

Demostraciones de PRYE

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Probabilidad de una unión finita

La probabilidad de una unión finita está dada por

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$P\left(\bigcup_{i=1}^{n+1} A_i\right) = P(A_{n+1}) + P\left(\bigcup_{i=1}^n A_i\right) - P\left(\bigcup_{i=1}^n (A_{n+1} \cap A_i)\right)$$

Probabilidad de una unión finita

$$\begin{aligned} & P\left(\bigcup_{i=1}^n A_i\right) \\ &= \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) \\ &+ \sum_{1 \leq i < j < k \leq n} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^n P\left(\bigcap_{i=1}^n A_i\right) \end{aligned}$$



Sobre eventos independientes

Dos eventos, A y B , de un espacio muestral, son llamados independientes cuando $P(A \cap B) = P(A)P(B)$.

Si A y B son independientes, entonces A^c y B son independientes; A^c y B^c son independientes.

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Usando la igualdad $P(B) = P(B \cap A) + P(B \cap A^c)$

$$\begin{aligned} & P(B \cap A^c) \\ = & \\ & P(B) - P(B \cap A) \\ = & \\ & P(B)(1 - P(A)) \\ = & \\ & P(B) P(A^c) \end{aligned}$$

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$$\begin{aligned} & P(A^c \cap B^c) \\ = & \\ & 1 - P(A \cup B) \\ = & \\ & 1 - [P(A) + P(B) - P(A \cap B)] \\ = & \\ & (1 - P(A))(1 - P(B)) \\ = & \\ & P(A^c) P(B^c) \end{aligned}$$

Binomial

$$X \sim B(n, p)$$

$$f(x) = P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \quad (0 \leq x \leq n)$$

- 1 Para todo $x \in \mathbb{Z}$ con $0 \leq x \leq n$, $P(X = x) \geq 0$.
- 2 $\sum_{x=0}^n P(X = x) = 1$.
- 3 $E[X] = np$.
- 4 $\text{Var}[X] = np(1 - p)$.

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Binomial

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$



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Geometría

$$X \sim \text{Geom}(p)$$

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Geometría

$$\sum_{k=0}^n p^k = \frac{1 - p^{n+1}}{1 - p}$$



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Hipergeométrica

$$X \sim Hg(N, K, n)$$

$$f(x) = P(X = x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

$$(\max\{0, n + K - N\} \leq x \leq \min\{K, n\})$$

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Hipergeométrica

$$\binom{m+n}{k} = \sum_{r=0}^k \binom{m}{r} \binom{n}{k-r}$$


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Poisson

$$X \sim \text{Pois}(\lambda)$$



$$f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (x \in \mathbb{N})$$

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Poisson

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$



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Normal

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)} \quad (x \in \mathbb{R})$$

- ❶ Para todo $x \in \mathbb{R}$, $f(x) \geq 0$.
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Normal

$$\int_{\mathbb{R}} e^{-x^2} dx = \sqrt{\pi}$$



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Chi-Cuadrado

$$X \sim \chi^2(\nu)$$

$$f(x) = \frac{1}{2^{\nu/2} \Gamma\left(\frac{\nu}{2}\right)} x^{(\nu/2)-1} e^{-x/2} \quad (x \geq 0)$$

- 1 para todo $x \in \mathbb{R}$, $f(x) \geq 0$
- 2 $\int_{-\infty}^{\infty} f(x) dx \geq 0$
- 3 $E[X] = \nu \quad (\nu \geq 0)$
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Chi-Cuadrado

$$\Gamma(x+1) = x\Gamma(x)$$



$$f(x) = \frac{1}{2^{v/2}\Gamma\left(\frac{v}{2}\right)} x^{(v/2)-1} e^{-x/2} \quad (x \geq 0)$$

- 1 para todo $x \in \mathbb{R}$, $f(x) \geq 0$
- 2 $\int_{-\infty}^{\infty} f(x)dx \geq 0$
- 3 $E[X] = v \quad (v \geq 0)$
- 4 $\text{Var}[X] = 2v \quad (v \geq 0)$

$$F \sim \mathbf{f}(u, v)$$

$$f(x) = \frac{\Gamma\left(\frac{u+v}{2}\right) \left(\frac{u}{v}\right)^{u/2}}{\Gamma\left(\frac{u}{2}\right) \Gamma\left(\frac{v}{2}\right)} \frac{x^{(u/2)-1}}{\left(1 + \frac{u}{v}x\right)^{(u+v)/2}} \quad (x \in \mathbb{R}^+)$$

- 1 Para todo $x \in \mathbb{R}$, $f(x) \geq 0$.
- 2 $\int_{\mathbb{R}} f(x) dx = 1$.
- 3 $E[F] = \frac{v}{v-2} \quad (v > 2)$
- 4 $\text{Var}[F] = \frac{2v^2(u+v-2)}{u(v-2)^2(v-4)} \quad (v > 4)$

$$F \sim \mathbf{f}(u, v)$$

$$f(x) = \frac{\left(\frac{u}{v}\right)^{u/2}}{B\left(\frac{u}{2}, \frac{v}{2}\right)} \frac{x^{(u/2)-1}}{\left(1 + \frac{u}{v}x\right)^{(u+v)/2}} \quad (x \in \mathbb{R}^+)$$

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$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$



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Gamma

$$X \sim \text{Gamma}(\alpha, \beta)$$

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad (x \in \mathbb{R}^+)$$

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Gamma

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$$



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Hipergeométrica a Binomial

Sea $X \sim Hg(N, K, n)$. Para n fijo, si N, K cumplen que

$$\frac{x-1}{K} < \epsilon_1$$

$$\frac{n-x-1}{N-K} < \epsilon_2$$

$$\frac{n-1}{N-n+1} < \epsilon_3$$

Entonces,

$$\left| \frac{Hg(N, K, n)(x)}{B\left(n, \frac{K}{N}\right)(x)} - 1 \right| < (\epsilon_1 + 1)^x (\epsilon_2 + 1)^{n-x} (\epsilon_3 + 1)^n - 1 \quad \text{☕}$$