Escuela de Ingeniería
Civil Informática

Cálculo Numérico
"Taller 4"

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Taller n°4

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1.

i	Y_i	$v_i\ (m/s)$
0	0,000	0,0
1	0,002	0,006130
2	0,004	0,011756
3	0,006	0,016180
4	0,008	0,019021

a) $h = 0{,}002$

$$f'(0) = \frac{1}{2(0{,}002)}\left[-3\cdot(0) + 4(0{,}006180) - 0{,}011756\right]$$

$$f'(0) = \frac{1}{0{,}004}\left[0 + 0{,}02472 - 0{,}011756\right] = \boxed{3{,}241}$$

b) $f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right] + \frac{h^2}{3} f^{(3)}(x_0)$

$f(x_0) = f^2(x_0) + E(x_0)$
$f(x_0+h) = f^2(x_0+h) + E(x_0+h)$
$f(x_0+2h) = f^2(x_0+2h) + E(x_0+2h)$

$E_t = \left| f'(x_0) - \frac{1}{2h}\left[-3f^2(x_0) + 4f^2(x_0+h) - f^2(x_0+2h)\right]\right.$

$E_t = \left| f'(x_0) - \frac{1}{2h}\left[-3(f(x_0) - E(x_0)) + 4(f(x_0+h) - E(x_0\right.\right.$

$- (f(x_0+2h) - E(x_0+2h))]$

$\left[E_t \le \frac{h^2}{3} n + \frac{\varepsilon}{h}\right]$

$E_t = \frac{2h}{3} n - \frac{\varepsilon}{h^2}$

$\frac{2h}{3} n - \frac{\varepsilon}{h^2} = 0$

$\boxed{h = \sqrt[3]{\frac{3\varepsilon}{2n}}}$

2b)

TOMANDO $h=2$

$$I \approx \frac{2}{3}\left[\ln(1) + 4\ln(3) + \ln(5)\right]$$

$$I \approx 4.002591$$ //.

2c) $$\int_1^5 \ln(x)\,dx = \left[x\cdot\ln(x) - x\right]\Big/_1^5$$

$$= (5\ln 5 - 5) - (\overset{0}{\cancel{\ln(1)}} - 1)$$

$$= 5\ln(5) - 5 + 1$$

$$= 5\ln(5) - 4$$

$$\approx 4.0471895621 7$$

$$\therefore\ \varepsilon_1 = \left|\frac{4.04718956217 - 4.002591}{4.04718956217}\right|$$

$$\varepsilon_1 = \left|\frac{0.04459856217}{4.04718956217}\right|$$

$$\varepsilon_1 = 0{,}0110$$

$$E''t = \frac{3n}{2} + \frac{2\varepsilon}{h^3}$$

$$E''t\left(\sqrt[3]{\frac{2\varepsilon}{2n}}\right) = \frac{3n}{2} + \frac{2\varepsilon}{\frac{3\varepsilon}{2n}}$$

$$= \frac{3n}{2} + \frac{4n}{3}$$

$$= \frac{17}{6}n > 0$$

Diremos entonces que:

$$h = \sqrt[3]{\frac{3\varepsilon}{2n}} \text{ es mínimo.}$$

2) $I = \int_1^5 \ln x \, dx$

$E_I < 0{,}01$

a) $\frac{h^5}{90} \left| f^{(4)}_{(\varepsilon)} \right| < 0{,}01$ $\qquad f^{(4)}_{(x)} = \frac{-6}{x^4}$

$\frac{h^5}{90} \cdot \frac{6}{\varepsilon^4} < 0{,}01$

$\frac{h^5}{\varepsilon^4} < 0{,}15$

$h^5 < 0{,}15 \, \varepsilon^4$

$\boxed{h < \sqrt[5]{0{,}15 \cdot \varepsilon^4}}$ $\quad \varepsilon \in [1, 5]$

Si: $\varepsilon = 5 \Rightarrow \boxed{h < 2{,}48}$ Para $E_I < 0{,}01$

3.- Evaluar numéricamente por regla de Simpson 3/8 la integral doble:

$$\int_0^1 \int_0^x \sqrt{x+y}\, dy\, dx$$

$a = 0$

$b = x$

$n = 3$

$h = \frac{x}{3}$

$X_0 = 0$

$f(X_0) = \sqrt{x}$

$X_1 = \frac{x}{3}$

$f(X_1) = \sqrt{\frac{4x}{3}}$

$X_2 = \frac{2x}{3}$

$f(X_2) = \sqrt{\frac{5x}{3}}$

$X_3 = X$

$f(X_3) = \sqrt{2X}$

Aplicando fórmula de Simpson 3/8

$$\int f(x)\, dx = \frac{3h}{8}\left[f(X_0) + 3f(X_1) + 3f(X_2) + f(X_3)\right]$$

$$= \frac{3}{8}\left[\sqrt{x} + 3\frac{\sqrt{4}}{3} + 3\frac{\sqrt{5x}}{3} + \sqrt{2x}\right]$$

$$\frac{x}{8}\left[\sqrt{x}\left(1 + 3\frac{\sqrt{4}}{3} + 3\frac{\sqrt{5}}{3} + \sqrt{2}\right.\right]$$

$$= \frac{x}{8}\left[\sqrt{x} \cdot 9{,}751298524\right]$$

$$= 1{,}218912316\, x\sqrt{x}$$

Ahora tenemos:

$$\int_0^1 1{,}218912316\, x\sqrt{x}$$

$a=0$
$b=1$
$n=3$
$h=0{,}\overline{33}$

$x_0=0$
$f(x_0)=0$

$x_1=0{,}3\overline{3}$
$f(x_1)=0{,}234579784$

$x_2=0{,}6\overline{6}$
$f(x)=0{,}663491824$

$x_3=0{,}99$
$f(x_3)=1{,}218912314$

ahora por Simpson

$$I=\frac{3\cdot 0{,}33\overline{3}}{8}\left[0+0{,}70373935 2+1{,}990475472+1{,}218912314\right]$$

$$I=0{,}124999999\left[3{,}913127138\right]$$

$$I=0{,}489140888$$

$$\therefore \int_0^1\int_0^x \sqrt{x+y}\,dy\,dx \simeq 0{,}489140888$$