

Varuna Specification

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1 Introduction

Marlin-style proof systems are preprocessing zkSNARKs making use of holography. Some notable features of the construction include: 1. the structured reference string (SRS) is universal and updatable 2. it targets rank-1 constraint satisfiability (R1CS) 3. it has a modular proving pipeline with solid theoretical foundations. In the following, we introduce several improvements to the original Marlin construction [CHM⁺20]. For a high level introduction to Marlin, we recommend [Inb23].

In section 2 we show a glossary of notation. In section 3 we briefly compare our construction to previous work. In sections 4, 5 and 6 we describe the non-interactive protocol. Changes related to batch proving are marked in [blue](#) for clarity. We finish by showing condensed diagrams of the interactive protocol for batch proving. It is similar to the diagrams in [CHM⁺20] (Figure 5 in Section 5 and Figure 7 in Appendix E, in the latest ePrint version), but with some changes: it shows not just the information-theoretic Interactive Oracle Proof (IOP) but also the use of a polynomial commitment scheme (the cryptography layer); and it aims to be fully up-to-date with the recent optimizations in <https://github.com/AleoNet/snarkVM/commit/6d64025f3f775fa164d70d9a8177ec93c97cd36e>. This document can act as a “bridge” between the codebase and the theory that our references describe.

2 Quick Preliminaries

\mathbb{F}	the finite field over which the R1CS instance is defined
x	public input
\hat{x}	polynomial of degree less than $ x $ that agrees with the instance x in $H[\leq x]$
w	secret witness
\bar{w}	shifted witness, defined as $\frac{w(X) - \hat{x}(X)}{v_{H[\leq x]}(X)}$
\mathcal{D}	the set consisting of the frequency of circuits occurring in one batch proof (This is relevant in the context of multi-circuit batch proofs)
\mathcal{M}	the set of all R1CS matrices: $\{M_i\}_{M \in \{A, B, C\}, i \in [\mathcal{D}]}$
R_i	constraint domain for R1CS matrices $\{A_i, B_i, C_i\}$ of a single circuit i (defined as a multiplicative subgroup of \mathbb{F} consisting of roots of unity)
R	$\arg \max_{R_i} R_i $
C_i	variable domain for R1CS matrices $\{A_i, B_i, C_i\}$ of a single circuit i (defined as a multiplicative subgroup of \mathbb{F} consisting of roots of unity)
C	$\arg \max_{C_i} C_i $
K_M	domain for non-zero entries (approx. the number of addition gates) in matrix M
K	$\arg \max_{K_M} K_M $
X	domain sized for input (not including witness)
$v_D(X)$	vanishing polynomial over domain D
$s_{D_1, D_2}(X)$	“selector” polynomial over domains $D_1 \supseteq D_2$, defined as $\frac{ D_2 ^{v_{D_1}}}{ D_1 ^{v_{D_2}}}$ (outputs 0 on all elements in $D_1 \setminus D_2$, but 1 on all elements of D_2)
L_a^S	lagrange polynomial, where $L_a^S(a) = 1$ and $L_a^S(b) = 0$ for all $a, b \in S$
A, B, C	R1CS instance matrices
$\text{row}_M, \text{col}_M$	LDEs of (respectively) row positions, column positions of matrix M^*
rowcol_M	LDE of the element-wise product of row and col , given separately for efficiency
rowcolval_M	LDEs of the element-wise product of rowcol_M and the values of non-zero elements of matrix M^*
b	bound on the number of queries
\mathcal{H}	a stateful, duplex sponge based hash function using the Poseidon permutation (originally proposed in [COS20])
transcript	the concatenation of preprocessing inputs, the public input and proof elements written up to that point in time
d	the supported degree bound of the trusted setup
\mathcal{P}	prover
\mathcal{V}	verifier
\mathcal{V}^p	\mathcal{V} with “oracle” access to polynomial p (via commitments provided by the indexer, later opened as necessary by \mathcal{P})
row	the rowcheck
lin	the lineal sumcheck
mat	the matrix sumcheck
Q	the query set, consisting of the queries to $g_A, g_B, g_C, g_1, \text{row}, \text{lin}, \text{mat}$

3 Comparing Varuna to Marlin

Varuna is a proving system based on Marlin. It adopts almost all of the improvements for the AHP noted in section 9.1 of the Marlin paper. Only the suggestion “Single low-degree extension for each matrix” is not implemented because although it reduces the proof size, it increases the prover time too much. Regarding the polynomial commitment scheme, Varuna adopts all of the improvements noted in section 9.2 of the Marlin paper. We use a slightly adjusted version of the [KZG10]-like scheme with improvements introduced in [MBKM19], [Gab19], [CHM⁺20] and [BCMS20]. More precisely, we use the

equation described in [Gab19] section 3 that avoids negative G1 powers.

We also separate out the rowcheck to more easily support custom gates, add support for rectangular matrices, and replace the lincheck by a lineval - allowing us to evaluate $z_M = z * M$ without materializing z_M as an oracle [Gab20].

After an offline phase, the proof requires five rounds of interaction between a prover and verifier. Besides making the protocol more efficient for single proofs, we also introduce batch-proving.

It is worth pointing out for clarity that snarkVM decomposes variables used in high level program operations into linear combinations before constraining them. This allows for reducing the number of actual variables in the system. Each linear combination corresponds to a partial or full row of an R1CS matrix, and consists of:

- constant and variable terms, representing the usage of the public 1 variable and other variables at 'compile time'.
- value term, representing the value of the assigned linear combination at 'proving time'.

A full R1CS row is "completed" when we introduce a multiplication between non-const linear combinations ($a*b=c$). A simple example using the public 1 variable and 3 new variables which have the assigned value 2:

- $a = \{constant : 1, terms = [var_1 : 1], value : 3\}$
- $b = \{constant : 2, terms = [var_2 : 1], value : 4\}$
- $c = \{constant : 5, terms = [var_3 : 1], value : 12\}$

Leads to a single row of R1CS:

- $A = [1, 1, 0, 0]$
- $B = [2, 0, 1, 0]$
- $C = [5, 0, 0, 1]$

4 Offline phase

The goal of the offline phase is to encode the R1CS linear relationships A , B and C as low degree polynomials. However, calculating $M(X, Y)$ naively by using X and Y as the row and column indices would cost $\Omega(R * C)$ operations. To overcome this issue, we implement section 5.3.1 from the original Marlin paper, with three differences:

1. the indexer also outputs univariate polynomials for $\{\text{rowcol}_M\}_{M \in \{A, B, C\}}$, $\{\text{rowcolval}_M\}_{M \in \{A, B, C\}}$ is used instead of val. These achieve savings for the proving time in Round 3 below.
2. we adopt an optimization from [Gab20]. We do not encode the R1CS matrices using bivariate derivatives, but rather using Lagrange polynomials, which simplifies the online phase of the protocol.
3. The prover will initialize $A' := \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $B' := \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $C' := \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. These random variables will ensure the scheme is zero-knowledge.

For each $M \in \mathcal{M}$, $\text{row}_M(X)$, $\text{col}_M(X)$, $\text{rowcol}_M(X)$ and $\text{rowcolval}_M(X)$ are obtained such that the following polynomials are low-degree extensions of M : $\hat{M}(X, Y) = \sum_{\kappa \in K} \text{val}_M(\kappa) \cdot L_{\text{row}_M(\kappa)}^R(X) \cdot L_{\text{col}_M(\kappa)}^C(Y)$. The polynomials encode the row index, column index and value of the **non-zero** entries of the matrix M^* . Constructing these polynomials for a circuit takes only $\Omega(|K| \log |K|)$ operations.

The preprocessing inputs which are added to the transcript consist of:

- the protocol name "VARUNA-2023"
- the instance batch sizes \mathcal{D}
- the circuit commitments from each circuit's verifying keys

We will also collect and prepare values from the universal setup for our proving and verification keys. In order to create commitments in the context of multi-circuit batch proofs, we take the union of the proving key's committer key.

5 Prover algorithm

In the following sections, we use indices i and j to refer to the circuits and instances proven in one batch.

Round 1

The prover:

1. initializes $A' := \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $B' := \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $C' := \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $z' := (x, w')$, and $w' := [w, \rho_A, \rho_B, \rho_C]$ where $\rho_A, \rho_B, \rho_C \xleftarrow{\$} \mathbb{F}$ satisfy $\rho_A \cdot \rho_B - \rho_C = 0$. These random variables ensure the scheme is zero-knowledge.
2. uses the full assignments $z_{i,j} = (x_{i,j}, w_{i,j})$ to compute the linear combinations $z_{A,i,j} := A_i z_{i,j}$, $z_{B,i,j} := B_i z_{i,j}$ and $z_{C,i,j} := C_i z_{i,j}$ and compute polynomials $\hat{z}_{M,i,j}(X) \in \mathbb{F}^{|z_{M,i,j}|+b}[X]$ which agrees with $\bar{z}_{M,i,j}$ on R_i and is random elsewhere.
3. **commits** to $\{[w_{i,j}]\}$, $[m]$:
 - $\hat{w}_{i,j}(X) \in \mathbb{F}^{|w_{i,j}|+b}[X]$ which agrees with $\bar{w}_{i,j}$ on $R_i[\geq |x_{i,j}|]$ and is random elsewhere
 - $m(X) \in \mathbb{F}^{<2|C|+2b-2}[X]$ such that $\sum_{\kappa \in C} m(\kappa) = 0$. This is also needed to make the scheme zero knowledge. It is sufficient for the masking polynomial to be sampled with respect to the biggest C , as it serves to hide queries to g_1 and h_1 .

The prover computes:

- batch combiners $\tau_{i,1} = 1$ and $\tau_{i,j} = \mathcal{H}(\mathbf{transcript})[2+k]$ for $i \in [|\mathcal{D}|]$, $j \in [\mathcal{D}_i - 1]$ and $k = 0$ initially but incrementing by 1 for each invocation. These are used to bundle the sumcheck problems into \mathcal{D} sumcheck problem.
- batch combiners $\nu_1 = 1$ and $\nu_i = \mathcal{H}(\mathbf{transcript})[2 + \sum_i \mathcal{D}_i - 1 + k]$ for $k \in [|\mathcal{D}| - 1]$. These are used to bundle the sumcheck problems into 1 sumcheck problem.

Round 2

Next, the prover will run a rowcheck in order to convince the verifier that each $z_{i,j} A_i \cdot z_{i,j} B_i = z_{i,j} C_i$. This is done through a BatchZeroCheck. Note that we can combine checks for different circuits and instances using batch combiners and selector polynomials: $\sum_i \nu_i s_{R,R_i} \sum_j \tau_j (z_{A,i,j} \cdot z_{B,i,j} - z_{C,i,j}) = h_0(X) v_R(X)$.

The prover **commits** to $[h_0]$.

The prover computes:

- challenge $\alpha = \mathcal{H}(\mathbf{transcript})[0]$. This is used to reduce the lincheck problems to sumcheck problems
- randomizers $\eta_A = 1$, $\eta_B = \mathcal{H}(\mathbf{transcript})[1]$ and $\eta_C = \mathcal{H}(\mathbf{transcript})[2]$. These are used to bundle the sumcheck problems into $\sum_i \mathcal{D}_i$ sumcheck problems

The rest of the protocol is for the prover to convince the verifier that $\hat{z}_{M,i,j}$ are obtained as linear combinations from $z_{i,j}$, without revealing $w_{i,j}$. These are essentially $3 * \sum_i \mathcal{D}_i$ lineval problems.

Round 3

The prover runs a batch univariate sumcheck to assert that $m(X) + \sum_i \nu_i s_{C,C_i}(X) \sum_j \tau_j \sum_M \sum_{c \in C} \eta_M \hat{M}_i(\alpha, c) \hat{z}_{i,j}(c) - \sigma/|C| = 0$. Here, $\sigma = \sum_i \nu_i s_{C,C_i}(X) \sum_j \tau_j \sum_M \eta_M \sigma_{M,i,j}$

The prover computes and **commits** to $[g_1(X)]$ and $[h_1(X)]$ and sends $\{\sigma_{M,i,j}\}$ to the verifier.

The prover computes the next challenge $\beta = \mathcal{H}(\mathbf{transcript})[0]$ to evaluate the polynomial identity.

Round 4

What is left, is for the prover to prove that each $\hat{M}(\alpha, \beta)$ evaluates to σ_M . This can be reformulated as new rational sumchecks:

$$\text{for each } M \in \mathcal{M}, \text{ RatSumcheck for } \frac{V_R(\alpha) V_C(\beta) \text{rowcolval}_M(\kappa)}{|R||C|(\alpha - \text{row}_M(\kappa))(\beta - \text{col}_M(\kappa))} = \sigma_M \text{ over } K_M$$

The prover computes and **commits** to $\{[g_M(X)]\}$ and sends $\{[\sigma_M]\}$ to the Verifier adhering to:

$$a_M(X) - b_M(X)(X g_M(X) + \sigma_M/|K_M|) = h_M(X) v_{K_M}(X) \quad (***)$$

The prover then computes randomizers $\delta_1 = 1$, $\delta_{i,j} = \mathcal{H}(\mathbf{transcript})[k]$ for $i \in [|\mathcal{D}|]$, $j \in [\mathcal{D}_i - 1]$ and $k = 0$ initially but incrementing by 1 for each invocation.

Round 5

When doing a batch zero check (to conclude the rational sumchecks), we could multiply all terms $h_M(X)v_{K_M}(X) = a_M(X) - b_M(X)(Xg_M(X) - \sigma_M/|K_M|)$ by an appropriate selector polynomial, and divide by $v_K(X)$. Instead, we perform the following optimization. Substituting in the selector we get that:

$$\frac{h_M(X)v_{K_M}(X)s_{K/K_M}}{v_K(X)} = h_M(X)\frac{|K_M|}{|K|}$$

The prover **commits** to $[h_2(X)]$:

$$\text{let } h_2(X) := \sum_{M \in \mathcal{M}} \delta_M h_M(X) |K_M| / |K|$$

Next, the prover computes a challenge $\gamma = \mathcal{H}(\text{transcript})[0]$.

Completing the proof

The prover uses a polynomial commitment scheme to prove that previous commitments $\{\text{cm}_{g_M}\}$ and cm_{g_1} open to their evaluations at the challenges β , respectively γ , in order to enforce their degree bounds. However, the prover still needs to commit to various terms to prove the rowcheck, lineal sumcheck and matrix sumcheck problems. This can be done in an efficient way by constructing linear combinations, which we call virtual commitments (as outlined in section 9.2 of [CHM⁺20] and more formally introduced in [CBBZ23]). The prover proves that the following virtual commitments evaluate to 0 at the challenges α, β, γ :

- $\text{vcm}_{\text{rowcheck}}$ is a linear combination of cm_{h_0} and sums $\{\sigma_{i,j,M}\}$.
- vcm_{lin} is a linear combination of $\text{cm}_m, \{\text{cm}_{\hat{w}_{i,j}}\}, \text{cm}_{h_1}$ using evaluations $g_1(\beta)$ and sums $\{\sigma_M\}_{M \in \mathcal{M}}$ and $\{\sigma_{i,j,M}\}_{M \in \{A,B,C\}}$ as factors.
- vcm_{mat} is a linear combination of cm_{h_2} and for each $M \in \mathcal{M}$, index commitments to $\text{row}_M, \text{col}_M, \text{rowcol}_M, \text{rowcolval}_M$, using evaluations $\{g_M(\gamma)\}$, and sums $\{\sigma_M\}$ as factors

The prover then runs **PC.Open** to construct a batch opening proof, using i.a. the (virtual) commitments, commitment randomness (to make the opening zero-knowledge), Q and \mathcal{H} . In the process, the prover computes a randomizer for each linear combination of (virtual) commitments checked in Q and a challenge for each element of those linear combinations.

6 Verifier algorithm

Computing challenges, randomizers and batch combiners

The verifier:

1. adds the first round's commitments to the transcript, computes the batch combiners $\{\nu_i\}, \{\tau_{i,j}\}$
2. adds the second round's commitments to the transcript, computes challenges and matrix randomizers $\alpha, \{\eta_M\}_{M \in \{A,B,C\}}$
3. adds the third round's commitments to the transcript, computes the challenge β
4. adds the fourth round's commitments to the transcript, computes randomizers $\{\delta_M\}_{M \in \mathcal{M}}$.
5. adds the fifth round's commitments to the transcript and computes the challenge γ

Completing the verification

After checking degree bounds on the verifying key, the verifier will verify the proof using a polynomial commitment scheme.

The verifier then runs **PC.Verify**, using i.a. the (virtual) commitments, commitment randomness, Q and \mathcal{H} . In the process, the verifier computes a randomizer for each linear combination of (virtual) commitments checked in Q and a challenge for each element of those linear combinations.

Compared to [Gab19], we use inverses to efficiently check equality: $e([-W]_1, [x]_2)e([-v-W^z]_1, [1]_2) \prod_{i \in [4]} e([CM_i]_1, [x^{d_i-d}]_2) == 1$:

- CM_i is a linear combination of the (virtual) commitments which the verifier queries
- W is a linear combination of the proof's commitments to witness polynomials
- v is a linear combination of the random commitment polynomials at Q
- z is a linear combination of Q
- degree bound d_i is:
 - d for $\text{cm}_{\text{mat}}, \text{cm}_{\text{lin}}, \text{cm}_{\text{row}}$.
 - $C - 2$ for cm_{g_1}
 - $K_M - 2$ for cm_{g_M}

7 Analysis

The following was adapted from Marlin [CHM⁺20] section 5.3.3.

7.1 Soundness

For the single circuit case, we argue that the soundness error is at most: $\{\frac{2|R|}{|\mathbb{F}\setminus R|} + \frac{2|C|}{|\mathbb{F}\setminus C|} + \frac{3|K|}{|\mathbb{F}|}\}$

Suppose that a given \hat{w} does not contain an encoding of a valid witness w for a given index. We know that either $\hat{z}_A \cdot \hat{z}_B \neq \hat{z}_C$ or one of σ_M does not actually sum to the correct linear combination of $z \cdot M$.

In the first case, the polynomial identity $\hat{z}_A \cdot \hat{z}_B - \hat{z}_C = h_0(X)v_R(X)$ does not hold, so given Schwartz-Zippel the probability that the verifier still accepts is at most $\frac{2|R|}{|\mathbb{F}\setminus R|}$.

In the second case, we have to account for the soundness errors of the outer sumcheck and inner rational sumchecks, which are again bounded by the maximum degree in the respective polynomial equation divided by the size of the set from which the test point is chosen. The outer sumcheck has soundness error at most $\frac{2|C|}{|\mathbb{F}\setminus C|}$. The inner rational sumchecks have soundness error of at most $\frac{3|K|}{|\mathbb{F}|}$.

Soundness for the multi-circuit sumcheck follows from creating random linear combinations of single-circuit sumchecks, as also noted in section 3.1 of [CBBZ23].

7.2 Zero-Knowledge

Just like the Marlin paper, we only sketch the intuition because a full proof (which includes constructing a simulator) is similar to the non-holographic setting described in [BCR⁺19].

The first message of the prover includes an encoding of the witness, which is protected against up to b queries outside of R because the encodings are b -wise independent over $\mathbb{F} \setminus R$. The second message includes the polynomial $h_0(X)$, which in fact is b -wise independent everywhere on \mathbb{F} , noting any witness-dependent polynomial requires a hiding bound. Subsequent commitments from the prover do not reveal any further information because they are produced for a sumcheck instance that is shifted by a random polynomial (the polynomial $m(X)$).

However, we still have to prevent the summed (randomly evaluated) assigned witness $\sigma_{i,j,M}$ from leaking information. We can be sure of this because during round 1 the prover added 3 random elements to the witness, one for each matrix M .

This leads to (perfect) zero knowledge with query bound b and a query checker C that rejects any query to \hat{w} that lies in R .

7.3 Efficiency

Proof size

- Marlin: $13\mathbb{G}_1 + 8\mathbb{F}$
- Varuna: $9\mathbb{G}_1 + 10\mathbb{F}$
- Varuna batch i circuits and j instances: $(5 + j + 3i)\mathbb{G}_1 + (1+9i)\mathbb{F}$

Prover time Because the inner sumchecks are amortized across all instances, asymptotically, addition gates of additional instances become almost free, nearing the prover time of [Gro16].

Verifier time

- Marlin: 2 pairings + $O(|x| + \log m)$
- Varuna: 2 pairings + $O(|x| + \log m)$
- Varuna batch i circuits and j instances: 2 pairings + $O(\sum_{i,j} |x_{i,j}| + \log R_{max})$

8 Diagram including multi-circuit batching

$$\mathcal{P}(\mathbb{F}, R, C, K, \{A_i\}, \{B_i\}, \{C_i\}, \{x_{i,j}\}, \{w_{i,j}\}) \quad \mathcal{V}^{\{\text{row}_M, \text{col}_M, \text{rowcol}_M, \text{rowcolval}_M\}_{M \in \{A, B, C\}}}(\mathbb{F}, R, C, K, \{x_{i,j}\})$$

$$z_{i,j} := (x_{i,j}, w_{i,j}), z_{M,i,j} := M_i z_{i,j}$$

$$\text{sample } \hat{w}_{i,j}(X) \in \mathbb{F}^{<|w_{i,j}|+b}[X] \text{ and } \hat{z}_{M,i,j}(X) \in \mathbb{F}^{<|R_i|+b}[X]$$

$$\text{sample mask polynomial } m(X) \in \mathbb{F}^{<2|C|+2b-2}[X] \text{ such that } \sum_{\kappa \in C} m(\kappa) = 0$$

$$\begin{array}{c} \xrightarrow{\text{commitments } \{\text{cm}_{\hat{w}_{i,j}}\}, \text{cm}_m} \xrightarrow{\{\nu_i\}, \{\tau_{i,j}\}} \mathbb{F} \\ \xleftarrow{\{\nu_i\}, \{\tau_{i,j}\}} \end{array}$$

BatchZeroCheck for $\sum_i \nu_i s_{R,R_i} \sum_j \tau_j (\hat{z}_{A,i,j} \cdot \hat{z}_{B,i,j} - \hat{z}_{C,i,j}) = 0$ over R

find $h_0(X)$ such that

$$\sum_i \nu_i s_{R,R_i} \sum_j \tau_j (\hat{z}_{A,i,j} \cdot \hat{z}_{B,i,j} - \hat{z}_{C,i,j}) = h_0(X) v_R(X) \quad (*)$$

$$\begin{array}{c} \xrightarrow{\text{commitments } \text{cm}_{h_0}} \xrightarrow{\eta_A, \eta_B, \eta_C, \alpha} \mathbb{F} \\ \xleftarrow{\eta_A, \eta_B, \eta_C, \alpha} \mathbb{F} \setminus R \end{array}$$

BatchUniSumcheck for $m(X) + \sum_i \nu_i s_{C,C_i}(X) \sum_j \tau_j \sum_M \eta_M \hat{M}_i(\alpha, X) \hat{z}_{i,j}(X) - \sigma/|C| = 0$ over C

find $g_1(X) \in \mathbb{F}^{|C|-1}[X]$ and $h_1(X)$ such that

$$m(X) + \sum_i \nu_i s_{C,C_i}(X) \sum_j \tau_j \sum_M \eta_M \hat{M}_i(\alpha, X) \hat{z}_{i,j}(X) = h_1(X) v_C(X) + X g_1(X) + \sigma/|C| \quad (**)$$

$$\begin{array}{c} \xrightarrow{\text{commitments } \text{cm}_{g_1}, \text{cm}_{h_1}, \text{ and claimed sums } \{\sigma_{i,j,M}\}} \xrightarrow{\beta \leftarrow \mathbb{F} \setminus C} \\ \xleftarrow{\beta \in \mathbb{F}} \end{array}$$

for each $M \in \mathcal{M}$, **RatSumcheck** for $\frac{V_R(\alpha) V_C(\beta) \text{rowcolval}_M(\kappa)}{|R||C|(\alpha - \text{row}_M(\kappa))(\beta - \text{col}_M(\kappa))} = \sigma_M$ over K_M

$$\text{let } a_M(X) := v_{R_i}(\alpha) v_{C_i}(\beta) \text{rowcolval}_M(X)$$

$$\text{let } b_M(X) := |R||C|(\beta - \text{col}_M(X))(\alpha - \text{row}_M(X))$$

$$= |R||C|(\alpha\beta - \alpha\text{col}_M(X) - \beta\text{row}_M(X) + \text{rowcol}_M(X)) \text{ (over } K_M)$$

find $g_M(X), h_M(X) \in \mathbb{F}^{|K_M|-1}[X]$ and $\sigma_M \in \mathbb{F}$ such that

$$a_M(X) - b_M(X)(X g_M(X) + \sigma_M/|K_M|) = h_M(X) v_{K_M}(X) \quad (***)$$

$$\xrightarrow{\text{commitments } \{\text{cm}_{g_M}\}, \text{ and claimed sums } \{\sigma_M\}} \xrightarrow{\{\delta_M\} \leftarrow \mathbb{F}}$$

$$\text{let } h_2(X) := \sum_{M \in \mathcal{M}} \delta_M h_M(X) |K_M|/|K|$$

$$\xrightarrow{\text{commitment } \text{cm}_{h_2}} \xrightarrow{\gamma \leftarrow \mathbb{F}}$$

To verify $(***)$, \mathcal{V} will need to check the following:

$$\underbrace{\sum_{M \in \mathcal{M}} \delta_M s_{K,K_M}(\gamma) (a_M(\gamma) - b_M(\gamma)(\gamma g_M(\gamma) + \sigma_M/|K_M|)) - h_2(\gamma) v_K(\gamma)}_{\text{mat}(\gamma)} \stackrel{?}{=} 0$$

Compute $\hat{x}_{i,j}(X) \in \mathbb{F}^{<|x_{i,j}|}[X]$ from input $x_{i,j}$

To verify $(**)$, \mathcal{V} will need to compute $\sigma = \sum_i \nu_i s_{C,C_i}(X) \sum_j \tau_j \sum_M \eta_M \sigma_{i,j,M}$ and check the following:

$$\underbrace{m(\beta) + \sum_{i \in [D]} \nu_i s_{C,C_i}(\beta) \sum_{j \in [D_i]} \tau_{i,j} \sum_{M \in A, B, C} \eta_M \sigma_{i,j,M} (\hat{x}_{i,j}(\beta) + v_{X_{i,j}}(\beta) \hat{w}_{i,j}(\beta)) - h_1(\beta) v_C(\beta) - \beta g_1(\beta) - \sigma/|C|}_{\text{lin}(\beta)} \stackrel{?}{=} 0$$

To verify $(*)$, \mathcal{V} will need to check the following:

$$\underbrace{\sum_i \nu_i s_{R,R_i}(\alpha) \sum_j \tau_j (\sigma_{i,j,A} \cdot \sigma_{i,j,B} - \sigma_{i,j,C}) - h_0(\alpha) v_R(\alpha)}_{\text{rowcheck}(\alpha)} \stackrel{?}{=} 0$$

$$\{v_{g_M} := g_M(\gamma), v_{g_1} := g_1(\beta)\}$$

$$\begin{array}{c} \xrightarrow{\{v_{g_M}\}, v_{g_1}} \xrightarrow{\xi_1, \dots, \xi_{|Q|} \leftarrow F} \\ \xleftarrow{\xi_1, \dots, \xi_{|Q|}} \end{array}$$

use PC.Open with randomness $\xi_1, \dots, \xi_{|Q|}$ to construct a batch opening proof π of the following:

$$(\text{cm}_{g_M}, \text{vcm}_{\text{mat}}) \text{ at } \gamma \text{ evaluate to } (v_{g_M}, 0) \quad (***)$$

$$(\text{cm}_{g_1}, \text{vcm}_{\text{lin}}) \text{ at } \beta \text{ evaluate to } (v_{g_1}, 0) \quad (**)$$

$$(\text{vcm}_{\text{rowcheck}}) \text{ at } \alpha \text{ evaluates to } (0) \quad (*)$$

$$\xrightarrow{\pi}$$

verify π with PC.Verify evaluations $\{v_{g_M}\}, v_{g_1}$ and commitments $\{\text{cm}_{g_M}\}, \text{vcm}_{\text{mat}}, \text{cm}_{g_1}, \text{vcm}_{\text{lin}}, \text{vcm}_{\text{rowcheck}}$

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