Question 3.- Say whether the following is true or false and support your answer by a proof: For any integer n, the number n^2+n+1 is odd.

We will proof that such statement is true but, before that we ought to recall some facts we have seen during the course:

1.- $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})$ s.t. Odd(n) => n = 2m+1
Odd(x) meaning that x has the property of being odd
2.- $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})$ s.t. Even(n) => n = 2m
Even(x) meaning that x has the property of being even
3.- The product of an even integer and an odd integer will be another even integer
3.1.- Let's take any two p,q integers s.t. $Odd(p) \wedge Even(q)$
3.2.- $(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z})(p=2m+1 \wedge q=2n)$
3.3.- The product of p,q
(2m+1)2n = 4mn + 2n = 2(2mn+n)
3.4.- As (2mn + n) is an integer itself, 2(2mn+n) is an even integer

On that basis, we can proceed with the proof.

Proof: We will compare the number n^2+n+1 with the form of odd numbers seen (2m+1)

1.- We equate
$n^2+n+1 = 2m + 1$
2.- 1 is canceled in both sides of the equation and factorize the left side
$n^2+n = 2m \Rightarrow n(n+1) = 2m$
3.- n(n+1) is the product of two consecutive numbers so
$(Odd(n) \wedge Even(n+1)) \vee (Even(n) \wedge Odd(n+1))$
4.- Thus n(n+1) is the product of an even number and an odd number, resulting in another even number that can take the form 2m
5.- We have confirmed the equation and thus the truth of the statement presented in the question.
QED.