Question 8.- Prove (from the definition of a limit of a sequence) that if the sequence$\{a_n\}^{\infty}_{n=1}$ tends to limit L as $n \to \infty$, then for any fixed number M >0, the sequence$\{Ma_n\}^{\infty}_{n=1}$ tends to the limit M L

We have to prove from the definition of a limit of a sequence which, formally, can be expressed as:

$$(\forall\, \varepsilon > 0)(\exists n \in \mathbb{N})(\forall\, m \geq n)(|a_n - L| < \varepsilon)$$

Proof: The definition shows a relation between arbitrary bug numbers in the sequence, the limit and a variable ε that gets closer and closer to 0 s.t. the bigger the n, the closer ε to 0.

1.- By definition, the relation can be express also as follows:

$$(|a_n| < L + \varepsilon)$$

The absolute value of a_n gets closer and closer to L, thus being ε closer and closer to 0. We have done nothing but rephrase the relation between a_n, L and ε.

2.- So, for any fixed number M >0 so the sequence $\{Ma_n\}$, the product by M has to be done in both sides of the inequality for the relation to hold

$$(|a_n| < L + \varepsilon) \Rightarrow (M\,|a_n| < M\,(L + \varepsilon)) \Leftrightarrow (M\,|a_n| < ML + M\varepsilon)$$

Since ε gets closer and closer to 0, Mε gets also closer and closer to 0, so the limit is ML

$$(M\,|a_n| < ML + \varepsilon)$$

3.- So, from the definition of a limit of a sequence and the properties of relations between variables, it has been proven the statement. QED.