Chesskell: Modelling a Two-Player Game at the Type-Level

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Why do type systems exist?

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Type systems exist because:

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Type systems exist because: we want to avoid errors.

(Cardelli, "Type systems")

Type Errors

A type system can prevent certain errors from occurring at all:

```
not 5
```

The above will not compile, preventing an error.

Type Errors cont.

You have a website, where you sell books.

Type Errors cont.

You have a website, where you sell books.

For some reason, you use Java to build the server:

```java
int noOfPages = -1;
```

This is obviously an error. But it compiles!

Type-Level Programming

Recent developments to Haskell have focused on performing computation at the type level with *type families* (Schrijvers et al., "Towards open type functions for Haskell", Eisenberg, Vytiniotis, et al., "Closed type families with overlapping equations").

Haskell is NOT a dependently typed language; types and values are separated.

Type Erasure

In fact, Haskell programs undergo *type erasure*.

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```haskell
x :: Int
x = 3
```

Haskell type-level programming involves circumventing type erasure.

Complex Type-Level Computation

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Mezzo - musical composition (Szamozvancev and Gale, "Well-typed music does not sound wrong (experience report)")

BioShake - Bioinformatics workflows (Bedő, "BioShake: a Haskell EDSL for bioinformatics workflows")

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What issues do we run into when implementing a complex rule set at the type level?

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What issues do we run into when implementing a complex rule set at the type level?

Is Haskell's type system mature enough for Chess?

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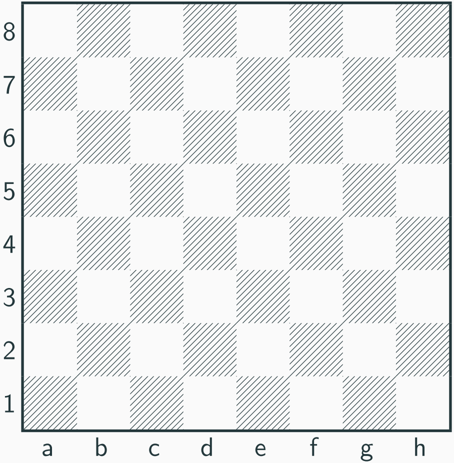
- It has a *well-defined ruleset*.

Why Chess?

- It has a *well-defined ruleset*.

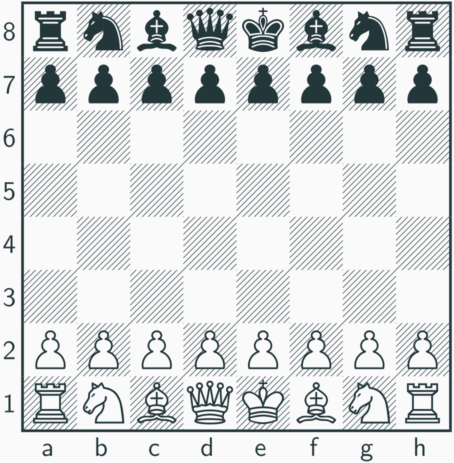
A note on Chess

A Chess game takes place on a board.



A note on Chess cont.

There are two *Teams*; Black and White.



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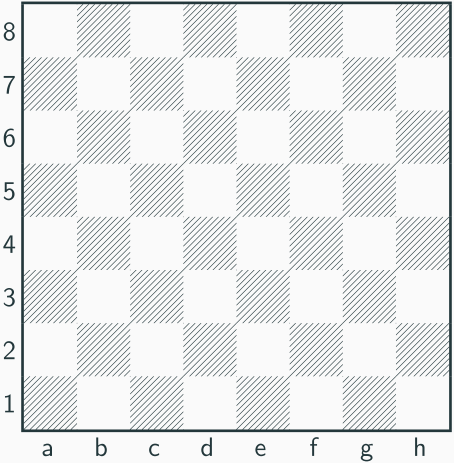
Each Team has 16 *Pieces*; 8 Pawns, 2 Rooks, 2 Bishops, 2 Knights, a Queen, and a King.



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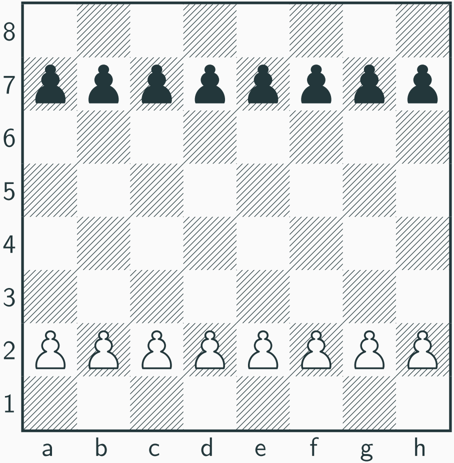

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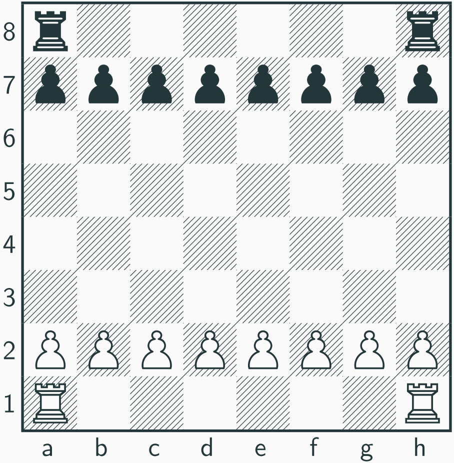

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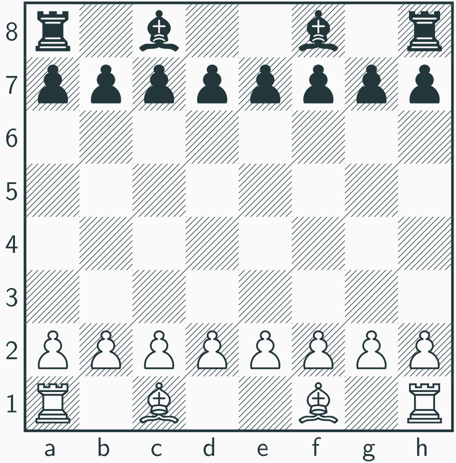

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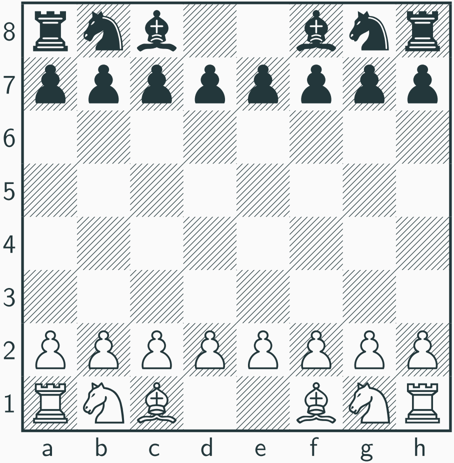

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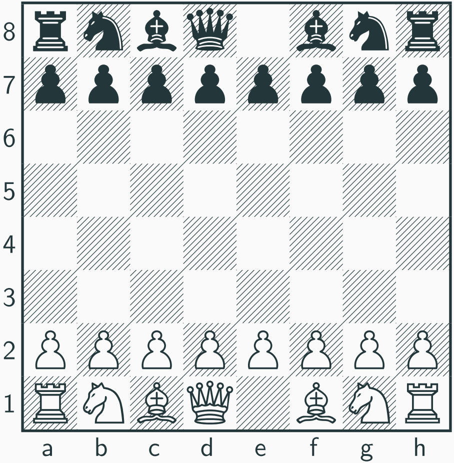

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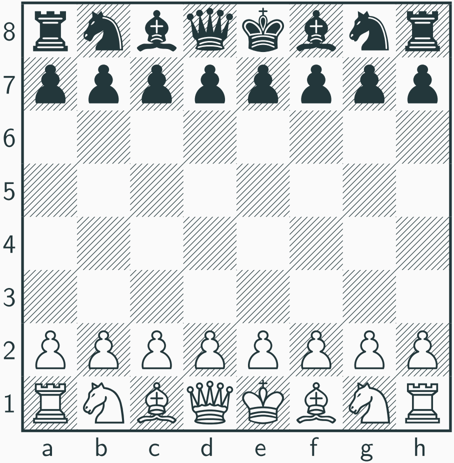
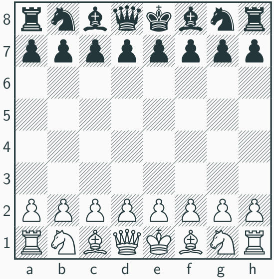
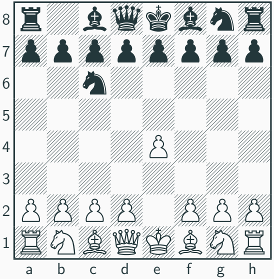


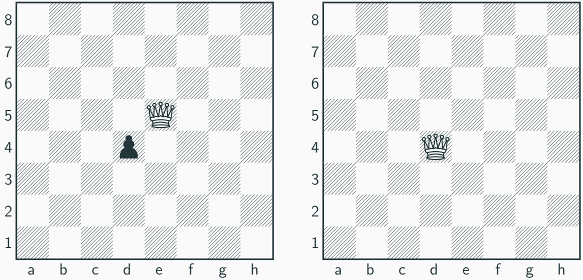
Each piece has different movement rules, allowing them to move around the 8x8 board.





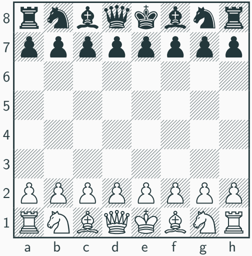
A note on Chess cont.

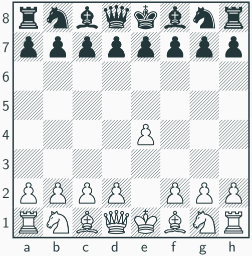
Pieces can remove other pieces from the board via *capture*; which almost always involves moving to the other piece's square.



A Short Example

Below is a valid move by a White Pawn:





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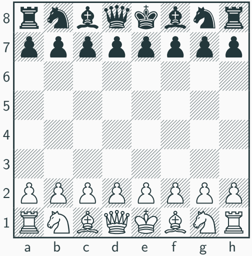
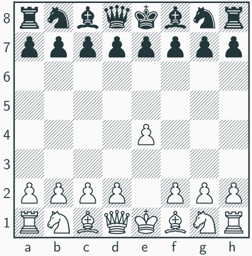
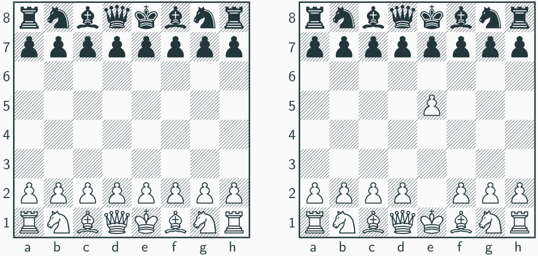





```
chess
    pawn e2 to e4
end
```

A Short Example cont.

Below is an *invalid* move by a White Pawn:



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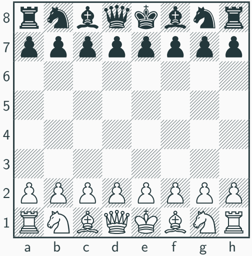
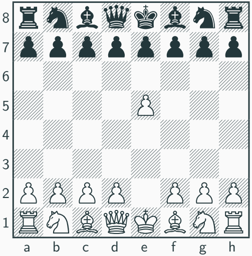
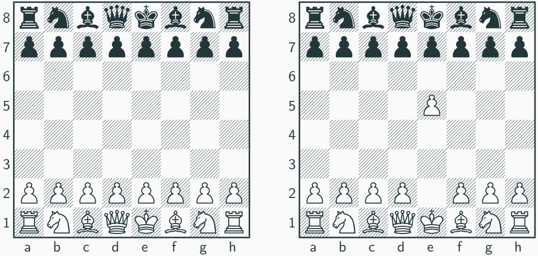





```
chess
    pawn e2 to e5
end
```

A Short Example cont.

Below is an *invalid* move by a White Pawn:




```
-- Fails to compile with type error:
--      * There is no valid move from E2 to E5.
--      The Pawn at E2 can move to: E3, E4
chess
    pawn e2 to e5
end
```

A Little Terminology

In Haskell, values have *types*, and types have *kinds*.

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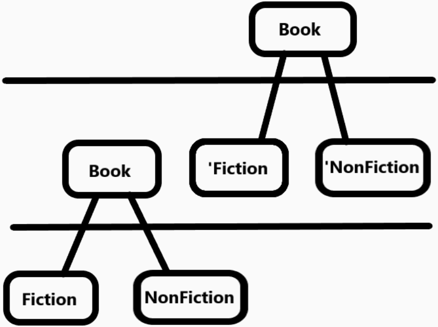
```
data Book = Fiction | NonFiction
```



A Little Terminology cont.

In Haskell, you compute on values with *functions*.

```
factorial :: Int -> Int
factorial 0 = 1
factorial x = x * factorial (x - 1)
```

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```
factorial :: Int -> Int
factorial 0 = 1
factorial x = x * factorial (x - 1)
```

But you have to use type families to compute on types:

```
type family Factorial (x :: Nat) :: Nat where
    Factorial 0 = 1
    Factorial x = Mult x (Factorial (x - 1))

type family Mult (x :: Nat) (y :: Nat) :: Nat where
    Mult 0 y = 0
    Mult 1 y = y
    Mult x y = y + (Mult (x - 1) y)
```

Problems with Type Families?

Lots of idiomatic Haskell code relies on functions being *first-class*; partial application, mapping, etc.

```
x = map (+ 2) [1,2,3]
-- = [3,4,5]
```

Problems with Type Families?

Lots of idiomatic Haskell code relies on functions being *first-class*; partial application, mapping, etc.

```
x = map (+ 2) [1,2,3]
-- = [3,4,5]
```

But type families can't be partially applied!

```
-- Type error: type family (+) was expecting 2
    arguments, got 1
type X = Map (+ 2) '[1,2,3]
```

Introducing First Class Families

Thanks to Li-yao Xia, we have First Class Families!

It relies on a data type `Exp`, and a type family `Eval`, to create a type-level interpreter:

```
type Exp a = a -> *
type family Eval (e :: Exp a) :: a
```

Making a First Class Family

```haskell
type family And (x :: Bool) (y :: Bool) :: Bool where
    And True  True  = True
    And True  False = False
    And False True  = False
    And False False = False
```

becomes:

```haskell
data And :: Bool -> Bool -> Exp Bool
type instance Eval (And True  True)  = True
type instance Eval (And True  False) = False
type instance Eval (And False True)  = False
type instance Eval (And False False) = False
```

Type-Level Mapping

With the below definition of Map:

```
data Map :: (a -> Exp b) -> f a -> Exp (f b)
type instance Eval (Map f '[])        = '[]
type instance Eval (Map f (x ': xs)) = Eval (f x) ':
    Eval (Map f xs)
```

And a definition of a type-level (+):

```
data (:+) :: Nat -> Nat -> Exp Nat
type instance Eval (Z     :+ y) = y
type instance Eval ((S x) :+ y) = S (x :+ y)
```

We can now map over a type-level list:

```
Eval (Map (:+ 2) '[1,2,3])
-- = '[3,4,5]
```

Representing Movement

Each turn of movement is expressed as a single First Class Family:

```
data Move :: Position -> Position -> BoardDecorator
    -> Exp BoardDecorator
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data Move :: Position -> Position -> BoardDecorator
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```

Thanks to First Class Families, we can extend this with rule-checking naturally; using a type-level version of the function composition operator, `(.)`:

```
PostMoveCheck2 . PostMoveCheck1 . Move fromPos toPos
    . PreMoveCheck2 . PreMoveCheck1
```

The Board type

To avoid repeated length checks, we use *length-indexed vectors* with a type-level implementation of Peano natural numbers:

```
data Vec (n :: Nat) (a :: Type) where
    VEnd   :: Vec Z a
    (:->)  :: a -> Vec n a -> Vec (S n) a
```

Since a Chess board is always an 8x8 grid, we use vectors of vectors:

```
type Eight = (S (S (S (S (S (S (S (S Z))))))))
type Row   = Vec Eight (Maybe Piece)
type Board = Vec Eight Row
```

In the codebase, we use a wrapper data structure (named `BoardDecorator`) to hold additional useful information.

Using the Type-Level Model

To interact with this type level model, the output of each `Move` call is piped to the next one:

```
x = Move a1 a2 StartBoard
y = Move e3 e4 x
z = -- ...
```

Using the Type-Level Model cont.

Below is a simplified representation of what happens for the game:
`chess pawn a1 to a2 king e2 to e1 end`

```
(MoveWithCheck King e2 e1 . MoveWithCheck Pawn a1 a2)
    StartBoard
```

Using the Type-Level Model cont.

Below is a simplified representation of what happens for the game:

`chess pawn a1 to a2 king e2 to e1 end`

```
(MoveWithCheck King e2 e1 . MoveWithCheck Pawn a1 a2)
    StartBoard

data MoveWithCheck :: PieceName -> Position ->
    Position -> Exp Board
type instance Eval (MoveWithCheck name fromPos toPos
    board)
     -- If there is a piece of that type at fromPos
     = If (IsPieceAt name fromPos board)
          -- then
          (Move fromPos toPos board)
          -- else
          (TypeError -- ...)
```

Interacting with Type-Level model at the value level

The core idea is wrapping the `BoardDecorator` type in a `Proxy`, so that it can be passed around within a value by functions:

```
data Proxy a = Proxy

edslMove :: SPosition from
         -> SPosition to
         -> Proxy (b :: BoardDecorator)
         -> Proxy (Eval (Move from to b))
edslMove (x :: SPosition from) (y :: SPosition to) (z
    :: Proxy (b :: BoardDecorator))
     = Proxy @(Eval (Move from to b))
```

But this would still look similar to Haskell syntax; we need a new approach.

Creating the EDSL

Ideally, the EDSL should look like existing chess notation:

```
1. e4 e5 2. Nf3 Nc6 3. Bb5 a6
```

Can achieve using Continuation Passing Style, inspired by Dima Szamozvancev's Flat Builders work (Szamozvancev, "Well-typed music does not sound wrong").

Chess Continuations

We define some important continuations:

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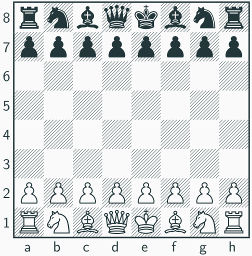
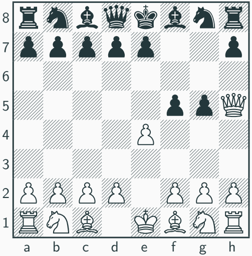
All of the above continuations can be chained together like so:

```
game = chess pawn a1 to a2 bishop e4 to d5 end
```

A Longer Example

Below is a short game, ending in checkmate by White:





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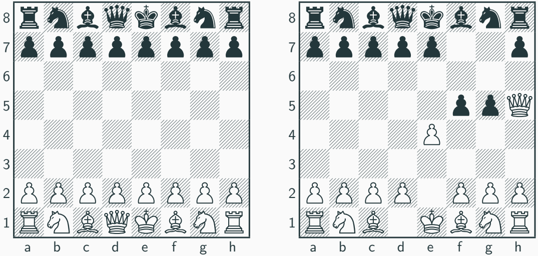



```
game = chess
    pawn e2 to e4
    pawn f7 to f5
    queen d1 to f3
    pawn g7 to g5
    queen f3 to h5
end
```

A Longer Example

What about a piece trying to move after Checkmate, when the game ends?

```
game = chess
    pawn e2 to e4
    pawn f7 to f5
    queen d1 to f3
    pawn g7 to g5
    queen f3 to h5
    pawn g5 to g4
end
```

A Longer Example

What about a piece trying to move after Checkmate, when the game ends?

```
-- Below results in the following type error:
    -- * The Black King is in check after a Black
   move. This is not allowed.
    -- * When checking the inferred type
    --      game :: Data.Proxy.Proxy (TypeError ...)
game = chess
    pawn e2 to e4
    pawn f7 to f5
    queen d1 to f3
    pawn g7 to g5
    queen f3 to h5
    pawn g5 to g4
end
```

A Longer Example cont.

What about if the White Queen tries to move through another piece, mid-game?

```
game = chess
    pawn e2 to e4
    pawn f7 to f5
    queen d1 to d3  -- Invalid move
    pawn g7 to g5
    queen f3 to h5
end
```

A Longer Example cont.

What about if the White Queen tries to move through another piece, mid-game?

```
-- Below results in the following type error:
     -- * There is no valid move from D1 to D3.
     -- The Queen at D1 can move to: E2, F3, G4, H5,
    ...
     -- * When checking the inferred type
     -- game :: Data.Proxy.Proxy (...)
game = chess
     pawn e2 to e4
     pawn f7 to f5
     queen d1 to d3  -- Invalid move
     pawn g7 to g5
     queen f3 to h5
end
```

A Longer Short Example

We also developed a shorthand syntax!

The below game:

```
game = chess
    pawn e2 to e4
    pawn f7 to f5
    queen d1 to f3
    pawn g7 to g5
    queen f3 to h5
end
```

becomes:

```
game = chess
    p e4 p f5
    q f3 p g5
    q h5
end
```

Demo

Testing Overview

Combination of:

- Unit testing with assertions, based on whether a code snippet compiles or fails to compile;
- Unit tests of custom-made board scenarios, to test out specific behaviour;
- EDSL tests of custom board scenarios, for the same purpose;
- EDSL testing with famous Chess games, written out in Chesskell notation.

Unit Testing

Unit tests rely on two main assertions; `shouldTypecheck`, and `shouldNotTypecheck`, which succeed or fail based on whether a specific code snippet fails with a type error or not.

We created unit tests for individual type families, to determine if they have the behaviour they should:

```
oppositeTeamTest1 :: White :~: Eval (OppositeTeam
    Black)
oppositeTeamTest1 = Refl

-- ...

it "1: OppositeTeam Black = White" $
    shouldTypecheck oppositeTeamTest1
```

(Note that a value with type `x :~: y` will only compile if `x` and `y` can be unified.)

Unit Testing cont.

We also created unit tests for every FIDE Law of Chess that could be tested in this manner:

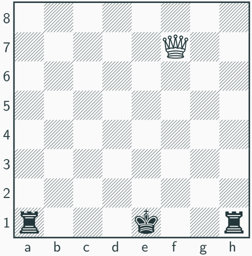
```
whiteBishopCannotTakeOwnTeam :: Proxy (a ::
    BoardDecorator)
whiteBishopCannotTakeOwnTeam = Proxy @(Eval (Move (At
    C Nat1) (At D Nat2) WhiteStartDec))

-- ...

it "1: A White Bishop cannot take a piece on the same
    team" $
     shouldNotTypecheck whiteBishopCannotTakeOwnTeam
```

Scenario Testing

We created custom Chess test boards, paired with unit tests, to model specific behaviour:




```
blackCanCastleLeft :: '(True, False) :~: CanCastle
    Black BlackCastleLeftDec
blackCanCastleLeft = Refl
-- ...
shouldTypecheck blackCanCastleLeft
```

EDSL Scenario

The EDSL was similarly tested with scenarios, to ensure that rule-breaking moves did not compile:

```
didntPromoteBlack = create
        put _Wh _P at h7
        put _Bl _P at a2
    startMoves
        pawn h7 promoteTo _B h8
        pawn a2 to a1
    end
```

EDSL Scenario

The EDSL was similarly tested with scenarios, to ensure that rule-breaking moves did not compile:

```
-- Below fails with the following type error:
    -- * Promotion should have occurred at: a1. Pawns
    must be promoted when they reach the opposite end
    of the board.
    -- * When checking the inferred type:
    --     didntPromoteBlack :: Data.Proxy.Proxy
    (TypeError ...)
didntPromoteBlack = create
        put _Wh _P at h7
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    startMoves
        pawn h7 promoteTo _B h8
        pawn a2 to a1
    end
```

EDSL Game Testing

Testing for EDSL correctness primarily consisted of writing out the first few moves of some famous game, and then making variations with errors:

```
loopVsGandalf = chess
        p e4 p c5
        n f3 p d6
        p d4 p d4
        n d4 n f6
        n c3 p a6
    end
```

EDSL Game Testing

Testing for EDSL correctness primarily consisted of writing out the first few moves of some famous game, and then making variations with errors:

```
loopVsGandalfError = chess
        p e4 p c5
        n f3 p d6
        p d4 p d4
        n d4 n f6
        n c3 p a7
    end
```

EDSL Game Testing

Testing for EDSL correctness primarily consisted of writing out the first few moves of some famous game, and then making variations with errors:

```
loopVsGandalfError = chess
        p e4 p c5
        n f3 p d6
        p d4 p d4
        n d4 n f6
        n c3 p a7   -- Pawn moves to same place!
    end
```

Compile-time and memory issues

Compile-time and memory issues came up time and again throughout development; putting a hard limit on the length of Chesskell games.

With some games, GHC will run out of memory (>25GB) and crash.

Through testing, it seems the upper limit is **12 moves maximum**; while all 6-move games tested have compiled, most 8- and 10-move games do as well.

Compile-time and memory issues cont.

In fact, we discovered a difference in behaviour between type applications (Eisenberg, Weirich, and Ahmed, "Visible type application") and type signatures:

```
-- Compiles, but would hang
chess :: Spec (Proxy StartDec)
chess cont = cont (Proxy @StartDec)

-- Would fail to compile
chess :: Spec (Proxy StartDec)
chess cont = cont (Proxy :: Proxy StartDec)
```

Compile-time and memory issues cont.

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chess cont = cont (Proxy @StartDec)

-- Would fail to compile
chess :: Spec (Proxy StartDec)
chess cont = cont (Proxy :: Proxy StartDec)
```

So we filed a GHC bug report:
https://gitlab.haskell.org/ghc/ghc/-/issues/18902

Project Management

The project was managed successfully, making use of:

- Spiral methodology;
- Git and GitHub for version control;
- Weekly supervisor meetings;
- A Trello board to track upcoming tasks and completed features (Figure 1);
- Extensive unit and integration testing.

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The implementation of Chesskell was feature-complete by the 4th of December; since then, work has gone into optimisation, testing, and write-ups.

Project Management cont.

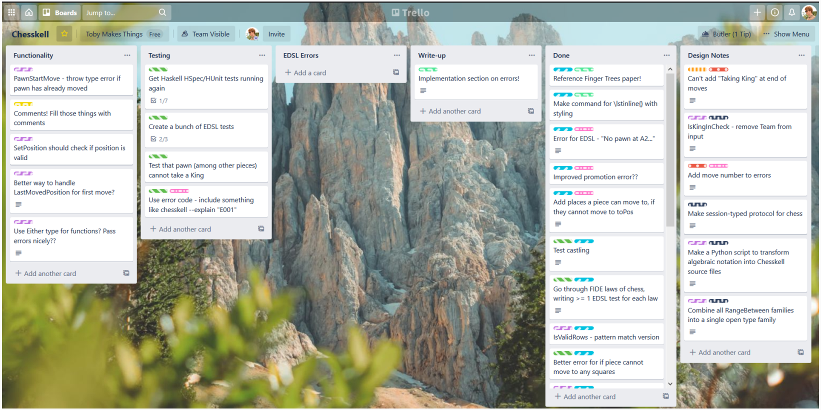

Figure 1: The Trello board used to track development.

Further Work

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- A session-typed version of Chesskell;
- Further optimisations to try and increase the move limit;
- An automated tool to transform from Algebraic Notation into Chesskell notation.

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- A first draft of a Haskell Symposium paper about the development of Chesskell, including our findings on compile-time and memory usage issues.

Furthermore, Chesskell is unique and has never been done before. Though there is room for further work and improvement, Chesskell is a success!

Q&A

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