p-adic Locally Analytic Representations

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1 Reductive Groups over Fields

2 Locally Analytic Groups

Let L be a p-adic field, and K/L an extension with valuation extended.

An **L-locally analytic group** (of dimension d?) is a topological group G such that:

- it is a **locally L-analytic manifold** admits an atlas consists of charts $G \supset U \to K^d$ where the transition maps given by locally analytic functions (i.e. locally given by power series over L); and
- the multiplication is locally analytic.

A vector space V over a topological field K is called **locally convex**, if it has a fundamental system of open 0-neighbourhoods consisting of $\mathcal{O}_K$-submodules. (Well let's just say this is a kind of good space so that we can talk about locally analytic functions, and normed spaces are locally convex).

Let G be a L-analytic group, V be a locally convex Hausdorff vector space over K. We let $C^{\mathrm{an}}(G, V)$ be the set of locally analytic maps $G \to V$. For a representation $\rho : G \to \mathrm{GL}(V)$, we say that $v \in V$ is a **locally analytic vector**, if

$$g \mapsto \rho(g)v : G \to V$$

is locally analytic.