pdf(domain, population_mean_m,␣
 ↪population_std_sigma / 5**(1/2)))*i,
                   color = orange, label="estimated")
    ax2.legend()
    ax2.margins(0)
    cnt+=1


plt.subplot((len(bucket)//3)+1, 3,cnt)
plt.title(populatoin distribution : red, \nmean : {}, \nstd : {}.
 ↪format(population_mean_m, population_std_sigma))
plt.xlabel(sampling mean)
plt.ylabel(appearance / number of sample)
plt.xlim(population_range)
plt.plot(domain, norm.pdf(domain, population_mean_m, population_std_sigma),␣
 ↪color = red, label="population")
plt.plot(domain, norm.pdf(domain, population_mean_m, population_std_sigma /␣
 ↪5**(1/2)), color = orange, label="estimated")
```

[303]:

```
[<matplotlib.lines.Line2D at 0x20555f74978>]
```



As the figure shown, when sampling count n (number of sample X bar, X1 X2 X3 … Xn) is increase :

- distribution of X bar is going to bell-shape
- mean value of X bar is going to population mean
- standard deviation of X bar is going to population standard variation / root n

CLT Demonstrated

1.7.3 Bonus Track

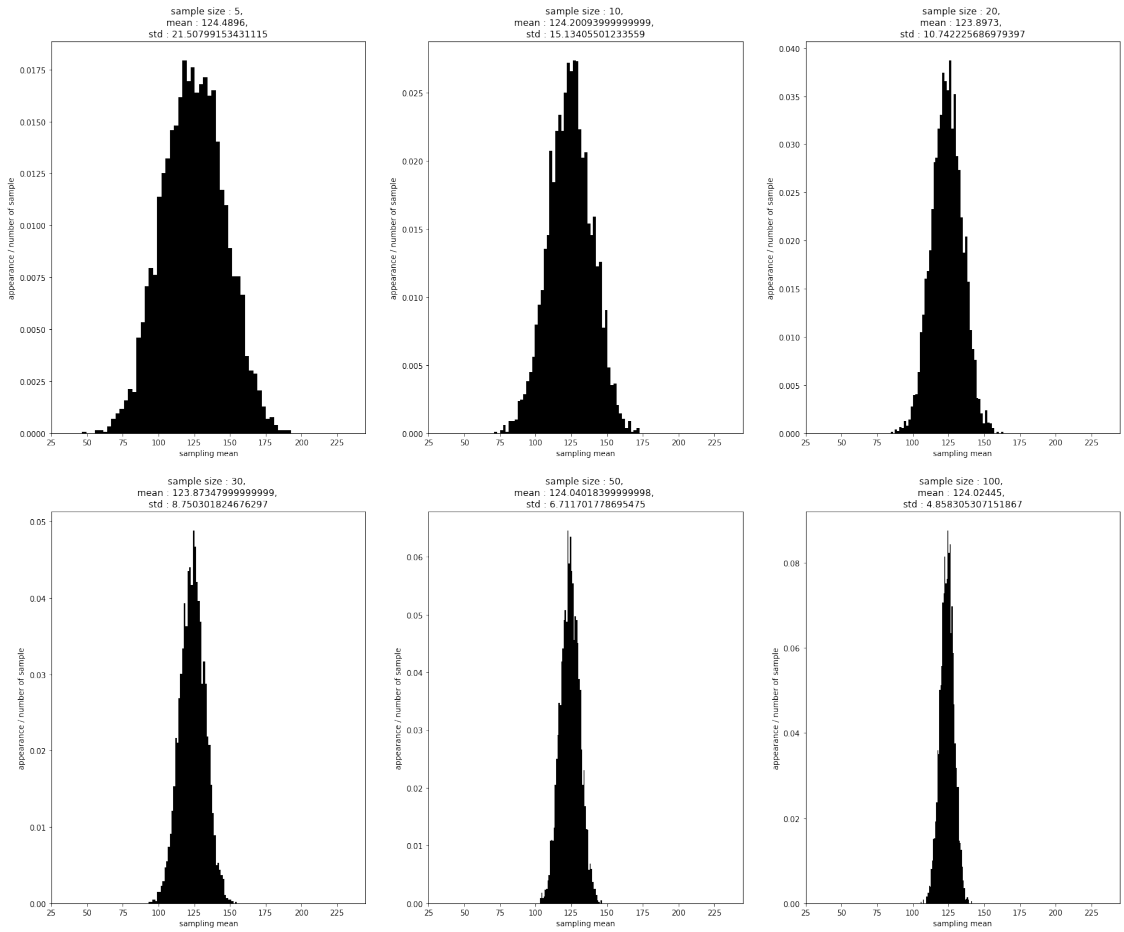
[304]:
```
bucket2 = {}

# sampling : 5000
# sample sizes : 5
```

```
bucket2[5] = Xbar(5000, 5)

# sampling : 5000
# sample sizes : 10
bucket2[10] = Xbar(5000, 10)

# sampling : 5000
# sample sizes : 20
bucket2[20] = Xbar(5000, 20)

# sampling : 5000
# sample sizes : 30
bucket2[30] = Xbar(5000, 30)

# sampling : 5000
# sample sizes : 50
bucket2[50] = Xbar(5000, 50)

# sampling : 5000
# sample sizes : 100
bucket2[100] = Xbar(5000, 100)

for i in bucket2:
    print(len(bucket2[i]))
```

```
5000
5000
5000
5000
5000
5000
```

[305]:

```
plt.figure(figsize = [25,32])

cnt = 1
for i in bucket2 :
    plt.subplot(len(bucket2)//2, 3, cnt)
    plt.title(sample size : {}, \nmean : {}, \nstd : {}.format(i, np.
 ↪array(bucket2[i]).mean(), np.array(bucket2[i]).std()))
    plt.xlabel(sampling mean)
    plt.ylabel(appearance / number of sample)
    plt.xlim(population_range)
    plt.hist(np.sort(bucket2[i]), color = black, bins = 50, density=True)
    cnt+=1
```


sample size : 5,
mean : 124.4896,
std : 21.50799153431115
sample size : 10,
mean : 124.20093999999999,
std : 15.13405501233559
sample size : 20,
mean : 123.8973,
std : 10.742225686979397
sample size : 30,
mean : 123.87347999999999,
std : 8.750301824676297
sample size : 50,
mean : 124.04018399999998,
std : 6.711701778695475
sample size : 100,
mean : 124.02445,
std : 4.858305307151867
appearance / number of sample
sampling mean