

ELEC 341 – 2022-201

# Final Exam 50 Marks

## Instructions

Open book.

Open computer.

- Google OK.
- No electronic communication.
  - No messaging
  - No file sharing

Refer to **APE Resources** folder on **Canvas**:

- Read ***Ape-341-2022.pdf*** for instructions.
- Use ***makeMat341.p*** (**LATEST** version ) to create your MAT file.
- Use ***xfDS222.p*** to generate your Data-Sheet curve.
- Use ***xfCheck222.p*** to check your System Identification results.

Do not begin Control section until System Identification is correct.

Fix **ALL ERRORS** reported by ***makeMat341.p***. **Corrupt files will not be graded.**

If you do not answer any questions, use ***makeMat341.p*** to create blank variables.

Worksheets are generic and may contain extra elements that do not exist in the problems.

Worksheets are for scratch use only. Do not hand in.

Figure 1

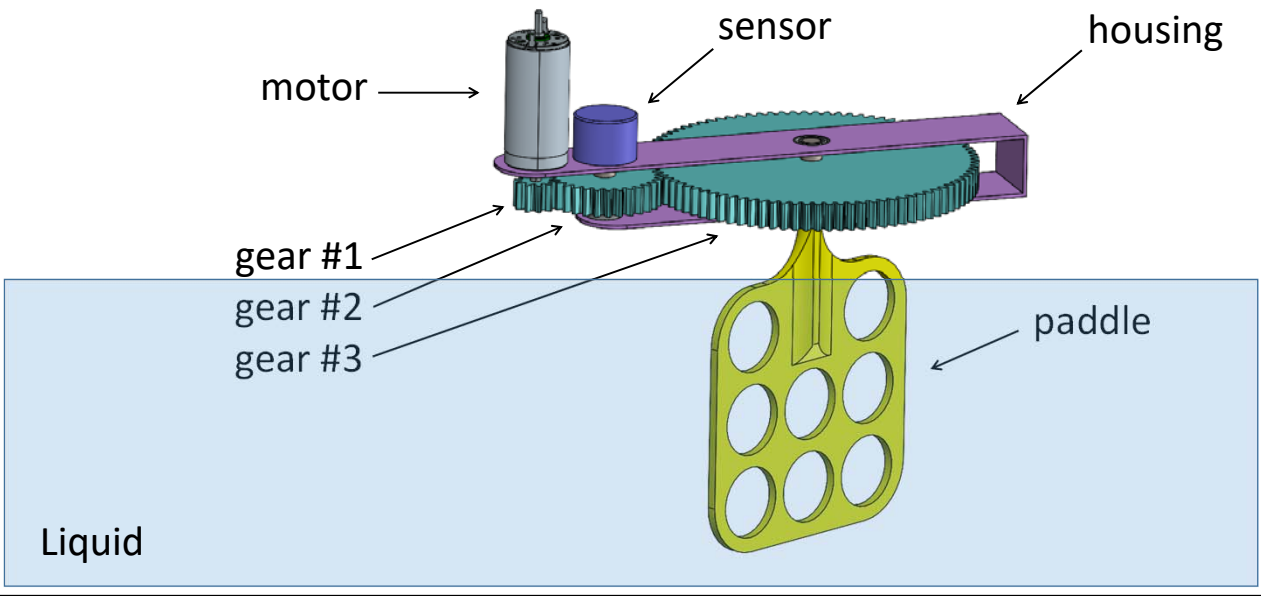


Figure 2

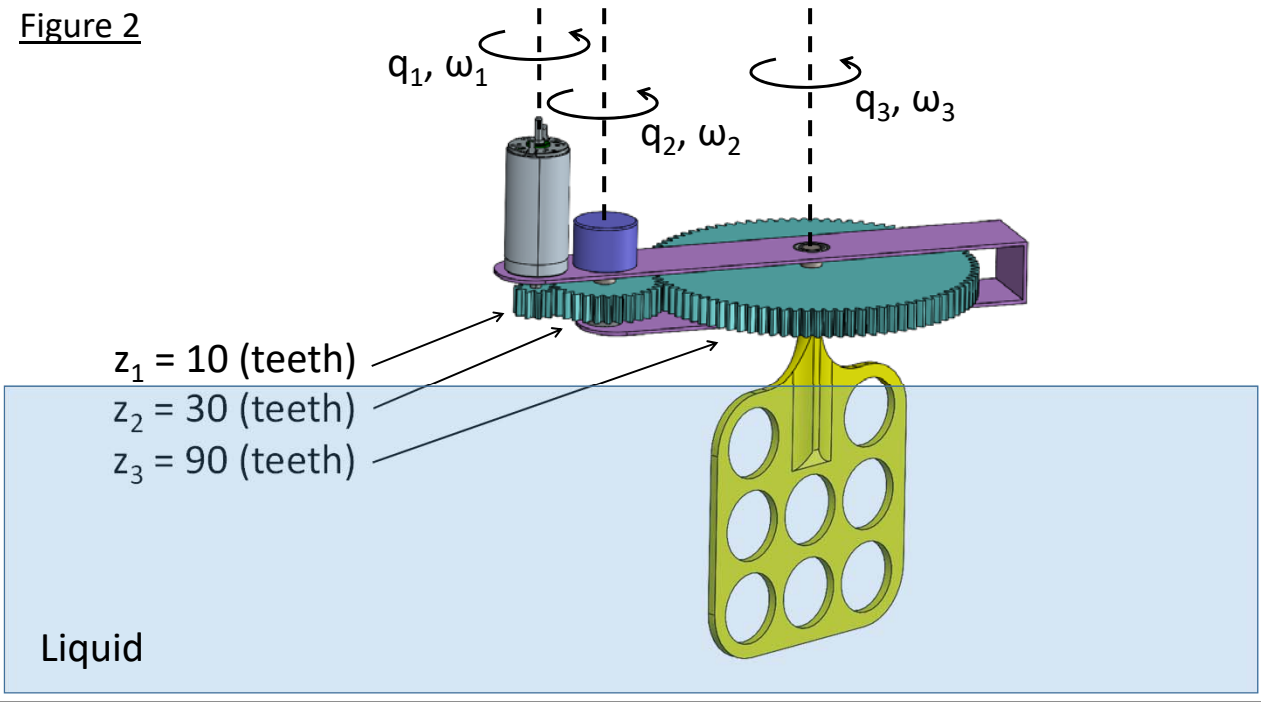


Figure 3

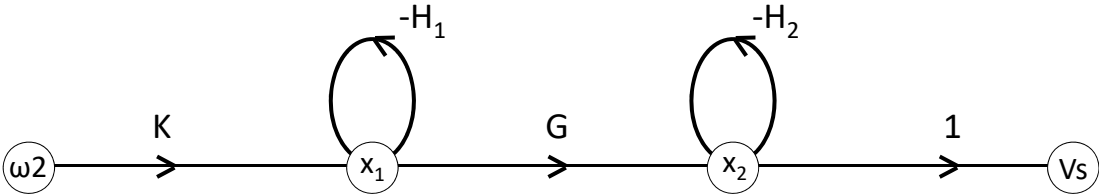


Table 1: System

Parameter	Value	Physical Units
CF	$\#C \times 20$	Hz
Ns	$\#D$	Control cycles
Nd	$\#D + \#E$	Control cycles

Table 2: Sensor

Parameter	Value
K	100,000
G	$s + (\#A \times 25)$
H1	$(s / 15) + \#B$
H2	$s^2 + (\#C \times 50 \times s) + (\#D \times \#E \times 5,000)$

Table 3: Motor

Parameter	Value	Physical Units
Rw	$\#A / 2$	$\Omega$
Lw	$\#B \times 10$	mH
Km	$(\#C + \#D) \times 10^{-2}$	Nm/A
Bm	$(\#E + \#F) \times 10^{-6}$	Nms/rad
Jm	$\#G \times 3 \times 10^{-6}$	Nms <sup>2</sup> /rad

Table 4: Mechanism

Parameter	Value	Physical Units
Bg	$\#D$	Nms/rad
Bp	$\#E \times 20$	Nms/rad
J1	$\#F \times 2 \times 10^{-6}$	Nms <sup>2</sup> /rad
J2	$\#G \times 2 \times 10^{-5}$	Nms <sup>2</sup> /rad
J3	$\#H \times 2 \times 10^{-4}$	Nms <sup>2</sup> /rad

The mixer in **Figure 1** rotates a mixing paddle at a precise speed.  
The motor is attached to Gear #1 which rotates at speed  $\omega_1$ .  
The sensor is attached to Gear #2 which rotates at speed  $\omega_2$ .  
The paddle is attached to Gear #3 which rotates at speed  $\omega_3$ .

The mechanism consists of 3 gears and a paddle (see **Table 4**).  
Gear #1 does not have any additional friction since it is connected rigidly to the motor.  
**Each** of gears 2 & 3 have friction **Bg** (bearing & gear tooth friction combined).  
The damping between the paddle and the liquid is **Bp**.  
The inertia of gears 1, 2 and 3 are **J1**, **J2**, and **J3**, respectively.

The motor is a standard DC motor (see **Table 3**).  
The windings have resistance **Rw** and Inductance **Lw**.  
The motor constant is **Km**.  
The bearings have friction **Bm**.  
The rotor has inertia **Jm**.

The sensor is described in **Figure 3 & Table 2**

The controller parameter are shown in **Table 1**.  
The controller operates at a control frequency **CF**.  
A weighted sum filter with **Ns** samples is used to filter the sensor signal.  
A weighted sum filter with **Nd** samples is used to filter the FDD of the error signal.

## SYSTEM IDENTIFICATION

1. 5 mark(s) Amplifier

A voltage amplifier receives controller Voltage **Vc** and delivers motor Voltage **Vm**.  
The step response of **Ga** (where **Vc = 1V u(t)**) is provided by running **xfDS222.p**.  
Develop a 2nd order approximation with the same **DC GAIN**, **OVERSHOOT** and **PEAK TIME**.  
**Ga = Vm / Vc**  
**Vm** = motor input voltage (V)  
**Vc** = controller output voltage (V)  
• Q1.Ga (V/V) LTI Object

2. 5 mark(s) Sensor

The sensor is modeled by the signal flow graph shown in Figure 3 with the parameters in Table 2.  
Compute the transfer function **Hs**:  
**Hs = Vs /  $\omega_2$**   
**Vs** = sensor output voltage (V)  
 **$\omega_2$**  = gear #2 speed (rad/s)  
• Q2.Hs (Vs/rad) LTI Object

**COW:**

The sensor is a **3rd order** system. If you got a lower-order transfer function, you made a mistake.  
If you got a higher-order transfer function, try using `minreal()` to cancel redundant poles & zeros.  
DO NOT include a tolerance. If nothing cancels, they are not redundant and you made a mistake.

**3. 5 mark(s) Effective Impedance**

Compute the total effective inertia **J** that is present at the motor.

Compute the total effective damping **B** that is present at the motor.

Compute the total effective spring rate **K** that is present at the motor.

- Q3.J (Nms<sup>2</sup>/rad) Scalar
- Q3.B (Nms/rad) Scalar
- Q3.K (Nm/rad) Scalar

**COW:** Use *WORKSHEET A* to track impedances.

**4. 10 mark(s) G**

Compute the forward path that includes the mixer (motor & mechanism) **Gm**.

Compute the forward path that includes the entire system (amplifier, motor & mechanism) **Gs**.

$$\mathbf{Gm} = \omega_1 / \mathbf{Vm}$$

$$\mathbf{Gs} = \omega_1 / \mathbf{Vc}$$

$\omega_1$  = motor speed (rad/s)

**Vm** = motor voltage (V)

**Vc** = controller output voltage (V)

- Q4.Gm (rad/Vs) LTI Object
- Q4.Gs (rad/Vs) LTI Object

**COW:**

Use *WORKSHEET B* to track signals & systems. Add any missing elements to the system.

Do not add any elements to the controller. The controller model is complete.

**5. 5 mark(s) GH**

Compute the open-loop transfer function that includes both the system (amplifier, motor & mechanism) and sensor **GH**.

$$\mathbf{GH} = \mathbf{Vs} / \mathbf{Vc}$$

**Vs** = sensor output voltage (V)

**Vc** = controller output voltage (V)

- Q5.GH (V/V) LTI Object

**COW:**

Is the output of **Gs** the same as the input of **Hs**?

Check Worksheet B. Label the signals.

Run **xfCheck222.p** - **ALL** checks must **PASS** to continue this exam.

## CONTROL

Overshoot must be kept small to prevent splashing.

When overshoot is small, rise, peak, and settle times all become similar, so it is sufficient to optimize settle time only.

Design a controller to control the speed of the **paddle** (gear #3).

- $I/P = \text{Desired } \textcolor{red}{SPEED} \omega_{3d} \text{ (rad/s)}$
- $O/P = \text{Actual } \textcolor{red}{SPEED} \omega_{3a} \text{ (rad/s)}$

**COW:**

*Don't forget the motor moves gear #1.*

*Don't forget the sensor measures gear #2.*

### Requirements

- $GOS \leq 5\%$
- $T_s \leq 300 \text{ (ms)}$
- $Ess = 0\%$

### Goals

- minimum GOS
- minimum  $T_s$

**6. 5 mark(s) Filter Design**

A WS filter with **Ns** (see Table 1) samples is required to compensate sensor noise.

A WS filter with **Nd** (see Table 1) samples is required to compensate FDD noise.

Use a linear approximation to compute each delay coefficient, **Nshat** and **Ndhat**.

Compute the feedback gain **Kf** and dynamics **Df**.

**Nshat** = delay introduced by sensor filter (# of control cycles)

**Ndhat** = delay introduced by FDD filter (# of control cycles)

**Kf** = **paddle** speed / sensor voltage

- Q6.Nshat (pure) Scalar
- Q6.Ndhat (pure) Scalar
- Q6.Kf (rad/Vs) Scalar
- Q6.Df (pure) LTI Object

**COW:** *Don't forget to include the sensor WS filter delay when you compute **Df**.*

**7. 5 mark(s) Partial Dynamics**

Compute the PID Controller Partial Dynamics **Dp**. Include the derivative filter.

Compute the initial gain for marginal stability **K0** using partial dynamics **Dp**.

Compute the associated cross-over frequency **wxo**.

- Q7.Dp (pure) LTI Object
- Q7.K0 (V/rad) Scalar
- Q7.wxo (rad/s) Scalar

8. 5 mark(s) Initial Controller

Find the zeros **Z** that maximize phase margin **PM**.  
Use a search resolution no larger than **0.1 rad/s**.  
Use the zeros **Z** to compute the full dynamics **D**.

- Q8.Z (rad/s) 1x2 Vector
- Q8.PM (rad/s) Scalar
- Q8.D (V/rad) LTI Object

**COW:** Don't forget to include the FDD WS filter delay when you compute **D**.

9. 5 mark(s) Heuristic Tune

Tune the PID gains to meet all the RCGs.

- Q9.K (V/rad) Scalar
- Q9.Kp (pure) Scalar
- Q9.Ki (sec<sup>-1</sup>) Scalar
- Q9.Kd (sec) Scalar

**COW:**  
Heuristic Tuning is easier when you start from a desired Phase Margin so start by tuning **K**.  
When tuning, make sure it is **paddle** speed you are controlling, and not some other joint.

All **LEFT-HALF-PLANE** poles & zeros have **NEGATIVE** real components.

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Page

**WORKSHEET A**  
(Please Print Out)

A circuit diagram showing a series R-L circuit. It includes a DC voltage source (battery), a switch, a resistor, an inductor, and a motor (represented by a circle with a cross). The motor is connected to ground. The circuit is drawn with red lines.

A mechanical system diagram showing three masses (represented by rectangles) connected in series. The top mass is connected to a fixed support (hatched area) by a spring and a damper. The middle mass is connected to the top mass by a spring and a damper. The bottom mass is connected to the middle mass by a spring and a damper. The bottom mass is also connected to ground by a spring and a damper. The system is shown with displacement coordinates and arrows indicating motion.

**WORKSHEET B**  
(Please Print Out)

A block diagram of a control system. The input is  $\omega_{3d}$ . The output is  $\omega_{3a}$ . The system is divided into two parts by a dashed line: the Controller and the System. The Controller contains blocks K, D, Dh, and Kh. The System contains blocks Ga, Gm, and Hs. The feedback signal is  $V_s$ . The control signal is  $V_c$ .