Question 6.- Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7

Rephrasing the statement:

$$(\exists n \in \mathbb{N})((prime(n) \wedge prime(n+2) \wedge prime(n+4)) \Rightarrow n=3)$$

For:

$$(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})(prime(n) \, iff \; m|n \Rightarrow (m=n) \vee (m=1 \wedge m \neq n))$$

And the only even prime number is 2, for every other even number is divisible, at least, by 1, 2 and itself. We

Proof: Every three consecutive odd numbers have at least one of the divisible by 3, and the only number divisible by three that is a prime number is 3 itself.

1.- We can have the following limitations as true:

- Any prime triple would consist in odd numbers, since there is only one prime that is even.
- By definition, such odd numbers will be consecutive odd numbers.
- 1 is not a prime number by definition.
- 3 is the only number divisible by 3 that is also a prime.
- By combining the previous statements, the only three consecutive odd numbers that can contain a number divisible by 3 is the triple 3, 5, 7. The triple of the statement.

2.- At this point we see that this question can be proven by finding that for any three consecutive odd numbers, which take the form n, n+2, n+4 for odd(n), one of them is divisible by 3. This has been already proven in question 5 for any integer. Here is my proof:

https://github.com/PrxSaavedra/Intr_Math_Thinking/blob/master/Q5.pdf

3.- Since any prime triple will take the form of three consecutive odd numbers and, for any three consecutive odd numbers, one will be divisible by 3. Since, by definition, in all prime triples, such number divisible by 3 need to be prime itself, being 3 the only case. And also by definition, 1 is not a prime number, the only prime triple including 3 is the question prime triple 3, 5, 7. Taking all that into consideration, any three consecutive numbers different from the one in the question will include one number divisible by 3 that is not 3, thus not being a prime number. So there is no prime triple other that 3, 5, 7. QED.