Example 6.101

Solve the equation $\sqrt{x+2}+4=x$

Solution. First, we isolate the radical expression on one side of the equation. (This will make it easier to square both sides.)

$$\sqrt{x+2} = x-4 \qquad \text{Square both sides of the equation.}$$

$$\left(\sqrt{x+2}\right)^{\mathbf{2}} = (x-4)^{\mathbf{2}}$$

$$x+2 = x^2-8x+16 \qquad \text{Subtract } x+2 \text{ from both sides.}$$

$$x^2-9x+14=0 \qquad \text{Factor the left side.}$$

$$x=2 \quad \text{or} \quad x=7$$

Check

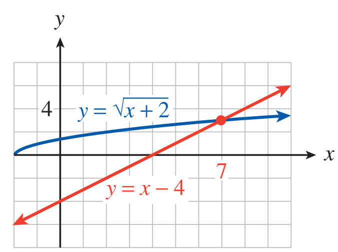
Does $\sqrt{\mathbf{2}+2}+4=\mathbf{2}$? No; 2 is not a solution.

Does $\sqrt{\mathbf{7}+2}+4=\mathbf{7}$? Yes; 7 is a solution.

The apparent solution 2 is extraneous. The only solution to the original equation is 7. We can verify the solution by graphing the equations

$$y_1=\sqrt{x+2} \quad \text{and} \quad y_2=x-4$$

as shown at right. The graphs intersect in only one point, $(7,3)$, so there is only one solution, $x=7$.



Caution 6.102 When we square both sides of an equation, it is *not* correct to square each term of the equation separately. Thus, in Example 6.101, p. 430, the original equation is *not* equivalent to

$$(\sqrt{x+2})^2+4^2=x^2 \qquad \text{Incorrect!}$$

This is because $(a+b)^2 \neq a^2+b^2$. Instead, we must square the *entire* left side of the equation as a binomial, like this,

$$(\sqrt{x+2}+4)^2=x^2$$

or we may proceed as shown in Example 6.101, p. 430.

Checkpoint 6.103 **Practice 2.** Solve $2x-5=\sqrt{40-3x}$

$x=$__

Hint:

- Square both sides.
- Solve the quadratic equation.
- Check for extraneous roots.

Answer. 5

Solution. $x=5$