Assignment 2

CS6510: Applied Machine Learning
IIT-Hyderabad
Jan-Apr 2019

Max Marks: 40
Due: 10 Mar 2019 11:59 pm

This homework is intended to cover programming exercises in the following topics:

- SVMs, Neural Networks, Ensemble Classifiers

Instructions

- Please use Google Classroom to upload your submission by the deadline mentioned above. Your submission should comprise of a single file (PDF/ZIP), named <Your_Roll_No>_Assign2, with all your solutions.
- For late submissions, 10% is deducted for each day (including weekend) late after an assignment is due. Note that each student begins the course with 7 grace days for late submission of assignments. Late submissions will automatically use your grace days balance, if you have any left. You can see your balance on the CS6510 Marks and Grace Days document.
- You have to use Python for the programming questions.
- Please read the department plagiarism policy. Do not engage in any form of cheating - strict penalties will be imposed for both givers and takers. Please talk to instructor or TA if you have concerns.

Questions: Theory

(No programming required)

1. **Kernels:** ***(5 marks)*** Let k_1 and k_2 be valid kernel functions. Comment about the validity of the following kernel functions, and justify your answer with proof or counter-examples as required:

   (a) $k(x, z) = k_1(x, z) + k_2(x, z)$

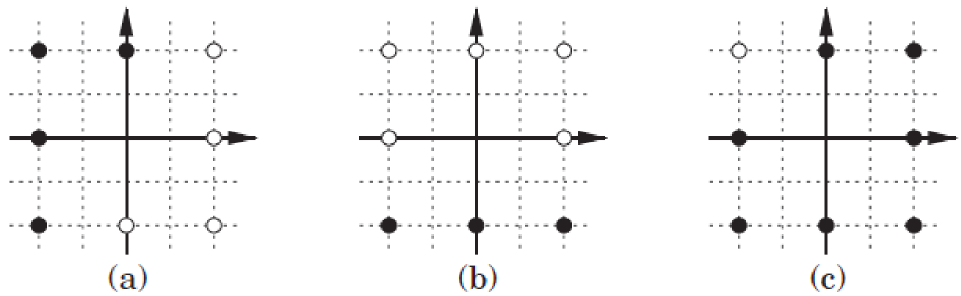
   (b) $k(x, z) = k_1(x, z)k_2(x, z)$

   (c) $k(x, z) = h(k_1(x, z))$ where h is a polynomial function with positive co-efficients

(d) $k(x,z) = \exp(k_1(x,z))$

(e) $k(x,z) = \exp\left(\frac{-\|\mathbf{x}-\mathbf{z}\|_2^2}{\sigma^2}\right)$

2. **Perceptron Classifier:** ***(3 marks)*** Solve the three simple classification problems shown in the figure below by drawing a decision boundary. Find weight and bias values that result in single-neuron perceptrons with the chosen decision boundaries:



3. **Neural Networks:** ***(2+2=4 marks)*** Given the following training data/label inputs:

$$\{\mathbf{x}_1 = [11]; y_1 = 1\}, \{\mathbf{x}_2 = [1-1]; y_2 = -1\}$$

and that these inputs are used to train a perceptron with no bias using the LMS (Widrow-Hoff/Delta learning) rule, answer the following questions:

(a) What does the mean error surface look like? In particular, comment on its curvature along the $\mathtt{w}_1$ and $\mathtt{w}_2$ axes, as well as where the surface is centered on (the minimum).

(b) What can you say about the eigenvalues of the Hessian of the error function?

Questions: Programming

4. **SVMs:** ***(3 + 2 + 4 + 3 = 12 marks)*** In this question, you will be working on a soft-margin SVM. You may find it helpful to review the Scikit Learn's SVM documentation: `http://scikit-learn.org/stable/modules/svm.html`.

We will apply soft-margin SVM to handwritten digits from the processed US Postal Service Zip Code data set. The data (extracted features of intensity and symmetry) for training and testing are available at:

- http://www.amlbook.com/data/zip/features.train
- http://www.amlbook.com/data/zip/features.test

We will train a one-versus-one (one digit is class +1 and another digit is class -1) classifier for the digits '1' (+1) and '5' (-1).

(a) Consider the linear kernel $K(\mathbf{x}_n, \mathbf{x}_m) = \mathbf{x}_n^T \mathbf{x}_m$. Train using the provided training data and test using the provided test data, and report your accuracy over the entire test set, and the number of support vectors.

(b) In continuation, train only using the first $\{50, 100, 200, 800\}$ points with the linear kernel. Report the accuracy over the entire test set, and the number of support vectors in each of these cases.

(c) Consider the polynomial kernel $K(\mathbf{x}_n, \mathbf{x}_m) = (1 + \mathbf{x}_n^T \mathbf{x}_m)^Q$, where Q is the degree of the polynomial. Comparing $Q = 2$ with $Q = 5$, comment whether each of the following statements is TRUE or FALSE.

   i. When $C = 0.0001$, training error is higher at $Q = 5$.
   ii. When $C = 0.001$, the number of support vectors is lower at $Q = 5$.
   iii. When $C = 0.01$, training error is higher at $Q = 5$.
   iv. When $C = 1$, test error is lower at $Q = 5$.

(d) Consider the radial basis function (RBF) kernel $K(\mathbf{x}_n, \mathbf{x}_m) = e(-||\mathbf{x}_n - \mathbf{x}_m||^2)$ in the soft-margin SVM approach. Which value of $C \in \{0.01, 1, 100, 10^4, 10^6\}$ results in the lowest training error? The lowest test error? Show the error values for all the C values.

Deliverables:

- Code
- Brief report (PDF) with your solutions for the above questions

5. **SVMs (contd):** ***(3 + 4 = 7 marks) (5 marks)*** GISETTE (`https://archive.ics.uci.edu/ml/datasets/Gisette`) is a handwritten digit recognition problem. The problem is to separate the highly confusible digits '4' and '9'. This dataset is one of five datasets of the NIPS 2003 feature selection challenge. The dataset for this problem is large, so please budget time accordingly for this problem.

   (a) *Standard run:* Use all the 6000 training samples from the training set to train the model, and test over all test instances, using the linear kernel. Report the train error, test error, and number of support vectors.

   (b) *Kernel variations:* In addition to the basic linear kernel, investigate two other standard kernels: RBF (a.k.a. Gaussian kernel; set $\gamma = 0.001$), Polynomial kernel (set `degree` $=$ 2, `coef0` $= 1$; e.g, $(1 + \mathbf{x}^T\mathbf{x})^2$). Which kernel yields the lowest training error? Report the train error, test error, and number of support vectors for both these kernels.

Deliverables:

- Code
- Brief report (PDF) with your solutions for the above questions

6. **Random Forests:** ***(5 + 2 + 2 = 9 marks)***

   (a) Write your own random forest classifier (this should be relatively easy, given you have written your own decision tree code) to apply to the *Spam* dataset [`data`, `information`]. Use 30% of the provided data as test data and the remaining for training. Compare your results in terms of accuracy and time taken with Scikitlearn's built-in random forest classifier.

   (b) Explore the sensitivity of Random Forests to the parameter m (the number of features used for best split).

(c) Plot the OOB (out-of-bag) error (you have to find what this is, and read about it!) and the test error against a suitably chosen range of values for m.

Deliverables:

- Code
- Brief report (PDF) with your solutions for the above questions