

Question 2.- Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

We need to find out if

$$\forall n \in \mathbb{Z} (5 | n + [n+1] + [n+2] + [n+3] + [n+4])$$

Such statement is true in the set of integers for any n.

Proof: We will establish the form of divisible by 5 in the set of integers and compare it with the sum of any five consecutive integers.

1.- By definition, $(\forall p \in \mathbb{Z})(\exists q \in \mathbb{Z})(5 | p \Rightarrow p = 5q)$

2.- Let's define m as the integer resulting of summing any five consecutive integers

$$m = n + [n+1] + [n+2] + [n+3] + [n+4] \Rightarrow m = (n + n + n + n + n) + (1 + 2 + 3 + 4) \Rightarrow m = 5n + 10 = 5(n+2)$$

3.- Let's compare m with the form of any integer divisible by 5.

$$5 | m \text{ if } (\exists q \in \mathbb{Z}) \text{ s.t. } m = 5q$$

$$5(n+2) = 5q \Rightarrow q = n+2$$

4.- As n+2 is an integer, q is an integer and m is divisible by 5.

So the sum of any five consecutive integers is divisible by 5 and adopt the form of 5 times the third of those integers. Thus being true the statement of the question. QED.