ELECTRONIC CIRCUIT DESIGN

LECTURE NOTES AND
LABORATORY EXERCISES
(DRAFT 0.2.0)

by

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Chapter 1

Electronic Principles

1.1 Symbols and Notation

Fig. 1.1 shows the circuit symbols for npn and pnp transistors where “b” is base, “c” is collector and “e” is emitter. These transistors can be distinguished from one another by the direction of the arrow, which should never be omitted, on the emitter pin. The arrow indicates the direction of positive current flow under normal operating conditions, that is from the p-region to the n-region of the transistor with the p-region at higher potential relative to the n-region. When the current flows out of the emitter, that indicates an npn transistor whilst a pnp transistor has the current flow *into* the emitter.

(a) npn (b) pnp

Figure 1.1: Bipolar junction transistor symbols, pin and voltage notation.

In Fig. 1.1, the arrows point in the direction of increasing voltage. The first letter in the subscripted quantity is the pin where the voltage is measured with respect to the pin indicated in the second subscript letter. For example, V_{BE} is the voltage at the base of the transistor with respect to the emitter. Two-letter subscripted quantities will be interpreted as such unless the letters used are the same, e.g. V_{CC} and V_{EE}. Identical two-letter subscripts will be used for voltage supplies. Thus, V_{CC} would be a voltage supply to the collector; V_{EE} would be voltage supplied to the emitter. If the second letter in the subscript is omitted, the indicated voltage is measured with respect to a ground reference node common to the whole circuit. There are a few situations where these rules are not observed and they will be pointed out explicitly in due course.

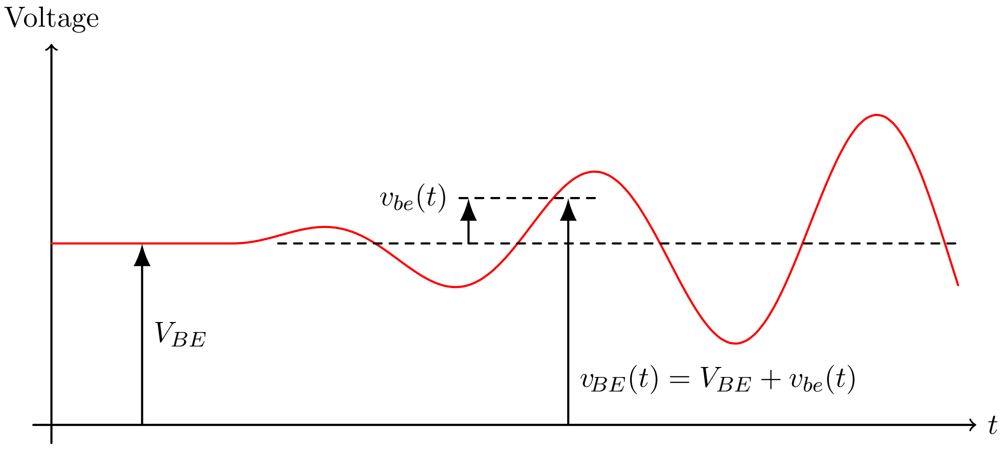
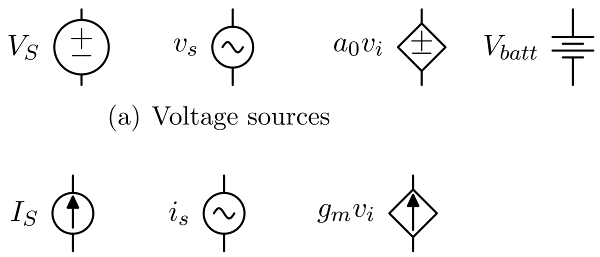


Figure 1.2: Variable naming convention for mixed ac and dc analog signals.

In naming current and voltage variables, the following convention will be adopted:

- uppercase variable and uppercase subscript: dc or average, e.g. V_{BE},
- lowercase variable and lowercase subscript: ac component, e.g. v_{be},
- lowercase variable and uppercase subscript: total, e.g. v_{BE}.

The usage of this convention is exemplified in Fig. 1.2. In addition, the voltage and current source circuit symbols that will be used are shown in Fig. 1.3.



(a) Voltage sources

(b) Current sources

Figure 1.3: Voltage and current source circuit symbols.

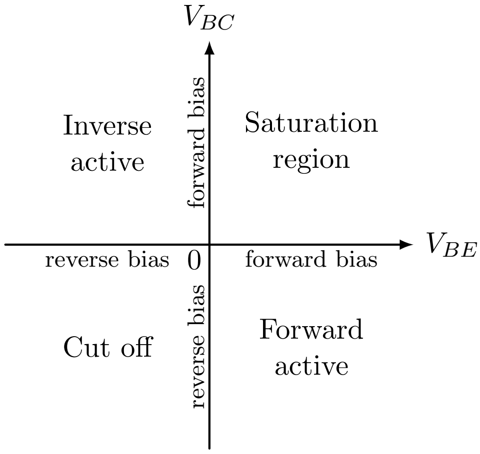
In many of the circuits that will be discussed, the npn type shall be used as representative transistor for the circuit. Often the transistor circuit will also work with pnp types provided that terminal voltages and currents necessary for proper functioning are properly accounted. This may entail checking direction of voltage drops and terminal currents.

1.2 Bipolar Junction Transistors

(Reading data sheet)

1.2.1 Modes of operation

The mode of operation in a BJT is a function of the bias on the two pn junctions. There are four modes as shown in Table 1.4.



V_{BE}	V_{BC}	Mode
F	R	Forward active
F	F	Saturation
R	F	Inverse active
R	R	Cut off

Figure 1.4: BJT modes of operation for npn type where “F” denotes that the junction is forward-biased whilst “R” indicates reverse bias.

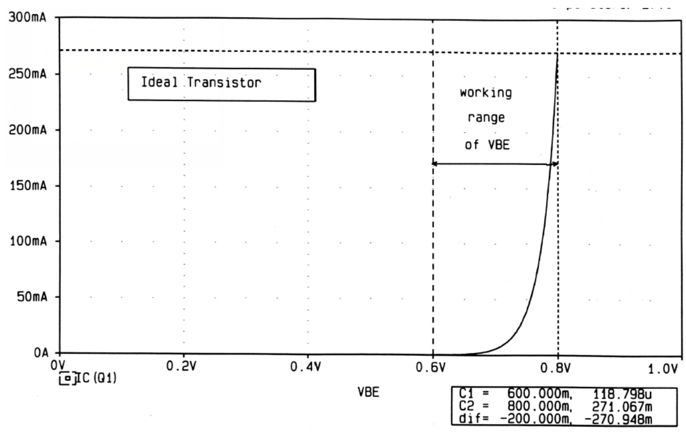
Unless specified otherwise, it may be assumed that the transistor is in the forward active mode.

1.2.2 Ideal Electrical Characteristics

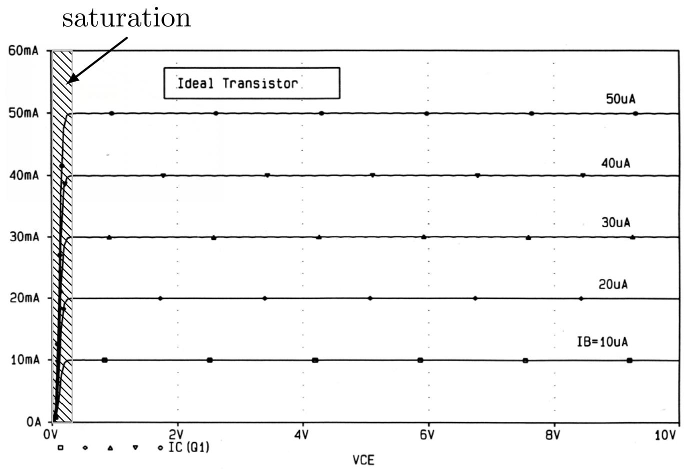
1.2.3 Simple DC model

When the transistor is in the forward active mode, an approximate dc model suitable for calculations by hand is shown in Fig. 1.6.

In the model, it is assumed that the base-emitter voltage drop is 0.7 V and that the collector current is only linearly dependent on the base current, i.e. inde-

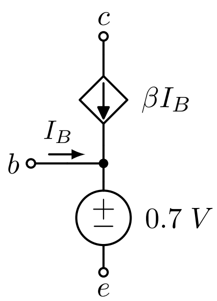


(a) I_C vs V_{BE}

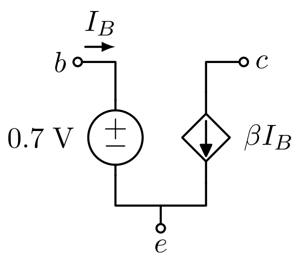


(b) I_C vs V_{CE} with I_B as parameter

Figure 1.5: SPICE simulation of quasi-ideal bipolar junction transistor common emitter characteristics. Refer to Fig. 1.7 for circuit diagram.



(a) Form I



(b) Form II

Figure 1.6: Simple dc models of a BJT in forward active mode.

pendent of collector voltage.

$$I_C = \beta I_B \tag{1.1}$$

The constant of proportionality is called the transistor's “beta”. The base-emitter voltage given is typically used but in some circumstances maybe specified differently, e.g. 0.6 V. These assumptions are acceptable for hand calculations; more accurate (and complex) models can be obtained by using a computer program like SPICE which stands for Simulation Program with Integrated Circuit Emphasis.

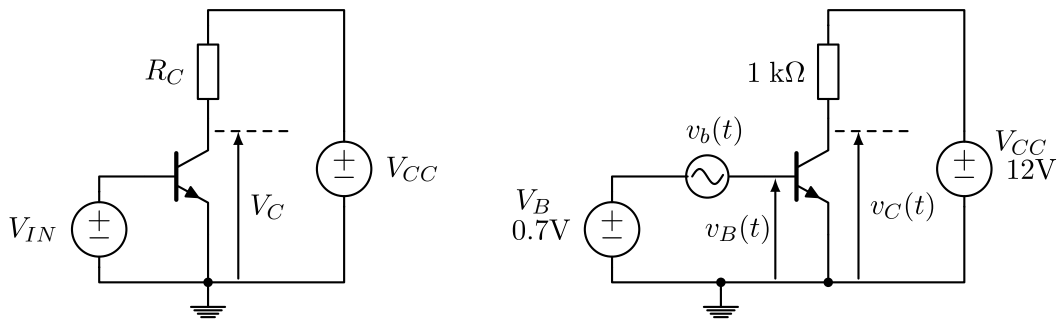
1.2.4 Biasing

Biasing a transistor typically entails the application of dc voltages across its terminals so that the transistor will be in a desired mode of operation. A biasing circuit is used to provide a stable dc voltage and current operating point or quiescent (Q) point. For bias calculations, the simple dc model is usually sufficient.

There are four commonly used biasing circuits, namely

- fixing base-emitter voltage V_{BE} (crude biasing) — not practical
- fixing the base current I_B or fixed base bias is better than crude biasing but still not practical
- fixing the emitter current I_E or auto-biasing which is commonly used for discrete circuits
- using a circuit called a current mirror is usually the preferred method in linear integrated circuits

A crude approach to the biasing of an npn transistor for forward active mode is shown in Fig. 1.7 where V_{IN} is a variable dc voltage source. The voltage sources



(a) Biasing circuit (b) Small-signal source at base

Figure 1.7: Crude biasing arrangement for forward active mode operation of an npn transistor.

V_{IN} and V_{CC} supply the base current and collector current respectively. In order for the base-emitter pn junction to be forward biased, V_{IN} needs to be approximately 0.7 V. The collector voltage is given by

$$V_C = V_{CC} - I_C R_C. \tag{1.2}$$

To ensure that the base-collector pn junction is reverse-biased, the collector voltage should be larger than the base voltage, i.e. $V_C > V_B$. This could be arranged by an appropriate choice of V_{CC} and R_C. Thus, the transistor will operate in the forward active mode. The two currents which enter the transistor through the base and collector leaves via the emitter which means that

$$\begin{aligned} I_E &= I_B + I_C \\ &= I_B + \beta I_C \\ I_E &= (1+\beta) I_B. \qquad (1.3) \end{aligned}$$

Alternatively

$$\begin{aligned} I_E &= I_B + I_C \\ &= \frac{I_C}{\beta} + I_C \\ I_E &= \frac{1+\beta}{\beta} I_C \qquad (1.4) \\ I_C &= \frac{\beta}{1+\beta} I_E \qquad (1.5) \end{aligned}$$

If β is large then

$$I_C \approx I_E \tag{1.6}$$

.

Fig. 1.9 shows biasing by fixing the base current of the transistor. The variable resistor R_B allows in-circuit adjustment of I_B to suit the transistor's characteristics. From the dc model, it is clear that

$$I_B = \frac{V_{CC} - 0.7}{R_B} \tag{1.7}$$

from which the collector current can be obtained from $I_C = \beta I_B$. However this requires knowledge of the value of β. The value of β varies between transistor models and manufacturers. While a typical value for β maybe 120, it could be 40 or an order larger, say 400. The value of β also varies with temperature and is also dependent on I_C and V_{CE}.

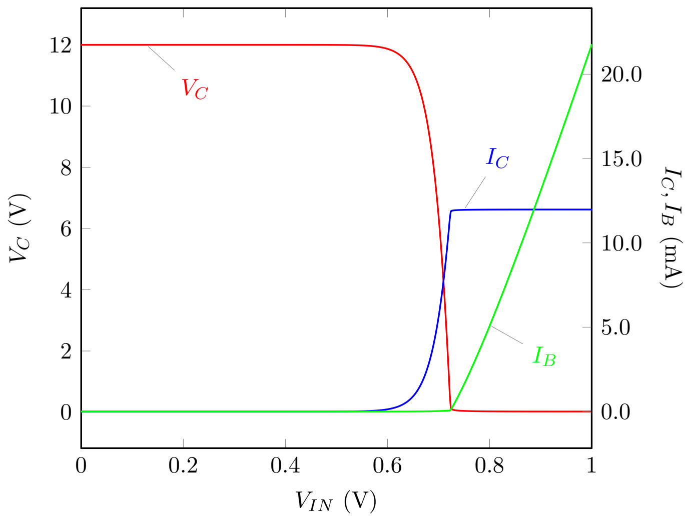
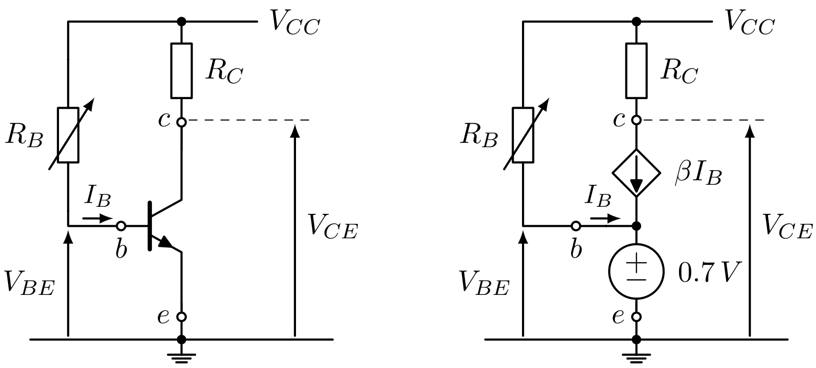


Figure 1.8: Variation of V_{CE} (V_C), I_C and I_B as the base-emitter voltage is swept in a crude biasing circuit. Q is the operating point.



(a) Biasing circuit (b) Simple dc model

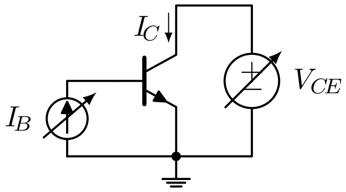
Figure 1.9: Fixed base current biasing.

When the collector characteristics for a given transistor are available, it can used to design an appropriate operating point and base resistor using the load line method. Consider a load line superimposed on the I_C-V_{CE} characteristic curves as shown in Fig. 1.10. The load line is a straight line governed in this particular circuit by the equation:

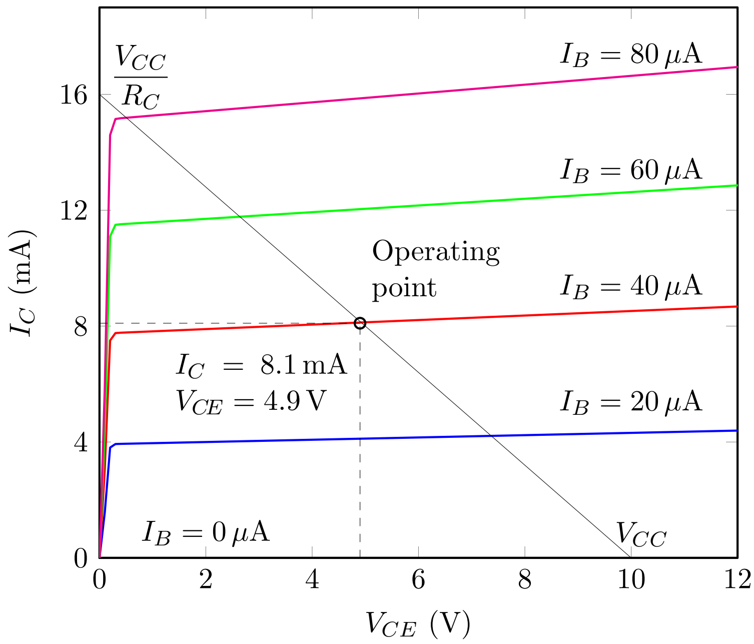
$$V_{CE} = V_{CC} - I_C R_C. \tag{1.8}$$

The load line cuts the V_{CE} axis at V_{CC} and the I_C axis at $\frac{V_{CC}}{R_C}$ as can be verified from the load line equation. The gradient of the line is determined by R_C. Thus, the line can be manipulated by choosing values for V_{CC} and R_C.

The transistor I_C-V_{CE} characateristic curves as shown in Fig. 1.10 were derived



(a) Circuit used for obtaining the characteristic curves



(b) I_C vs V_{CE}

Figure 1.10: I_C-V_{CE} characteristic curves as obtained from a SPICE simulation of the 2N2222 model, with load line superimposed.

with the collector-emitter voltage and base current as independent variables which is depicted as a variable current source at the base and a variable voltage source at the collector. For various combinations of base current and collector-emitter voltage the collector current is recorded and the curves in Fig. 1.10 are obtained. In the active region, the collector current is directly proportional to the base current[1].

For operation in the active region, several reasonable choices in load lines and operating points may be available (see Fig. 1.11). Suppose that V_{CC} has been fixed to 12 V. For load line A, there are three choices (Q_1, Q_3 and Q_4) for operating points depending on the desired collector current. Operating points Q_2 and Q_4

[1]The collector current is also slightly dependent on V_{CE} but we will ignore this here and address it later.

have the same base current. For load line A, the collector resistance is

$$\frac{V_{CC}}{R_C} = 16 \times 10^{-3} \implies R_C = \frac{12}{16 \times 10^{-3}} = 750\,\Omega$$

whereas for load line B the collector resistance would be doubled since the maximum collector current is halved.

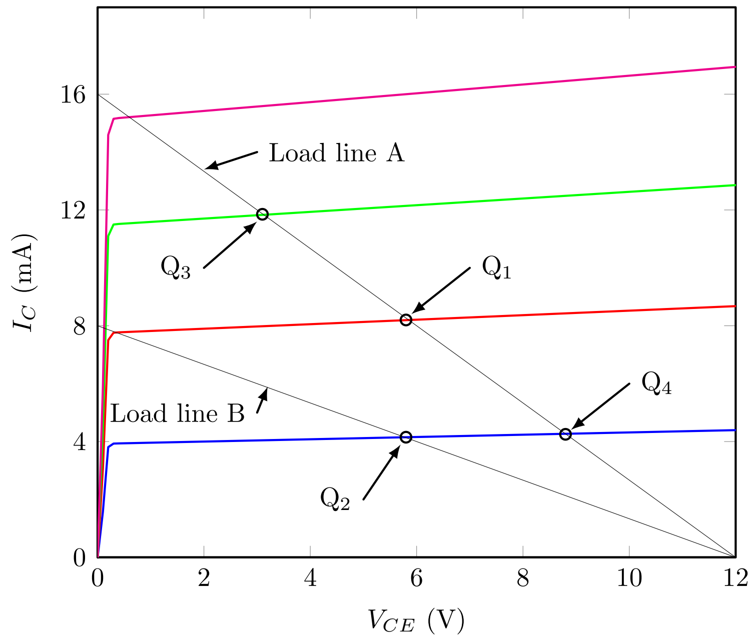
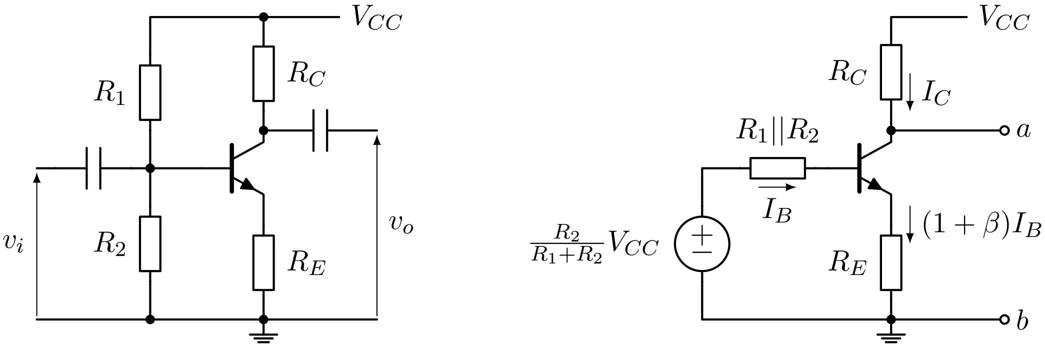


Figure 1.11: Selecting load line and operating point.

The general structure of an auto-biased BJT circuit is shown in Fig. 1.12. When the BJT is connected in the manner shown, it is called common emitter. The autobias method typically utilises four resistors so that the collector current can be fixed by designing for an appropriate emitter current. (Assuming that $\beta = \infty$ then $I_C = I_E$.) The dc biasing signals are separated from the input and output circuitry via ac coupling capacitors, so that they don't interfere with dc signals in those circuitries. For dc analysis of the biasing network, these capacitors are removed since capacitive impedance is infinite at dc, the common emitter circuit can be analysed in isolation. The two base resistors R_1 and R_2 can be replaced by a voltage source $\frac{R_2}{R_1+R_2}V_{CC}$ in series with resistance $R_1||R_2$.

In order to select appropriate resistor values for the circuit shown in Fig. 1.12 the following procedure could be used. It is assumed that the I_C-V_{CE} are available so that a load line may be drawn. For this particular biasing, the maximum



(a) Auto-biasing (b) dc analysis

Figure 1.12: Auto-biased common emitter stage with decoupling capacitors.

collector current in the load line equation is given by

$$I_C = \frac{V_{CC}}{R_C + R_E}.$$

The load line cuts the horizontal or V_{CE} axis at V_{CC}. Once the supply voltage (V_{CC}) is chosen, the collector and emitter resistances can be selected to yield a desired collector current at the operating point Q. This will also determine the dc base current at Q. Assuming that β is reasonably large, then $I_E = I_C$ where the latter is specified by Q. The base resistors R_1 and R_2 can then be chosen to satisfy the base current at Q, which can be verified by applying KVL in the base-emitter loop of the dc analysis circuit. While the steps described are specific to the common emitter circuit, the general procedure is similar for other BJT amplifier circuits.

1.2.5 DC Analysis

The forward-biased base-emitter voltage (V_{BE}) and thermal voltage (V_T) are usually specified. Typically,

$$V_{BE} = 0.7 \text{ V} \text{ or } V_{EB} = 0.7 \text{ V} \tag{1.9}$$

$$V_T = 26 \text{ mV} \tag{1.10}$$

For dc analysis of transistor terminal currents and voltages:

$$I_C = \beta I_B \tag{1.11}$$

$$I_E = (1+\beta) I_B \tag{1.12}$$

where $\beta = h_{fe}$. For an npn transistor

$$V_E = V_B - V_{BE} \tag{1.13}$$

whilst for a pnp transistor

$$V_E = V_B + V_{EB} \tag{1.14}$$

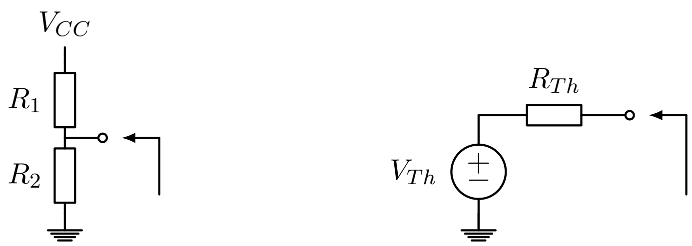
where V_E is the emitter voltage and V_B is the base voltage.

Obtaining the dc equations typically involve invoking Ohm's law and Kirchoff's laws especially KVL. One must also be familiar with the formula for current division

$$I_1 = \frac{R_2}{R_1 + R_2} I \tag{1.15}$$

– not just for dc calculations.

It is often necessary to deal with the typical biasing network shown in Fig.1.13. Apply Thevenin's theorem to convert this network into its equivalent form.



(a) Voltage divider (b) Thevenin equivalent

Figure 1.13: Thévenin equivalent circuit for biasing network

where

$$V_{Th} = \frac{R_2}{R_1 + R_2} V_{CC} \tag{1.16}$$

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} \tag{1.17}$$

1.3 Bipolar Junction Transistor Models

1.3.1 The Ebers-Moll Model

The base emitter pn junction could be more realistically modelled as a diode instead of a constant dc source as shown in Fig. 1.6. The Ebers-Moll model of the BJT takes this into account. It is an equivalent circuit model that is based on the physics of two interacting pn junctions. The Ebers-Moll model for the static (dc) condition is discussed further.

The diode equation is given by

$$I_D = I_S(e^{\frac{V_D}{nV_T}} - 1) \tag{1.18}$$

where V_T is the thermal voltage defined as

$$V_T = kT/q \tag{1.19}$$

and where

k = Boltzmann's constant = 1.38×10^{-23} J/K
T = temperature in Kelvin
q = charge on an electron = 1.6×10^{-19} C
n = ideality factor ($1 < n < 2$)
I_S = scaling current
I_D = diode terminal current
V_D = diode terminal voltage.

At room temperature $V_T \approx 26$ mV. Consequently, the model shown in Fig. 1.14 is obtained and by current considerations

$$I_D = I_E$$
$$I_C = \frac{\beta}{1+\beta} I_E$$
$$\alpha = \frac{\beta}{1+\beta} \tag{1.20}$$
$$\therefore I_C = \alpha I_D \tag{1.21}$$

For a given transistor, the parameters α and β are usually measured quantities. A more general model that is valid for any connection of the transistor takes into account both the base-emitter and base-collector pn junctions. In the Ebers-Moll model each pn junction is modelled by a diode. The nearness of the two pn junctions causes current in one diode to control a current source in parallel with the other diode. The injection version of the Ebers-Moll model utilises two different α-parameters, namely α_F and α_R for forward and reverse operations of the emitter and collector diodes, denoted as DE and DC respectively. This model is shown in Fig. 1.3.1. An alternative formulation of the Ebers-Moll model from carrier

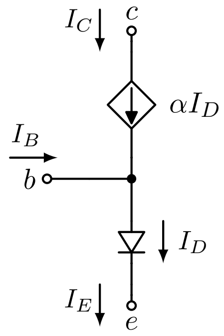


Figure 1.14: Forward active mode BJT model with diode representing base-emitter pn junction.

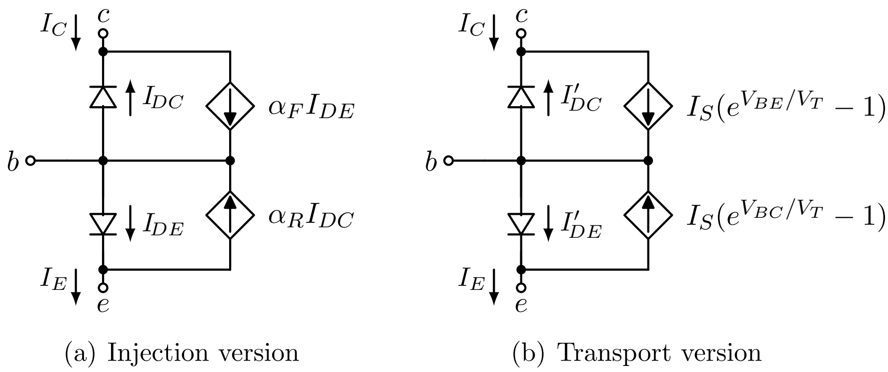


Figure 1.15: Ebers-Moll model of a BJT.

transport considerations is shown in Fig. 1.3.1. The currents in the controlled sources are determined by the diode voltages. The diode currents are given by

$$I'_{DC} = \frac{I_S}{\alpha_R}(e^{V_{BE}/V_T} - 1) \tag{1.22}$$

$$I'_{DE} = \frac{I_S}{\alpha_F}(e^{V_{BC}/V_T} - 1) \tag{1.23}$$

The collector-base current source is controlled by the base-emitter voltage (V_{BE}) whilst the emitter-base current source is controlled by the base-collector voltage (V_{BC}). In the expressions shown in Fig. 1.3.1, note the absence of the adjustment factor n. When the diode is forward biased, it is assumed that $V_D = 0.7$ V. Hence, $V_D/V_T >> 1$ and the exponential dominates the expression. Conversely, if the diode is reverse biased, V_D/V_T is a large negative number and the exponential term is negligible compared to the unity term (-1). Furthermore, the scaling current I_S of the controlled source is much smaller than the diode current in the parallel branch.

For this particular case, the controlled source can be omitted as shown in Fig. 1.16 where the reverse-biased diode current has been defined as $I_D = -I_{CBO}$. When the latter is neglected as well (remove the reverse biased diode from the circuit), then we arrive at the diode-based model as shown in Fig. 1.14 where $I_D = \frac{I_S}{\alpha}(e^{V_{BE}/V_T} - 1)$. It is often convenient to use a common-emitter configuration

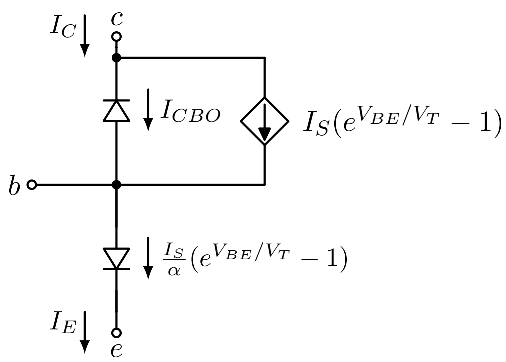


Figure 1.16: Ebers-Moll model reduction when base-emitter diode is forward biased and base-collector diode is reverse biased.

of the model. (See Fig. 1.17.) The input voltage is applied between the base and the emitter, and the output voltage is taken between the collector and the emitter. (The emitter is "common" to both the input and the output, hence the naming convention.)

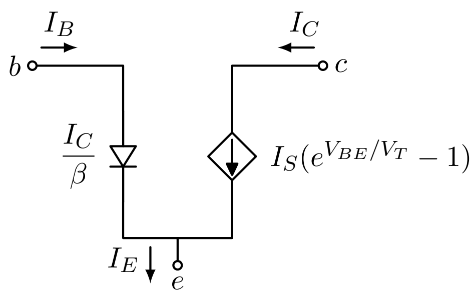


Figure 1.17: Common emitter configuration of the Ebers-Moll model.

1.3.2 BJT h-parameters

The BJT h-parameters are measured small-signal parameters of real-world transistors subject to an operating point that expresses sensitivity to small changes at an input port. For a hybrid or h-parameter characterization a current and a voltage are selected as excitations. (See Appendix **??**.) Consider the h-parameters for the common emitter circuit shown in Fig. 1.18 where the base current and common-emitter voltage will be excited (independent variables) and the collector current and base-emitter voltage used as response quantities (dependent variables). The biasing circuit is not shown but it is taken for granted that the operating point is set by I_B and V_{CE} (independent variables) which in turn determines I_C and V_{BE} (dependent quantities). The h-parameters vary with collector current and

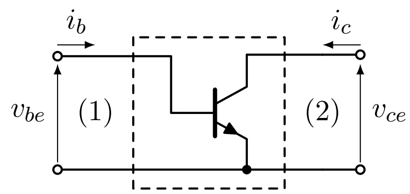


Figure 1.18: Common emitter BJT configuration for definition of h-parameters.

temperature. Typical values given by data sheets at $I_C = 5$ mA and $V_{CE} = 5$ V:

$$h_{fe} = 100$$
$$h_{ie} = 1\,\mathrm{k}\Omega$$
$$h_{oe} < \frac{1}{50\,\mathrm{k}\Omega}$$
$$h_{re} < 10^{-4}$$

The parameters h_{fe} and h_{re} are ratios; h_{ie} is an impedance and h_{oe} is an admittance. The gain parameter h_{fe} could be smaller or larger typically by a factor of four from the value given above. The input impedance h_{ie} could vary by an order,i.e. a factor of ten.

The four h-parameters are defined by the equations

$$v_{be} = h_{ie}i_b + h_{re}v_{ce} \tag{1.24}$$
$$i_c = h_{fe}i_b + h_{oe}v_{ce} \tag{1.25}$$

where

$$h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_{ce}=0} \tag{1.26}$$

$$h_{oe} = \left.\frac{i_c}{v_{ce}}\right|_{i_b=0} \tag{1.27}$$

$$h_{ie} = \left.\frac{v_{be}}{i_b}\right|_{v_{ce}=0} \tag{1.28}$$

$$h_{re} = \left.\frac{v_{be}}{v_{ce}}\right|_{i_b=0} \tag{1.29}$$

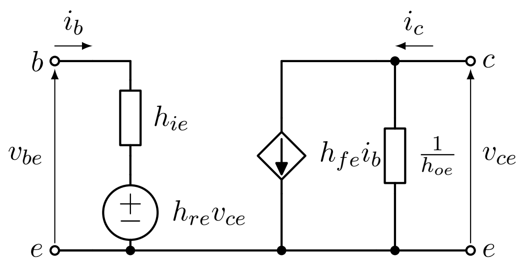


Figure 1.19: Small-signal equivalent circuit of common-emitter BJT as defined by the h-parameters equations.

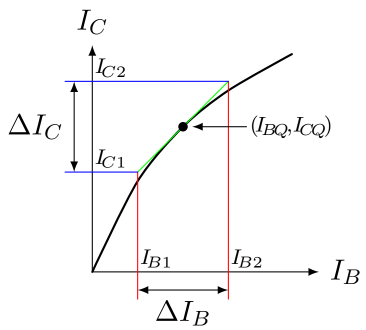


Figure 1.20: The small-signal parameter h_{fe} can be obtained from the slope of the I_C-I_B curve at the operating point.

The parameter is the small-signal current gain at a particular Q (see Fig. 1.20) that could be obtained from

$$h_{fe}\big|_{(\text{atQ})} = \left.\frac{dI_C}{dI_B}\right|_{\text{at}(I_{B1},I_{C1})} \approx \frac{\Delta I_C}{\Delta I_B} \tag{1.30}$$

Often it is assumed that $h_{fe} \equiv \beta$, though it is more appropriate to say that β is a dc parameter whereas h_{fe} is a small-signal ac parameter. In similar manner, the other h-parameters can be obtained from instantaneous measurements of terminal currents and voltages as shown in Fig. 1.21. Thus, h_{oe} is the slope of I_C vs V_{CE} at a given I_B, that is

$$h_{oe} = \frac{\Delta I_C}{\Delta V_{CE}} \tag{1.31}$$

The parameter h_{ie} is the reciprocal of the I_B-V_{BE} slope at Q

$$h_{ie} = \frac{\Delta V_{BE}}{\Delta I_B} \tag{1.32}$$

The parameter h_{re} is a voltage ratio for some constant I_B

$$h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}} \tag{1.33}$$

In practice, h_{re} is very small.

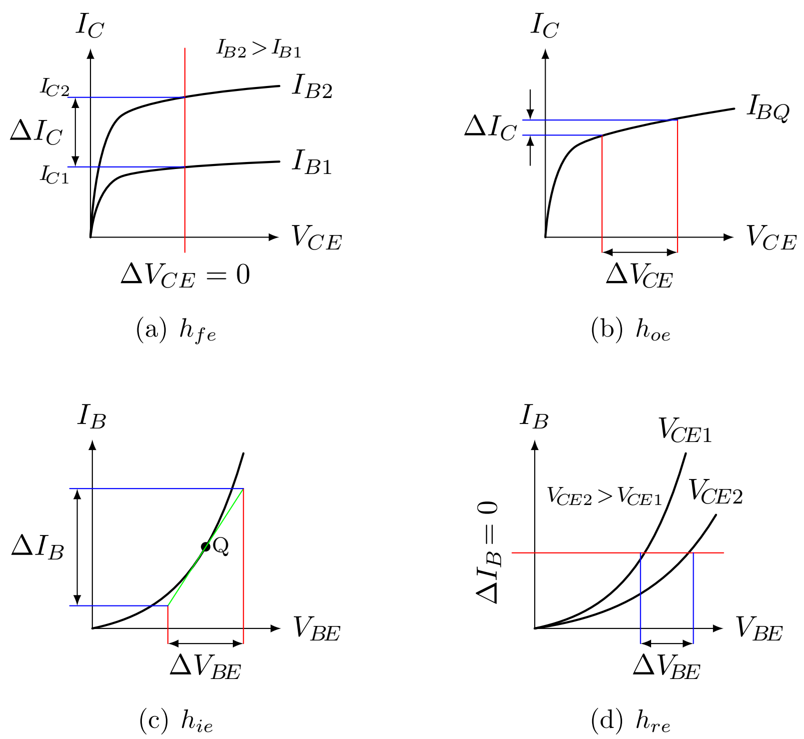


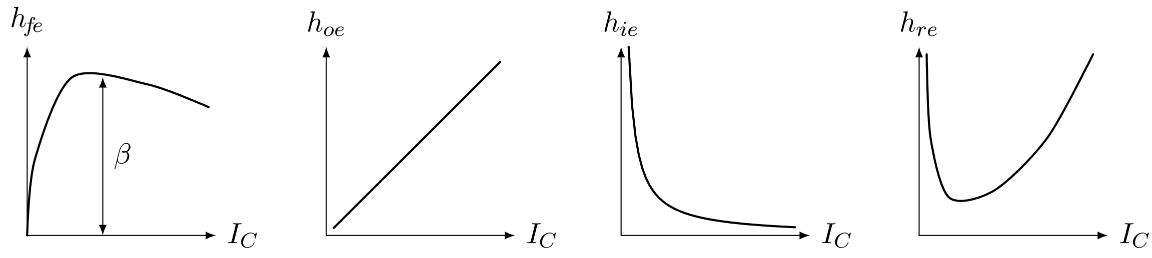
Figure 1.21: Obtaining h-parameters from measurements of I_C, V_{CE}, I_B and V_{BE}.

Consider an analysis of the common-emitter amplifier using the complete set of h-parameters. (See Fig. 1.23) From the small-signal ac equivalent circuit the following equations can be obtained:

$$i_c = h_{fe} i_b + h_{oe} v_{ce} \tag{1.34}$$

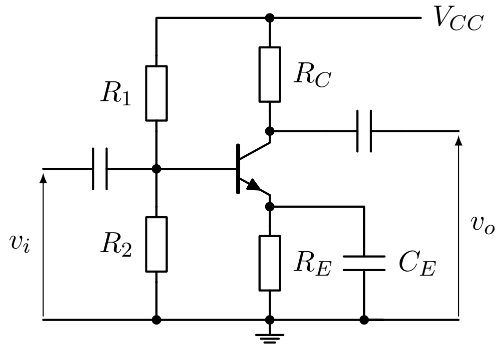
$$v_{be} = h_{ie} i_b + h_{re} v_{ce} \tag{1.35}$$

$$v_{ce} = -i_c R_c \tag{1.36}$$

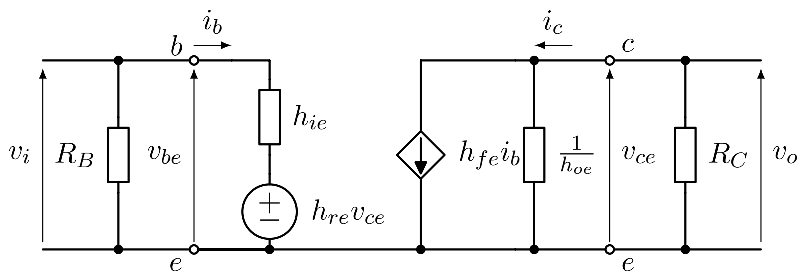


(a) h_{fe} (b) h_{oe} (c) h_{ie} (d) h_{re}

Figure 1.22: Variation of h-parameters with instantaneous collector current.



(a) Common emitter amplifier



(b) h-parameter small-signal equivalent circuit

Figure 1.23: Analysis of common-emitter amplifier using complete set of h-parameters.

$$
\begin{aligned}
i_c &= h_{fe} i_b - h_{oe} i_c R_c \\
i_c(1 + h_{oe} R_c) &= h_{fe} i_b \\
\therefore i_c &= \frac{h_{fe}}{1 + h_{oe} R_c} i_b
\end{aligned}
\tag{1.37}
$$

$$\begin{aligned} v_{be} &= h_{ie} i_b - h_{re} \frac{h_{fe}}{1 + h_{oe} R_c} R_c i_b \\ &= i_b \left(h_{ie} - \frac{h_{re} h_{fe} R_c}{1 + h_{oe} R_c} \right) \\ &= i_b \left(\frac{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_c}{1 + h_{oe} R_c} \right) \end{aligned} \tag{1.38}$$

Hence the voltage gain can be expressed as

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{v_{ce}}{v_{be}} \\ &= \left(\frac{-h_{fe}}{1 + h_{oe} R_c} R_c \right) \left(\frac{1 + h_{oe} R_c}{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_c} \right) \\ &= \frac{-h_{fe} R_c}{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_c} \end{aligned} \tag{1.39}$$

The input resistance looking into the base of the transistor is given by

$$\begin{aligned} R_{inb} &= \frac{v_{be}}{i_b} \\ &= \frac{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_c}{1 + h_{oe} R_c} \end{aligned} \tag{1.40}$$

1.3.3 Base Width Modulation

Discussion on Early voltage, origins and SPICE model

Figure 1.24: Early voltage

1.3.4 Hybrid-π Model

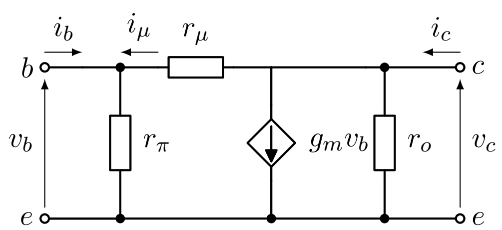


Figure 1.25: Hybrid-π equivalent circuit model.

The hybrid-π model (Fig. 1.25) is derived from small-signal considerations of a dynamic Ebers-Moll model, that is one where charge storage is taken into account.

The intrinsic parameters of the model are r_π and g_m. In addition, the resistance r_μ models the effect of the collector voltage on base current and r_o models the effect of the collector voltage on the collector current in the active mode. The hybrid-π model is connected to the h-parameters model through a set of defining equations. Refer to Fig. 1.19 and Fig. 1.25. Firstly note that $v_b \equiv v_{be}$ and $v_c \equiv v_{ce}$. For the hybrid-π model, we have

$$v_b = (i_b + i_\mu) r_\pi \tag{1.41}$$

$$v_c = (i_c - g_m v_b - i_\mu) r_o \tag{1.42}$$

$$i_\mu = \frac{v_c - v_b}{r_\mu} \tag{1.43}$$

Eliminating i_μ from these equations and rearranging yields

$$v_b = i_b \frac{r_\pi r_\mu}{r_\pi + r_\mu} + v_c \frac{r_\pi}{r_\pi + r_\mu} \tag{1.44}$$

$$i_c = i_b \frac{r_\pi r_\mu}{r_\pi + r_\mu} \left(g_m + \frac{1}{r_\mu} \right) + v_c \left(\frac{r_\pi}{r_\pi + r_\mu} \left(g_m + \frac{1}{r_\mu} \right) + \frac{1}{r_\mu} + \frac{1}{r_o} \right) \tag{1.45}$$

Therefore, comparing with h-parameter equations

$$v_b = i_b h_{ie} + v_c h_{re} \tag{1.46}$$

$$i_c = i_b h_{fe} + v_c h_{oe} \tag{1.47}$$

gives

$$h_{ie} = \frac{r_\pi r_\mu}{r_\pi + r_\mu} = r_\pi || r_\mu \tag{1.48}$$

$$h_{re} = \frac{r_\pi}{r_\pi + r_\mu} = \frac{r_\pi || r_\mu}{r_\mu} \tag{1.49}$$

$$h_{fe} = (r_\pi || r_\mu)(g_m + \frac{1}{r_\mu}) \tag{1.50}$$

$$h_{oe} = \frac{r_\pi || r_\mu}{r_\mu} \left(g_m + \frac{1}{r_\mu} \right) + \frac{1}{r_\mu} + \frac{1}{r_o} \tag{1.51}$$

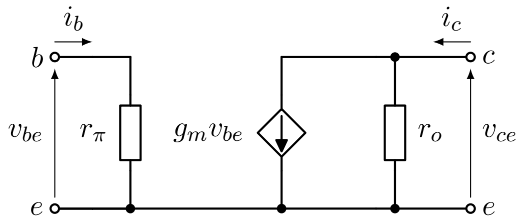
While the hybrid-π model shown in Fig. 1.25 may appear rather complicated, a simplified version often suffices for hand calculations. From the h-parameters, the value of h_{re} is very small and can be neglected. Thus, taking $h_{re} = 0$ means that $h_{ie} \equiv r_\pi$. (Since $r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}$ therefore h_{ie} is inversely proportional to instantaneous collector current.)

The resistance r_μ can also be ignored (having infinite resistance) in many simple situations. In other words, it is assumed that the base is electrically isolated from the collector. (For example, typical values of $h_{re} = 10^{-10}$ and $h_{ie} = 10^3$ would give $r_\mu = \frac{h_{ie}}{h_{re}} = 10^{13}\ \Omega$.) If it is also assumed that $h_{oe} \approx 0$ then $r_o = \frac{1}{h_{oe}} = \infty$ and the output resistance can be removed. With these simplifications, the simplified hybrid-π model is obtained as shown in Fig. **??** together with the simplified h-parameters form.

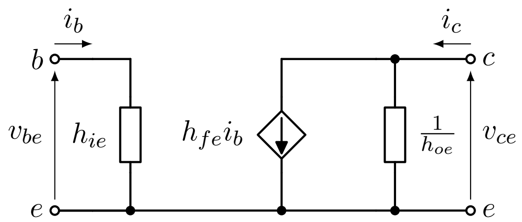
While a careful distinction has been between these two circuit models, parameters from the one of them may appear in the other. This should not cause undue confusion, provided the above assumptions are borne in mind. Furthermore, it is useful to know (assuming $h_{re} = 0$) that

$$g_m = \frac{h_{fe}}{h_{ie}} \tag{1.52}$$

.



(a) Simplified hybrid-π



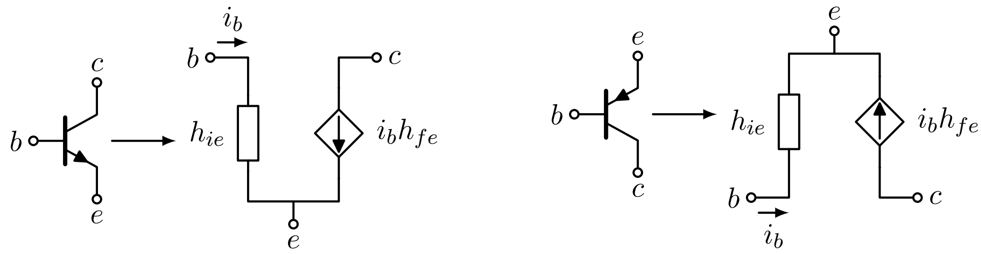
(b) Simplified h-parameters

Figure 1.26: Simplified hybrid-π and h-parameters equivalent circuits. In each model, the output resistance may also be ignored in some circumstances.

1.3.5 Simplified Small-Signal Equivalent Circuit

Valid at low frequencies. Assuming that $h_{re} = 0$ and $h_{oe} = 0$, the small-signal h-parameters model is shown in Fig.1.27.

Often h_{ie} is not provided. The value can be obtained using:



(a) NPN model

(b) PNP model

Figure 1.27: Simplified h-parameters model of a BJT

$$g_m = \frac{I_C}{V_T} \tag{1.53}$$

$$g_m = \frac{h_{fe}}{h_{ie}} \tag{1.54}$$

where I_C is the *dc* collector current.
The hybrid-π model is related to the h-parameters model by

$$r_\pi = h_{ie} \tag{1.55}$$

$$g_m v_{be} = i_b h_{fe} \tag{1.56}$$

and equation (1.54).

Chapter 2

BJT Amplifiers

2.1 Linear Amplifiers

When the gain of an amplifier is stated, this is usually taken as the power gain which is defined as the ratio of the output signal power to the input signal power:

$$A = \frac{p_o}{p_i} \tag{2.1}$$

where subscripts 'o' and 'i' mean output and input respectively. We may also refer to voltage gain A_v and current gain A_i which are defined as:

$$A_v = \frac{v_o}{v_i} \tag{2.2}$$

$$A_i = \frac{i_o}{i_i} \tag{2.3}$$

Consider the circuit shown in Fig. 2.1 where the dc base voltage, that is the circuit operating point, has been set to $V_B = 0.7\,\mathrm{V}$. A small-signal ac source ($v_b(t)$) is connected in series with the base dc voltage source so that the total base voltage is given by

$$v_B(t) = \overline{V}_B + v_b(t). \tag{2.4}$$

The total voltage at the collector is then

$$v_C(t) = \overline{V}_C + v_c(t). \tag{2.5}$$

The instantaneous variations in the collector voltage as a function of instantaneous changes in the base voltage as shown in Fig. 1.8 is depicted again in Fig. 2.2 expanded about the region around the Q-point. The relationship between the two quantities is almost linear. For small changes in the base voltage, a straight-line

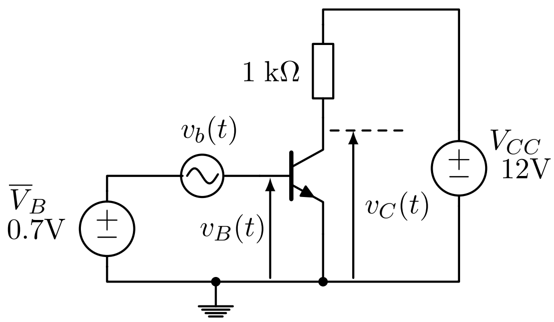


Figure 2.1: Linear BJT amplifier from the crude biasing example.

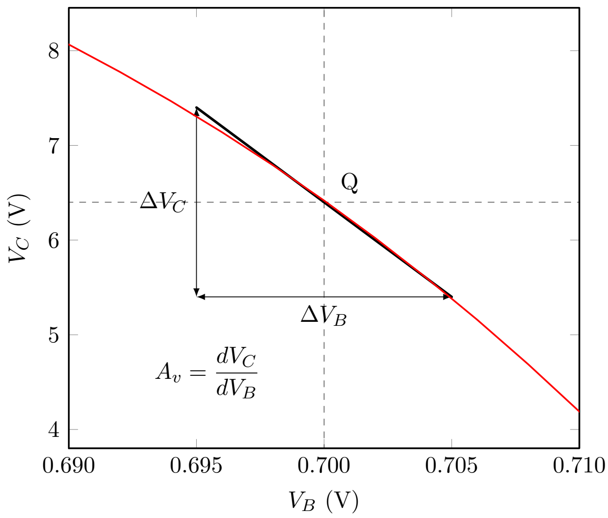


Figure 2.2: Definition of voltage gain as the slope of the instantaneous collector voltage with respect to the instantaneous base voltage.

approximation would be acceptable. We will adopt this so-called small-signal operating condition and also take it for granted that ac sources are sinusoidal and that all signals are in steady state.

The voltage gain A_v is defined as the ratio of the changes in these small-signals:

$$A_v = \frac{dV_C}{dV_B}. \tag{2.6}$$

In other words, if the base voltage changes by ΔV_B then the collector voltage will vary approximately by $\frac{dV_C}{dV_B}\Delta V_B$. The change in the collector voltage is approximately given since as can be seen from the graph, it is actually a little bit smaller (the red curve is below the straight line). Moreover, since the plot is curved, the voltage gain is also affected by the choice of operating point. In this example, the voltage gain is

$$A_v \approx \frac{7.4 - 5.4}{0.695 - 0.705} = -200.$$

The minus sign means that if the input signal was a sine wave, the output signal will be a sine wave shifted by 180° as shown in Fig. 2.3. Note that the ratio of the instantaneous values at the operating point, that is the ratio of the dc values, is *not* the gain.

$$A_v \neq \frac{\overline{V}_C}{\overline{V}_B} = \frac{6.4}{0.7} = 9.14$$

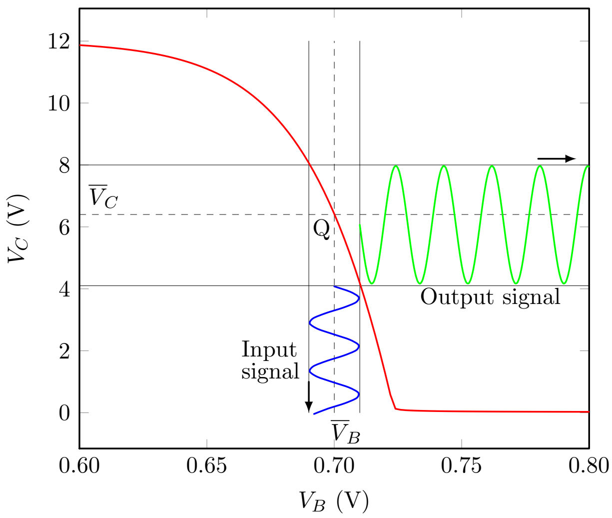
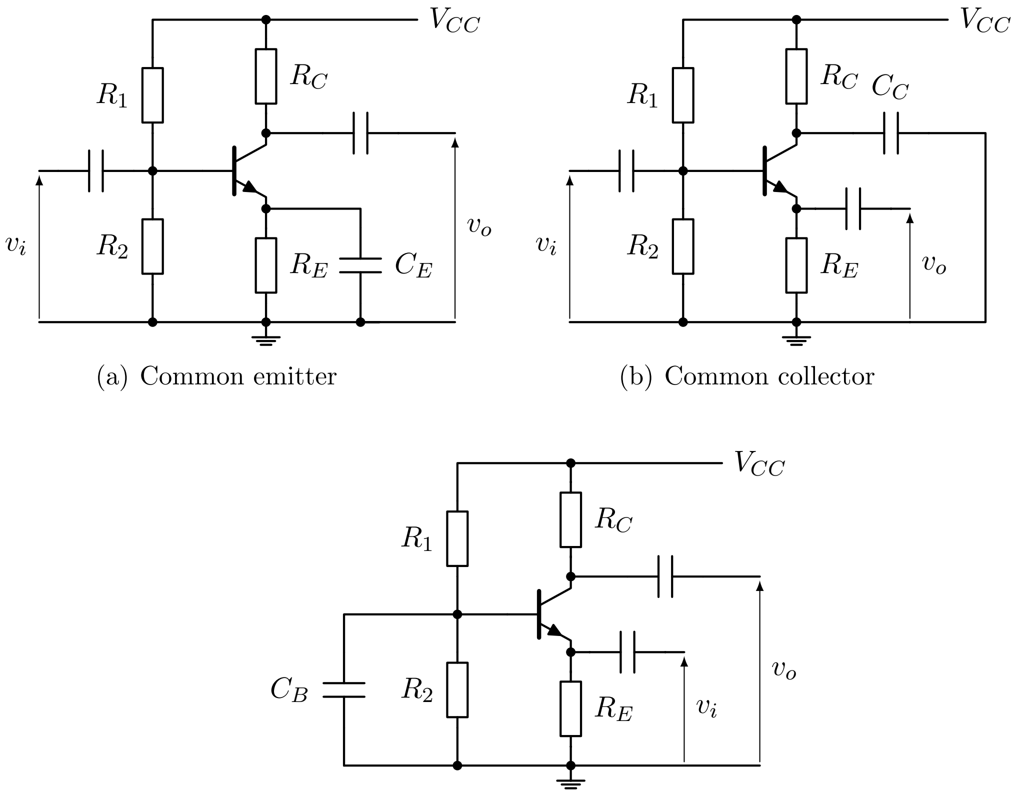


Figure 2.3: Instantaneous variation in collector voltage V_C with respect to base voltage V_B. Superimposed upon the plot are the operating point or quiescent point and time-dependent input ($v_B(t)$) and output ($v_C(t)$) signals. Note the difference in scale along the axes.

2.2 Auto-biased BJT Amplifier Circuits

Using the auto-biasing scheme, there are three common BJT amplifier circuits as depicted in Fig. 2.4. These circuits are called common emitter, common base and common collector depending on the transistor pin that is grounded possibly via a resistance and/or capacitance.

In the circuits shown, the pins are connected to ground by so-called bypass or decoupling capacitors. There are also capacitors connected to the pins which have been designated as input and output. These capacitors function as ac coupling so that in a multistage amplifier, dc levels of different stages will not be corrupted. From a dc point of view (remove all capacitors), all three circuits are identical for the purposes of calculating dc biasing. These circuits have different voltage gain, current gain, input resistance and output resistance, and as such will be the subject of our investigation. (For the common collector circuit, the resistor R_C is usually omitted.)



(a) Common emitter

(b) Common collector

(c) Common base

Figure 2.4: Auto-biased amplifier stages with ac coupling capacitors (unmarked) and bypass or decoupling capacitors (C_E, C_C and C_B). The port marked v_i is input; the port marked v_o is output.

For analysis in the small-signal steady-state sinusoidal domain, the simplified Ebers-Moll model is adopted. In the Ebers-Moll model,

$$i_C(t) = I_S(e^{v_{BE}(t)/V_T} - 1)$$

which can be simplified by assuming that $e^{v_{BE}(t)/V_T} >> 1$ to obtain

$$i_C = I_S e^{v_{BE}/V_T} \tag{2.7}$$

where for convenience time-dependence of ac or total quantities are implicitly assumed and (t) dropped from expressions. Also, a total variable is the sum of a dc (or average) part and a small-signal ac part, i.e.

$$\begin{aligned}
i_C &= I_C + i_c \\
v_{BE} &= V_{BE} + v_{be} \\
v_C &= V_C + v_c \\
i_B &= I_B + i_b
\end{aligned}$$

Consider small changes about the operating point defined by (I_C, V_{BE}, I_B, V_C). Differentiating (2.7) with respect to v_{BE}:

$$\begin{aligned}
\left.\frac{di_C}{dv_{BE}}\right|_{v_{BE}=V_{BE}} &= \left.\frac{I_S}{V_T}e^{v_{BE}/V_T}\right|_{v_{BE}=V_{BE}} \\
&= \frac{I_S}{V_T}e^{V_{BE}/V_T} \\
&= \frac{I_C}{V_T} \qquad (2.8)
\end{aligned}$$

Let us define the right-hand side expression as a transconductance

$$g_m \triangleq \frac{I_C}{V_T} \qquad (2.9)$$

which can be obtained from the dc value of the collector current (I_C).

In the small-signal regime about the operating point

$$\begin{aligned}
\Delta i_C &= i_c, \\
\Delta v_{BE} &= v_{be}
\end{aligned}$$

and

$$\frac{di_C}{dv_{BE}} = \frac{\Delta i_C}{\Delta v_{BE}} = \frac{i_c}{v_{be}}$$

Therefore,

$$\frac{di_C}{dv_{BE}} = \frac{i_c}{v_{be}} = \frac{I_C}{V_T} = g_m$$

and

$$i_c = g_m v_{be}. \qquad (2.10)$$

The collector voltage is

$$\begin{aligned}
v_C &= V_{CC} - i_C R_C \\
&= V_{CC} - (I_C + i_c)R_C \\
&= (V_{CC} - I_C R_C) - i_c R_C. \qquad (2.11)
\end{aligned}$$

Thus the dc component of the collector voltage is

$$V_C = V_{CC} - I_C R_C$$

and the ac part is

$$v_c = -i_c R_C. \tag{2.12}$$

Using (2.10)

$$v_c = -g_m v_{be} R_C. \tag{2.13}$$

The collector current is related to the base current by $i_c = \beta i_b$ which means that

$$\begin{aligned} i_b &= \frac{i_c}{\beta} \\ i_b &= \frac{g_m v_{be}}{\beta} \\ \Rightarrow v_{be} &= \frac{\beta}{g_m} i_b \end{aligned} \tag{2.14}$$

where (2.10) has been used. This is an Ohmic expression that relates the base-emitter voltage to the base current. Hence, the proportionality constant is an ac input resistance which is defined as

$$r_\pi = \frac{\beta}{g_m}. \tag{2.15}$$

Taken together, these results allow the construction of a simplified small-signal ac equivalent circuit (see Fig. 2.5) which will be referred to as the simplified π-model.

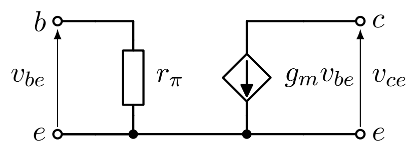


Figure 2.5: Simplified small-signal ac equivalent circuit of a BJT.

2.2.1 Common-Emitter Amplifier

The small-signal ac equivalent circuit of the common-emitter amplifier is shown in Fig. 2.6. The BJT has been replaced by its simplified small-signal ac equivalent circuit that was discussed in the previous section. For ac analysis, superposition is used to eliminate all dc sources. The parameters g_m and r_π are obtained from dc using (2.9) and (2.15) respectively. For the latter, β need to be known.

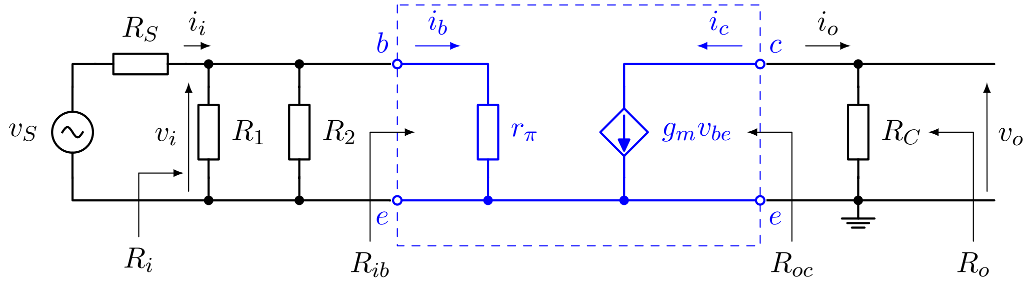


Figure 2.6: Small-signal equivalent circuit of common emitter BJT amplifier.

The voltage gain is

$$A_v = \frac{v_o}{v_i} = \frac{i_o R_c}{v_{be}} = \frac{-g_m \cancel{v_{be}} R_c}{\cancel{v_{be}}} = -g_m R_c \tag{2.16}$$

The current gain is

$$A_i = \frac{i_o}{i_b} = \frac{-g_m v_{be}}{v_{be}/r_\pi} = -\beta \tag{2.17}$$

The input resistance is

$$R_{ib} = \frac{v_{be}}{i_b} = r_\pi \tag{2.18}$$

The output resistances are

$$R_{oc} = \infty$$

$$R_o = R_c$$

Assuming typical values for $I_C = 25$ mA and $V_T = 26$ mV

$$g_m = \frac{2.5}{0.026} \approx 100\,\text{mA/V}$$

Taking $\beta = 100$ and $R_c = 1\,\text{k}\Omega$

$$A_v = -100 \times 1 = -100$$

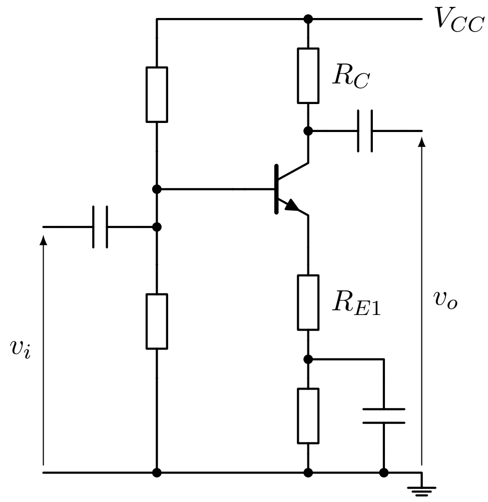
$$A_i = -100$$

$$\text{Powergain} = A_v \times A_i = 10^4$$

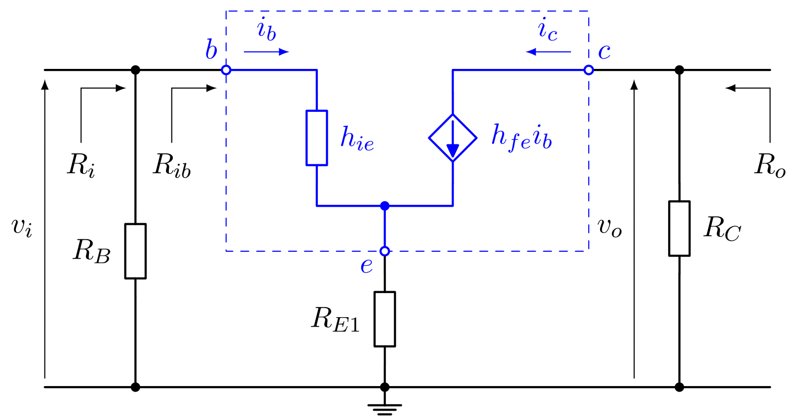
$$R_{ib} = \frac{100}{100} = 1\,\text{k}\Omega$$

$$R_o = 1\,\text{k}\Omega$$

A variation of the common-emitter amplifier circuit is shown in Fig. 2.7. An additional resistor (R_{E1}) is connected in series with the emitter resistor but is not shunted by a capacitor. In other words, R_{E1} is "undecoupled" from the emitter



(a) Actual circuit



(b) Small-signal equivalent circuit

Figure 2.7: Common emitter amplifier with un-decoupled emitter resistance.

circuit. For small-signal equivalent circuit analysis, assume that the shunt emitter capacitance is ideal, i.e. it shorts the emitter resistor, and that $h_{re} = 0$ and $h_{oe} = 0$. The input and output voltages are given by

$$v_o = -i_c R_c = -h_{fe} i_b R_c \tag{2.19}$$

$$v_i = i_b h_{ie} + i_b(1 + h_{fe})R_{E1} \tag{2.20}$$

From the second equation, the input resistance can be determined:

$$R_{ib} = \frac{v_i}{i_b} = h_{ie} + (1 + h_{fe})R_{E1} \tag{2.21}$$

$$R_i = R_{ib}||R_B \tag{2.22}$$

(The input resistance R_{ib} is high, similar to that of the emitter follower circuit that will be covered shortly.) The voltage gain is

$$\begin{aligned} A_v = \frac{v_o}{v_i} &= \frac{-h_{fe} i_b R_c}{i_b h_{ie} + i_b(1+h_{fe})R_{E1}} \\ &= \frac{-R_C}{R_{E1} + \frac{h_{ie}+R_{E1}}{h_{fe}}} \\ &\approx -\frac{R_C}{R_{E1}} \end{aligned} \tag{2.23}$$

where in the last line it is assumed that $R_{E1} \gg \frac{h_{ie}+R_{E1}}{h_{fe}}$ usually.

The output resistance R_o of an amplifier (see Fig. 2.8) is defined as an equivalent Thevenin resistance looking *into* the output of the amplifier. In general, R_o is not the same as the load resistance $R_L = \frac{V_o}{I_o}$.

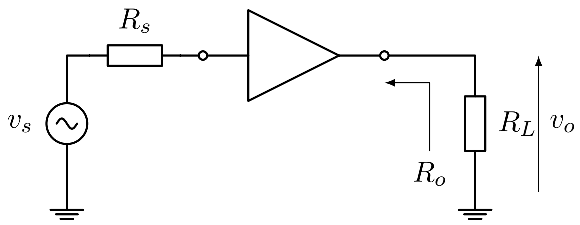


Figure 2.8: Definition of output resistance R_o of an amplifier.

The output resistance can be determined by three different methods (see Fig. 2.9):

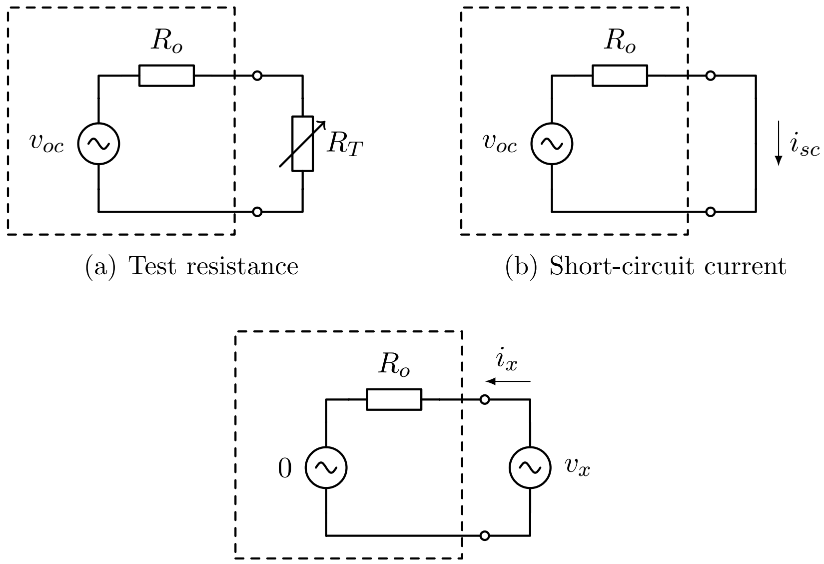
1. Tune an external test load resistance at the output, until the resultant output voltage is half the open-circuit voltage (v_{oc}) whereupon $R_o = R_{test}$.
2. Calculate the open circuit voltage and short circuit current. $R_o = v_{oc}/i_{sc}$
3. Injection of source (current or voltage) at output and short-circuiting all voltage signal sources. $R_o = v_x/i_x$

Consider the application of method 1 in determining the output resistance of the undecoupled common emitter amplifier. From voltage gain with no external load connected, i.e. open circuit,

$$v_o \approx -\frac{R_C}{R_E} v_i \tag{2.24}$$

which means that the open-circuit voltage is

$$v_{oc} = -\frac{R_C}{R_E} v_i \tag{2.25}$$



(a) Test resistance (b) Short-circuit current

(c) Injection

Figure 2.9: Methods for determining amplifier output resistance. The amplifier respresented by its Thevenin equivalent circuit (inside dashed box).

When an external test load (R_T) is placed at the output, it is in parallel with R_C according to the small-signal equivalent circuit. Consequently the voltage gain equation gives

$$v_o = -\frac{R_C||R_T}{R_E}v_i \tag{2.26}$$

Suppose R_T is tuned until $v_o = \frac{1}{2}v_{oc}$ and let the value of R_T which satisfies the voltage condition be R_T'. Substitute these into the above equation:

$$\begin{aligned}
\frac{1}{2}v_{oc} &= -\frac{R_C||R_T'}{R_E}v_i \\
-\frac{R_C}{2R_E}v_i &= -\frac{R_C R_T'}{(R_C + R_T')R_E}v_i \\
\frac{R_C + R_T'}{R_T'} &= 2 \\
R_T' &= R_C \qquad (2.27)
\end{aligned}$$

Therefore the output resistance $R_o = R_C$.

2.2.2 Common-Collector Amplifier

The small-signal equivalent circuit of the common-collector amplifier or *emitter follower* is shown in Fig. 2.10. The voltage gain can be obtained as follows:

$$
\begin{aligned}
v_i &= i_b r_\pi + (g_m v_{be} + i_b) R_E \\
v_{be} &= i_b r_\pi \Rightarrow g_m v_{be} = g_m i_b r_\pi = \beta i_b \qquad (2.28) \\
\Rightarrow v_i &= i_b r_\pi + (\beta i_b + i_b) R_E \\
&= (r_\pi + (1+\beta) R_E) i_b \qquad (2.29) \\
v_o &= (1+\beta) i_b R_E \\
\therefore A_v &= \frac{v_o}{v_i} \\
&= \frac{(1+\beta) R_E}{r_\pi + (1+\beta) R_E} \\
&= 1 - \frac{r_\pi}{r_\pi + (1+\beta) R_E}
\end{aligned}
$$

Assuming typical values $r_\pi = R_E = 1\,\text{k}\Omega$ and $\beta = 100$:

$$A_v = 1 - \frac{1}{1 + (1+100)1} \approx 1$$

Thus the voltage at the output is approximately the same as the voltage at the input.

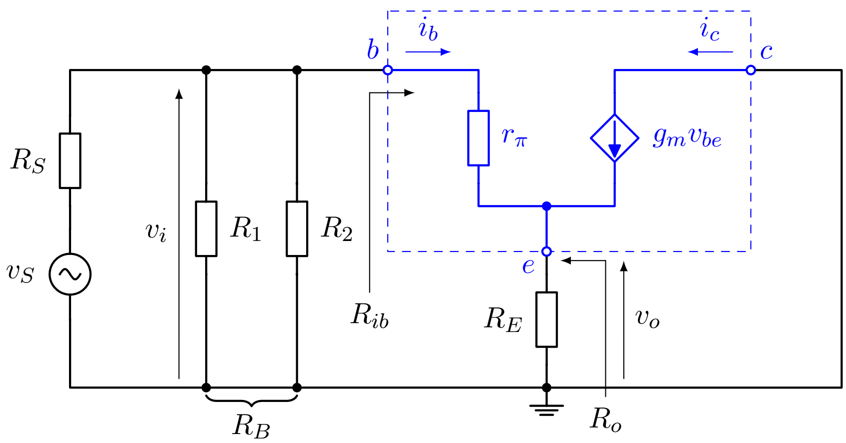


Figure 2.10: Small-signal ac equivalent circuit of common collector amplifier.

Using (2.29), the resistance looking into the base is

$$R_{ib} = \frac{v_i}{i_b} = r_\pi + (1+\beta) R_E$$

which is typically $R_{ib} = 1 + (101) \times 1 \approx 100\,\text{k}\Omega$.

The current gain is

$$A_i = \frac{i_o}{i_b} = \frac{i_b(1+\beta)}{i_b} = 1 + \beta = (\text{typically}100)$$

and the power gain is

$$A_p = A_v \times A_i = (\text{typically}100)$$

The output resistance of the common-collector amplifier can be obtained using method 2. For this exercise, the small-signal ac analysis will be repeated using the simplified h-parameters equivalent circuit model as shown in Fig. 2.11 where the input circuit consisting of the voltage source, source resistance and the biasing resistances have been replaced by its Thevenin equivalent. The Thevenin parameters are defined by

$$v_s' = \frac{R_B}{R_B + R_s} v_s \tag{2.30}$$

$$R_s' = R_s || R_B \tag{2.31}$$

The output voltage is given by

$$v_o = i_b(1 + h_{fe})R_E \tag{2.32}$$

By KVL, the Thevenin equivalent input voltage source is also

$$\begin{aligned} v_s' &= i_b(R_s' + h_{ie}) + (1 + h_{fe})i_b R_E \\ &= i_b(R_s' + h_{ie} + (1 + h_{fe})R_E) \end{aligned} \tag{2.33}$$

Therefore

$$v_o = \frac{(1 + h_{fe})R_E}{R_s' + h_{ie} + (1 + h_{fe})R_E} v_s' \tag{2.34}$$

where the proportionality constant is typically ≈ 1.

The input resistance is given by

$$R_i = R_{ib} || R_B \tag{2.35}$$

which is somewhat easier to discern from Fig. 2.10. The resistance looking into the base is

$$R_{ib} = \frac{v_i}{i_b} = \frac{i_b(h_{ie} + (1 + h_{fe})R_E)}{i_b} \tag{2.36}$$

$$= h_{ie} + (1 + h_{fe})R_E \tag{2.37}$$

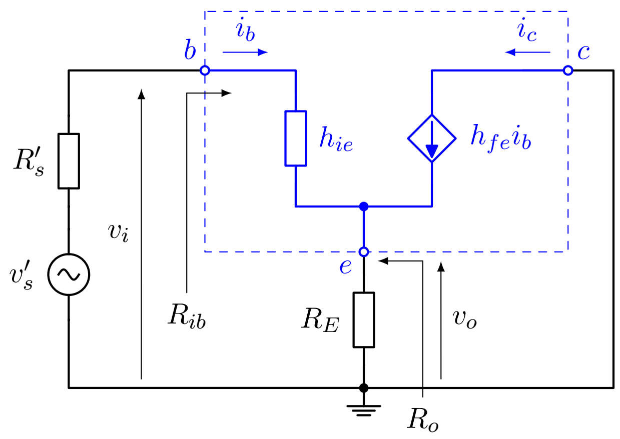


Figure 2.11: Small-signal analysis using simplified h-parameters equivalent circuit.

Using typical values of transistor parameters and $R_1 = 10$ k, $R_2 = 10$ k ($R_B = 5$ k) yields $R_{ib} = 102$ k and $R_i = 4.8$ k. While the value of R_{ib} is relatively high, the smaller R_B spoils the high input resistance. This problem can be ameliorated by "bootstrapping" as discussed in later section.

For determining the output resistance using method 2, note that the open-circuit voltage is given by (2.34). Using method 2 for determining the output resistance, let i_{sc} be the short circuit current through the output when it is short-circuited. Under this condition, let the base current $i_b = i_{b,sc}$. Hence by KCL

$$i_{sc} = i_{b,sc}(1 + h_{fe}) \tag{2.38}$$

By KVL

$$i_{b,sc} = \frac{v'_s}{R'_s + h_{ie}} \tag{2.39}$$

Thus

$$i_{sc} = \frac{v'_s(1 + h_{fe})}{R'_s + h_{ie}} \tag{2.40}$$

Therefore the output resistance is

$$\begin{aligned} R_o &= \frac{v_{oc}}{i_{sc}} \\ &= \frac{\frac{v'_s(1+h_{fe})R_E}{R'_s+h_{ie}+(1+h_{fe})R_E}}{\frac{v'_s(1+h_{fe})}{R'_s+h_{ie}}} \\ &= \frac{R_E(R'_s + h_{ie})}{R'_s + h_{ie} + (1 + h_{fe})R_E} \end{aligned} \tag{2.41}$$

This expression is in the form of two resistances in parallel and can be rewritten as

$$\frac{1}{R_o} = \frac{1}{R_E} + \frac{1+h_{fe}}{R_s' + h_{ie}} \tag{2.42}$$

that is

$$R_o = R_E || \left(\frac{R_s' + h_{ie}}{1+h_{fe}} \right) \tag{2.43}$$

However, $R_E \gg$ than the second resistive term and the output resistance can be taken as

$$R_o \approx \frac{R_s' + h_{ie}}{1+h_{fe}} \tag{2.44}$$

Using typical values: $h_{ie} = 1$ k, $h_{fe} = 100$, $R_E = 1$ k and $R_s = 0$ gives $R_o = 1\text{k}\Omega || \frac{1}{101}\ \text{k}\Omega \approx 9.7\Omega$. For an ideal buffer, a high input resistance (R_{ib} high), unity voltage gain ($\frac{v_o}{v_i} \approx 1$) and low output resistance (R_o small) are desirable characteristics.

2.2.3 Common-Base Amplifier

The small-signal equivalent circuit of the common-base amplifier is shown in Fig. 2.12.

Voltage gain

$$\begin{aligned} v_i &= -v_{be} \\ v_o &= -i_c R_c = -g_m v_{be} R_c \\ \therefore A_v &= \frac{v_o}{v_i} = \frac{-g_m \cancel{v_{be}} R_c}{-\cancel{v_{be}}} = g_m R_c \end{aligned}$$

which is the same as the collector amplifier gain except for a change in sign; typically $100 \times 1 \approx 100$.

Current gain

$$A_i = \frac{i_o}{-i_e} = \frac{-\beta i_b}{-i_e} = \frac{\beta i_b}{(1+\beta)i_b} = \frac{\beta}{1+\beta}$$

where emitter current flows out of the emitter, a negative sign has been prefixed in the denominator to get correct gain; typically $\frac{100}{101} \approx 1$.

Power gain

$$A_p = A_v \times A_i = 100 \times 1 = 100$$

In order to determine the output resistance, the injection method will be applied (method 3). Recall that in this method, all independent input voltage sources are zeroed and a voltage source is applied to the output of the circuit. The common

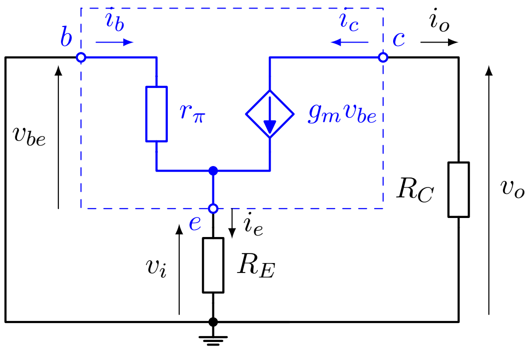


Figure 2.12: Small-signal equivalent circuit of common-base amplifier.

base amplifier typically has high output resistance. Thus in selecting an appropriate small-signal equivalent circuit for determining output resistance, the intrinsic output resistance of the BJT cannot be neglected. Therefore, unlike the earlier equivalent circuit, the output resistance $\frac{1}{h_{oe}}$ is reinstated as shown in Fig. 2.13.

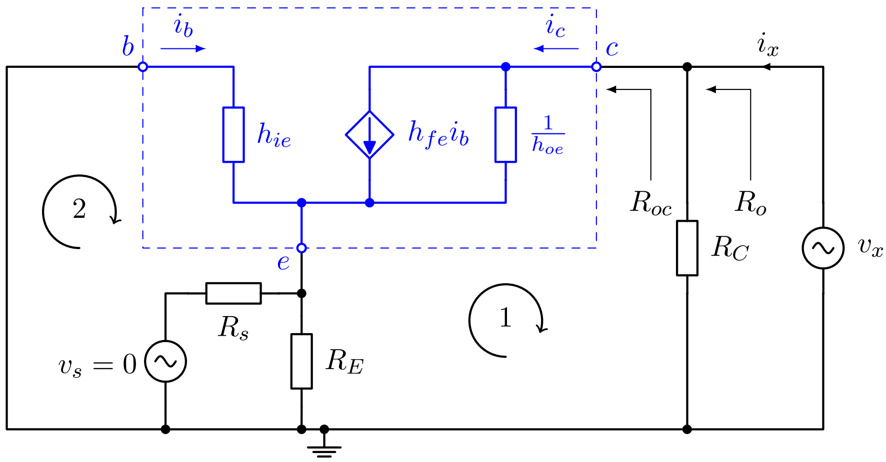


Figure 2.13: Small-signal h-parameters equivalent circuit of common base amplifier for output resistance analysis.

Applying KVL on loop 1 yields:

$$
\begin{aligned}
v_x &= (i_b + i_c)R'_E + (i_c - h_{fe}i_b)\tfrac{1}{h_{oe}} \\
&= i_c\left(R'_E + \frac{1}{h_{oe}}\right) + i_b\left(R'_E - \frac{h_{fe}}{h_{oe}}\right) \qquad (2.45)
\end{aligned}
$$

Applying KVL on loop 2:

$$\begin{aligned} -i_b h_{ie} - (i_b + i_c)R'_E &= 0 \\ i_b(h_{ie} + R'_E) &= -i_c R'_E \\ i_b &= \frac{R'_E}{h_{ie} + R'_E} i_c \end{aligned} \tag{2.46}$$

Substituting (2.46) into (2.45):

$$v_x = i_c \left(R'_E + \frac{1}{h_{oe}} - \frac{R'_E}{h_{ie} + R'_E} \left(R'_E - \frac{h_{fe}}{h_{oe}} \right) \right) \tag{2.47}$$

Therefore the resistance looking into the collector is

$$\begin{aligned} R_{oc} &= \frac{v_x}{i_c} \\ &= \frac{1}{h_{oe}} \left(1 + \frac{h_{fe} R'_E}{h_{ie} + R'_E} \right) + R'_E \left(1 - \frac{R'_E}{h_{ie} + R'_E} \right) \\ &= \frac{1}{h_{oe}} \left(1 + \frac{h_{fe}}{1 + \frac{h_{ie}}{R'_E}} \right) + \frac{R'_E h_{ie}}{h_{ie} + R'_E} \end{aligned} \tag{2.48}$$

The second term is clearly $R'_E || h_{ie}$. Since

$$(R'_E || h_{ie}) \ll \frac{1}{h_{oe}} \left(1 + \frac{h_{fe}}{1 + \frac{h_{ie}}{R'_E}} \right) \tag{2.49}$$

$$\therefore R_{oc} \approx \frac{1}{h_{oe}} \left(1 + \frac{h_{fe}}{1 + \frac{h_{ie}}{R'_E}} \right) \tag{2.50}$$

where $R'_E = (R_s || R_E)$.

Suppose $R_s \ll h_{ie}$,

$$\Rightarrow R_{oc} = \frac{1}{h_{oe}} \left(1 + \frac{h_{fe}}{\infty} \right) = \frac{1}{h_{oe}}$$

If $R'_E \gg h_{ie}$,

$$\Rightarrow R_{oc} = \frac{1}{h_{oe}} \left(1 + \frac{h_{fe}}{1 + 0} \right) = \frac{1 + h_{fe}}{h_{oe}}$$

Taking typical values of $\frac{1}{h_{oe}} = 100\ \mathrm{k}\Omega$ and $h_{fe} = 100$, bounds the output collector resistance by

$$10\ \mathrm{k} < R_{oc} < (101 \times 100\ \mathrm{k} = 10.1\ \mathrm{M}\Omega)$$

The upper bound on R_{oc} can be a very high value. The output resistance of the amplifier is $R_o = R_{oc} || R_C$.

A high R_{oc} is desirable when the collector load is a resonant circuit such as used in a tuned amplifier that will be discussed later. (The impedance at resonance of the tuned circuit will be in parallel with R_{oc}, therefore the latter needs to be much higher so that the Q will not be affected.) In practice, the common base amplifier is used in a cascode configuration such as the one shown in Fig. 2.14 where a common emitter amplifier draws a common base amplifier thereby screening the output from the input.

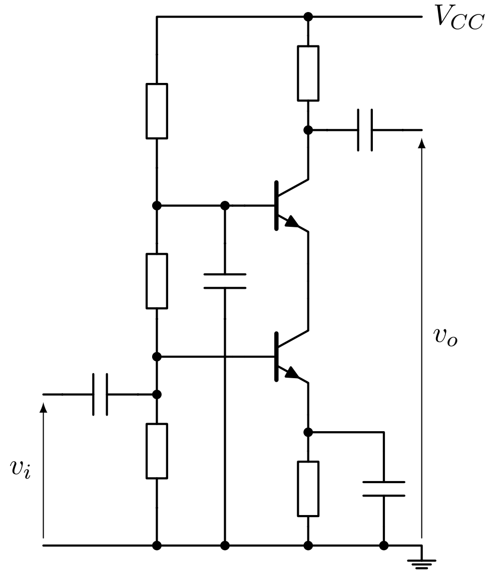


Figure 2.14: Cascode circuit.

2.3 Cascaded Stages

The output of a single amplifier can be fed into the input of a second amplifier to create a so-called cascaded circuit. Aside from improvement in overall gain of the cascaded circuit, it may also benefit from more desirable input-output characteristics inherited from the type of amplifier used in the input and output stages. The general case for N cascaded stages is shown in Fig. 2.15. The input impedance of the following stage (for example stage 2) contributes to the load impedance of the previous stage (for example stage 1). Interstage coupling can be ac or dc. AC coupling is effected by coupling capacitors.

A two-stage common emitter amplifier is shown in Fig. 2.17 where a pnp bjt is used in the second stage. The small-signal equivalent circuit is obtained in the usual manner; the ac equivalent circuit for the pnp based second stage is

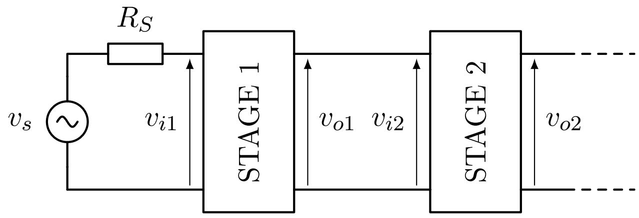
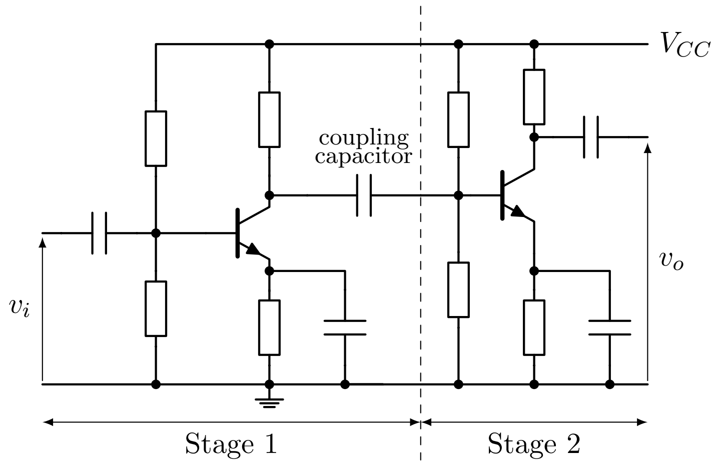
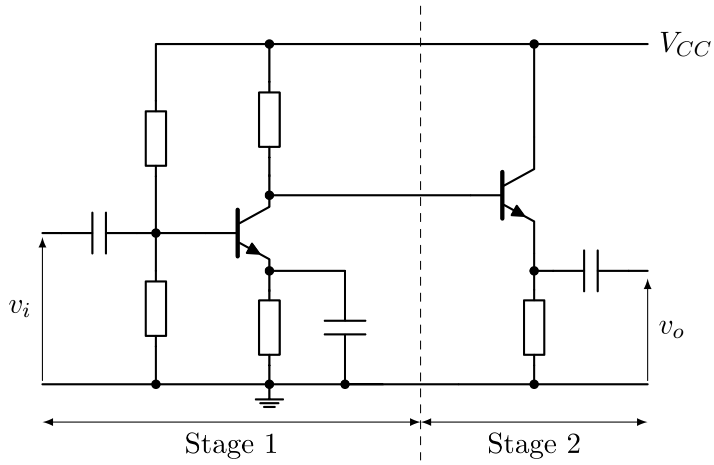


Figure 2.15: N-stage cascaded amplifier with stage 1 and stage 2 shown explicitly.



(a) AC interstage coupling



(b) DC interstage coupling

Figure 2.16: Two stage common emitter amplifier (a) and common emitter buffered by common collector (b) with different interstage coupling mechanisms.

identical to the npn stage. The voltages in the circuit can be related to the circuit parameters by

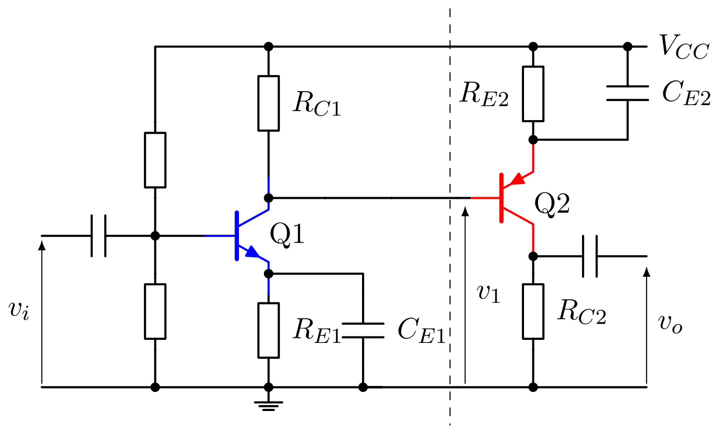
$$v_1 = -g_{m1}(R_{C1}||r_{\pi 2})v_i$$

$$v_o = -g_{m2}R_{C2}v_1$$

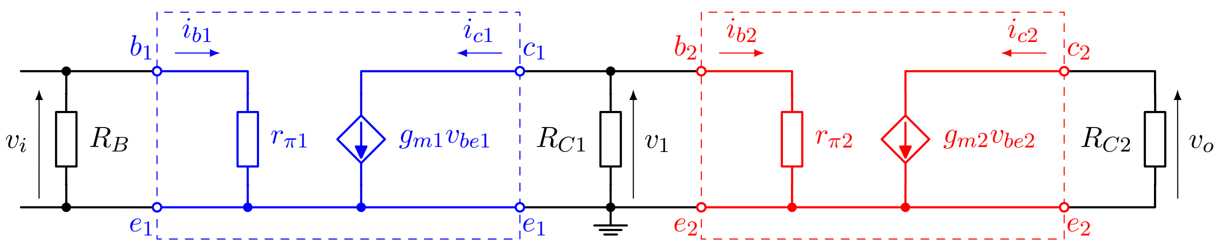
The total voltage gain is given by

$$\frac{v_o}{v_i} = \frac{v_o}{v_1} \times \frac{v_1}{v_i}$$

In this example, the output of the first stage is directly coupled to the second stage input without using an ac coupling capacitor. Quiescent dc levels are arranged so that both transistors are biased as desired. This method is often used in integrated circuits where high coupling capacitors would be difficult to fabricate.



(a) NPN-PNP two stage



(b) Equivalent circuit

Figure 2.17: Multistage common emitter amplifier.

2.4 Bootstrapped Emitter Follower

The bootstrapped emitter follower circuit (see Fig. 2.18) is the same as the common collector amplifier insofar as the voltage gain is concerned, but aims to improve the input resistance by reducing the shunting effect of the high input base resistance by the relatively lower biasing resistances. This is accomplished by the addition of a bootstrap capacitor (C_{boot}) that connects the emitter to the base circuit. The technique of feeding part of the output (at the emitter in this case) to the input (at the base here) is called bootstrapping.

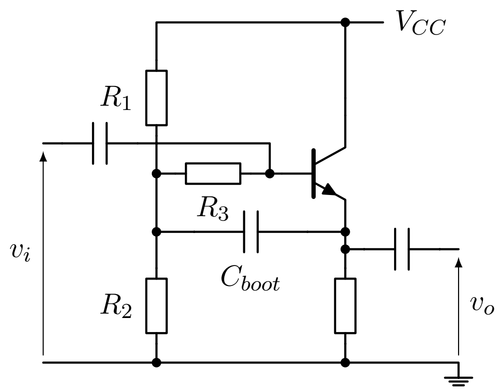


Figure 2.18: Bootstrapped emitter follower.

The small-signal equivalent circuit of the bootstrapped emitter follower is shown in Fig. 2.19. Let the resistances in parallel at the emitter be

$$R'_E = R_1||R_2||R_E \tag{2.51}$$

The input voltage is given by

$$v_i = i_b h_{ie} + (i_i + h_{fe} i_b) R'_E \tag{2.52}$$

The output voltage equation is

$$v_o = (i_i + h_{fe} i_b) R'_E \tag{2.53}$$

By current division

$$i_b = \frac{R_3}{R_3 + h_{ie}} i_i \tag{2.54}$$

Substituting (2.54) into (2.52)

$$v_i = i_i \frac{R_3}{R_3 + h_{ie}} h_{ie} + \left(i_i + i_i \frac{R_3}{R_3 + h_{ie}} h_{fe} \right) R'_E \tag{2.55}$$

Hence, the input resistance is

$$R_i = \frac{v_i}{i_i} = \frac{R_3}{R_3 + h_{ie}} \left(h_{ie} + \left(\frac{R_3 + h_{ie}}{R_3} + h_{fe} \right) R'_E \right) \tag{2.56}$$

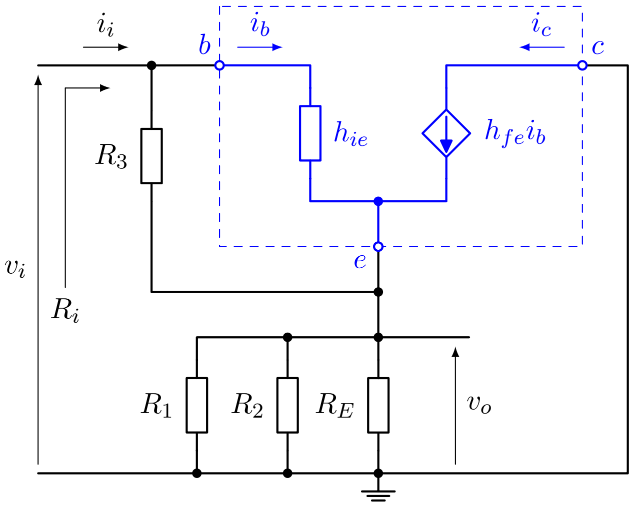


Figure 2.19: Small-signal equivalent circuit of bootstrapped emittter follower.

Example values with $R_3 = 1$ k, $R_E = 1$ k, $R_1 = R_2 = 10$ k, $h_{ie} = 1$ k and $h_{fe} = 100$ gives $R'_E = 0.833$ k and

$$R_i = \frac{1}{1+1}\left(1 + \left(\frac{1+1}{1} + 100\right)0.8333\right) = 43\text{k}\Omega$$

Consider what happens to the input resistance if the bootstrap capacitor is removed. The small-signal equivalent circuit without a bootstrap capacitor is shown in Fig. 2.20. Using the numerical example values, the resistance looking into the base is

$$R_{ib} = h_{ie} + (1 + h_{fe})R_E = 102\text{k}\Omega$$

Therefore the input resistance is

$$R_i = R_{ib}||(5 + 1) = 5.6\text{k}\Omega$$

which is much smaller than the input resistance with the bootstrap capacitor in situ.

The bootstrap capacitor provides a feedback path from the emitter to the base of the transistor via resistor R_3. Miller's theorem is useful in analysis of circuits containing feedback impedances. Before presenting the theorem, let us redraw the bootstrap equivalent circuit as depicted in Fig. 2.21.

First consider the circuit without the feedback resistance R_3. The output voltage is

$$v_o = i_b(1 + h_{fe})R'_E \tag{2.57}$$

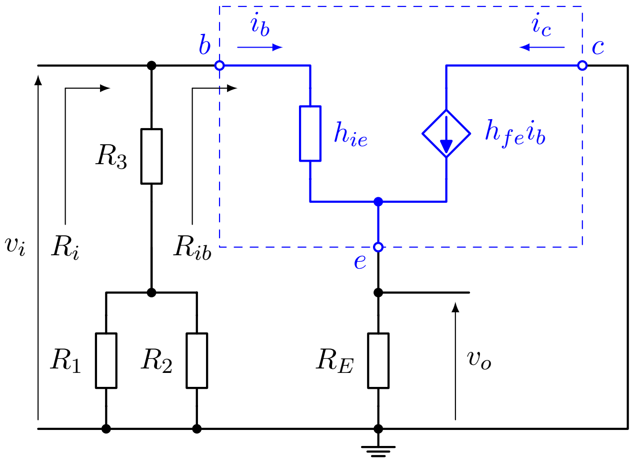
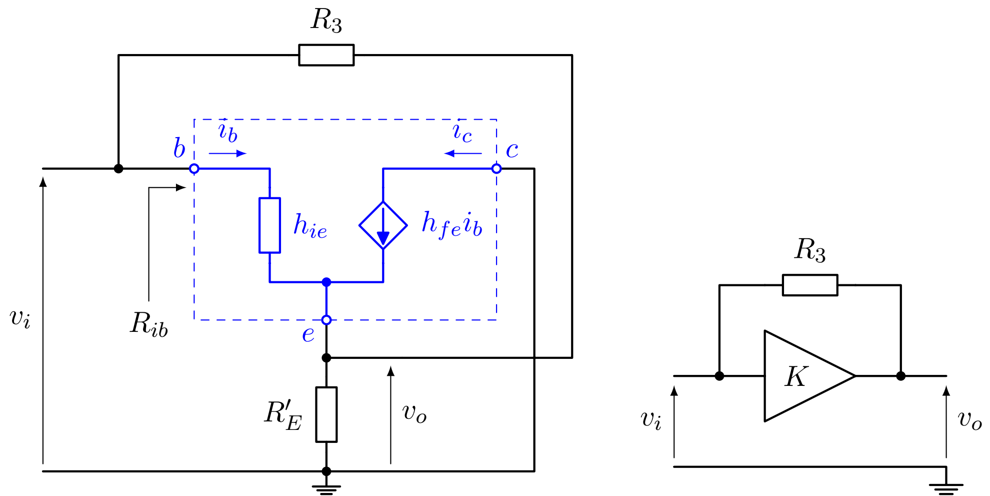


Figure 2.20: Small-signal equivalent circuit of bootstrapped emitter follower circuit with bootstrap capacitor removed.



(a) Equivalent circuit (b) Symbol

Figure 2.21: Bootstrapped emitter follower equivalent circuit redrawn as an amplifier with feedback.

The input voltage is

$$v_i = (h_{ie} + (1 + h_{fe})R'_E)i_b \tag{2.58}$$

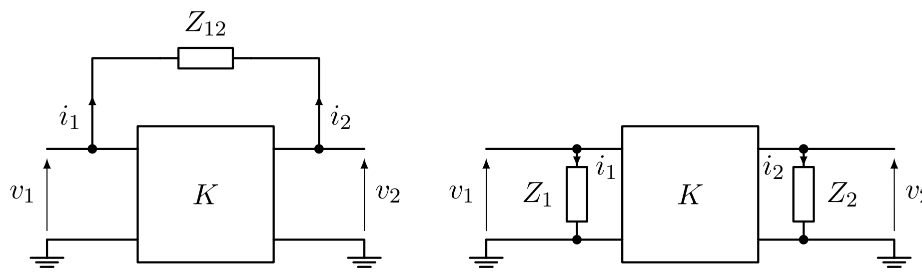
The input resistance looking into the base is

$$R_{ib} = \frac{v_i}{i_b} = h_{ie} + (1 + h_{fe})R'_E \tag{2.59}$$

The voltage gain is

$$\begin{aligned} K = \frac{v_o}{v_i} &= \frac{(1+h_{fe})R'_E}{(1+h_{fe})R'_E + h_{ie}} \\ &= \frac{(1+h_{fe})R'_E + h_{ie} - h_{ie}}{(1+h_{fe})R'_E + h_{ie}} \\ &= 1 - \frac{h_{ie}}{R_{ib}} \end{aligned} \tag{2.60}$$

The second term is typically small and the voltage gain is almost unity. Let $\varepsilon = \frac{h_{ie}}{R_{ib}}$.



(a) Linear two-port (b) Miller equivalent

Figure 2.22: Application of Miller's theorem for linear two-port network having feedback impedance.

The application of Miller's theorem for analysing linear two-port networks having a feedback impedance will now be presented. Refer to Fig. 2.22. Miller's theorem allows the feedback impedance that connects port 1 and port 2 to be replaced by a pair of Miller impedances (Z_1 and Z_2) that are connected from port 1 and port 2 terminals to ground respectively. The approach can be validated by consideration of the currents in the feedback path:

$$i_1 = \frac{v_1 - v_2}{Z_{12}} = \frac{v_1 - v_2}{v_1}\frac{v_1}{Z_{12}} = (1-K)\frac{v_1}{Z_{12}} = \frac{v_1}{\frac{Z_{12}}{(1-K)}} \tag{2.61}$$

where K is the open-loop voltage gain, that is $K = \frac{v_2}{v_1}$. Hence, the branch current i_1 can be represented as being shunted by an equivalent Miller impedance which appears in the denominator of the rightmost expression. Thus,

$$Z_1 = \frac{Z_{12}}{1-K} \tag{2.62}$$

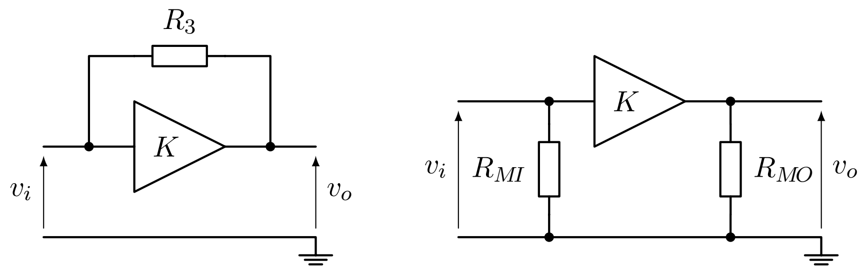
Similarly, consideration of the branch current i_2 yields

$$i_1 = \frac{v_2 - v_1}{Z_{12}} = \frac{v_2 - v_1}{v_1}\frac{v_1}{v_2}\frac{v_2}{Z_{12}} = (K-1)\frac{1}{K}\frac{v_2}{Z_{12}} = \frac{v_2}{\frac{Z_{12}}{\frac{(K-1)}{K}}} \tag{2.63}$$

and

$$Z_2 = \frac{Z_{12}}{\frac{K-1}{K}} = Z_{12}\frac{K}{K-1} \tag{2.64}$$

The Miller equivalent circuit is valid provided that the conditions that existed in the network when K was determined are not changed. For conveninence, we may also refer to the Miller impedances as Z_{MI} and Z_{MO} corresponding to Z_1 and Z_2, the input port Miller impedance and the output port Miller impedance. As an application example of the Miller technique consider the bootstrap emitter follower once more. (See Fig. 2.23; also note that since we're dealing with resistances we use the symbol R instead of the more general symbol Z for impedances.)



(a) Bootstrapped emitter follower (b) Miller equivalent

Figure 2.23: Application of Miller's theorem for the bootstrapped emitter follower circuit.

The Miller resistances are given by:

$$R_{MI} = \frac{R_3}{1-K} \tag{2.65}$$

$$R_{MO} = R_3\frac{K}{K-1} \tag{2.66}$$

Using the numerical values given earlier,

$$K = 1 - \frac{h_{ie}}{R_{ib}} = 1 - \frac{1}{1+101\times 0.833333} = 0.988258$$

$$R_{MI} = \frac{1}{1-0.988258} = 85.1666 \text{ k}\Omega$$

Therefore the input resistance is

$$R_i = R_{MI}||R_{ib} = 85.1666||(1+101\times 0.8333) = 42.6 \text{ k}\Omega$$

The equivalent Miller output resistance is

$$R_{MO} = 1\times\frac{0.988258}{0.988258-1} = -84.2 \text{ k}\Omega$$

A negative resistance can be interpreted as a voltage source. Here, R_{MO} is in parallel with R_o of the emitter follower. Hence, the overall effect is a slight increase in the total output resistance [1].

2.5 Frequency Response

2.5.1 High Frequency Equivalent Circuit Model

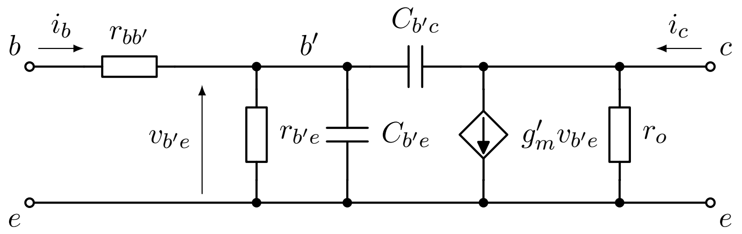


Figure 2.24: Hybrid-π small-signal BJT equivalent circuit with junction capacitances and base spreading resistance included.

For circuit analysis at high frequencies the hybrid-π model needs to be modified to improve accuracy. Two junction capacitances ($C_{b'c}$ and $C_{b'e}$) and a base spreading resistance $r_{bb'}$ are added to the model as shown in Fig. 2.24. (The resistance r_μ is omitted because the reactance of $C_{b'c}$ is usually much smaller.) The base spreading resistance models the high resistance region between the base contact and the active region of the transistor. An external base node is defined and labelled as b and a fictitious internal base node is created called b'. Thus, prior to the model discussed in this section, it is assumed that $r_{bb'} = 0$ and $b = b'$. The resistance $r_{bb'}$ is typically of the order of 10 Ω to 100 Ω and is not negligible. $r_{b'e}$ is dependent on collector current since $g'_m = \frac{I_C}{V_T}$.

$$r_{b'e} = \frac{h_{fe}}{g'_m} = \frac{h_{fe}}{I_C} V_T \tag{2.67}$$

With $h_{fe} = 100$, and $I_C = 1, 10$ and 100 mA, gives $r_{b'e} = 2.6$ k, 260Ω and 26Ω respectively. Clearly, for the last two values $r_{bb'}$ would have a sizeable impact. The overall transconductance parameter is given by

$$g_m = \frac{h_{fe}}{h_{ie}} = \frac{h_{fe}}{r_{b'e} + r_{bb'}} \tag{2.68}$$

[1]The total resistance of two resistances, say R_1 and R_2, connected in parallel is $\frac{R_1 R_2}{R_1 + R_2}$. Suppose R_2 is negative; the denominator is reduced and therefore the overall resistance increases, but would still be positive.

Taking the reciprocal

$$\begin{aligned}
\frac{1}{g_m} &= \frac{r_{b'e}}{h_{fe}} + \frac{r_{bb'}}{h_{fe}} \\
&= \frac{1}{g'_m} + \frac{r_{bb'}}{h_{fe}} \\
\therefore r_{bb'} &= \left(\frac{1}{g_m} - \frac{1}{g'_m}\right) h_{fe} \qquad (2.69)
\end{aligned}$$

The inclusion of two capacitances originate from capacitance due to the base-collector pn junction ($C_{b'c}$) and capacitance due to the base-emitter pn junction ($C_{b'e}$) which when forward biased can be capacitively large. At low frequencies the capacitances can be ignored (open circuit), the two base resistances are in series, i.e.

$$r_{bb'} + r_{b'e} = r_\pi = h_{ie} \qquad (2.70)$$

and the simpler hybrid-π model is obtained.

A figure of merit for the high frequency performance of the transistor is the unity gain bandwidth product (f_T) or transition frequency. This parameter is defined as the frequency at which the current gain of the BJT drops to unity under short circuit load condition.

At high frequencies, the 3-dB breakpoint is dependent on the intrinsic BJT input capacitances (C_i) and resistances (R_i). Under short circuit load condition, the input capacitance C_i is determined by

$$\begin{aligned}
C_i &= C_{b'e} + C_{MI} \\
&= C_{b'e} + (1 + g'_m(R_L || r_o))C_{b'c} \\
&\text{Short circuit load} \Rightarrow R_L = 0 \\
\therefore C_i &= C_{b'e} + C_{b'c} \approx C_{b'e} \qquad (2.71)
\end{aligned}$$

where C_{MI} is the equivalent Miller capacitance [2] for $C_{b'c}$.

Assume that the short circuit current is wholly supplied by the dependent source, i.e.

$$|i_{sc}| = |i_c| \approx |g'_m v_{b'e}| \qquad (2.72)$$

The short circuit current gain can be obtained as follows:

$$v_{b'e} = i_b(r_{b'e} || C_{b'e}) = i_b \frac{r_{b'e}}{1 + j\omega r_{b'e} C_{b'e}} \qquad (2.73)$$

[2]For purely capacitive impedances, $C_{MI} = C_{12}(1 - K)$ where K is the gain of the two-port.

$$\begin{aligned} i_c &= g'_m v_{b'e} \\ &= \frac{h_{fe}}{r_{b'e}} i_b \frac{r_{b'e}}{1 + j\omega r_{b'e} C_{b'e}} \\ \frac{i_c}{i_b} &\triangleq h_{fe}(\omega) = \frac{\hat{h}_{fe}}{1 + j\omega r_{b'e} C_{b'e}} \end{aligned} \tag{2.74}$$

where $\hat{h}_{fe}$ is the dc value of the frequency dependent function $h_{fe}(\omega)$.

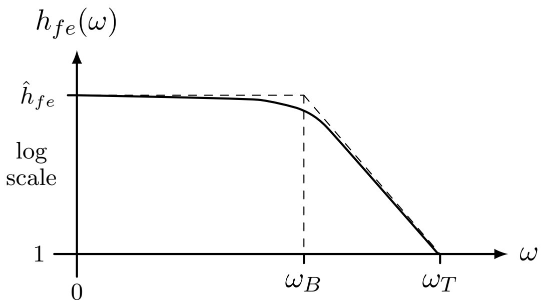


Figure 2.25: Frequency dependence of the short circuit current gain showing the 3-dB breakpoint frequency ω_B and unity gain bandwidth product f_T (or ω_T).

The 3-dB breakpoint occurs at ω_B where $\omega r_{b'e} C_{b'e} = 1$. Therefore

$$\omega_B = \frac{1}{r_{b'e} C_{b'e}} \tag{2.75}$$

If $\omega \gg \omega_B$

$$h_{fe}(\omega) \approx \frac{\hat{h}_{fe}}{j\omega r_{b'e} C_{b'e}} \tag{2.76}$$

By definition, $|h_{fe}(\omega)| = 1$ at the frequency ω_T. Hence, neglecting the unity term in denominator, and taking magnitudes

$$\frac{\hat{h}_{fe}}{\omega_T r_{b'e} C_{b'e}} = 1$$

$$\Rightarrow \omega_T = \frac{\hat{h}_{fe}}{r_{b'e} C_{b'e}} \tag{2.77}$$

Thus

$$\omega_T = \hat{h}_{fe} \omega_B \tag{2.78}$$

Now

$$g'_m = \frac{\hat{h}_{fe}}{r_{b'e}} \Rightarrow g'_m r_{b'e} = \hat{h}_{fe}$$

$$\Rightarrow \omega_T = \frac{g'_m r_{b'e}}{r_{b'e} C_{b'e}} = \frac{g'_m}{C_{b'e}} \tag{2.79}$$

But g'_m is a function of collector current I_C,

$$g'_m = \frac{I_C}{V_T}$$

hence ω_T is a function of collector current (see Fig. 2.26). It is usually prudent

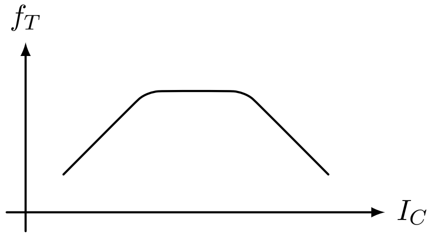


Figure 2.26: Variation of unity gain bandwidth product with respect to collector current.

to design amplifiers and oscillators well below f_T. Moreover, the high frequency model is valid up to about $0.3 f_T$.

2.5.2 Effect of Input Impedance

Circuit and intrinsic BJT capacitances have significant effect on the the frequency response of amplifiers. At "midband" these capacitances have no effect on gain. Consider an amplifier fed via a coupling capacitor (C_C), having an input capacitance and resistance of C_i and R_i respectively as shown in Fig. 2.27. Typically C_C is a large capacitance and C_i is relatively small. Let f_L and f_U be the lower and upper cutoff frequencies respectively, i.e.the frequencies where the gain drop by 3-dB with respect to the gain at midband.

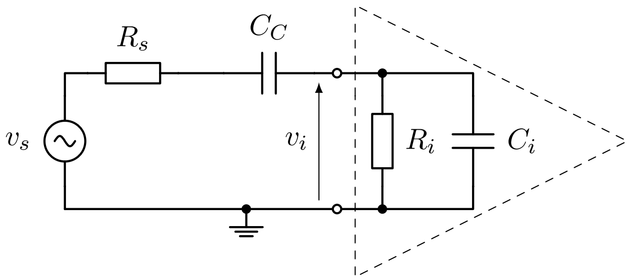


Figure 2.27: Frequency response model for a wideband amplifier.

At midband, C_C is approximately short circuit ($Z_C \approx 0$) and C_i is open circuit ($Z_i = \infty$). Therefore the midband gain is

$$A_0 = \frac{v_i}{v_s} = \frac{R_i}{R_i + R_s} \tag{2.80}$$

At low frequencies, the impedance of C_C is not negligible though Z_i may still be considered ∞. The frequency dependence at low frequencies is

$$\begin{aligned} A(\omega) &= \frac{v_i}{v_s} \\ &= \frac{R_i}{R_i + R_s + \frac{1}{j\omega C_C}} \\ &= \frac{R_i}{R_i + R_s} \frac{1}{1 + \frac{1}{j\omega C_C (R_i + R_s)}} \\ &= A_0 \frac{1}{1 - j\frac{\omega_L}{\omega}} \\ \therefore \frac{A(\omega)}{A_0} &= \frac{1}{1 - j\frac{f_L}{f}} \end{aligned} \tag{2.81}$$

where

$$\omega_L = \frac{1}{C_C(R_i + R_s)} \tag{2.82}$$

At high frequencies, $Z_C = 0$ but the impedance of C_i is no longer infinite. Let

$$Z_i = R_i || C_i = \frac{R_i}{1 + j\omega C_i R_i} \tag{2.83}$$

Hence the gain is

$$\begin{aligned} A(\omega) &= \frac{v_i}{v_s} \\ &= \frac{Z_i}{Z_i + R_s} \\ &= \frac{R_i/(1 + j\omega R_i C_i)}{R_i/(1 + j\omega R_i C_i) + R_s} \\ &= \frac{R_i}{R_i + R_s + j\omega R_i R_s C_i} \\ &= \frac{R_i}{R_i + R_s} \frac{1}{1 + j\omega \frac{R_i R_s C_i}{R_i + R_s}} \\ &= A_0 \frac{1}{1 + j\frac{\omega}{\omega_U}} \\ \therefore \frac{A(\omega)}{A_0} &= \frac{1}{1 + j\frac{f}{f_U}} \end{aligned} \tag{2.84}$$

where

$$\omega_U = \frac{R_i + R_s}{R_i R_s C_i} = \frac{1}{(R_i || R_s) C_i} \tag{2.85}$$

A Bode plot of the frequency response of a wideband amplifier is shown in Fig. 2.28. The response curve has been normalised by dividing the gain function

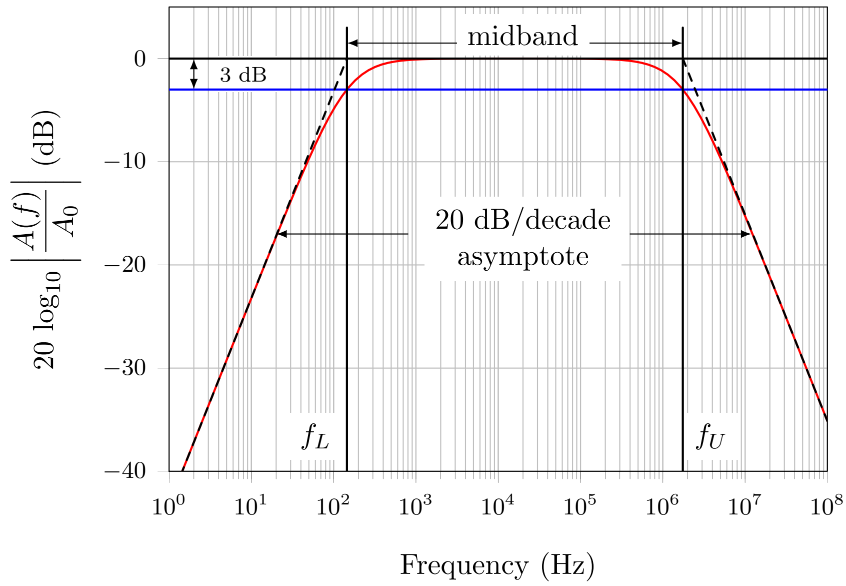


Figure 2.28: Bode plot of the frequency response of a wideband amplifier normalised with respect to the gain at midband due to the effects of source resistance, coupling capacitance and amplifier input impedance.

$|A(f)|$ with the gain at midband (A_0). At $f = f_L$,

$$\left|\frac{A(f)}{A_0}\right| = \frac{1}{\sqrt{2}} \tag{2.86}$$

which in dB is $20\log\frac{1}{\sqrt{2}} \approx= -3$. Similarly, at $f = f_U$ the normalised gain is also $\frac{1}{\sqrt{2}}$. Thus, f_L and f_U are the lower and upper frequencies respectively where the gain drops by 3dB with respect to the gain at midband.

When $f \ll f_L$,

$$\left|\frac{A(f)}{A_0}\right| \approx \frac{1}{f_L/f} = \frac{f}{f_L} \tag{2.87}$$

the gain is proportional to frequency. Similarly, for $f \gg f_U$,

$$\left|\frac{A(f)}{A_0}\right| \approx \frac{1}{f/f_U} = \frac{f_U}{f} \tag{2.88}$$

and the gain is inversely proportional to frequency. Hence, at these frequency extremes, if the frequency changes by a factor of ten or one *decade*, the gain changes by $20\log_{10}10 = 20$ dB. In other words, the gain drops asymptotically by 20 dB/decade.

2.5.3 Effects due to Output and Bypass Capacitances

Consider the small-signal equivalent circuit of a common emitter amplifier shown in Fig. 2.29.

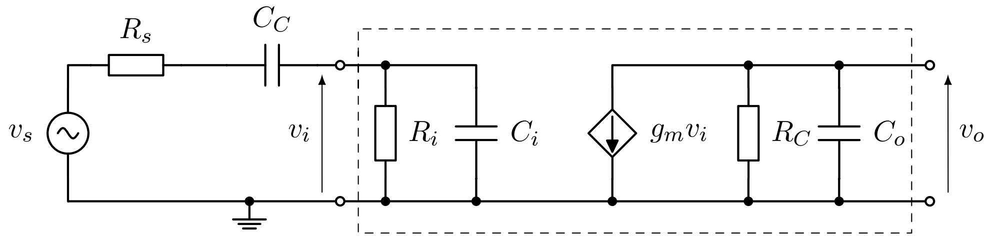


Figure 2.29: Small-signal model of common emitter amplifier for determining effect due to output capacitance on frequency response.

At high frequencies, assume that C_C is short circuit. Let $Z_C = R_C || C_o$,

$$Z_C = \frac{R_C}{1 + j\omega R_C C_o} \tag{2.89}$$

The voltage gain is given by

$$\begin{aligned}
\frac{v_o}{v_i} &= -g_m Z_C \\
&= \frac{-g_m R_C}{1 + j\omega R_C C_o} \\
\text{Let } A_{vo} &\triangleq -g_m R_C \\
\therefore \frac{v_o}{v_i} &= \frac{A_{vo}}{1 + j\frac{\omega}{\omega_3}}
\end{aligned} \tag{2.90}$$

where $\omega_3 = \frac{1}{R_C C_o}$ Thus, at high frequencies the the gain magnitude drops by a further 20 dB/decade for a total asymptotic drop rate of 40 dB/decade. The overall normalised gain magnitude of this common emitter amplifier is shown in Fig. 2.30.

Consider the frequency response of common emitter amplifier including the effect due to the emitter bypass capacitance (see Fig. 2.31) on the frequency response. For the response at low frequencies, C_C and C_E cannot be ignored. However, it is assumed that R_1 and R_2 are sufficiently large so that they can be neglected in determining the amplifier's input resistance R_{IN}. Furthermore, assume that the output coupling capacitance C_{C2} is negligible at low frequencies.

Let $Z_E = R_E || C_E$:

$$Z_E = \frac{R_E}{1 + j\omega C_E R_E} \tag{2.91}$$

The input voltage is given by

$$v_i = i_b \left(\frac{1}{j\omega C_C} + h_{ie} + (1 + h_{fe}) Z_E \right) = i_b \, Z_{IN} \tag{2.92}$$

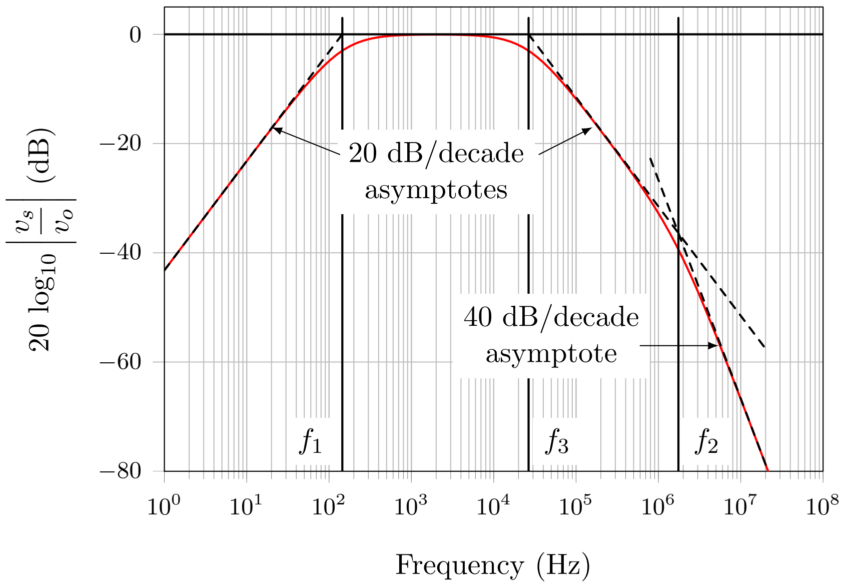


Figure 2.30: Normalised gain magnitude of a common emitter amplifier with the output impedance taken into account.

The output voltage is

$$v_o = -i_b h_{fe} R_C \tag{2.93}$$

Therefore the voltage gain is

$$\frac{v_o}{v_i} = -\frac{h_{fe} R_C}{Z_{IN}} \tag{2.94}$$

When $1 \ll \omega C_E R_E$,i.e. $\omega \gg \frac{1}{R_E C_E}$,

$$Z_E \approx \frac{R_E}{j\omega R_E C_E} = \frac{1}{j\omega C_E} \tag{2.95}$$

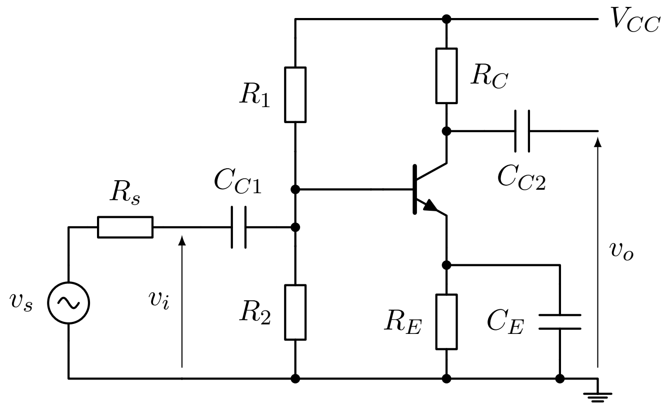
Using (2.92),

$$\begin{aligned} Z_{IN} &= \frac{1}{j\omega C_C} + h_{ie} + (1 + h_{fe}) Z_E \\ &= \frac{1}{j\omega C_C} + h_{ie} + \frac{1 + h_{fe}}{j\omega C_E} \\ &= h_{ie} + \frac{1}{j\omega}\left(\frac{1}{C_C} + \frac{1 + h_{fe}}{C_E}\right) \end{aligned} \tag{2.96}$$

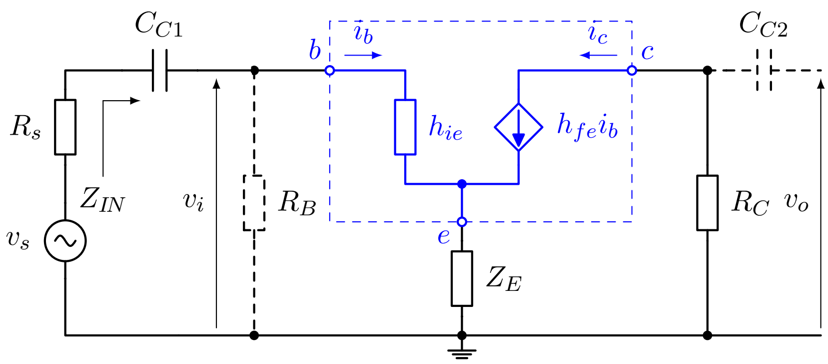
The second term can be written as an effective capacitance formed by a series connection of C_C and $\frac{C_E}{1+h_{fe}}$,

$$\frac{1}{C_{eff}} = \frac{1}{C_C} + \frac{1 + h_{fe}}{C_E} \tag{2.97}$$

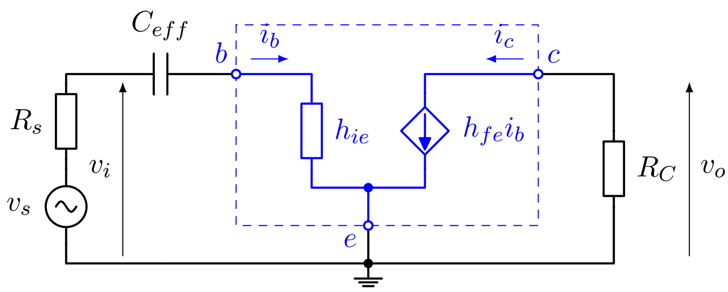
which in turn is in series with h_{ie} resulting in Z_{IN}.



(a) Actual circuit



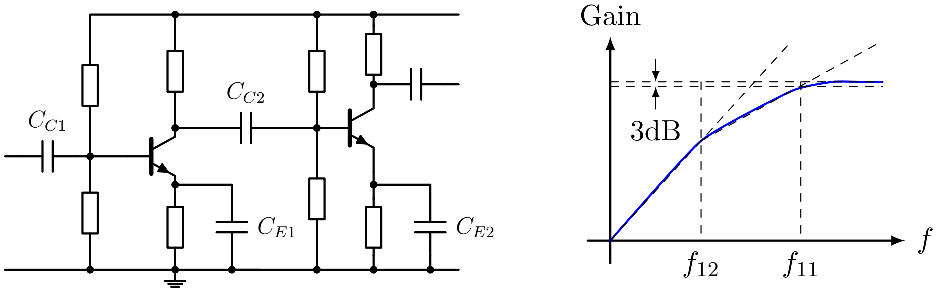
(b) Small-signal equivalent circuit



(c) Simplified equivalent circuit at low frequencies.

Figure 2.31: Common emitter amplifier with emitter bypass capacitance.

In amplifiers that consist of several cascaded stages, there may be more than one capacitance that determines the low frequency behaviour. The 3-dB breakpoint can be examined independently and the overall 3 dB point is given by the highest frequency. For example, in Fig. 2.32, the frequency breakpoints are f_{11} due to C_{C1} and C_{E1} and f_{12} due to C_{C2} and C_{E2}. The overall 3-dB breakpoint is due to f_{11} since it is the highest in this example.



(a) Cascaded CE amplifier (b) 3-dB breakpoints

Figure 2.32: Low frequency 3-dB breakpoints for cascaded stages.

Example: Consider the common emitter amplifier shown in Fig. 2.31 with the following component values

$$
\begin{array}{lll}
I_C = 5 \text{ mA} & & \\
R_C = R_{ext} = 2\text{k}\Omega & R_E = 1\text{k} & R_s = 0 \\
C_{C1} = C_{C2} = 10\mu\text{F} & C_E = 1000\mu\text{F} & \\
C_{be} = 100 \text{ pF} & C_{bc} = 3 \text{ pF} & \\
h_{fe} = 100 & r_{bb'} = 100\Omega &
\end{array}
$$

where R_{ext} is an external load resistance connected the output. It is assumed that R_1 and R_2 are negligibly small. For the small-signal analysis, the high frequency equivalent circuit will be used. (See Fig. 2.24) The high frequency model parameters can be determined as follows:

$$g'_m = \frac{I_C}{V_T} \approx 200\text{mS} \tag{2.98}$$

$$r_{b'e} = \frac{h_{fe}}{g'_m} = \frac{100}{0.2} = 500\Omega \tag{2.99}$$

At low frequencies, $C_{b'e}$ and $C_{b'c}$ can be neglected. The ac equivalent circuit at low frequencies is shown in Fig. 2.33. At midband, the capacitances are short circuits. The voltage gain can be obtained as follows:

$$
\begin{aligned}
\frac{v_{b'e}}{v_s} &= \frac{r_{b'e}}{r_{b'e} + r_{bb'}} = \frac{500}{500 + 110} = 0.83 \\
\frac{v_o}{v_{b'e}} &= -g'_m R_c || R_{ext} = -0.2(2000||2000) = -200 \\
\frac{v_o}{v_s} &= \frac{v_o}{v_{b'e}} \frac{v_{b'e}}{v_s} = -200 \times 0.83 = -167 \\
\left| \frac{v_o}{v_s} \right| &= 44.5 \text{ dB}
\end{aligned}
$$

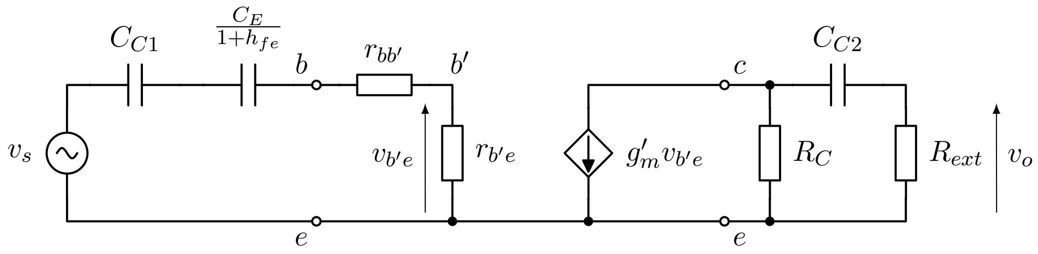


Figure 2.33: Common emitter amplifier equivalent circuit for determining low frequency response.

The low frequency breakpoints are determined by the series connections of components in the base loop at the input and in the collector loop at the output. In the latter case, the dependent source is taken as open circuit. Their effects will be considered separately. The input RC circuit time constant can be determined by:

$$C_{eff} = \left(\frac{1}{10\mu} + \frac{1000\mu}{101}\right) = 5\mu\text{F}$$

$$r_{b'e} + r_{bb'} = 600\Omega$$

$$\therefore RC = 600 \times 5 \times 10^{-6} = 3 \text{ ms}$$

Thus, the 3-dB breakpoint due to the input circuit is

$$f_{11} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 3 \times 10^{-3}} = 53 \text{ Hz}$$

By voltage division, the output voltage is given by

$$v_o = \frac{R_{ext}}{R_{ext} + \frac{1}{j\omega C_{C2}}} v_c \tag{2.100}$$

where v_c is the collector voltage, which can be written as

$$v_c = -g'_m v_{b'e} \times R_C || \left(R_{ext} + \frac{1}{j\omega C_{c2}}\right) \tag{2.101}$$

Combining these two results, yields

$$v_o = -g'_m v_{b'e} \times R_C|| \left(R_{ext} + \frac{1}{j\omega C_{c2}} \right) \times \frac{R_{ext}}{R_{ext} + \frac{1}{j\omega C_{C2}}}$$

$$\begin{aligned} \frac{v_o}{v_{b'e}} &= -g'_m \frac{R_C \cancel{\left(R_{ext} + \frac{1}{j\omega C_{C2}}\right)}}{R_C + R_{ext} + \frac{1}{j\omega C_{C2}}} \times \frac{R_{ext}}{\cancel{\left(R_{ext} + \frac{1}{j\omega C_{C2}}\right)}} \\ &= -g'_m \frac{R_C R_{ext}}{R_C + R_{ext}} \times \frac{1}{1 + \dfrac{1}{j\omega(R_C + R_{ext})C_{C2}}} \\ &= -g'_m (R_C||R_{ext}) \frac{1}{1 + \dfrac{1}{j\omega(R_C + R_{ext})C_{C2}}} \end{aligned} \quad (2.102)$$

The first factor ($-g'_m(R_C||R_{ext})$) is the midband gain obtained earlier. The second factor is the effect of the output circuit time constant on the low frequency response. The output RC circuit time constant is

$$(R_C + R_{ext})C_{C2} = (2000 + 2000) \times 10^{-5} = 40 \text{ ms}$$

The low frequency 3-dB breakpoint due to the output circuit is

$$f_{12} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 40 \times 10^{-3}} = 4 \text{ Hz}$$

Therefore, the low frequency 3-dB breakpoint is 53 Hz since $f_{11} > f_{12}$.

At high frequencies, the intrinsic transistor capacitances cannot be ignored. However, the coupling capacitances and emitter bypass capacitance are assumed to be negligible. With these in mind, the high frequency equivalent circuit model shown in Fig. 2.34 is obtained.

The capacitances $C_{b'e}$ and $C_{b'c}$ determine the high frequency fall off. The "bridging" capacitor $C_{b'c}$ can be removed by applying Miller's theorem. Without $C_{b'c}$,

$$\frac{v_o}{v_{b'e}} = K = -g'_m R_L \quad (2.103)$$

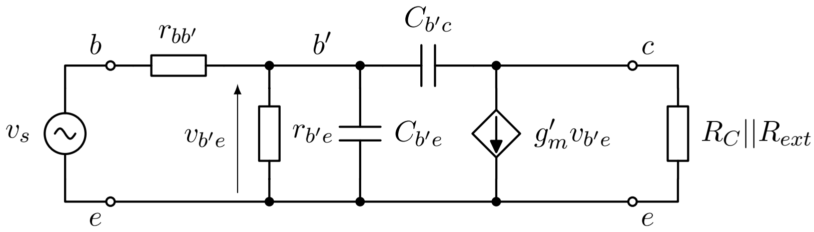
where $R_L = R_C||R_{ext} = 2000||2000 = 1000\Omega$. Thus, the Miller capacitance are

$$C_{MI} = (1 - K)C_{b'c} = (1 + g'_m R_L)C_{b'c} = 201 \times 3 = 603\text{pF}$$

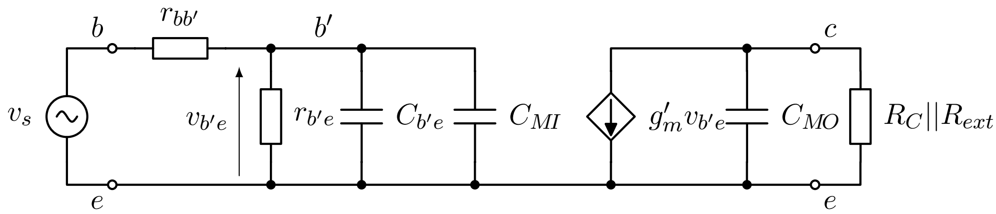
$$C_{MO} \approx C_{b'c} = 3\text{pF}$$

where the C_{MO} approximation is made since $K \gg 1$. The total input capacitance is

$$C_i = C_{b'e} + C_{MI} = 100 + 603 = 703\text{pF}$$



(a) Small-signal model



(b) Miller equivalent

Figure 2.34: Small-signal equivalent circuit of CE amplifier at high frequencies.

The output voltage is

$$v_o = -g'_m v_{b'e} \frac{R_L}{1 + j\omega R_L C_{MO}} \tag{2.104}$$

Therefore,

$$\frac{v_o}{v_{b'e}} = \frac{-g'_m R_L}{1 + j\omega R_L C_{MO}} \tag{2.105}$$

which implies a 3-dB breakpoint at

$$f_3 = \frac{1}{2\pi R_L C_{MO}} = \frac{1}{2\pi \times 1000 \times 3 \times 10^{-12}} = 53.1\text{MHz}$$

At the input,

$$v_{b'e} = \frac{Z_I}{Z_I + r_{bb'}} v_s \tag{2.106}$$

where

$$Z_I = \frac{r_{b'e}}{1 + j\omega r_{b'e} C_i} \tag{2.107}$$

Therefore,

$$\begin{aligned}
\frac{v_{b'e}}{v_s} &= \frac{\dfrac{r_{b'e}}{1 + j\omega r_{b'e} C_i}}{\dfrac{r_{b'e}}{1 + j\omega r_{b'e} C_i} + r_{bb'}} \\
&= \frac{r_{b'e}}{r_{b'e} + r_{bb'}(1 + j\omega r_{b'e} C_i)} \\
&= \left(\frac{r_{b'e}}{r_{b'e} + r_{bb'}}\right) \frac{1}{1 + j\omega \left(\dfrac{r_{b'e} r_{bb'}}{r_{b'e} + r_{bb'}}\right) C_i}
\end{aligned} \tag{2.108}$$

Hence, another high frequency 3-dB breakpoint is located at

$$f_2 = \frac{1}{2\pi(r_{bb'}||r_{b'e})C_i} = \frac{1}{2\pi(100||500)703 \times 10^{-12}} = 2.72\text{MHz}$$

Since $f_2 < f_3$, the 3-dB breakpoint past midband occurs at f_2. Hence, the amplifier bandwidth is

$$\text{BW} = 2.72\text{MHz} - 53\text{Hz} \approx 2.72\text{MHz}$$

A sketch of the Bode plot of the amplifier's magnitude response is shown in Fig. 2.35.

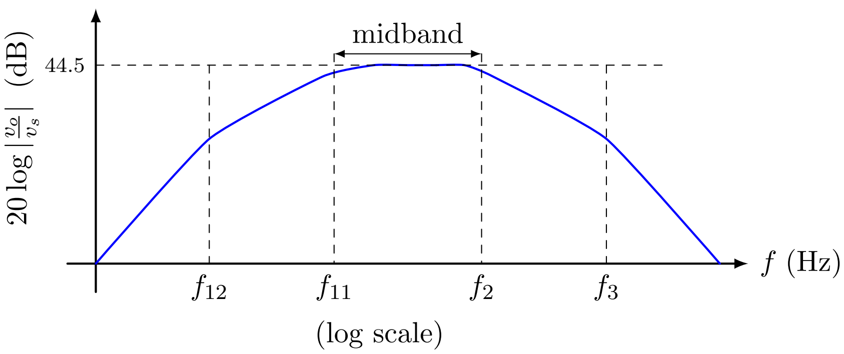


Figure 2.35: Frequency response of common emitter amplifier example.