Question 7.- Prove that for any natural number n,

$2 + 2^2 + 2^3 + . . . + 2^n = 2^{n+1} - 2$

We will proof that the sequence of adding powers of two is equal to 2 to n+1 minus 2 for any natural number n. That is true, as we will see by induction.

Proof: By induction

1.- The first step in a proof by induction is proving for the first case. Since we are talking about natural numbers. We prove the statement for n=1 by doing simple operations.

$$2^1 = 2^{1+1} - 2 \Leftrightarrow 2 = 2^2 - 2 = 4 - 2 = 2$$

2.- The second step in a proof by induction is assuming the statement for n and proving it for n+1.

We assume it is true for n	$2 + 2^2 + 2^3 + . . . + 2^n = 2^{n+1} - 2$
We check for n+1	$2 + 2^2 + 2^3 + . . . + 2^n + 2^{n+1} = 2^{(n+1)+1} - 2$
We substitute the left side by the assumption	$(2^{n+1} - 2) + 2^{n+1} = 2^{(n+1)+1} - 2$
Rearrange the equation	$2^{n+1} + 2^{n+1} - 2 = 2^{n+2} - 2$
Add 2 to both sides	$2^{n+1} + 2^{n+1} = 2^{n+2}$
Operate left side	$2 * 2^{n+1} = 2^{n+2}$
Multiply powers of same base	$2^1 * 2^{n+1} = 2^{n+1+1} = 2^{n+2}$

3.- We have proven the statement for the first case of n and, assuming it true for n, for n+1, thus finishing our proof by induction and proving the statement of the question is true. QED.