

Question 1: Say whether the following is true or false and support your answer by a proof

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

The statement refers to a relation between a pair of natural numbers (m, n) and will be true if there is at least one of such pair so that  $3m + 5n = 12$

Let's remember that  $0 \notin \mathbb{N}$

We will find, by contradiction, that there is no such pair of numbers in the set of naturals

Proof: By contradiction. Let's assume  $3m + 5n = 12$  has solutions for a pair (m, n) in the set of natural numbers.

1.- Let's clear m

$$3m + 5n = 12 \Rightarrow 3m = 12 - 5n \Rightarrow m = \frac{(12 - 5n)}{3}$$

2.-  $m \in \mathbb{N}$  iff

2.1.-  $12 - 5n > 0$  so that m is positive. Such inequation is only true in the natural numbers for  $n=1$  or  $n=2$ . So  $m > 0 \Rightarrow n < 3$

and

2.2.-  $3 \mid 12 - 5n$ , so  $\exists p \in \mathbb{N}$  such that  $3p = 12 - 5n$  and, by obtaining common factor,  $12 - 5n = 3(4 - 5p)$ . Given that p is a natural number,  $3p \geq 3 \Rightarrow n \geq 3$

3.- Taking both conditions,  $m \in \mathbb{N}$  iff  $n < 3 \wedge n \geq 3$  but that is a contradiction.

For  $3m + 5n = 12$  to be true in the set of natural numbers for both m and n, n would need to be bigger or equal to 3 and smaller than 3 at the same time, which is not possible. So by contradiction, there is no solution for the equation in  $\mathbb{N}$ , thus being false the statement of the question 1. QED.