MINI PROJECT 4

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ELEC 301

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Part A – An Active Filter

Given transfer Function:

$$H(s) = A_M \frac{1/(RC)^2}{s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}}$$

Values of C and A_M

We have to find the poles of the transfer function; therefore it follows that.

$$\omega_n = \frac{1}{RC} \ and \ \zeta = 3 - A_M$$

Which defines the system behavior as,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

To obtain a 2nd order Butterworth filter with given 3dB frequency of 10kHz,

$$2\pi \times 10^3 rad\ s^{-1} = \frac{1}{RC}$$

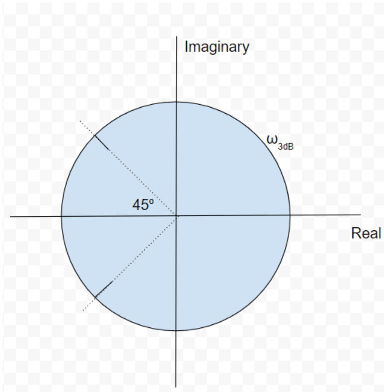
Which yields.

$$C = 1.6\ nF$$

And using the s-plane (Plot 1), we can find the 2nd order Butterworth filter poles.

Complex pole damping factor,

$$\zeta = \sqrt{2} = 3 - A_M$$



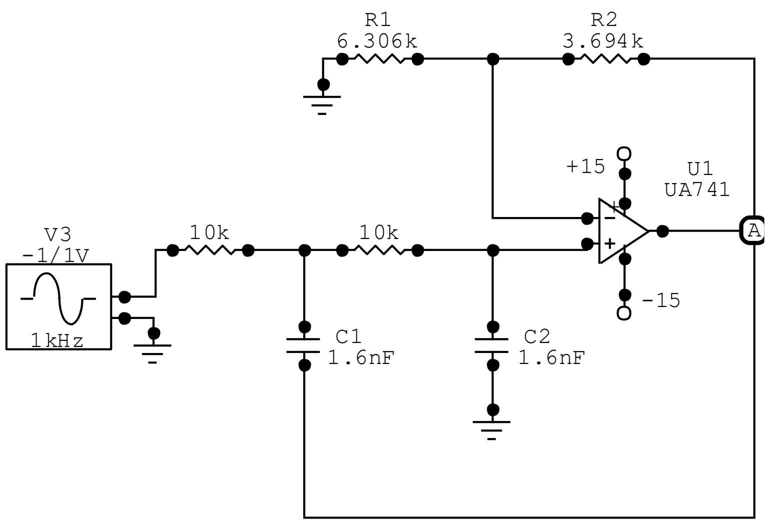
Plot 1: Location of poles on s-plane

Therefore,

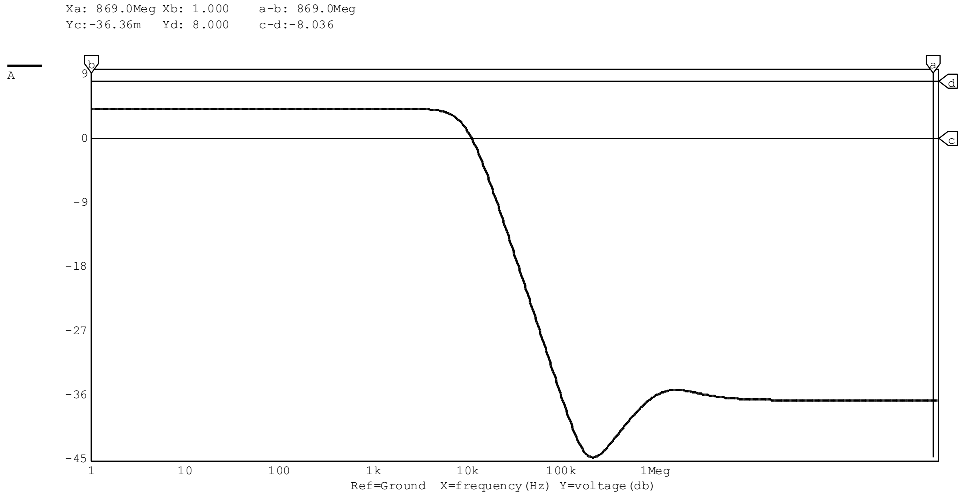
$$A_M = 1 + \frac{R_2}{R_1} = 1.586 \; and \; R_1 + R_2 = 10k$$

$$R_1 = 6.306 \; k\Omega \; and \; R_2 = 3.694 \; k\Omega$$

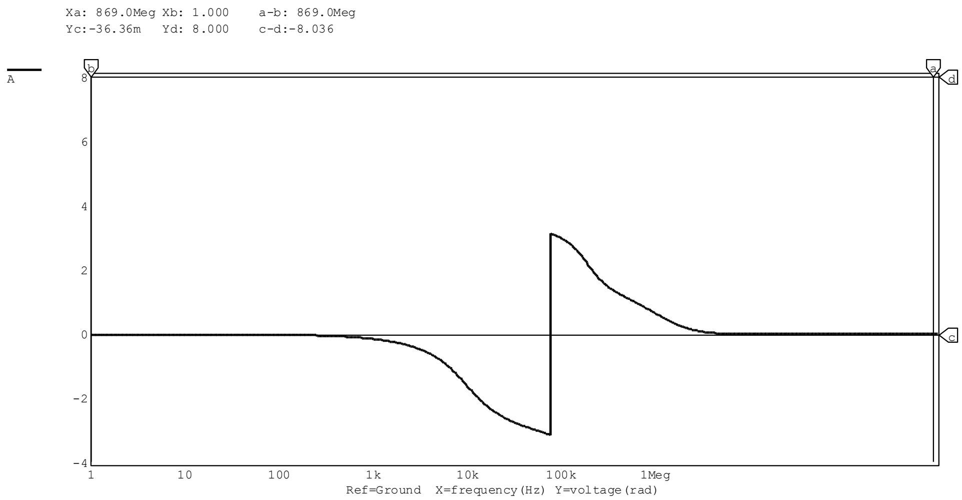
Thus,



Circuit 1: 2nd Order Butterworth filter model



Plot 2: Magnitude Bode Plot for 2nd order Butterworth filter.



Plot 3: Phase Bode Plot for 2nd order Butterworth filter.

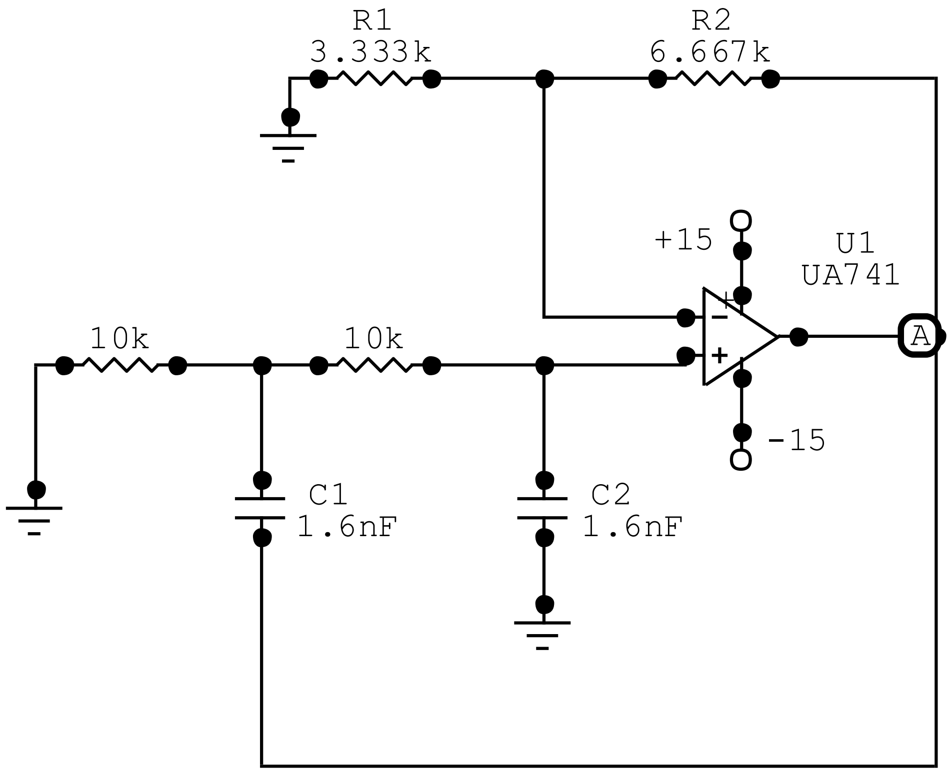
Oscillating Gain

The pole locations are found from the roots of the denominator of transfer function,

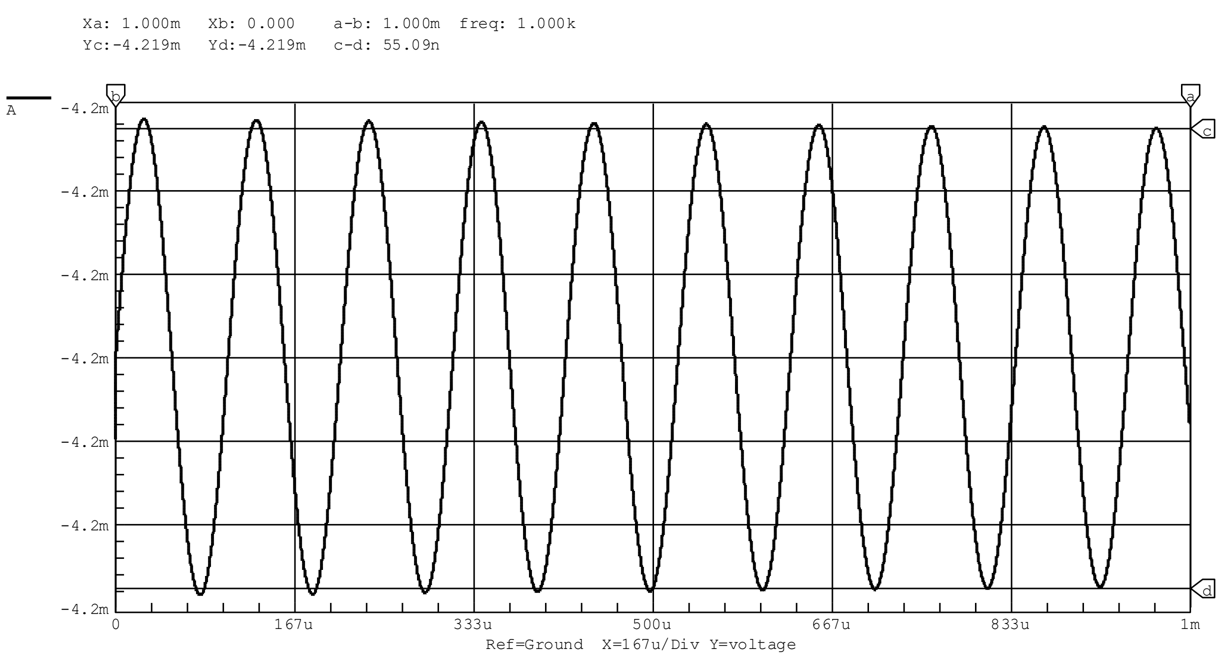
$$s^2 + s\frac{3 - A_M}{RC} + \frac{1}{RC^2} = 0$$

So, when $A_M = 3$ and $R_1 + R_2 = 10k$ yields,

$$R_1 = 3.333k\Omega \; and \; R_2 = 6.667 \; k\Omega$$



Circuit 2: Oscillating Circuit



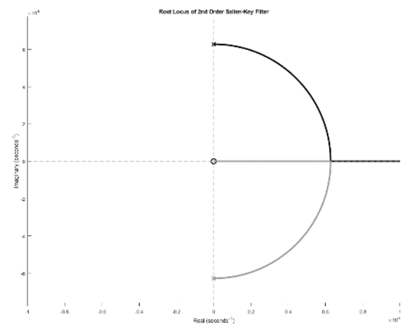
Plot 4: Oscillating Transient Output of Circuit 2

Oscillation frequency at marginally stable gain,

$$f = \frac{9}{0.942\ ms} = 9.554\ kHz$$

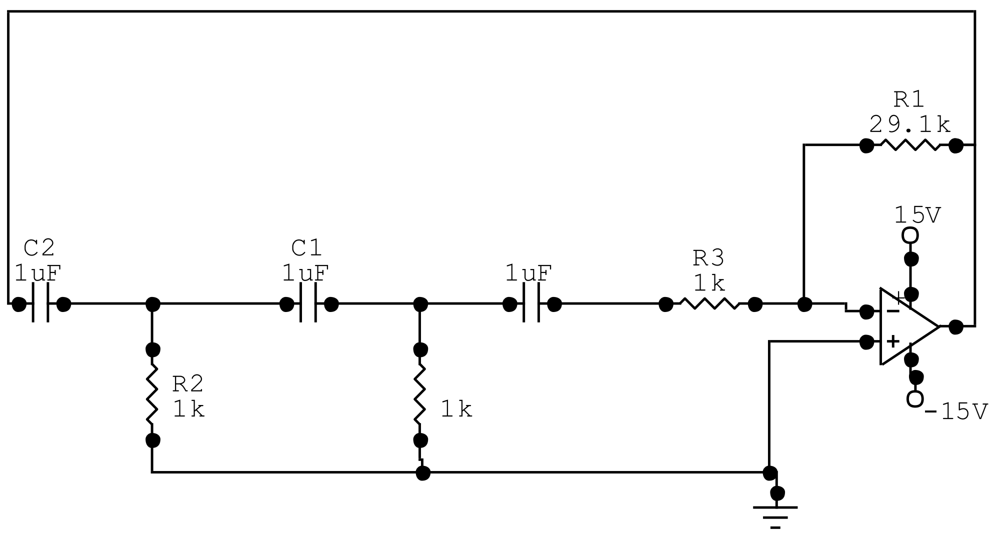
As A_M increases, the poles move along the root locus (the circle) from the pole location as shown in figure 4a to the right until it lands on the imaginary axis as shown in figure 4b. After that point, the response is an undamped oscillation. Increasing A_M further will cause the filter to go unstable as the poles have now crossed to the right-hand plane.

Plot 6: Root Locus for A_M=1.585



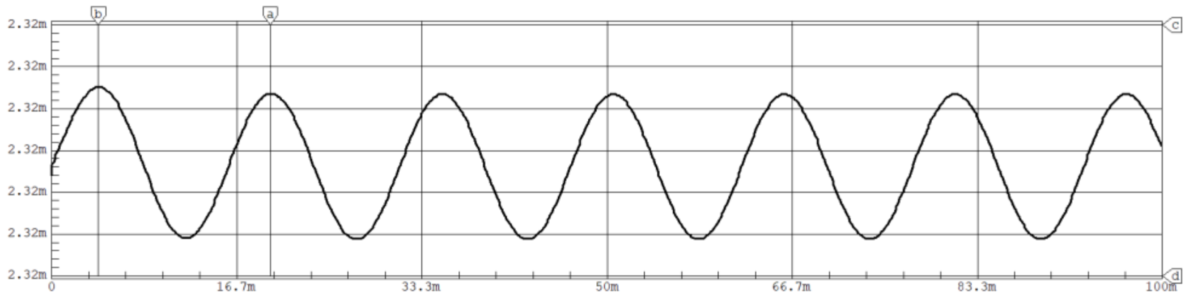
Plot 5: Root Locus for A_M=3

Part B – A Phase Shift Oscillator



Circuit 3: Phase Shift Oscillator Circuit

The Oscillator Gain $= -\frac{29.1k}{1k} = -29.1$, which satisfy the gain condition for non-inverting amplifier. The transfer function of the oscillators contains a pair of complex poles situated on the imaginary axis with a denominator, which results in the generation of a finite signal even in the absence of an input signal. Initially, the feedback resistor had a resistance of 29R (29kΩ) but the signal produced eventually decayed. To prevent this, the resistance was increased to 29.1kΩ.



Plot 7: Oscillating Circuit Output

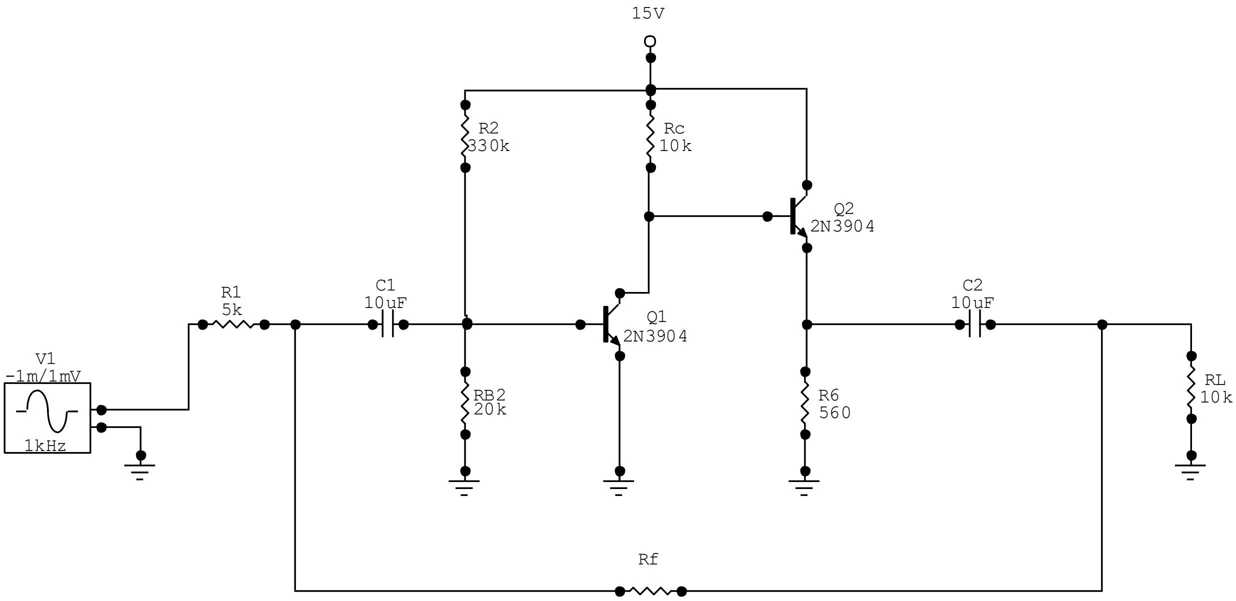
$$f_{calculated} = \frac{1}{2\pi\sqrt{6}RC}$$

Using the above formula from class notes, Measured and Calculated frequencies for different values of R and C are written in table below.

	Original	**Halved**	**Double**
R	$1\ k\Omega$	$500\ \Omega$	$2\ k\Omega$
C	$1\ \mu F$	$0.5\ \mu F$	$1\ \mu F$
$f_{measured}$	$257\ Hz$	$257\ Hz$	$257\ Hz$
$f_{calculated}$	$259.9\ Hz$	$64.97\ Hz$	$16.24\ Hz$

The frequencies listed in Table, which were both calculated and measured, are quite similar. However, because our SPICE program measures frequency graphically, the accuracy of these measurements is limited to the nearest hertz. The differences between the measured and calculated frequencies for the halved component values could potentially be attributed to non-idealities of the operational amplifier that were not considered when deriving the frequency formula.

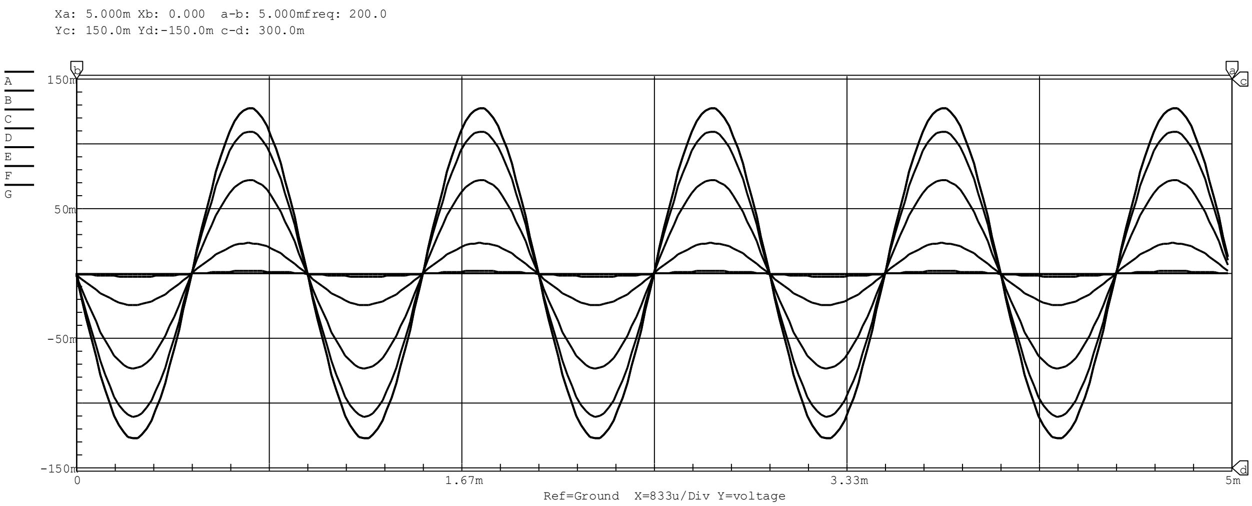
Part C – A Feedback Amplifier



Circuit 4: Feedback Amplifier

Largest Open Gain

Using Parameter Sweep on RB2 on *CircuitMaker*,



Plot 8: Transient Analysis with increasing values of RB2

Thus, we obtain $R_{B2} = 20\ k\Omega$ which can yield maximum Open Loop Gain.

DC Bias Values

Measured DC Bias Values of each Transistor is given in Table Below:

	V_C	V_B	V_E	I_C	I_B	I_E
Q1	$1.900\ V$	$0.654\ V$	$0.000\ V$	$1.295\ mA$	$10.77\ \mu A$	$1.305\ mA$
Q2	$15.00\ V$	$1.900\ V$	$1.260\ V$	$2.190\ mA$	$15.40\ \mu A$	$2.210\ mA$

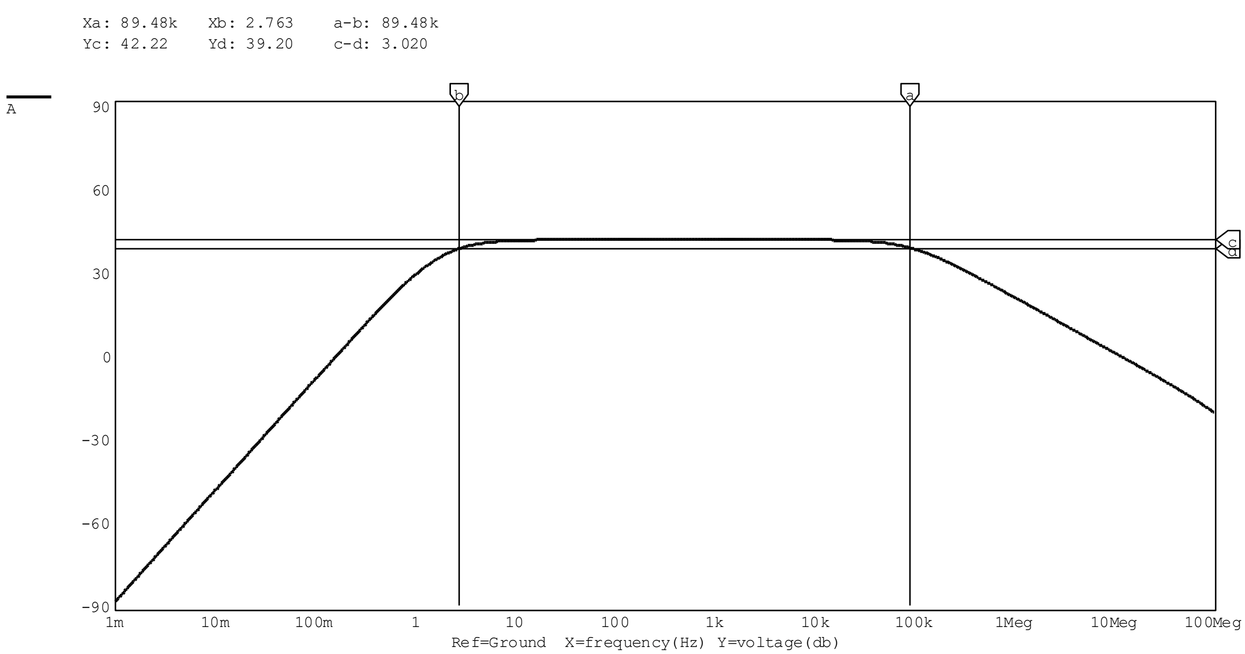
Hybrid π models of transistor is calculated with,

$$g_m = \frac{I_C}{V_T}, r_\pi = \frac{h_{FE}}{g_m}, h_{FE} = \frac{I_C}{I_B}$$

	g_m	r_π	h_{FE}
Q1	0.052	$2.32\ k\Omega$	120
Q2	0.088	$1.62\ k\Omega$	142

Open-Loop Frequency Response

With $R_f = 10{,}000\ G\Omega$ (infinite resistance), the open loop frequency response from the circuit,



Plot 9: Open-Loop Frequency Response

Measured Values from Plot 6 are,

$$\omega_{L3dB} = 2.763\ Hz \qquad \omega_{H3dB} = 89.48\ kHz$$

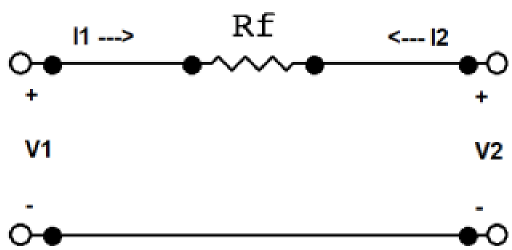
$$A_M = 42.22\ dB = -129.121927361 \frac{V}{V}\ (-ve\ cause\ of\ inverting\ amplifier)$$

Applying the test voltages,

$$R_{input} = \frac{2mV}{782.9nA} = 2.55\ k\Omega$$

$$R_{output} = \frac{707.1\mu V}{11.18\mu A} = 63.247\ \Omega$$

Prediction of Closed Loop Response



Circuit 5: Feedback Network

Since shunt-shunt topology is being utilized by the feedback network, the y-parameters to represent the feedback network can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus,

$y_{11} = \frac{I_1}{V_1}|_{V_2=0} = \frac{1}{R_f} = 10^{-5}\ \mho$

$y_{12} = \frac{I_1}{V_2}|_{V_1=0} = -\frac{1}{R_f} = -10^{-5}\ \mho$

$y_{22} = \frac{I_2}{V_2}|_{V_1=0} = \frac{1}{R_f} = 10^{-5}\ \mho$

$y_{21} \approx 0$, as the feed-forward gain is very small.

Voltage Gain

$A_M = -129.121927361 \frac{V}{V}$, but due to shunt-shunt topology, the unit of gain should be in terms of

$$\frac{V}{I} \approx \frac{V_{out}}{i_i} = R_S \left(\frac{V_{out}}{V_i}\right)$$

$$A_{open-loop} = 5k\Omega \times -129.121927361 = -645.61\ kV/A$$

Therefore, closed-loop gain calculated by,

$$A_f = \frac{A}{1 + A\beta} = -\frac{645.61}{1 + (-645.61) \times (-10^{-5})} = -86.59\ kV/A$$

Or,

$$A_f = -\frac{98.47}{5\ k\Omega} = -17.32\ V/V$$

3dB frequency Points

Due to Feedback, there exists a bandwidth extension with a factor of $(1 + A\beta)$,

$$(\omega_{L3dB})_{feedback} = \frac{\omega_{L3dB}}{1 + A\beta} = 0.371\ Hz$$

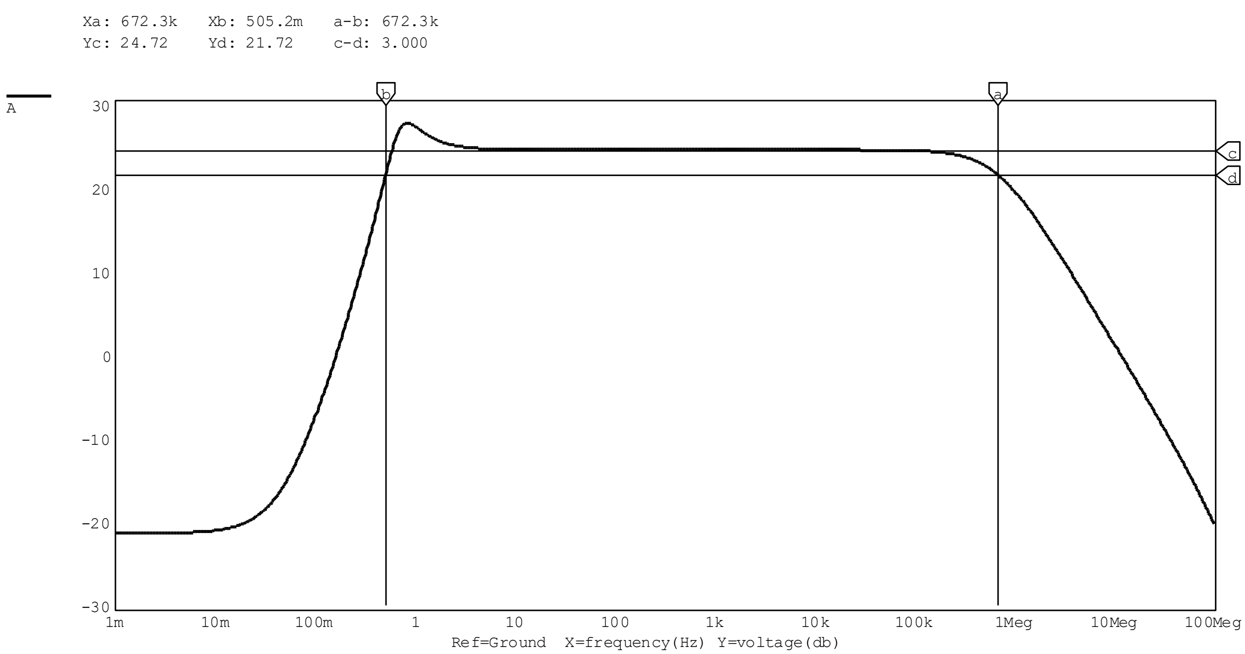
$$(\omega_{H3dB})_{feedback} = \omega_{H3dB}(1 + A\beta) = 667.17\ kHz$$

Impedance Values

$$(R_{input})_{feedback} = \frac{R_{input}}{1 + A\beta} = 360.4\ \Omega$$

$$(R_{output})_{feedback} = \frac{R_{output}}{1 + A\beta} = 8.585\ \Omega$$

Measured Closed-Loop Frequency Response



Plot 10: Bode Plot for Feedback Amplifier with Rf=100kΩ

The values measured from the bode are compared to our calculated values in following table:

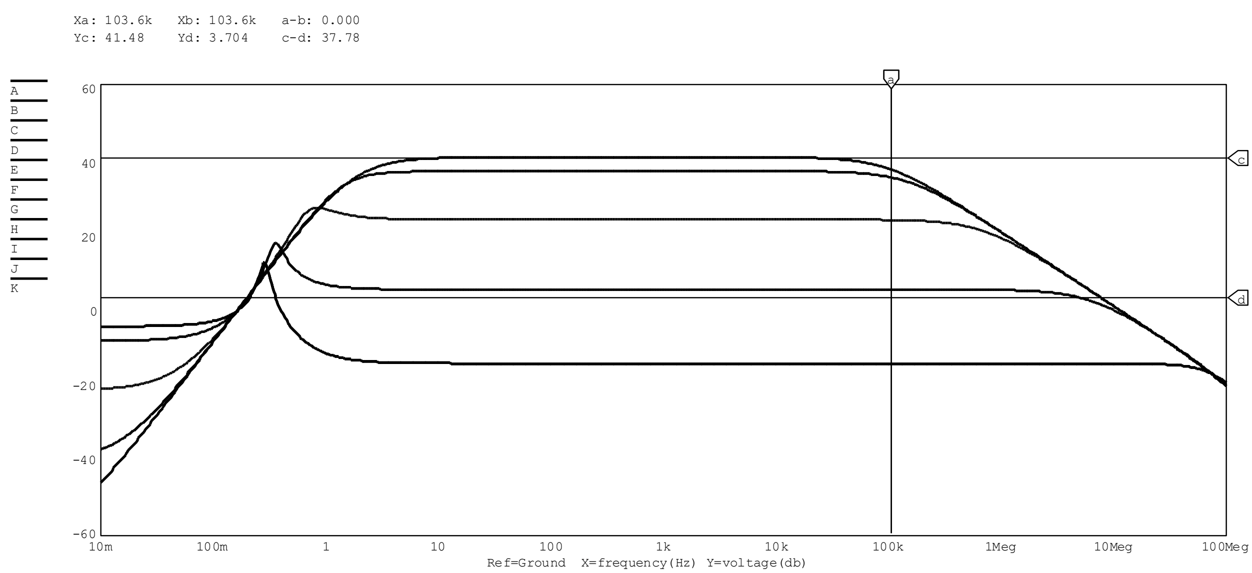
	Measured	**Calculated**
$\boldsymbol{A_M}$	$-17.21\,V/V$	$-17.32\,V/V$
$\boldsymbol{\omega_{L3dB}}$	$0.505\,Hz$	$0.371\,Hz$
$\boldsymbol{\omega_{H3dB}}$	$672.3\,kHz$	$667.17\,kHz$
$\boldsymbol{R_{input}}$	$242.63\,\Omega$	$360.4\,\Omega$
$\boldsymbol{R_{output}}$	$8.657\,\Omega$	$8.585\,\Omega$

*Values of impedances were measured using the same methods described above.

Input impedance has a higher error.

Closed-Loop frequency response with different values of R_f

Closed frequency response is measured over the same range of frequencies (10mHz to 100MHz), for $R_f = 1k\Omega, 10k\Omega, 100k\Omega, 1M\Omega$ and $10M\Omega$.



Plot 11:Closed loop magnitude Bode plots for Rf = 1kΩ, 10kΩ, 100kΩ, 1MΩ, from lowest to highest midband gain respectively.

Feedback factor β, can be calculated by solving, $A_f = \frac{A}{1+A\beta}$

The units of A_f are V/A whereas, the bode plot gives A in unit of V/V. But as solved earlier, we can just multiply $A \times R_S$ to change units from V/V to V/A.

Moreover, calculated $\beta = -\frac{1}{R_f}$ (from feedback network)

R_f (Ω)	A_f (dB)	A_f (V/V)	A_f (kV/A)	Measured β	Calculated β
1k	-14.07	-0.198	-0.990	-1.001e-3	-1e-3
10k	5.892	-1.971	-9.853	-1.015e-4	-1e-4
100k	24.72	-17.21	-86.09	-1.002e-5	-1e-5
1M	37.82	-77.80	-389.01	-9.993e-7	-1e-6
10M	41.60	-120.2	-601.13	-9.069e-8	-1e-7

As seen from the table, $\beta_{measured} \approx \beta_{Calculated}$

I/O Impedance with Varying R_f

Using the methods described above, we calculate the I/O impedances of our amplifier for $R_f = 10k\Omega$, 100kΩ and 1MΩ.

R_f	$(R_{input})_{feedback}$	$(R_{output})_{feedback}$
$10\ k\Omega$	26.13 Ω	1.11 Ω
$100\ k\Omega$	238.09 Ω	8.657 Ω
$1\ M\Omega$	1.30 $k\Omega$	37.39 Ω

Additionally, to "amount of feedback (1+Aβ)" can be estimated using the formula,

$$(R_{input/output})_{feedback} = \frac{R_{input/output}}{1 + A\beta}$$

$$\text{amount of feedback} = \frac{R_{input/output}}{(R_{input/output})_{feeback}}$$

For predicted "amount of feedback", we are using the measured $A_{open-loop} = -645.61\ kV/A$ from part 2 and the calculated values of β from part 3 as described by instructor.

$$R_{input} = \frac{2mV}{782.9nA} = 2.55\ k\Omega \qquad R_{output} = \frac{707.1\mu V}{11.18\mu A} = 63.247\ \Omega$$

R_f	$(1 + A\beta)_{input}$	$(1 + A\beta)_{output}$	$(1 + A\beta)_{predicted}$
10 $k\Omega$	97.73	56.97	65.56
100 $k\Omega$	10.72	7.31	7.4561
1 $M\Omega$	1.965	1.69	1.645

From table 5, it appears that the amount of gain calculated from measured output impedance is consistently closer to predicted values as feedback resistance R_f increases and amount of gain values decrease and become closer to predicted values.

De-sensitivity Factor

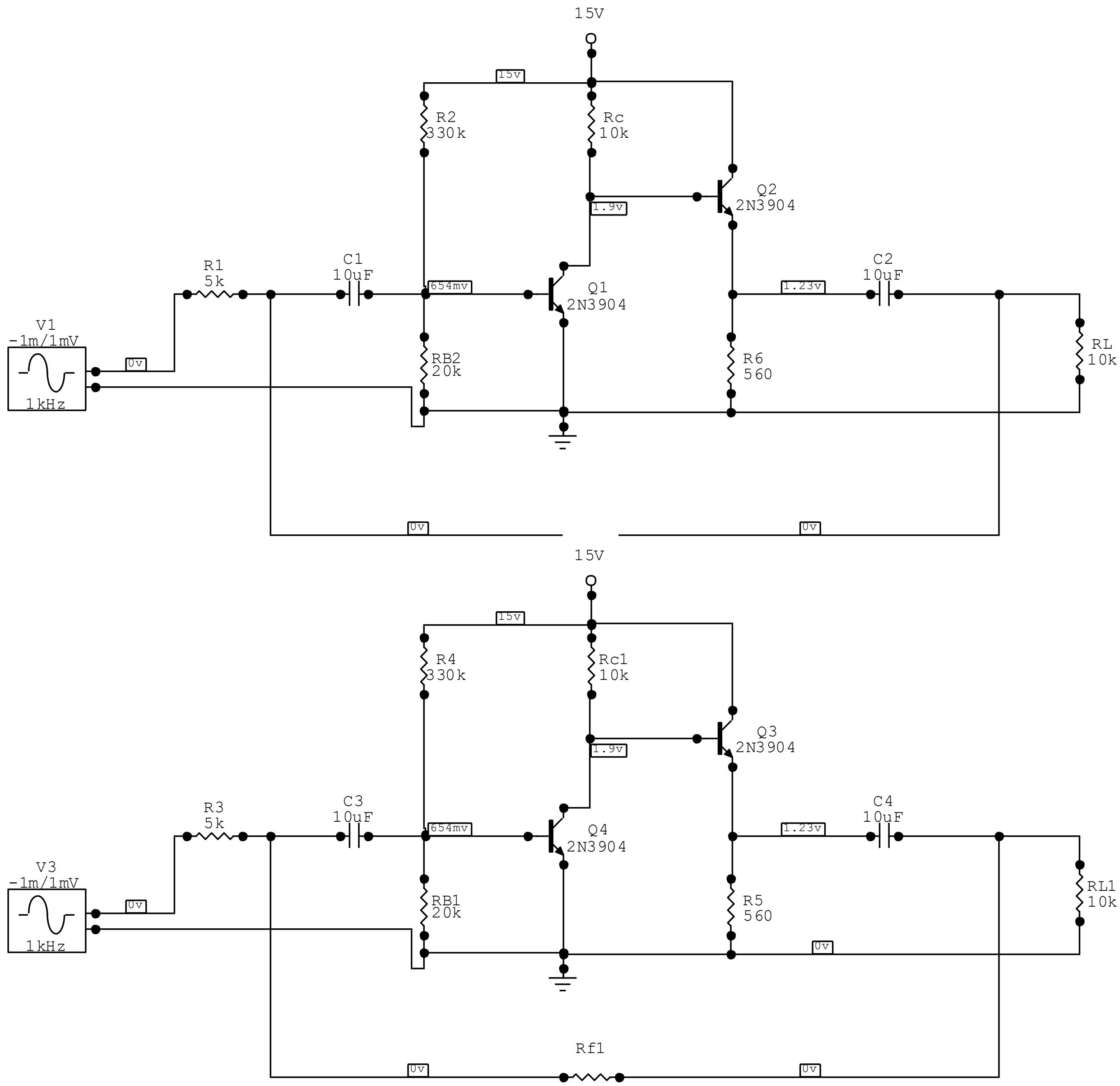
Gain de-sensitivity can be found by,

$$\frac{dA_f}{dA_{open-loop}} = \frac{1}{(1 + A\beta)^2}$$

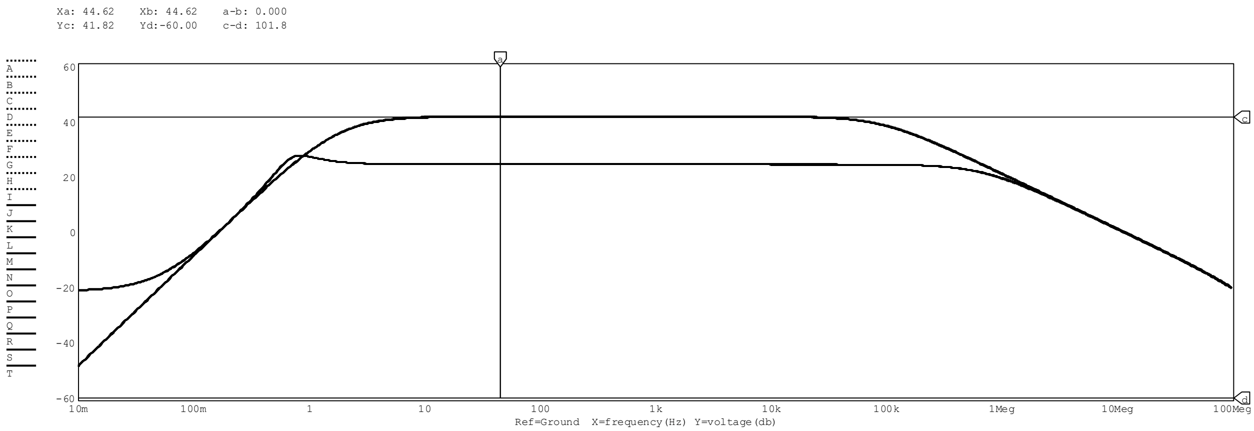
$$\frac{dA_f}{dA_{open-loop}} = \frac{1}{(1 + A\beta)^2} \times \frac{A_f}{A_{open-loop}}$$

$$\frac{\frac{dA_f}{A_f}}{} \times 100 = \left(\frac{1}{1 + A\beta}\right)\frac{dA}{A} \times 100$$

Thus, de-sensitivity factor is given by $1 + A\beta$



Circuit 6: Feedbavk Circuit with Rf = inf (top) and Rf = 100kΩ (below)



Plot 12:Bode Plot for midband gain for Rf=inf (top) and Rf=100kΩ (below)

R_C	$A\|_{R_f=\infty}$	$A\|_{R_f=100k\Omega}$	$(1+A\beta)\|_{R_f=\infty}$	$(1+A\beta)\|_{R_f=100k}$
$9.9\ k\Omega$	$-126.76\ V/V$	$-17.21\ V/V$	1	7.29
$10\ k\Omega$	$-127.49\ V/V$	$-17.23\ V/V$	1	7.35
$10.1\ k\Omega$	$-128.23\ V/V$	$-17.25\ V/V$	1	7.68

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On Comparison with the Predicted value of 7.45 from part 4 for $R_f = 100\ k\Omega$ the obtained values are very close.

Additionally, for $R_f = \infty$ the circuit is a open loop and therefore has $\beta = 0$, due to which $(1 + A\beta) = 1$.