

ELEC 341 – Graded Assignments

Assignment A5  
State-Space Representation  
  
10 Marks

Learning Objectives

State-Space Representation

Mechanical System

Electro-Mechanical System

Transmissions

Matlab

ss()

inv()

eye()

padarray() % may be useful

State Transition Matrix

A State-Space representation of the Mechanical Circuit in Assignment #3 allows you to compute positions and forces without an electrical equivalent.

Derive a state-space representation of “Mechanical Circuit”. Configure your matrices so a **UNIT** step applies **F0 = 1N**.

Use the state vector  $\bar{x}$  and output vector  $\bar{y}$ , exactly as shown.

All speeds in (m/s), all distances in (m), all forces in (N).

The state vector  $\bar{x}$  contains ALL POSSIBLE states. If any 2 states are dependent, remove the one further to the right.

$\bar{x} = [v0 \ v1 \ v2 \ v3 \ d0 \ d1 \ d20 \ d32]^T$

$\bar{y} = [d3 \ fk1]^T$

where:  $d20 = d2 - d0$

$d32 = d3 - d2$

where:  $fk1$  is a **separating** force (see Assignment 3)

Q1

4 mark(s)

State Matrices

Compute the **MECHANICAL A & B** state matrices.

• Q1.A

(mixed)

Matrix

• Q1.B

(mixed)

Matrix

Q2

1 mark(s)

Output Matrices

Compute the **C & D** output matrices.

• Q2.C

(mixed)

Matrix

• Q2.D

(mixed)

Matrix

There are multiple ways to get a transfer function from a set of State-Space matrices. You can compute the state transition matrix, or use the ‘ss’ function.

Q3

1 mark(s)

Distance TF

Compute the transfer function **G**.

**G**

Input = F0

Output = d3

• Q3.G

(rad/Vs)

LTI

Q4

1 mark(s)

Force TF

Compute the transfer function **G**.

**G**

Input = F0

Output = fk1

• Q4.G

(rad/Vs)

LTI

**COW:** Use **step()** to see the response of each transfer function.

Compare them to what you got in Assignment #3.


Change your C&D matrices to check everything else you computed in Assignment #3.

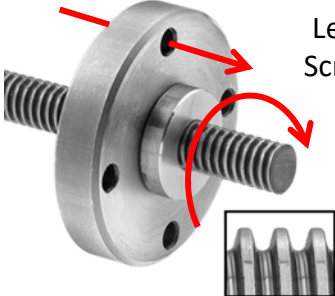
Make sure you apply the same input force.

Replace  $F_0$  in Assignment #3 with a “Linear Actuator”. A linear actuator works just like a motor, but the motion is in a straight line. Hydraulic linear actuators are commonly used in excavators and other heavy equipment.

Another type of linear actuator is an electric motor with a rotary  $\rightarrow$  linear transmission (like a lead-screw). When the motor turns the screw, the nut moves back & forth.

Linear Actuator  $\rightarrow$

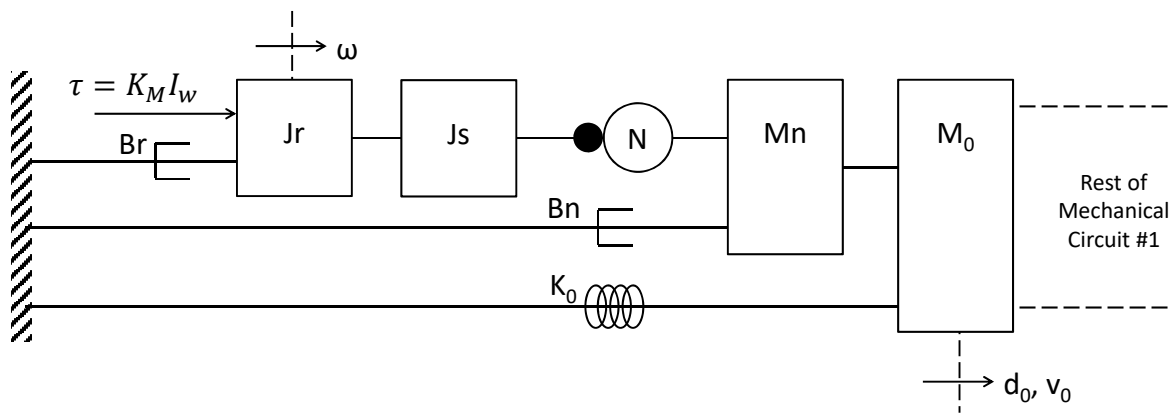




Lead Screw

The applied force in the Mechanical Circuit in Assignment #3, is replaced with the Motor in Assignment #3 and a lead-screw with the following properties.

$N$	$= 5$	(rev/m)	Transmission Ratio
$J_s$	$= \#A \times 20$	( $\mu\text{Nms}^2$ )	Screw Inertia
$M_n$	$= \#B / 2$	(Kg)	Nut Mass
$B_n$	$= \#C / 2$	(Ns/m)	Friction Between Screw & Nut



Q5

2 mark(s)

Actuated System

Modify the State-Space matrices to include the linear actuator in the State-Space model.  
Configure your matrices so a **UNIT** step applies **Vin = 150V**.  
Compute the transfer function **G**.

<b>G</b>	Input = Vin	Output = fk20
• Q5.G	(N/150V)	LTI

Add gravity to **each** mass.       **$g = 9.81 \text{ (m/s}^2\text{)}$**   
Assume the vertical direction is as depicted in Assignment #3.  
Gravity may be modeled as an additional input to each mass.  
The mass of the lead-screw nut is affected by gravity because it translates back and forth,  
but the inertia of the screw and motor are not, since they rotate.

Q6

2 mark(s)

Gravity

Compute the transfer function **G**.  
The unit-step response of **G** should show the effect of gravity only.

<b>G</b>	Input = gravity	Output = fk20
• Q6.G	(N/grav)	LTI

**COW:** By the principle of super-position, adding the step responses of Q5.G and Q6.G gives  
the total response of the motor and gravity.

