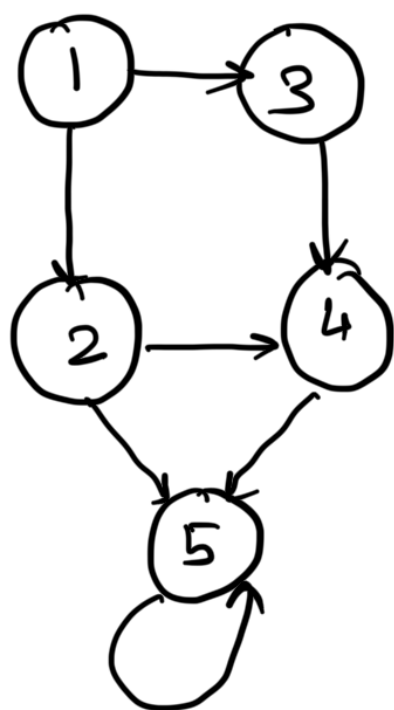


Graph: Data structure

- A graph is a non-linear data structure composed of 2 main components:

1. Vertices - Nodes \rightarrow Entity / Objects
2. Edges - Connection - pair of vertices

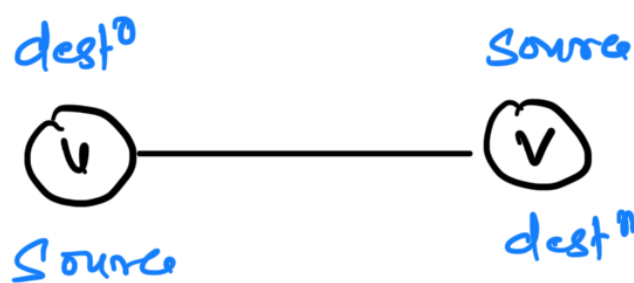
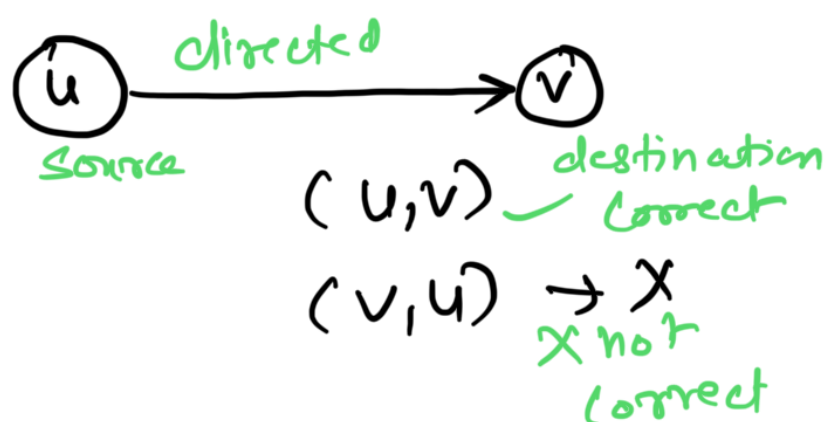
$$G(V, E)$$



Directed Graph

$$V = \{1, 2, 3, 4, 5\}$$

$$E = \{(1, 2), (1, 3), (3, 4), (2, 4), (4, 5), (2, 5)\}$$



Undirected Graph

(u, v)
 (v, u) } both are correct

These graphs have Symmetric relation

Components of Graph -

1. Vertex \rightarrow fundamental unit of Graph
 \rightarrow can be labeled or unlabeled

2. Edges \rightarrow Ordered pair, \rightarrow directed graph
 unordered pair \rightarrow undirected graph

Graph Properties -

1. Number of edges - Undirected Graph

The no. of possible pairs in a 'n' vertex graph is $n(n-1)$.

$$\boxed{\frac{n(n-1)}{2}} \leftarrow \begin{array}{c} (u,v) = (v,u) \\ \uparrow \\ \text{same} \end{array}$$

2. Number of edges - Directed Graph

The no. of possible pairs in a 'n' vertex graph is $n*(n-1)$

$$\boxed{\text{No. of edges} \leq n(n-1)} \quad \begin{array}{c} (u,v) \neq (v,u) \\ \uparrow \\ \text{not same} \end{array}$$

Types of Graph

1.

①

②

- No edges betⁿ the vertices

④

③

- Set of isolated nodes without any relation

Null graph

2.

⤿

- A graph with only

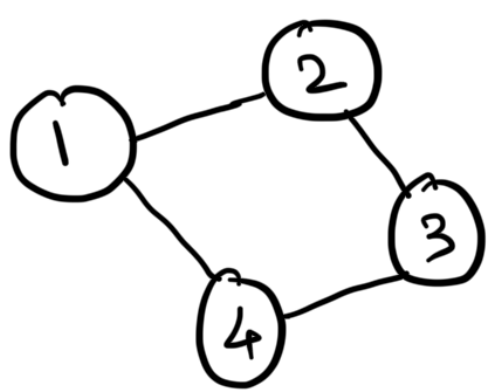


Trivial Graph

one vertex

- isolated vertex

3.

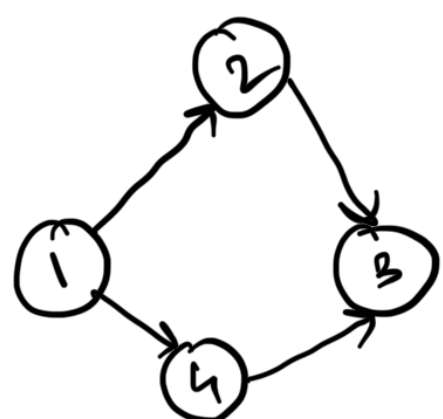


Undirected Graph

- Edges have no direction, but both connected vertices are treated equally

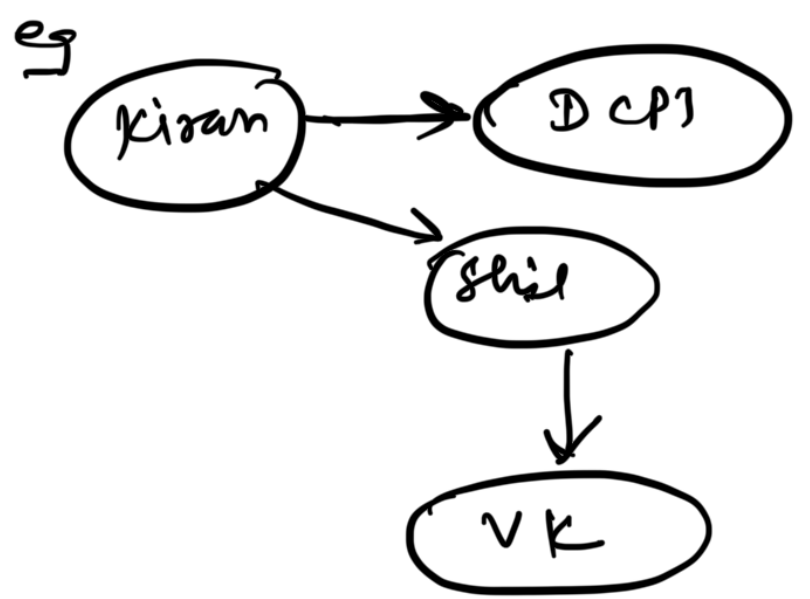
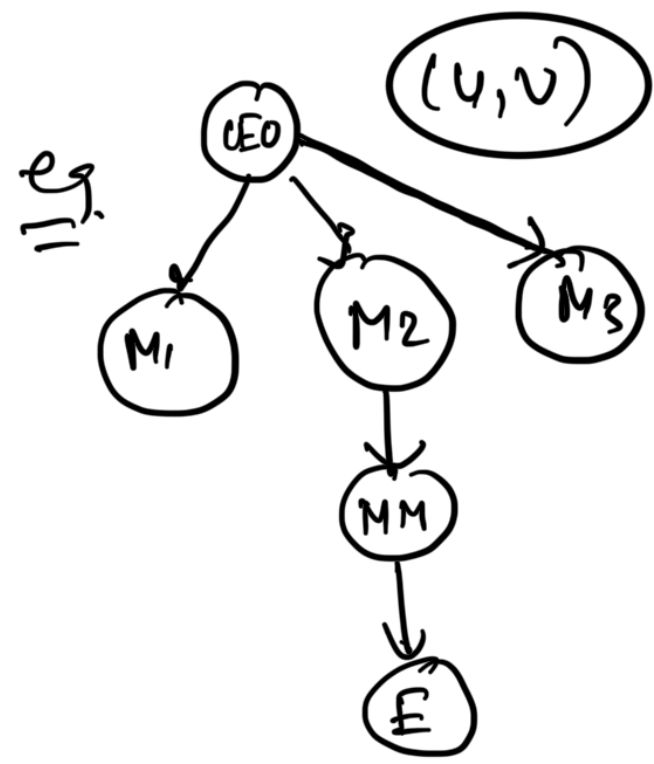
- Mutual Relationship
↓
Social Media
Social N/w

4.

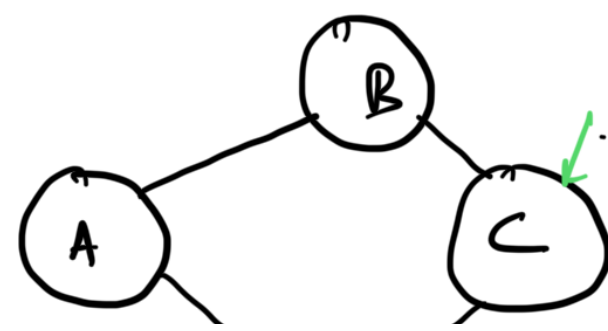


Directed Graph (Digraph)

- Edges have direction
- source to destination (u) (v)
- one vertex points to another vertex
- Twitter follow relationship



5



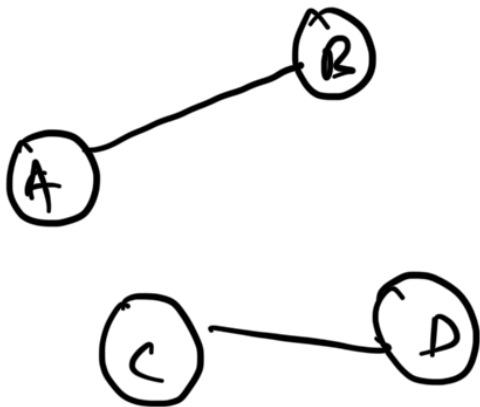
- Every node is reachable from any other node



Connected graph

- Network eg

⑥

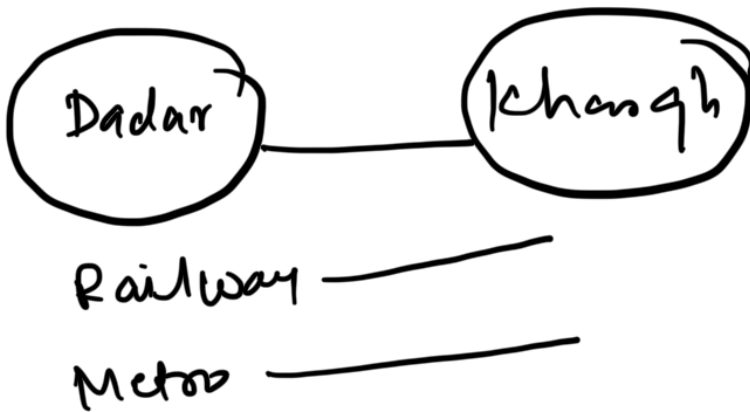


Dis connected graph

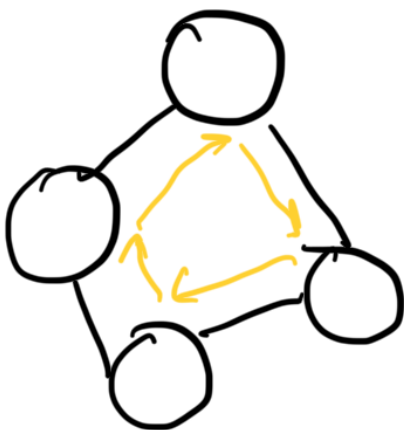
- Atleast one vertex is reachable

- eg. railway & Metro paths from same source to destination

eg



⑦

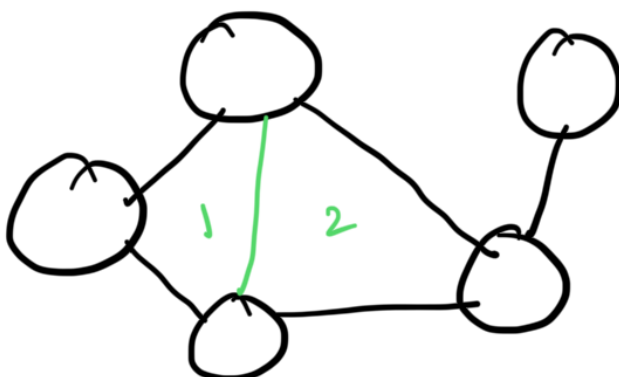


Cycle graph

- A graph where all vertices form a cycle

eg. Roundtrip

⑧



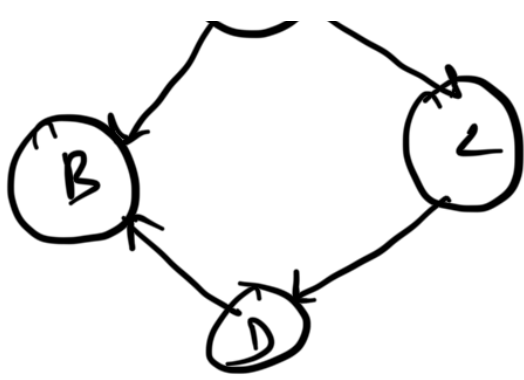
Cyclic graph

- A graph contains atleast 1 cycle

⑨

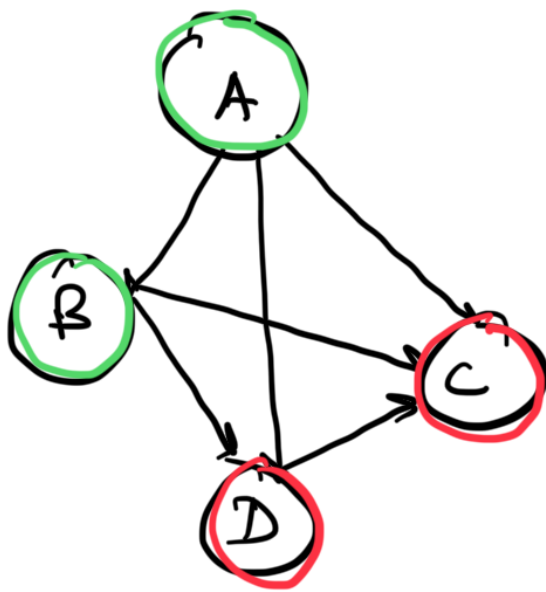


- Directed graph with



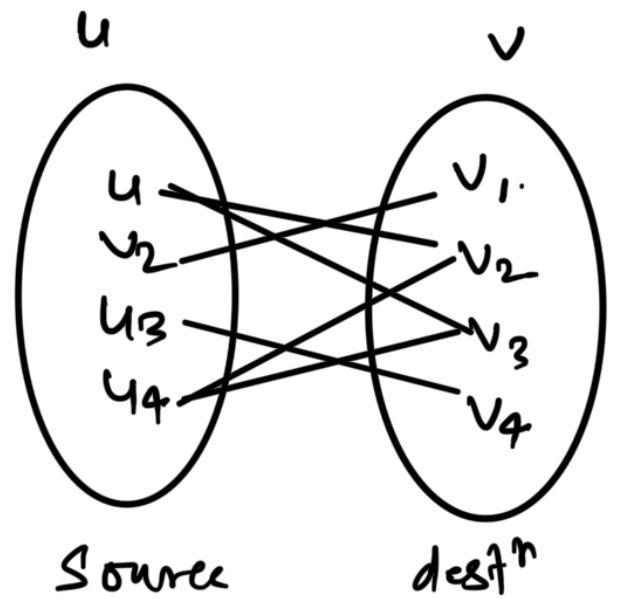
Directed Acyclic Graph
(DAG)

10

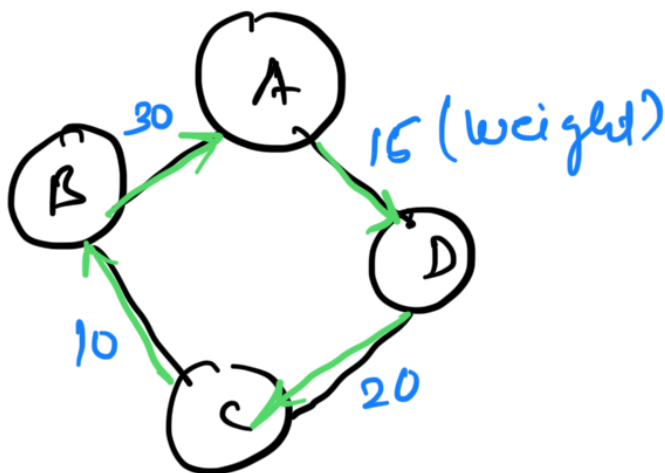


Bipartite Graph

- A graph which divides the vertices in two sets of vertex



11



Weighted Graph

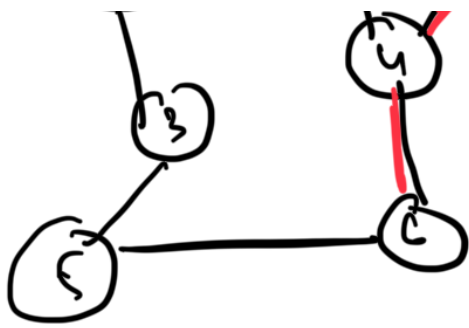
- Graph where edges carry weight
- weight \Rightarrow cost
Amount
distance
etc

Degree of the vertex:

Indegree(N), Outdegree(N)

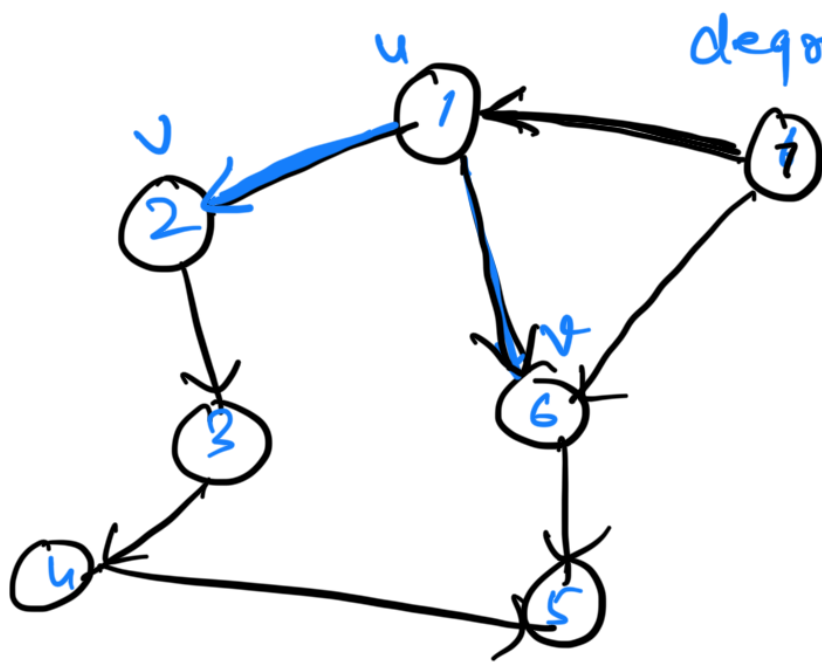


degree \Rightarrow source



$$\text{degree}(2) = \underline{2}$$

$$\text{degree}(4) = \underline{3}$$



$$\text{degree}(u) \Rightarrow \text{Outdegree}$$

$$(v) \Rightarrow \text{Indegree}$$

$$\text{degree}(1) = 2$$

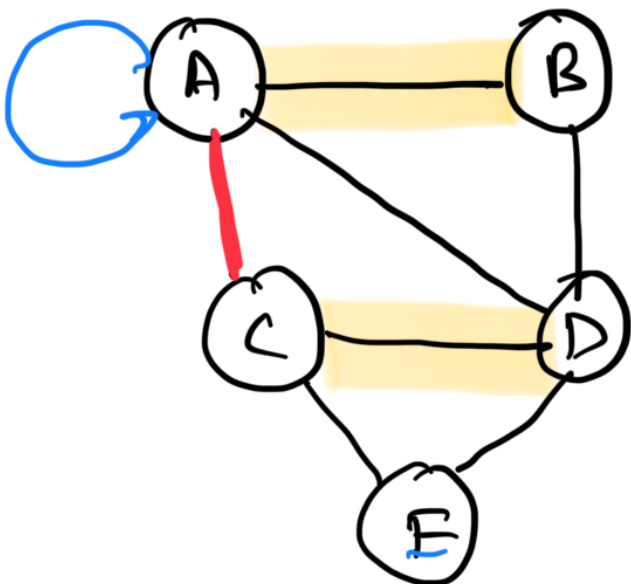
$$\text{degree}(1) = 1$$

Representation of Graph in data structures

1. Adjacency Matrix

2. Adjacency List

Adjacency Matrix (Row, Col) \Rightarrow (5x5)



$\rightarrow A$

$\rightarrow B$

$\rightarrow C$

$\rightarrow D$

$\rightarrow E$

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	0
C	1	0	0	1	1
D	1	1	1	0	1
E	0	0	1	1	0

JAR

Matrix consists only 0, & 1 & 0, 1, 3

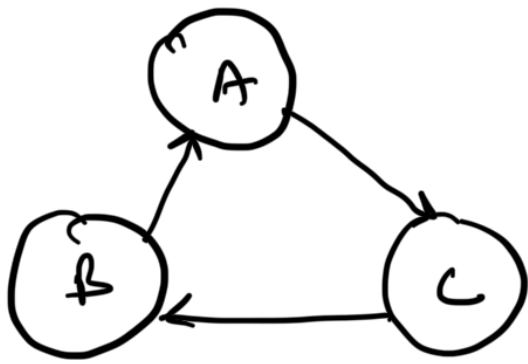
Undirected

Space Complexity = Rows \times Cols

$$= n \times n$$

$$= O(n^2)$$

Adjacency matrix \Rightarrow Symmetric matrix

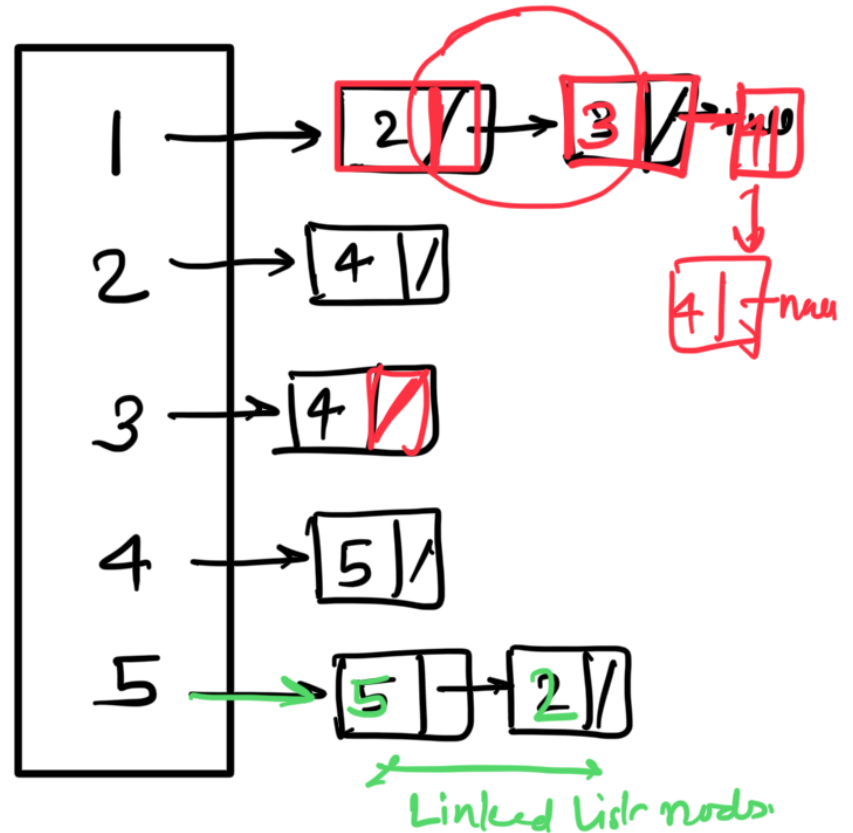
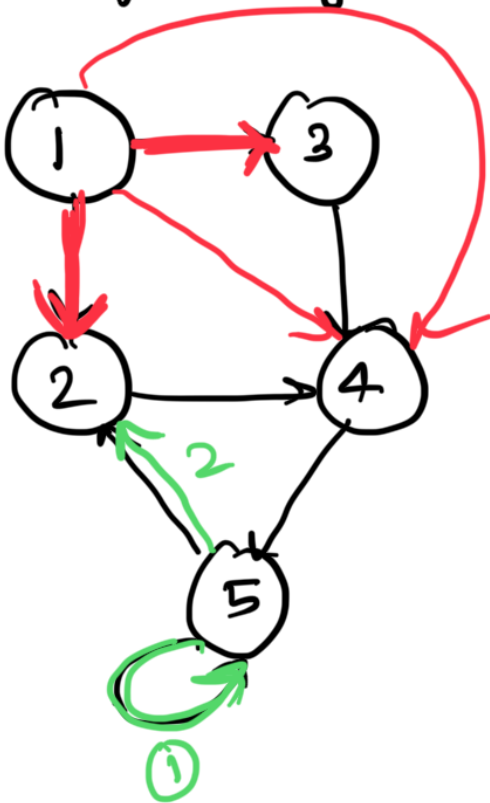


Directed graph

Adjacency Matrix

	A	B	C
A	0	0	1
B	1	0	0
C	0	1	0

2. Adjacency List



$$\begin{aligned}
 \text{Space Complexity} &= \text{No. of edges} \\
 &= |E| \\
 &= O(|E|) \\
 &= O(n)
 \end{aligned}$$