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Optimum design analysis of hybrid cable-stayed suspension bridges

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ABSTRACT

A design methodology to predict optimum post-tensioning forces and dimensioning of the cable system for hybrid cable-stayed suspension (HCS) bridges is proposed. The structural model is based on the combination of an FE approach and an iterative optimization procedure. The former is able to provide a refined description of the bridge structure, which takes into account geometric nonlinearities involved in the bridge components. The latter is utilized to optimize the shape of post-tensioning forces as well as the geometry of the cable system to achieve minimum deflections, lowest steel quantity involved in the cable system and maximum performance of the cables under live load configurations. Results are proposed in terms of comparisons with existing formulations to validate the proposed methodology. Moreover, parametric studies on more complex long span structures are also developed to verify existing cable-dimensioning rules and to analyze between HCS bridges and conventional cable-stayed or suspension systems.

Introduction

Cable supported bridges are typically employed to overcome medium or long spans, because of their structural, economical and aesthetic properties. Such characteristics arise from an enhanced combination of the structural components of the bridge, which are essentially girder, cable system and pylons. The cable elements can be arranged following cable-stayed or suspension configurations [1]. Moreover, in the case of hybrid cable-stayed suspension (HCS) bridges, the cable system is defined by presence of stays, hangers and main cable, leading to bridge structures, which, typically, present better performances than conventional ones based on pure suspension and cable-stayed configurations [2,3]. Cable supported bridges, especially for long spans, are defined through a large number of cable elements, which lead to highly statically indeterminate structures. As a result, post-tensioning stresses and cross-sections of the cables can be considered as design variables, which must be determined to identify the bridge configuration under dead and live loading, enforcing the lowest steel quantity involved in the cable-system and the optimum performance of the structural elements. The design procedure becomes more complicated in the cases of HCS bridges, because of the large number of variables involved in the solving procedure and the interaction between cable-stayed and suspension systems, which, typically, produces complex phenomena due to the geometric nonlinearities involved in the cable elements [4].

Most of existing cable supported bridges are designed by using traditional techniques, in which iterative methods based on simple design rules obtained by the designer's experience and expertise were utilized [5,6]. However, during the last decades, many research efforts are carried out with the aim to propose proper procedures to calculate the optimum configuration of the bridge. In particular, zero displacement methods (ZDMs) are based on the use of explicit constraint equations, which enforce the bridge structure under dead loading to remain practically undeformed [7,8]. The governing equations, expressed as a function of the internal forces of the cable system, introduce a determinate equation system, in which the unknown quantities are obtained, prescribing, at discrete points of the structure, zero displacement conditions. Alternatively to ZDMs, force equilibrium methods (FEMs) consider as control variables to be solved, the internal forces of the cable system, which are calculated to reduce bending moments and displacements of the girder, achieving a structural scheme of the girder approximately equivalent to a simply supported continuous beam [9–11]. Such methodologies are typically utilized in the framework of pure suspension and cable-stayed bridge schemes, since the resulting equations introduce a determinate equation system in terms of cable stresses. However, in the cases of HCS bridges, the presence of the two cable systems introduces additional variables in the solving procedure and thus further

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Nomenclature

$(\bar{\bullet})$	values arising from previous iteration step or trial values obtained by preliminary dimensioning rules
$(\cdot)^G$	stiffening girder variable
$(\cdot)^P$	pylon variable
$(\cdot)^S$	stay variable
$(\cdot)^H$	hanger variable
$(\cdot)^M$	main cable variable
A	cross-section area
ΔA	incremental cross-section area of the cable
b	half girder cross-section width
CS	cable-stayed bridge
d	girder depth
DL	dead load configuration
E	material elasticity modulus
f	main cable sag
g	self-weight per unit length
H	pylon height
H_{t0}	initial main cable horizontal axial force
HCS	hybrid cable-stayed suspension bridge
I_i	moment of inertia with respect to the i-axis
J	factor torsional stiffness
k	number of iterations
K^P	in plane flexural top pylon stiffness
l	lateral span length
l_m	portion of the main span without stays
L	central span length
L_i	ith cable length
L_T	girder total length
LL	live load configuration
N	number of cables
p	live load per unit length
Q	total cable steel quantity
r	stayed-suspension coupling parameter
ΔS_i	incremental initial stress in the ith cable
S_i	cable stress under self-weight loading
S_{gi}	initial design stress of the ith cable
S_a	allowable cable stress
ΔS_a	incremental allowable cable stress
SP	suspension bridge
U_i	component of the displacement field along i-axis
α	longitudinal stay geometric slope
δ_A	maximum allowable displacement
Δ	stay spacing step
ϕ	maximum longitudinal main cable geometric slope
Φ_i	stress performance factor of the ith cable
γ	material specific weight
Ω_i	strength performance factor of the ith cable
ξ_i	cable cross-section optimization factor
ψ_i	cable stress optimization factor
Ψ_i	component of the rotation field around i-axis

equations are required to impose the constraint equations on the bridge configuration [3].

Alternatively to direct methods, models, developed in the framework of structural optimization, are frequently adopted in the literature [12]. In particular, post-tensioning forces are determined by solving constrained minimization problems as a function of performance or objective scalar valued functions, constraint equations and control variables. For cable-stayed bridges, existing models based on optimization methods (OMs) utilize an objective scalar function, which is typically expressed in terms of norm of displacements [13,14]. From the minimization of the objective function, such methods are able to calculate the optimum set of post-tensioning forces, which achieves minimum deflections and a uniform bending moment distribution under the effect of dead loading. Advanced techniques for cable-stayed bridges, proposed in [15], are based on a multicriterion optimization procedure, in which in the minimization procedure also the characteristics of the girder cross-section are included. Moreover, refined formulations based on robust optimization algorithms can be recovered in [16], in which models based on the neural-network concepts are able to obtain the optimum structural configuration. However, models developed in the framework of OMs, especially in the cases of long span bridges, due to the presence of a large number of variables are affected by convergence problems in the solving procedure, which may lead to a local minimum of the objective function and unpractical results in the bridge definition.

In the framework of suspension bridges, the procedure to calculate the initial configuration under dead loads is relatively simple, because the main cable extremities are fixed at earth constrains. As a consequence, optimization techniques are, frequently, employed with the purpose to identify the structural behavior of the bridge with respect more complex external loads, such as aeroelastic [17] or seismic [18] phenomena. However, most of the previous methodologies are typically concerned to evaluate optimum post-tensioning forces in the dead load (DL) configuration, without achieving the complete optimization of the geometry, the stiffness of the structural elements and thus the costs of construction. Such tasks were investigated only recently by few methodologies, mainly developed in the framework of cable-stayed bridge schemes. In particular, Hassan [19] introduced a generalized formulation by means of a combined approach based on the finite element method and an optimization genetic algorithm, in which the distribution of the cable cross-sections is expressed by means of B-spline curves. Moreover, Baldomir et al. [20] have proposed an iterative approach, in which, initially, the post-tensioning cable forces in the DL configuration are determined by solving compatibility conditions arising from flexibility matrix of the structure. Subsequently, the optimization procedure is utilized to minimize the cross-sections of the cable system, on the basis of the maximum effects on stress and displacement variables evaluated on the live load (LL) configurations.

Finally, as far as HCS bridges are concerned, the evaluation of the optimum design in terms of post-tensioning forces as well as of cable cross-section design has been mostly analyzed in the context of cable-stayed or suspension configurations and, to the best Authors' knowledge, no work on the optimal design of HCS bridges is available from the literature. As a consequence, in order to reach a better understanding further investigations on the design of HCS bridges must be developed.

In the proposed formulation, a generalized approach based on two-step algorithm is developed, which evaluates post-tensioning cable forces in the DL configuration and optimum steel quantity involved in the cable system on the basis of LL results. The method, presented for HCS bridges, is quite general to be applied to several schemes based on small, medium or long spans and can be, easily, specialized for pure cable-stayed and suspension schemes. In order to prove the effectiveness of the proposed model, comparisons with existing formulations available from the literature and parametric studies in terms of cable system configurations are proposed. The outline of the paper is as follows. In Section 2, the formulation of the design methodology, bridge modeling, together with the description of the iterative procedure concerning the optimization technique is presented. In Section 3, numerical details of the design method are reported, whereas in Section 4, numerical comparisons and parametric results are proposed.

Formulation of the design methodology

Bridge modeling

The bridge scheme, reported in Fig. 1, is consistent with a long span bridge typology, in which the cable system is composed by the combination of suspension and cable-stayed configurations. Suspension and cable-stayed cable systems are based on earth and self-anchored schemes and consist of a double layer of cables arranged in the plane containing girder and pylon extremities. The hanger rods and the stays are hinged, at both ends, to the girder and main cable or to the girder and pylons, whereas the main cable is supported at the top pylon cross-sections consistently to a saddle connection.

The bridge structure is modeled by means of a 3D finite element approach based on a displacement-type finite element approximation implemented in a FE software, i.e. Comsol COMSOL Multiphysics [21]. In order to reduce the computational efforts in the numerical calculations, the FE model is based on beam elements for girder and pylons and truss elements for the cable system. Specifically, the bridge deck is replaced by a longitudinal spline with equivalent cross-section and material properties, whereas the pylons are composed by two columns linked at their top by horizontal beam elements. The bridge deck is connected to the suspension system by means of explicit constraint equations, which are imposed between the off-set nodes of the girder and those associated to the cable elements. The cable system, which is connected to the pylons and girder, is essentially defined by the combination of stays, hangers and main cable. In particular, the cable system is modeled according to the Multi Element Cable System (MECS) approach, in which each cable is discretized using multiple truss elements [22–24]. The stiffness reduction caused by sagging is accounted by allowing the cable to deform under applied loads. Large deformations are reproduced by using Green Lagrange formulation and the axial strain is calculated by expressing the global strains in tangential derivatives and projecting the global strains on the cable edge. Additional details on the approach here adopted to model nonlinear behavior of the cable elements can be found in [4,21,22,25].

Design methodology

The main aim of the optimization model is to evaluate optimum set of post-tensioning forces and cable cross-sections to satisfy structural and design requirements in either dead and live configurations. From the design point of view, it is required that, under the action of dead loads, the post-tensioning forces of the cables are calculated in such a way that the bridge should behave as a simply supported continuous beam, thus presenting reduced displacements of the girder. Moreover, the cross-sections of the cable

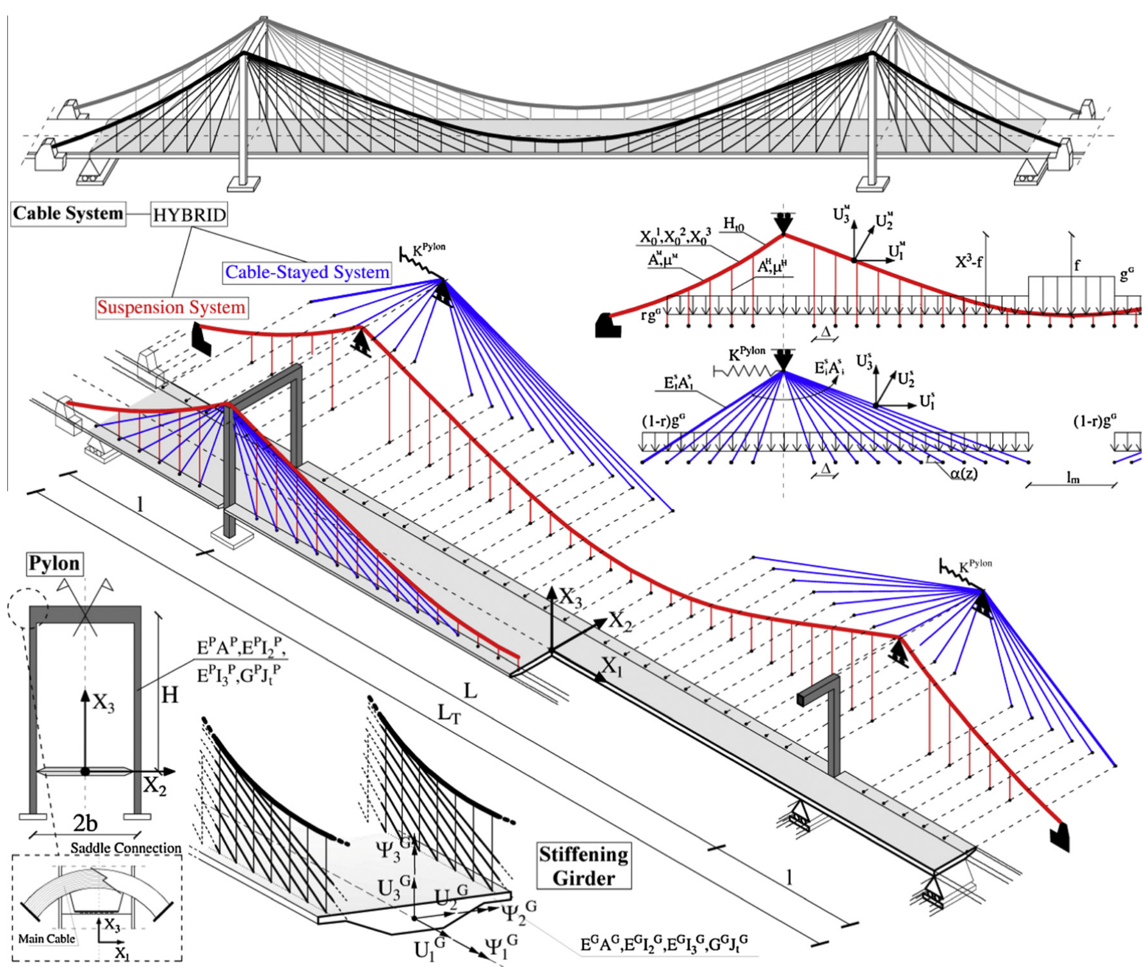

Fig. 1. Structural scheme of the hybrid cable-stayed suspension (HCS) bridge.

system elements are designed consistently to the maximum performance criterion, which, basically, consists to verify, under the worst LL combination, the equality condition between the maximum absolute or incremental applied stresses and the corresponding allowable value. Such task is developed by means of an iterative procedure defined by a two-step algorithm, defined on the results arising from DL and LL configurations. In particular, at first, an optimization modeling of the bridge is developed, aimed to reduce the steel quantity involved in the cable system and to verify displacement prescriptions in the DL configuration (optimization phase). Subsequently, the analysis is carried out in LL configuration to evaluate proper correction factors, which modify the cable system characteristics on the basis of maximum stresses and displacements under ultimate limit states (correction phase).

Without loss of generality, the analysis is developed for the HCS bridges even if the cases concerning pure cable-stayed and suspension bridge schemes can be easily derived as particular cases. With reference to the bridge scheme reported in Fig. 2, the control variables, evaluated during the optimization analysis, are represented by the cross-sections (A_i^S, A_i^H, A^M) and the post-tensioning forces of the cable system (S_i^S, S_i^H, S^M) and are designed by means of the following relationships:

$$\begin{aligned}&\underset{\sim}{S}_C=\left\{S_1^S,\ldots,S_{N^S}^S,S_1^H,\ldots,S_{N^H}^H,S^M\right\},\\&\underset{\sim}{A}=\left\{A_1^S,\ldots,A_{N^S}^S,A_1^H,\ldots,A_{N^H}^H,A^M\right\}\end{aligned}\tag{1}$$

where N^S is the number of stays, N^H is the number of hangers and the superscripts S, H and M refer to the stays, hangers and main cable, respectively.

During the optimization phase, the control or design variables, for a better convergence of the optimization procedure, are assumed to be expressed in terms of the optimization factors $(\xi_i, \psi_i) \in \Xi^S$ and $(\xi^M, \psi^M) \in \Xi^M$:

$$\begin{aligned}&A_i^S=\xi_i\bar{A}_i^S, S_i^S=\psi_i\bar{S}_i^S,\\&A^M=\xi^M\bar{A}^M, S^M=\psi^M\bar{S}^M,\end{aligned}\tag{2}$$

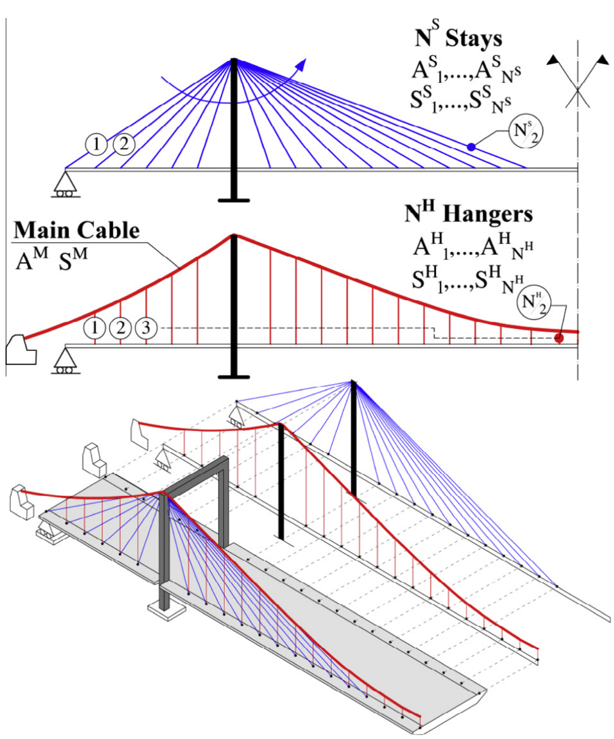

Fig. 2. Design and control variables to be determined in HCS bridge scheme.

where $i = 1, \ldots, N^S$, Ξ^S and Ξ^M are the domain spaces of the optimization variables associated to the cable-stayed system (S) and the main cable (M) and the quantities reported with the superscript $(\bar{\bullet})$ refer here and in the following to the values arising from previous iteration step or, in the case of the first step, assumed by trial variables obtained by preliminary dimensioning rules reported subsequently in Section 3. As a consequence, the factors (ξ_i, ψ_i) and (ξ^M, ψ^M) should be considered as a variable to be changed during the optimization procedure. Moreover, the cross-sections of the suspension system elements, i.e. A_i^H, are changed from their previous estimated values, i.e. $\bar{A}_i^H$, introducing additive incremental variables, i.e. ΔA_i^H, as a function of explicit constraint equations, which enforce the stresses in the hangers to be equal to the design quantity S_{gi}^H:

$$\underset{\sim}{L}_{S_1}\left[\left(\bar{A}_i^H+\Delta A_i^H\right),S_i^H\left(\underset{\sim}{\xi},\underset{\sim}{\psi}\right)-S_{gi}^H\right]=0,\quad \text{with } i=1\ldots N^H,\Delta A_i^H\in\Xi^H\tag{3}$$

where $\underset{\sim}{L}_{S_1}$, with $\underset{\sim}{L}_{S_1}:\Xi^H\to\Xi$, is the constrain operator, which ensures that the stress variables are equal to the design value, i.e. S_{gi}^H (discussed subsequently), Ξ^H and Ξ are the domain of the hanger cross-sections or the global bridge solution, respectively, and $(\underset{\sim}{\xi},\underset{\sim}{\psi})$ are the vectors collecting the optimization factors of the stays and main cable, i.e. (ξ_i, ψ_i) and (ξ^M, ψ^M), respectively. The displacement conditions to achieve the "*zero configuration*" are expressed in terms of explicit constraint relationships defined by the operator $\underset{\sim}{L}_U$, with $\underset{\sim}{L}_U:\Xi^S\times\Xi^H\times\Xi^M\to\Xi$, which reproduces the undeformed configuration on the basis of a proper set of post-tensioning stresses in the suspension system S_i^H, with $i=1\ldots N^H$, in the main cable S^M and the anchor stays (2) as follows:

$$\underset{\sim}{L}\left[\left(\bar{S}_j^H+\Delta S_j^H,\bar{S}_1^S+\Delta S_1^S,\bar{S}_{N^S}^S+\Delta S_{N^S}^S,\bar{S}^M+\Delta S^M\right),\underset{\sim}{U}\left(\underset{\sim}{\xi},\underset{\sim}{\psi},\xi^M,\psi^M\right)\right]=0,\tag{4}$$

where $j = 1, \ldots, N^H$, $\underset{\sim}{U}$ with $\underset{\sim}{U}^T=\left[U^G_{3(1\ldots N^H)},U_1^{P_L},U_1^{P_R},U_1^{M-P_L},U_1^{M-P_R}\right]$ is the vector containing the vertical displacements at the hangers/girder connections (N^H) and the horizontal displacements at the top pylon left (L) and right (R) cross-sections $(U_1^{P_L},U_1^{P_R})$ and at the intersection points of the left and right top pylon cross-sections with the main cable $(U_1^{M-P_L},U_1^{M-P_R})$.

The objective function, which is minimized during the optimization procedure, is represented by the scalar valued function Q, which describes the total steel quantity involved in the cable-system:

$$\underset{(\underset{\sim}{\xi},\underset{\sim}{\psi})}{MinQ}\left(\underset{\sim}{\xi},\underset{\sim}{\psi}\right)=\underset{(\underset{\sim}{\xi},\underset{\sim}{\psi})}{Min}\left(\sum_{i=1}^{N^S}L_i^SA_i^S+\sum_{i=1}^{N^H}L_i^HA_i^H+A^ML^M\right)\gamma\tag{5}$$

where γ is the specific weight of the cables and (L_i^S,L_i^H,L^M) are the lengths of the i-th stay, hanger or main cable, respectively. Additional conditions on the stress distribution in the cable system are imposed by explicit constraint equations, ensuring that the stresses in the stays and main cable under dead loading are fixed to a design quantity, i.e. (S_{gi}^S, S_g^M):

$$\underset{\sim}{L}_{S_2}\left[S_i^S\left(\underset{\sim}{\xi},\underset{\sim}{\psi}\right)-S_{gi}^S\right]=0,\quad \text{with } i=1,N_S\tag{6}$$

$$\underset{\sim}{L}_{S_3}\left[\max\left(S^M\left(\underset{\sim}{\xi},\underset{\sim}{\psi}\right)\right)-S_g^M\right]=0,\tag{7}$$

It is worth noting that the optimization procedure, defined by Eqs. (3)–(7), is concerned to determine the optimum bridge configuration, which involves the lowest steel quantity in the cable system and verifies constrain equations on design displacement and stress variables of the bridges. However, the initial post-tensioning stresses in the cable systems, namely $(S^H_{gi}, S^S_{gj}, S^M_g)$ with $i = 1 \dots N^S, j = 1 \dots N^H$, should be considered as known variables when the optimization problem is solved. To this aim, in order to calculate such quantities an iterative procedure, namely two-step algorithm, is required, going between the optimization and the correction steps, iteratively. In the former the optimization problem is considered by solving Eqs. (3)–(7), whereas in the latter the initial post-tensioning stresses are quantified on the basis of the LL results. In particular, in the framework of LL combinations, based on ultimate, fatigue and serviceability, i.e. ULS, FLS and SLS, the following conditions, concerning maximum and relative stresses and maximum absolute displacements should be verified:

$$\begin{aligned}
&\max\left[S^H_i(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{ULS} \leqslant S_A, \max\left[\Delta S^H_i(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{FLS} \leqslant \Delta S_A \quad \text{with } i = 1, N^H\\
&\max\left[S^S_i(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{ULS} \leqslant S_A, \max\left[\Delta S^S_i(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{FLS} \leqslant \Delta S_A \quad \text{with } i = 1, N^S\\
&\max\left[S^M(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{ULS} \leqslant S_A, \max\left[\Delta S^M_i(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_{FLS} \leqslant \Delta S_A
\end{aligned} \tag{8}$$

$$\max\left[|U^G_3|\right]_{SLS} \leqslant \delta^G_{3A}, \max\left[|U^P_1|\right]_{SLS} \leqslant \delta^P_{1A} \tag{9}$$

where S_A and ΔS_A are the maximum or incremental allowable values in the cables, δ^G_{3A} is the maximum vertical displacement of the girder and δ^P_{1A} is the maximum displacement of the pylon. It is worth noting that the maximum value of the internal stresses can be associated to the effects of live loads or produced by seismic or aeroelastic loading schemes. Similarly, proposed formulation can be generalized in such a way to be consistent with respect to enhanced reliability formulations such as the one reported for instance in [26]. The initial stresses for the cable elements $\underset{\sim}{S^S_g}, \underset{\sim}{S^H_g}, \underset{\sim}{S^M_g}$ are evaluated as a function of two sets of factors associated to stays, hangers and main cable, namely Φ_i and Ω_i, which are introduced to verify code prescriptions defined by Eqs. (8) and (9), ensuring that the predicted values should be equal in the worst loading combination to the corresponding permissible values, leading to the optimum utilization of the bridge components and thus the lowest steel quantity involved in the cable system. Such quantities must be considered as variables, which are solved for during the iteration procedure by using as secant approach as a function of the values arising from the previous iteration steps. The main purpose of the optimization factors is to modify the stiffness or the stress distribution of the cable elements, in such a way to verify prescriptions on bridge deformability, reducing the material volume involved in the cable system. In particular, the performance factors Φ_i optimize the stress distribution in the cable system, in such a way that the maximum value should be close to the strength of the material, reaching the optimum solution in terms of material utilization and volume involved in the cable system. On the contrary, variables Ω_i define the allowable level of strength and stiffness for each cable element, enforcing the pylons and girder to have lower displacements than the maximum permissible values. In the proposed modeling, the stiffness of the stays in the lateral or in the central spans are designed to reduce horizontal and vertical displacements of the girder and pylon, respectively, whereas that of hangers is designed to reduce vertical displacements of the girder. Moreover, the stiffness of the main cable is utilized to reduce maximum vertical displacements of the girder. Such choices can be considered reasonable, from the physical point of view, especially in the cases of long span bridges, in which girder deformability is mostly influenced by the stiffness of the cable system in relationship to the prevailing truss behavior of the bridge structures. Therefore, in order to satisfy prescriptions on allowable displacements and stresses, the evaluation of the performance factors Ω_i and Φ_i is defined by the following assumptions:

- the performance factors for the cable-stayed system elements, lateral and central stays, i.e. Ω^S_i, are defined to reduce the horizontal displacements of the pylons and vertical deflections of the girder, whereas for the hanger members the variables Ω^H_i are associated to reduce the vertical displacements of the girder (see Fig. 3);
- the factor Ω^M concerning the main cable dimensioning is modified to verify prescriptions on girder displacements with respect to the maximum absolute value observed in the girder deflections;
- for all cable elements $(\Phi^S_i, \Phi^H_i, \Phi^M)$ factors are defined enforcing that the maximum stresses should be equal to the allowable quantity;
- the definition of the performance factors is based on a linear approximation of the displacement and stresses, based on the secant description, whose path can be thought defined by the line connecting two states, represented by the final, i.e. the maximum allowable status, and the current solution arising from the last converged configuration, i.e. at $k-1$.

Therefore, on the basis of the previous remarks, introducing the following limit functions concerning the horizontal and vertical displacements of the pylon and girder and allowable stresses

$$g^P_{U^k_{1i}} = \frac{\delta^P_{1A}}{\max\,(U^P_{1LL})^k_i} - 1, \quad g^G_{U^k_{3i}} = \frac{\delta^G_{3A}}{\max\,(U^G_{3LL})^k_i} - 1, \quad g_{S^k_{Ai}} = \frac{(S^j_A)^{k-1}_i}{S_A} - 1, \quad j = S, H, M \tag{10}$$

the relationships, which quantify the performance factors Ω_i for the cable-stayed system, are defined by the following expressions:

$$(\Omega)^k_i = \begin{cases} \dfrac{|\Delta_{\max}|}{\max\limits_{LL}\left[(|\Delta|)^k_i\right]}\,\dfrac{(S_A)^{k-1}_i}{S_A} & \text{if} \quad g^k_{\Delta_i} \leqslant 0 \\[2ex] \dfrac{|\Delta_{\max}|}{\max\limits_{LL}\left[(|\Delta|)^k_{i-1},(|\Delta|)^k_{i+1}\right]}\,\dfrac{(S_A)^{k-1}_i}{S_A} & \text{if} \quad g^k_{S_{Ai}} < 0 \text{ and } g^k_{\Delta_i} > 0 \\[2ex] 1 & \text{if} \quad g^k_{S_{Ai}} \geqslant 0 \text{ and } g^k_{\Delta_i} > 0 \end{cases} \tag{11}$$

with

$$\begin{aligned}
&\Delta = U^P_1, \Delta_{\max} = \delta^P_{1A}, g_\Delta = g^P_{U_{1i}} \quad (Stays - lateral\,span)\\
&\Delta = U^G_3, \Delta_{\max} = \delta^G_{3A}, g_\Delta = g^G_{U_{3i}} \quad (Stays - main\,span)\\
&\Delta = U^G_3, \Delta_{\max} = \delta^G_{3A}, g_\Delta = g^G_{U_{3i}} \quad (Hangers)\\
&\Delta = \max(U^G_3), \Delta_{\max} = \delta^G_{3A}, g_\Delta = \max(g^G_{U^k_{3i}}) \quad (Main\,cable)
\end{aligned} \tag{12}$$

where k represents the number of iterations during the solving procedure. It is worth noting that in Eqs. (11) and (12), the expressions utilized in the case of negative values of the displacement functions, i.e. $g_{U^k_{3i}} \leqslant 0$ or $g_{U^k_{1i}} \leqslant 0$, indicate the need to increase the stiffness of the cable system, since the displacements do not verify girder and pylon displacement prescriptions. Contrarily, when the displacement functions are strictly positive the cable stiffness is released enforcing the equality with maximum permissible value, which is achieved only for positive values of the allowable stress function, i.e. $g_{S^k_{Ai}} \geqslant 0$. The factors concerning the stresses, namely Φ_i, can be derived by the ratios between the allowable stress and the maximum value observed in the LL combinations:

$$(\Phi^S)^k_i = \frac{(S_A)}{\max\limits_{LL}(S^S_{LL})^k_i}, (\Phi^H)^k_i = \frac{(S_A)}{\max\limits_{LL}(S^H_{LL})^k_i}, (\Phi^M)^k = \frac{(S_A)}{\max\limits_{LL}(S^M_{LL})^k}, \tag{13}$$

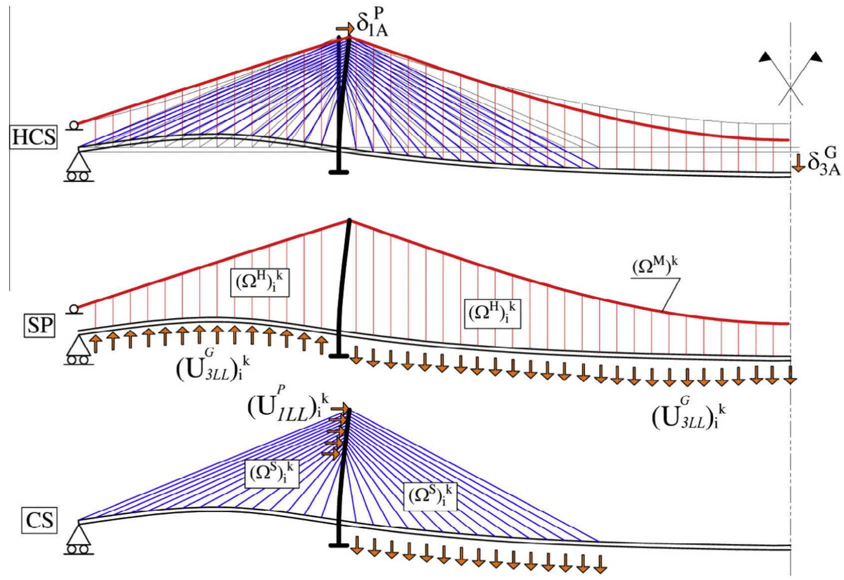

Fig. 3. Identification of the performance factors and design variables on the bridge scheme.

A synoptic representation of the variables reported in Eqs. (11)–(13) are reported in Fig. 3. Starting from Eqs. (11)–(13), the estimate the design initial stresses on the basis of the following relationships:

$$(S^S_{gi})^k = \left[\Phi^S\Omega^S\right]^k_i (S^S_g)^{k-1}_i, (S^H_{gi})^k = \left[\Phi^H\Omega^H\right]^k_i (S^H_g)^{k-1}_i, (S^M_g)^k = \left[\Phi^M\Omega^M\right]^k_i (S^M_g)^{k-1} \quad (14)$$

It is worth noting that Eq. (14) predict, by means of the factors $\Phi^{S,H,C}_i$, the initial stresses on the basis of a linear approximation of the stress and displacement increments between dead and live load configurations, enforcing the maximum stresses, reached in all cable-system elements or the displacements on the girder and pylon cross-sections, to be equal approximately to the corresponding allowable values. Moreover, the piecewise functions defined by Eqs. (11) and (12) modify the allowable stress levels, increasing the stiffness of the cable system and verifying prescriptions on maximum displacements on both girder and pylons. Finally, once the new values of the stress levels in the DL configuration are evaluated by means of Eqs. (14), the procedure goes back to find the new cross-sections of the cables and the post-tensioning forces on the basis of the optimization method by means of Eqs. (3)–(7). This procedure is repeated until achieving the convergence conditions of the algorithm defined on the basis of the following expression:

$$\max\left\{\sum_{i=1}^{N_S}\left[\frac{(S^S_{gi})^k-(S^S_{gi})^{k-1}}{(S^S_{gi})^{k-1}}\right], \sum_{i=1}^{N_H}\left[\frac{(S^H_{gi})^k-(S^H_{gi})^{k-1}}{(S^H_{gi})^{k-1}}\right], \frac{(S^M_g)^k-(S^M_g)^{k-1}}{(S^M_g)^{k-1}}\right\} \leqslant toll. \quad (15)$$

A synoptic representation of the optimization procedure is reported in Fig. 4. It is worth noting that the previous procedure can be easily specialized for the cases of pure cable-stayed and suspension bridge schemes. In such cases, since the number of

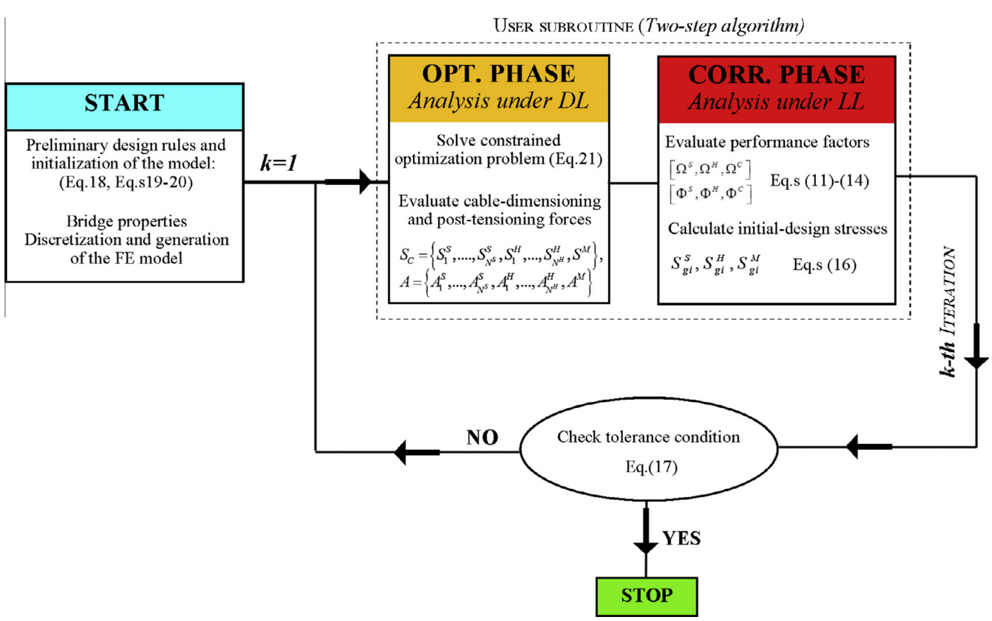

Fig. 4. Flowchart of the optimization procedure.

variables is equal to the number of equations, the optimization problem is transformed in terms of a determinate system equation formed by explicit relationships defined in terms of initial post-tensioning stresses, cross-sections of the cable system elements and design stresses under DL. More details concerning the main equations as well as the iteration procedure for the pure cable-stayed and suspension systems are reported in Appendix A. Moreover, the description of the model is presented with reference to the final configuration, but can be easily specialized also for the different steps involved in the bridge construction. As a matter of fact, in the loading scheme concerning the erection procedure, only the service prescriptions should be verified, which are less restrictive than the ones required by the ULS combinations. Moreover, in such analyses, the variables concerning the cable dimensioning can be considered as known quantities, which are identified on the basis of the design prescriptions defined on the final bridge configuration and with respect to ultimate loading conditions. The procedure to evaluate the bridge configuration during the iteration steps is reported in Appendix B.

Numerical procedure and iteration algorithm

The numerical algorithm is based on the following different steps, which are executed, iteratively:

- generation of the finite element model, evaluation of the initial post-tensioning forces and the cable cross-sections in the cable system by means of the optimizations procedure;
- calculation of the maximum stresses and displacements under live loads and prediction of the new set of the initial stress quantities on the basis of the performance factors.

At first, the generation of the FE model is defined on a 3D description, whose initial configuration, only for the first iteration, is designed on the basis of practical design rules, typically, accepted in the framework of cable-supported bridges [1,2,24]. In the framework of long span bridges, typically the ratio between sag and horizontal main cable projection, i.e. f/L, is in the range between 0.1 and 0.2. The main cable geometric profile (X_0^3) can be expressed by means of a parabolic approximation as a function of the positions defined by the top pylon cross-sections and the lowest cross-section of the main cable. In the dead load configuration, the main cable is subjected to the transferring stresses arising from the girder, which can be assumed differently distributed between the cable-stayed and the suspension systems as a function of a dead load factor (r), typically assumed in the range between 0.2 and 0.4 for design purposes [2]. Therefore, the configuration of the main cable can be expressed as function of the following relationships [27]:

$$X_0^3(X_1) = -\frac{M(X_1)}{H_{t0}}, \quad H_{t0} = \frac{1}{f}\left[(g^M + rg^G)\frac{L^2}{8} + (1-r)g^G\frac{l_m}{4}\left(L - \frac{l_m}{2}\right)\right] \tag{16}$$

where $M(X_1)$ is a fictitious bending moment due to distributed self-weight loads taken by the suspension system calculated as for a simply supported beam, l_m is the portion of the main span without the presence of stays, H_{t0} horizontal axial force, f is the main cable sag, g^M and g^G are the self-weight of the main cable and girder, respectively, and r is the initial design dead load distribution factor between cable stayed and suspension systems adopted to be equal to $r = 0.3$ in the analysis [2]. Moreover, only for first iteration i.e. for $k = 1$, the initial cable cross-sections for stays ($\bar{A}_i^S$), hangers ($\bar{A}_i^H$), main cable $\overline{A^M}$ and the initial stresses (S_g^S, S_g^H) are estimated from the equilibrium relationships based on the truss assumption of the cable stress distribution on the girder:

$$\overline{A_i^S} = \frac{(1-r)g^G\Delta_i}{S_g^S \sin\alpha_i}, \overline{A_i^H} = \frac{rg^G\Delta_i}{S_g^H \sin\alpha_i}, \overline{A^M} = \frac{H_{t0}}{S_A\cos(\phi)} \tag{17}$$

$$S_g^S = (1-r)\frac{g^G}{g^G+p}S_A, S_g^H = r\frac{g^G}{g^G+p}S_A \tag{18}$$

where p is the per unit length live loads. It is worth noting that Eqs. (17) and (18), based on practical relationships from an engineering point of view, provide an initial or trial estimate of the cross-section or stress quantities, which are modified from the initial values during the optimization procedure. However, since the model is essentially nonlinear, Eqs. (17) and (18) are able to provide reasonable values for the unknown quantities, helping the numerical procedure to increase the stability and convergence rates.

The evaluation of the post-tensioning forces and the optimum cable cross-sections is developed by solving the governing equations concerning equilibrium (EQ), optimization (OBJ) and constrain equations, (CE) defined by Eqs. (3)–(7), whose compact form can be expressed as follows:

$$\begin{aligned}
&OBJ \rightarrow \underset{(\underset{\sim}{\xi},\underset{\sim}{\psi})}{Min}\, Q(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U}),\\
&EQ \rightarrow \underset{\sim}{K}(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U})\,\underset{\sim}{U} = \underset{\sim}{F}(\underset{\sim}{\xi},\underset{\sim}{\psi}),\\
&CE, \underset{\sim}{L}_U = 0 \rightarrow \underset{\sim}{S}{}_0^H(\underset{\sim}{\xi},\underset{\sim}{\psi}) + \underset{\sim}{C}(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U})\Delta\underset{\sim}{S} = 0,\\
&CE, \underset{\sim}{L}_{S_1} = 0 \rightarrow \underset{\sim}{K}{}^H(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U})\underset{\sim}{U}{}^H - \Delta\underset{\sim}{A}{}^H\underset{\sim}{S}{}_g^H = 0,\\
&CE, \underset{\sim}{L}_{S_2} = 0 \rightarrow \underset{\sim}{K}{}^S(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U})\underset{\sim}{U}{}^S - \Delta\underset{\sim}{A}{}^S\underset{\sim}{S}{}_g^S = 0,\\
&CE, \underset{\sim}{L}_{S_3} = 0 \rightarrow \max_{i=1\ldots N^{MC}}\left[\underset{\sim}{K}{}^M(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U})\underset{\sim}{U}{}^M(\underset{\sim}{\xi},\underset{\sim}{\psi})\right]_i - \Delta A^M S_g^M = 0.
\end{aligned} \tag{19}$$

where $\Delta S^T = [\Delta S_1^H, \Delta S_2^H, \ldots, \Delta S_{N^S}^H, \Delta S_1^S, \Delta S_{N^S}^S, \Delta S^M]$, $\underset{\sim}{K}$ is the global stiffness of the structure, $\underset{\sim}{U}$ is the global displacement vector, $\underset{\sim}{F}$ is the external force vector associated to the dead loading configuration, $\underset{\sim}{S}{}_0^H$ and ΔS^H are the initial and incremental stress vectors of the hanger elements, $\underset{\sim}{C}$ is the corresponding flexibility matrix, $\underset{\sim}{K}{}^H, \underset{\sim}{K}{}^S, \underset{\sim}{K}{}^M$ are the matrixes collecting the stiffness coefficients of the hangers, cable-stayed and main cable, respectively, $\Delta\underset{\sim}{A}{}^H$ and $\Delta\underset{\sim}{A}{}^S$ are the vectors containing the cross-sections of the hangers and the stays and ΔA^M is the cross-section of the main cable element. It is worth noting that Eq. (19) introduce as a nonlinear constrained optimization problem, in which the unknown quantities are represented by displacements and cable dimensioning of the cable systems. However, since the structural behavior is essentially nonlinear an iterative integration procedure must be performed. At this aim, the solving procedure is defined in the framework of gradient-based solver algorithms based on SNOPT solver, in which the optimal solution is computed by the evaluation of the gradients of both the objective function and all constraints by using numerical differentiation [28]. The objective function and constrain conditions are interpolated by means quadratic and linear polynomial approximations, respectively.

Once the configuration under dead loads is evaluated, the analysis is developed to determine maximum effects on the bridge components produced by the live loads in terms of stresses and displacements. However, since the bridge behavior is essentially nonlinear, the analysis under live loads should be considered as a continuation from the previously converged configuration under the dead load, taking into account also the construction steps involved in the erection procedure. In particular, all the variables involved in the solving procedure or in the definition of the structural elements of the bridge are taken from the last converged solution, i.e. under dead loads. At this point, the solving procedure is defined by a restarting analysis, in which the initial values are scaled as a function of the current solution [21]. The solution is performed for a fixed number of loading conditions, i.e. N^{LL}, which collect, for all bridge components, maximum stress and displacement effects:

$$\underset{\sim}{K}(\underset{\sim}{\xi},\underset{\sim}{\psi},\underset{\sim}{U_i})\Delta\underset{\sim}{U} = G_i(\underset{\sim}{\xi},\underset{\sim}{\psi}) - P_0(\underset{\sim}{\xi},\underset{\sim}{\psi}) \qquad i = 1,\ldots,N^{LL} \tag{20}$$

with

$$\underset{\sim}{\xi} = \underset{\sim}{\xi}^{DL}, \underset{\sim}{\psi} = \underset{\sim}{\psi}^{DL}, \underset{\sim}{S}^{S,H} = \underset{\sim}{\overline{S}}^{S,H} + \Delta\underset{\sim}{S}^{S,H}, S^M = \overline{S}^M + \Delta S^M \tag{21}$$

where P_0 is the vector of nodal point forces corresponding to the increment in element displacements and stresses from the dead load to the live load configurations, ΔU are the incremental displacement vector, $(\Delta \underset{\sim}{S}^{S,H}, \Delta S^M)$ are the variable associated to the incremental value of internal stress evaluated starting from the last converged values, i.e. $(\underset{\sim}{\overline{S}}^{S,H}, \overline{S}^M)$, and G_i is the live load vector force of the i-th loading configuration. From the loading combinations defined by Eq. (20), the new estimates of initial stresses are derived on the basis of Eqs. (11)–(14). Subsequently, convergence conditions defined by Eq. (17) are checked and if they are not satisfied, the analysis goes back to evaluate the configuration of the bridge under dead loading with the new values of the initial stresses. It is worth noting that the iteration procedure as well as the optimization problem is developed by using an external subroutine, which combines LivelinkTM for Excel package and Comsol Multiphysics [21]. The algorithm was implemented by means of proper customized subroutines, which manage the parameters involved in the iterative procedure. However, the proposed formulation can be implemented in several computational frameworks, since it is based on data, which can be easily extracted and handled from quite standard commercial FE softwares.

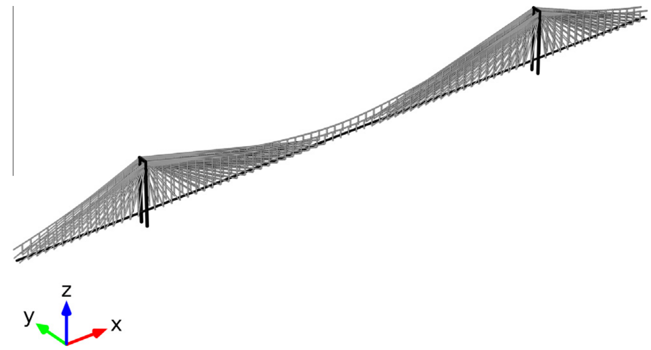

Fig. 5. Finite element modeling of the structural scheme.

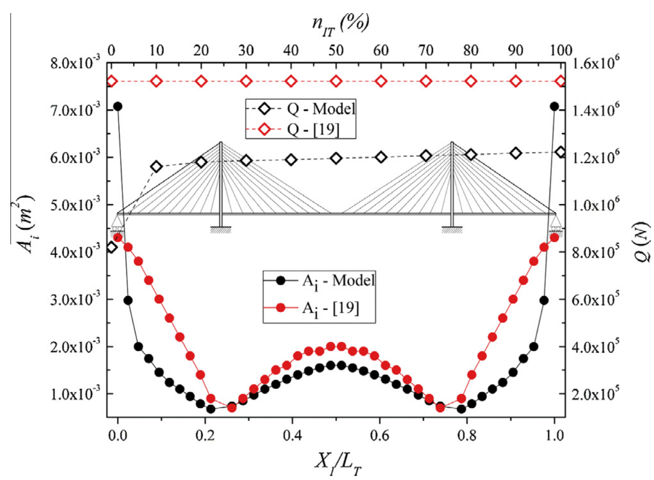

Fig. 6. Comparisons with results obtained in [19] in terms of cross-section distribution of the cables (A_i), total steel quantity (Q) as a function of the number of iterations.

Results

Validation of the proposed approach

In lack of results on HCS bridges available from the literature, comparisons with existing optimization techniques on cable-stayed bridges are performed to validate the proposed modeling. The structural scheme, reported in [19] refers to a geometry similar to that of the Quincy Bayview Bridge, located in Illinois (USA). The bridge is based on H-shaped concrete towers, 79 m high from the foundation structure, a double layer of cable system formed by 80 cables and a composite precast concrete-steel deck. The central and lateral spans are equal to 285.6 m and 128.1 m, respectively. The data utilized for the simulations are presented in Table 1, whereas the FE model utilized in the results is reported in Fig. 5. However, more details on the bridge structure can be recovered in [19].

At first, results in terms of distribution of optimized cable cross-sections are presented in Fig. 6, in which comparisons between the data predicted by proposed model and those obtained in [19] are analyzed. Moreover, in the same figure, the evolution of the total steel quantity as a function of the percentage number of iterations is also reported. The analyses show that the distribution of cable cross-sections determined by the proposed formulation is always below the cross-section values obtained in [19] and the corresponding percentage reduction of the total steel quantity is almost equal to 26%. The proposed formulation appears to be quite stable in reaching the optimum configuration, since a low number of iterations is required to obtain the convergence of the solution. The comparison in terms of cross-section distribution denotes, that, despite results obtained in [19], the proposed modeling finds the largest values of the cable areas in proximity of the anchor stays, in which, typically, maximum transferring stresses arising from stays of the central span are observed. Such results can be considered reasonable also with common design procedures and experimental evidences on cable-stayed bridges, since the anchor cables are responsible of both pylon and girder deformability, much more than adjoining elements. Additional analyses, reported in Fig. 7, are developed with the purpose to investigate the stress distribution

Table 1
Mechanical and geometric properties utilized for the comparisons with results obtained in [19].

Stiffening girder			Pylon			Cable and loads		
A^G	0.602	m^2	A^P	6.50–15.10	m^2	S_A	1.60	GPa
I^G_2	0.704	m^4	I^P_2	6.50–15.10	m^4	$\gamma^{S,H,M}$	84	kN/m^3
I^G_3	14.2	m^4	I^P_3	3.65–383.70	m^4	$E^{S,H,M}$	205	GPa
E^G	200	GPa	E^P	24.87	GPa	p	9	kN/m/lane
γ^G	77	kN/m^3	γ^P	24	kN/m^3	p/g^G	0.10–0.12	–

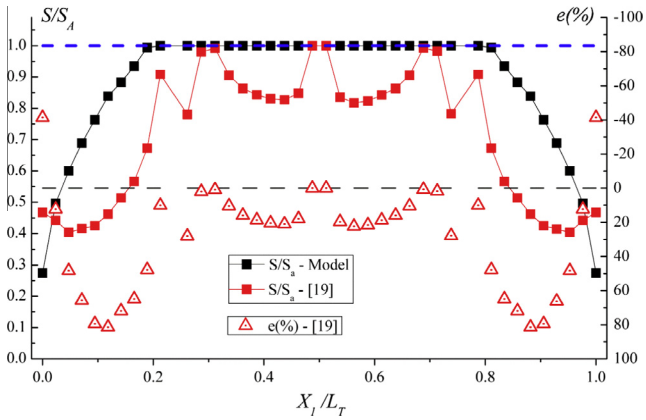

Fig. 7. Comparisons in terms of maximum stresses produced by live loads and percentage error (e) with the values determined in [19].

observed in the cable system produced by the application of LL. In particular, the envelope of maximum internal stresses for each element of the cable system is reported as function of the position of the cable along girder profile. A synoptic representation of the stress distribution also in relationship to the assumed loading combination is reported in Fig. 8. The results denote that the optimum solution, obtained by the proposed model, presents values of the stresses in the cables equal or mostly close to the allowable quantity; the comparison with respect to values obtained in [19] denote percentage errors ranging from −41.37 to 81.31 with an average value of 28.88. The equality to the permissible value is not verified for all the cables in the lateral span, whereas those of the central span under LL are designed to reach a value exactly equal to their strength. Such condition determines the best optimization of the cable-system cross-sections, which leads to strong reductions of the total steel quantity involved in the cable system. However, for those elements of the lateral span, including also the anchor stays, the stress rate is lower than the allowable value, since more a stiffness amount in such elements is required in order to verify prescriptions on maximum girder deflections.

Finally, in Figs. 9 and 10, comparisons with results developed in [19] are proposed in terms of envelope of vertical and horizontal displacements of girder and pylons, respectively. The analyses show how prescriptions on maximum deflections for both girder and pylons are below the permissible values. Moreover, in the same figures the evolution of the midspan vertical displacement (Fig. 9) the top pylon horizontal displacement (Fig. 10) and the cross-sections of the cables as well as the stress ratios between maximum stress and allowable value are reported as a function of the number of iteration steps. The analyses show how from the initial value the displacements are modified to verify bridge deformability; such task is performed reducing the maximum working stresses of the cables much below the allowable value and thus increasing the stiffness or the area of the elements of the cable system. Finally, the results show that the solutions present a convergent behavior toward the final optimum configuration.

Parametric studies

In order to prove the effectiveness and the applicability of the design procedure, further results, also in the framework of HCS bridges, are proposed for more complex structures involving several configurations of the cable system and a large number of variables such as those involved in long span bridges. In particular, the analysis is developed for three different bridge schemes based on HCS, pure cable-stayed or suspension systems, whose main span is equal for all cases to 1000 m. The deck is made of steel with aerodynamic cross section, 4 m depth and 20 m wide; the vertical moment of inertia (I_2^G), the transverse moment of inertia (I_3^G), the cross section area (A^G) and the torsional constant (J^G), the modulus of elasticity of steel (E^G) for the bridge deck are 3.41 m^4, 31 m^4, 2.1 m^2, 15 m^4, 2.1×10^8 kN/m^2, respectively. The towers are formed by H-shaped steel components, whose elements present vertical moment of inertial (I_2^P), transverse moment of inertia (I_3^P), cross section area (A^P), torsional constant (J^P), modulus of elasticity (E^P) and in plane flexural top pylon stiffness (K^P) are 20.57 m^4, 9.78 m^4, 1.97 m^2, 21.13 m^4, 2.1×10^8 kN/m^2 and 50 g^G respectively. Moreover, the aspect ratio (H/l) between pylon height and lateral span is equal to 0.4 for the HCS and pure suspension bridges or equal to 0.66 for the pure cable-stayed bridge. The stays and the

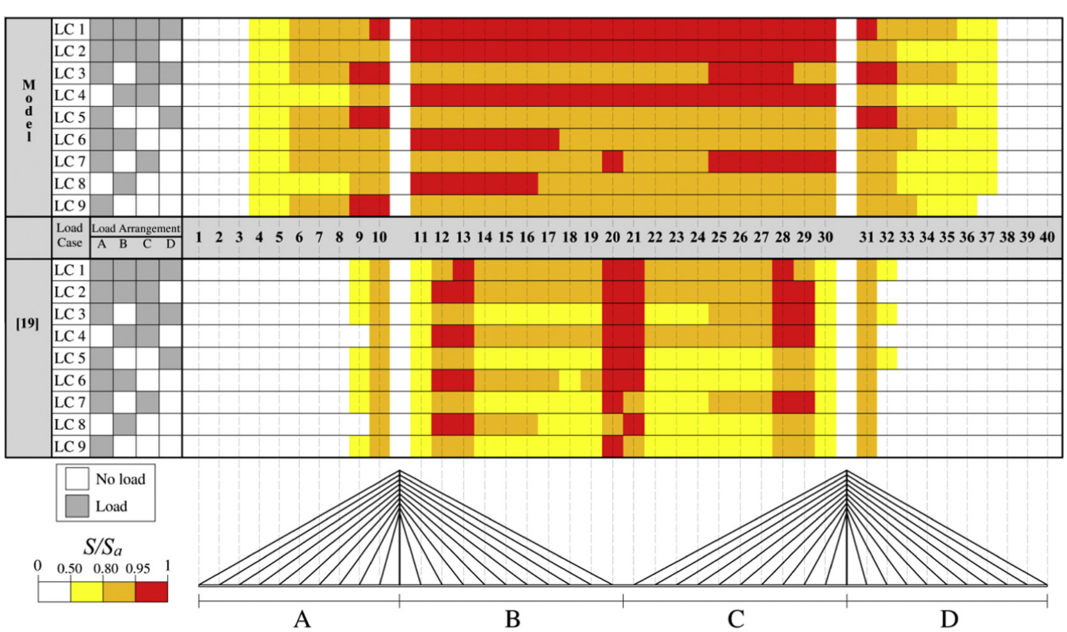

Fig. 8. Synoptic representation of the LL configurations, maximum stresses and comparison with data obtained in [19].

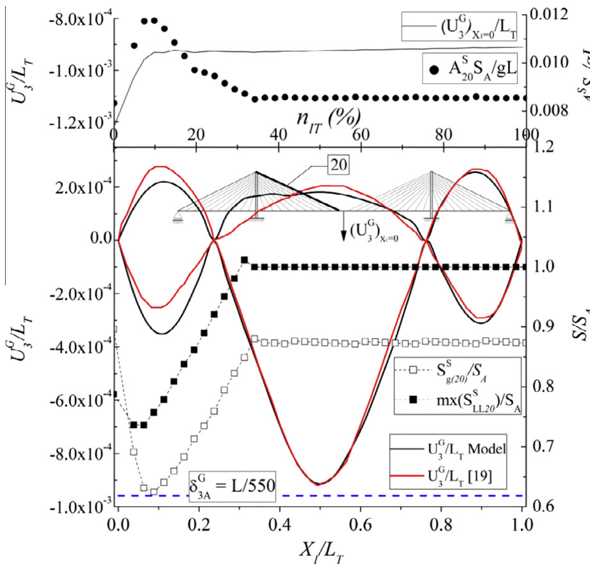

Fig. 9. Comparisons in terms of girder displacements produced by live loads with values determined in [19], convergence behavior of the cross-sections, maximum and initial post-tensioning stresses as a function of the percentage value of the iteration steps ($n_{IT\%}$).

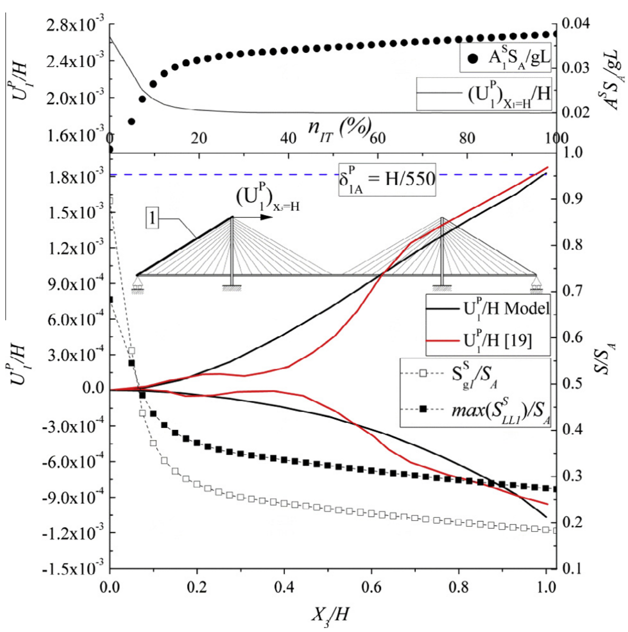

Fig. 10. Comparisons in terms of pylon displacements produced by live loads with values determined in [19], convergence behavior of the cross-sections, maximum and initial post-tensioning stresses as a function of the percentage value of the iteration steps ($n_{IT\%}$).

hangers present a distance equal to 20 m and an allowable stress (S_a) equal to 1.6×10^6 MPa and a minimum fatigue strength stress variation equal to 200 MPa (Strand). Dead loading of the girder including also permanent loads (g^G) are equal to 3.0×10^5 N/m, whereas the ratio between live and dead loads, defined consistently to the code prescriptions established in [29], is equal to 0.18 for ULS. In particular, results concerning the valuation of stress distribution are analyzed under ULS (factored) or FLS (unfactored), whereas results concerning displacements are considered under SLS (unfactored). The data utilized for the simulations are summarized in Table 2.

Table 2
Parametric study: mechanical and geometric properties of bridge.

Stiffening girder			Pylon			Cable and loads		
b	10	m	A^P	1.97	m^2	S_A	1.60	GPa
d	4	m	I^P_2	20.57	m^4	ΔS_A	200	MPa
A^G	2.1	m^2	I^P_3	9.78	m^4	$\gamma^{S,H,M}$	77	kN/m^3
I^G_2	3.41	m^4	E^P	210	GPa	$E^{S,H,M}$	205	GPa
I^G_3	31	m^4	$(H/l)_{HCS,SP}$	0.4	–	g^G	300	kN/m
J^G	15	m^4	$(H/l)_{CS}$	0.66	–	p/g^G	0.18	–
E^G	210	GPa	K^P	50 g^G	–			

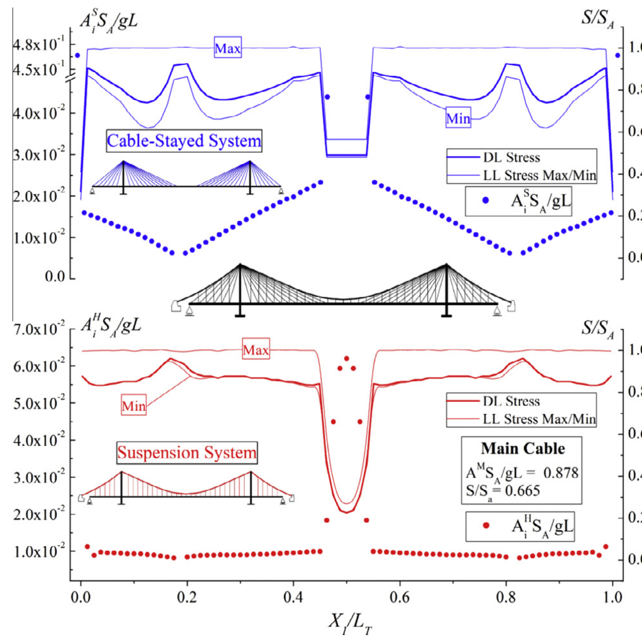

Fig. 11. HCS bridges: distribution of the cable cross-sections and envelope of the stresses in the cable-system in the stays, hangers and main cable.

At first, results concerning HCS bridges are presented in Fig. 11, in which the variability of the cross-sections of the cable system elements and their stress ranges observed under dead and live loads are reported. The cross-section distribution in the cable stayed-system presents its largest values in the anchor stays and in the longest stays of the main span, which, mostly, influence the deformability of the bridge. The hanger dimensioning presents a constant distribution of the cross-sections in the lateral and in the main spans, except in the midspan region, in which the cable-system of the HCS bridge behaves as a pure suspension scheme. As a matter of fact, in such region, the absence of the stays, produces a reduction of stiffness against vertical displacements, which are, mostly, influenced by the main cable characteristics. Such concept can be highlighted also analyzing the envelope of stresses, presented in the same figure. The results shows how most of the cable system elements, i.e. hangers and stays, are designed in such a way that under the application of LL, worst observed stress reaches the allowable value. However, for the anchor stays or in the midspan region, the design methodology predicts values of the cross sections, which produce maximum working stresses, locally, lower than the corresponding allowable value. Such predictions in terms of both stress or cross-sections can be explained due to the fact that the optimization procedure, in order to verify prescriptions on maximum displacements automatically modifies the cable stiffness of the most active elements, which produce the optimum stiffness effects and distribution on girder and pylon deflections.

Stress and displacement distributions as a function of the iteration steps to reach the optimum solution are presented in Figs. 12 and 13, respectively. Results concerning bridge deformability or stress distribution denote that from the initial configuration, girder displacements and maximum stresses in the cable system present a convergent behavior toward the final solution. In particular, such configuration is obtained by means of 25 iterations, in which the cross-sections of the cables as well as their initial stresses under DL are modified to verify code prescriptions on bridge deformability and cable strength. However, most of the optimization process is performed in the initial substeps, i.e. before 12 iterations, in which both displacements and stresses present values equal approximately to the corresponding design quantities, whose maximum error with respect to the final optimized values is lower than 7%. Moreover, maximum stresses in the cable system are strongly modified from the initial trial values, namely from $k = 1$, which, basically, correspond to the application of preliminary dimensioning rules available from the literature. From such values, the optimization method modifies the cable cross-sections according to stress and displacement requirements provided by the code prescriptions, leading to a reduced steel quantity involved in the cable system. Such behavior can be observed from the results presented in Fig. 14, in which the evolution of cable stresses and girder displacements at discrete points of the bridge are presented as a function of the number of iterations obtained in the solving procedure. The results show how the proposed method adjusts the maximum working stresses and thus the cross-sections of the cable system, providing improvements in the bridge deformability, which is reduced to verify prescription on maximum displacements.

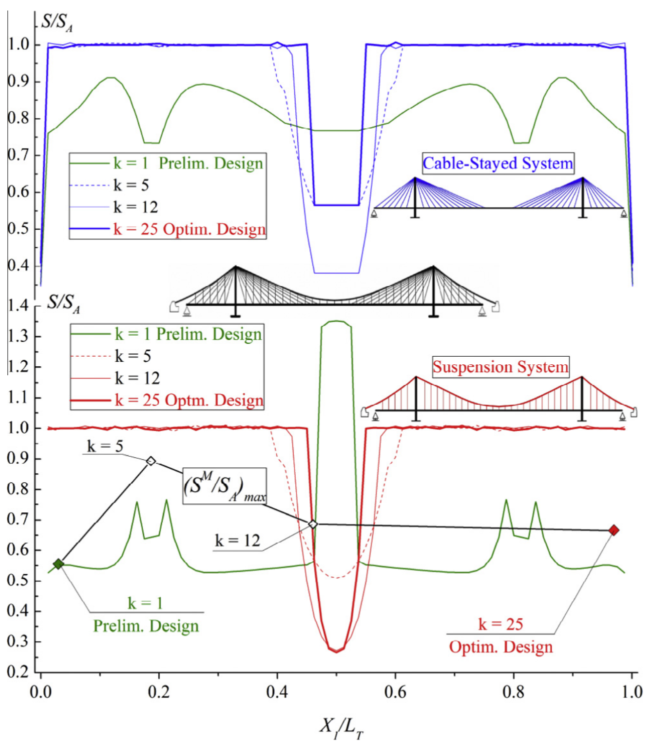

Fig. 12. HCS bridges: convergence behavior of the predicted maximum stresses in the cable system as a function of the iteration steps.

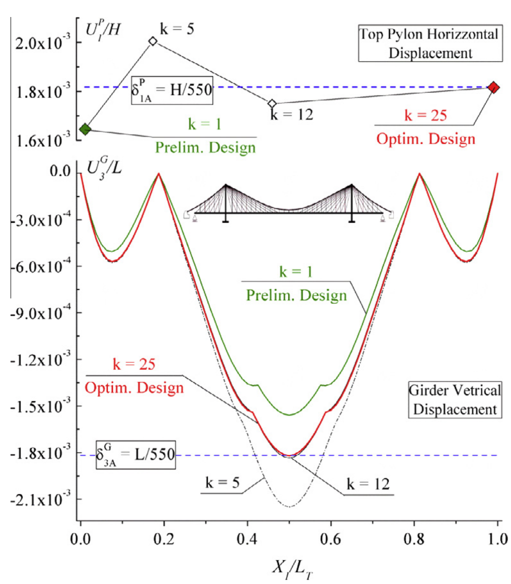

Fig. 13. HCS bridges: convergence behavior of the girder and pylon displacements in the cable systems as a function of the iteration steps.

Finally, a comparison in terms of cable-system configurations is developed, in which results obtained by the proposed methodology are analyzed with respect to long span bridge configurations based on HCS or pure cable-stayed (CS) or suspension (SP) cable systems. The key of these results is to analyze the applicability, in terms of convergence and stability, of the proposed formulation for different bridge schemes and to investigate the differences in terms of cable dimensioning between the bridge configurations. Results in terms of cross-section distribution and DL and LL stresses are reported in Figs. 15 and 16, respectively. The analyses denote that the cable cross sections of the HCS bridge scheme are always below the corresponding ones obtained from the configurations based on pure CS and SP schemes. For pure CS configuration, the distribution of the cross-sections is similar to that of the HCS bridge, since the largest values are predicted in proximity of the anchor stays and the longest stays of the central span. Analogous conclusions can be drawn for the distribution of the stresses in the cable elements, which are in both cases, under LL, equal o lower than the corresponding allowable value. Contrarily, the suspension system dimensioning presents larger value of required steel quantity than those observed in the case of HCS bridge for both main cable and hangers cross-sections, i.e. larger than 3.45 and 3.89 times, respectively. Moreover, the distribution of the hanger cross-sections is not constant as in the case of HCS bridge, since it presents a maximum value at a position on the girder profile equal to $X_1/L_T = 0.35$. Such prediction can be explained by the fact that SP bridges are mostly influenced by the characteristics of the main cable, which is typically affected by large displacements in the case of unsymmetrical loading distributions on the girder main span length. Similarly, results concerning SP system denote that the stress distribution presents at $X_1/L_T = 0.35$ a reduction of the maximum allowable stresses, since in that region, the bridge scheme is affected by large deformability and thus the cable dimensioning requires important contributions to verify prescriptions on maximum displacements. The analyses on HCS configurations do not denote such behavior, because of the presence of the cable-stayed system, which partially balances the reduced stiffness of the main cable, producing a regular distribution of cross-sections in the hanger cable-system. Finally, the working rate of the main cable cross-section is much larger than that observed in the case of SP dimensioning, since HCS bridges are characterized by an improved stiffness behavior because of the presence of the stays in the cable system. As a matter of fact, the stiffness behavior of the SP system is mainly influenced by main cable dimensioning, which is increased during the design procedure to verify deformability and stress prescriptions, leading to low values of the working

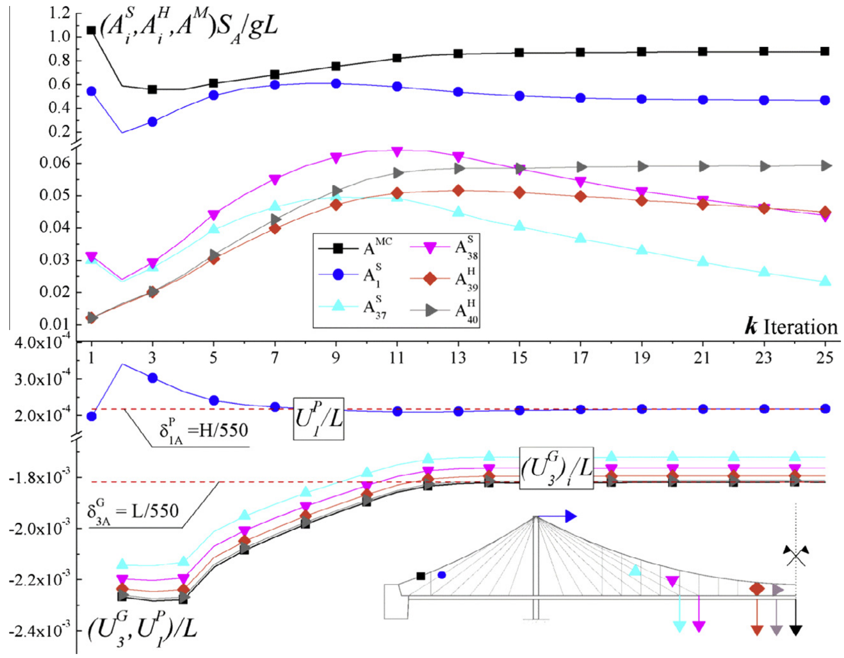

Fig. 14. HCS bridges: comparisons between maximum stresses in the cable system and vertical displacements in the girder at discrete points of the bridges as a function of the iteration steps.

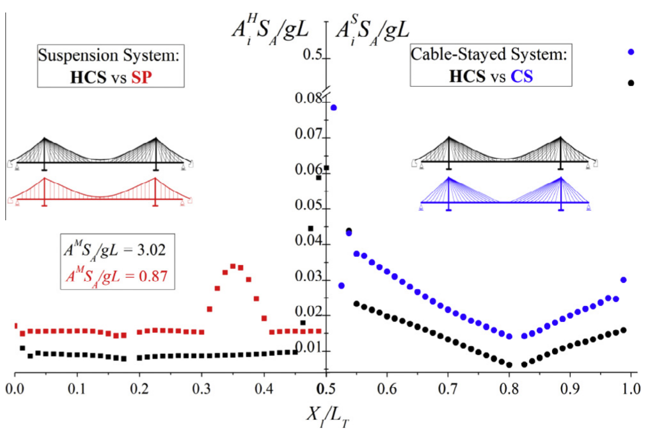

Fig. 15. Comparisons between predicted values of the cross-sections in the cable-system between hybrid cable-suspension (HCS), cable-stayed (CS) and suspension (SP) bridges.

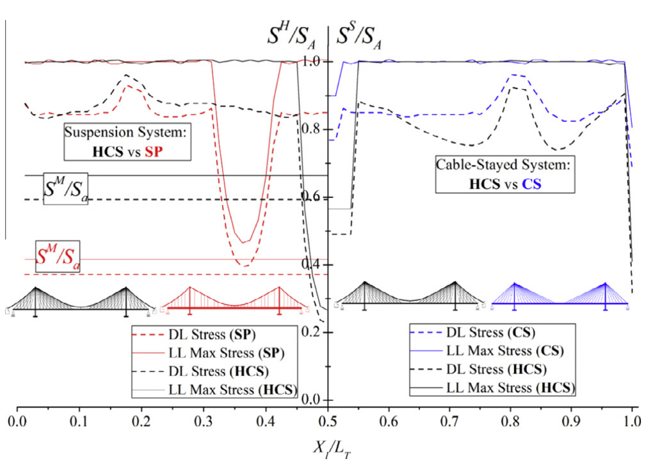

Fig. 16. Comparisons between predicted values of the maximum stresses in the cable-system between hybrid cable-suspension (HCS), cable-stayed (CS) and suspension (SP) bridges.

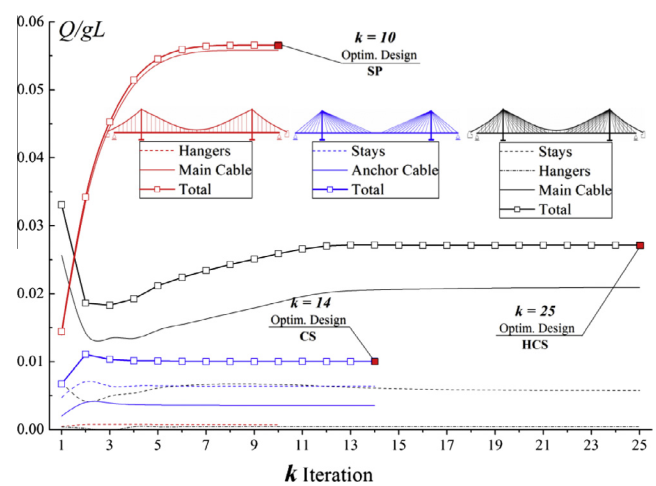

Fig. 17. Comparisons in terms of total steel quantity involved in the cable-between hybrid cable-suspension (HCS), cable-stayed (CS) and suspension (SP) bridges.

stress rates and thus large amount of steel quantity involved in such bridge component.

Finally, the evolution of the total required steel quantity involved in the cable system for each bridge scheme as a function of the number of iteration steps to obtain the optimum solution is reported in Fig. 17. Results denote for all bridge schemes a convergent behavior, since a limited number of iterations are required to derive the design configuration. The SP system presents the maximum steel quantity, which is mainly given by the contribution of the main cable, whereas the lowest configuration in terms of steel quantity is achieved by using CS system. The optimum solution of HCS bridge presents lower cross-sections of the main cable and stays than those observed in SP or in the CS systems. In particular, the main cable cross-section, which produces the most relevant contribution in the total steel quantity, from the pure SP system is reduced almost of 62%. Contrarily, the contribution arising from the hangers can be considered comparable, even if the corresponding steel quantity is much less of the one observed in the main cable. Finally, the configuration, which is characterized by the lowest steel quantity involved in the cable system is the one associated to the CS system. However, such results should be considered also in relationship to the geometry of the bridge scheme, which presents, typically, larger ratios between pylon and lateral span than those observed in SP or HCS bridge configurations. Therefore, the reduced values of total steel quantity in the cable-system are partially compensated by the costs involved in the pylon construction.

Conclusions

A combined model based on optimization design method and FE approach is proposed to quantify the optimum dimensioning of the cable-system and the post-tensioning stress in the DL configuration. The method is based on an two-step algorithm, in which two different phases based on the optimization and correction moduli are executed, iteratively. From the results, the following conclusions can be drawn:

- the proposed methodology can be considered a useful tool in the prediction of the required cable dimensioning and post-tensioning forces, since it is able by mean of a limited number of convergent iterations to provide the optimum design solution in terms of stress and displacements variables; however, in the present analysis, the optimum configuration is carried out with respect to the cable system elements only, without entering in the optimization of the pylon and girder characteristics;
- despite existing methodologies based on pure optimization procedure, the proposed method seems to be not affected by numerical convergence problems, since it is based on a hybrid two-step algorithm, in which the solution is enforced by using physically based expressions;
- the optimum solution is searched according to maximum utilization material criterion, for which under LL combinations, it is supposed that the worst stress should be equal to the corresponding maximum stress, leading, globally to a reduced steel quantity in the cable system;
- the increments of the cable system stiffness is achieved by using proper performance factors based on a secant description of the cable stiffness, which are able to reduce bridge deformability, thus verifying prescriptions on maximum deflections of girder and pylons;
- for the cable-stayed bridge scheme, the results show that cables which require larger values of steel quantity than the rest of the elements are those associated to the anchor stays or the longest elements in the midspan;
- the analyses for the investigated case on a long span bridge have shown that pure suspension or cable-stayed bridge schemes present values of total steel quantity larger or lower than that associated to the HCS configuration, with percentage errors equal to 108 and 76, respectively. However, for CS bridges, the reduced value of the involved steel quantity is compensated by the construction of the pylons, which present larger height with respect to conventional SP or HCS bridge schemes.

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Appendix A

The optimization model presented for HCS bridges is specialized here for the cases concerning pure cable-stayed and suspension bridge schemes. Since the number of variables of the cable system is strongly reduced, the governing equations introduce a determinate equation system which can be easily solved without the recourse of minimization techniques. In particular for the CS bridge scheme, the control variables are expressed by the post-tensioning stress and cable cross-section vectors, as follows:

$$\underset{\sim}{S}_C = \{S_1^S, \ldots, S_{N_s}^S\}, \quad \underset{\sim}{A} = \{A_1^S, \ldots, A_{N_s}^S\} \tag{A1}$$

The unknown quantities are derived in the DL configuration by solving the set of constraint equations concerned to enforce displacements of girder and pylons to be zero and the internal stresses to be equal to the initial design values, namely S_{gi}^S:

$$\underset{\sim}{L}_v\left[\left(\bar{S}_i^S + \Delta S_i^S, \bar{S}_1^S + \Delta S_1^S, \bar{S}_{N_S}^S + \Delta S_{N_S}^S\right), \underset{\sim}{U}\right] = 0, \tag{A2}$$

$$\underset{\sim}{L}_{S_3}\left[\left(\bar{A}_i^S + \Delta A_i^S\right), S_i^S\left(\underset{\sim}{\xi}, \underset{\sim}{\psi}\right) - S_{gi}^S\right] = 0, \tag{A3}$$

with $i = 1, \ldots, N_S$, $\underset{\sim}{U}^T = \left[U_{3(2\ldots N^S-1)}^G, U_1^{P_L}, U_1^{P_R}\right]$ is the vector containing vertical displacements of the stays except for the anchor ones $(U_{3,i}^G)$ and the horizontal displacements at the top pylon right and left cross-sections $(U_1^{P_L}, U_1^{P_R})$. Finally, at the kth iteration, optimization factors $(\Omega^S)_i^k$ and $(\Phi^S)_i^k$ are calculated in the LL combinations by using Eqs. (11) and (13). Similarly, the control variables for the SP bridge are defined by the following vectors:

$$\underset{\sim}{S}_C = \left\{S_1^H, \ldots, S_{N^H}^H, S^M\right\}, \quad \underset{\sim}{A} = \left\{A_1^H, \ldots, A_{N^H}^H, A^M\right\} \tag{A4}$$

whereas the constrain equations to impose zero vertical and horizontal displacements on the girder and pylons and, horizontal and internal stresses equal to the initial design value, S_{gi}^H:

$$\underset{\sim}{L}_{S_1}\left[\left(\bar{A}_i^H + \Delta A_i^H\right), S_i^H\left(\underset{\sim}{\xi}, \underset{\sim}{\psi}\right) - S_{gi}^H\right] = 0, \tag{A5}$$

$$\underset{\sim}{L}_U\left[\left(\bar{S}_j^H + \Delta S_j^H, \bar{S}^M + \Delta S^M\right), \underset{\sim}{U}\right] = 0, \tag{A6}$$

with $i = 1, \ldots, N^H$, and $U^T = [U_{3(1_k N^H)}^G, U_1^{M-P_{L,R}}]$. Finally, at the k-th iteration, optimization factors $(\underset{\sim}{\Omega}^S)_i^k$ and $(\Phi^S)_i^k$ are calculated in the LL combinations by using Eqs. (11) and (13).

Appendix B

In the case of HCS bridges, during the erection procedure, at the k-th construction step, the unknown quantities are represented by the post-tensioning forces of the cable system elements:

$$\underset{\sim}{S}_C = \{S_1^S, \ldots, S_k^S, S_1^H, \ldots, S_k^H, S^M\}, \tag{B1}$$

Moreover, the objective function D to be minimized during the optimization procedure consists of a scalar valued function defined by the square root of the vertical and horizontal displacements of girder and pylons, evaluated at the corresponding nodal points:

$$\underset{\underset{\sim}{(\psi)}}{Min}\, D(\underset{\sim}{\psi}) = \underset{\underset{\sim}{(\psi)}}{Min}(\|U\|) \tag{B2}$$

where $\|\bullet\|$ is the Euclidean norm of the $(\bullet)$ vector, $\underset{\sim}{U}$ with $\underset{\sim}{U}^T = [U_{3(1\ldots k)}^G, U_1^{P_L}, U_1^{P_R}, U_1^{M-P_L}, U_1^{M-P_R}]$ is the vector containing the vertical displacements at the hangers/girder connections at the k-th step erected element, horizontal displacements at the top pylon left (L) and right (R) cross-sections $(U_1^{P_L}, U_1^{P_R})$ and at the intersection points of the left and right top pylon cross-sections with the main cable $(U_1^{M-P_L}, U_1^{M-P_R})$. In addition, to Eqs. (B1) and (B2), constrain equations are necessary to guide the optimization procedure through a reasonable solution:

$$\max\left[S_i^H\left(\underset{\sim}{\psi}\right)\right]_k \leqslant S_A, \max\left[S_i^S\left(\underset{\sim}{\xi}, \underset{\sim}{\psi}\right)\right]_k \leqslant S_A, \max\left[S^M\left(\underset{\sim}{\xi}, \underset{\sim}{\psi}\right)\right]_k \leqslant S_A, \tag{B3}$$

For the pure suspension and cable-stayed bridge schemes, the solution during the erection procedure is derived by solving a determinate equation system. In particular, the unknown quantities are represented by the post-tensioning forces, which are evaluated reproducing the undeformed configuration on the basis of the following constrain relationships:

$$\underset{\sim}{L}_U^H\left[\underset{\sim}{S}_k^H, \underset{\sim}{U}\right] = 0, \quad \underset{\sim}{L}_U^S\left[\underset{\sim}{S}_k^S, \underset{\sim}{U}\right] = 0, \tag{B4}$$

where $(\underset{\sim}{S}_k^H, \underset{\sim}{S}_k^S$ are the vectors containing the stresses of the hangers or the stays elements and $\underset{\sim}{U}^{H(T)} = [U_{3(1)}^G, U_{3(2)}^G, \ldots, U_{3(k)}^G, U_1^{M-P},]$ contains the vertical displacements of the girder and the horizontal displacement of the pylon.

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