Question 10.- Given an infinite collection A_n, n=1,2,… of intervals of the real line, their *intersection* is defined to be

$$\bigcap_{n=1}^{\infty} A_n = \{x \mid (\forall n)(x \in A_n)\}$$

Give an example of a family of intervals A_n, n=1,2,…, such that $A_{n+1} \subset A_n$ for all *n* and the intersection consists of a single real number

Prove that your example has the stated property.

Actually, the example can be the same of question 9, but with the left side closed in the intervals.

Part 1.- Give an example of a family of intervals A_n, n=1,2,…, such that $A_{n+1} \subset A_n$ for all *n*

I propose the following family of intervals: $A_n = [\ 1, 1+\frac{1}{n}\ ]$

The family of intervals closed on both sides, from 1 to $1+\frac{1}{n}$

The property $A_{n+1} \subset A_n$ is fulfilled because as *n* gets arbitrarily bigger, $\frac{1}{n}$ gets smaller. So

$$(\forall n \in \mathbb{N})\left(\frac{1}{(n+1)} < \frac{1}{n}\right) \quad => \quad A_n = A_{n+1} \cup \left\{\frac{1}{(n+1)}, \frac{1}{n}\right\} \quad => \quad A_{n+1} \subset A_n$$

Part 2.- Prove that the intersection of the family of interval has no elements

1.- All the intervals are closed on the left side, thus 1 is included by the left side.

2.- We have already proven in class that for the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$. $\frac{1}{n} \rightarrow 0$ as $n \rightarrow \infty$

Thus for $\left\{1+\frac{1}{n}\right\}_{n=1}^{\infty}$. $1+\frac{1}{n} \rightarrow 1$ as $n \rightarrow \infty$

3.- So the right side of the intervals get smaller and closer and closer to 1, without reaching it, as *n* grows.

4.- Given that by the right side the intervals get smaller and closer to 1 without reaching it, but being 1 included by the left side as it is closed, the intersection of the family has only one element, the real number 1. QED