Asset Pricing & Portfolio Management

Post-graduation in Data Science for Finance

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Heuristic and Markowitz Modern Portfolio Management Investment Strategies

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1. Introduction

The financial market is a complex and ever-evolving field that captures the attention of investors and researchers worldwide. Analyzing stock market returns is crucial in investment decision-making, understanding asset behavior, and evaluating portfolio strategies. In this paper, we explore the empirical properties of financial market returns in the United States, using a diversified selection of stocks representing various sectors of the economy.

The chosen stocks for this analysis cover a range of economic sectors, reflecting the diversity and complexity of the U.S. stock market. They are as follows:

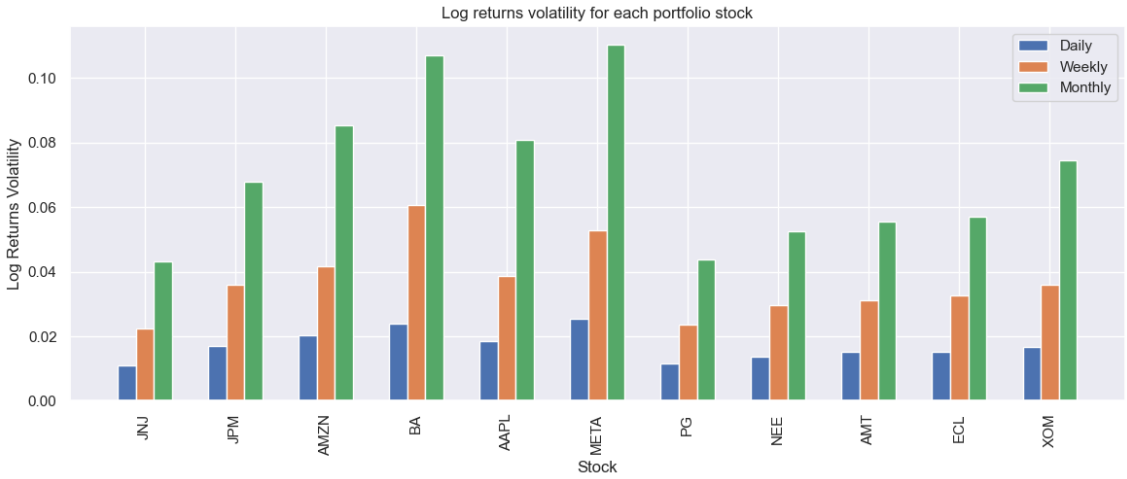
- Health Care: Johnson & Johnson (Ticker: JNJ)
- Financials: JPMorgan Chase & Co. (Ticker: JPM)
- Consumer Discretionary: Amazon.com Inc. (Ticker: AMZN)
- Industrials: Boeing Co. (Ticker: BA)
- Information Technology: Apple Inc. (Ticker: AAPL)
- Communication Services: Facebook Inc. (Ticker: META)
- Consumer Staples: Procter & Gamble Co. (Ticker: PG)
- Utilities: NextEra Energy Inc. (Ticker: NEE)
- Real Estate: American Tower Corp. (Ticker: AMT)
- Materials: Ecolab Inc. (Ticker: ECL)
- Energy: Exxon Mobil Corp. (Ticker: XOM)

This comprehensive selection of stocks will allow us to examine the performance and characteristics of returns in different sectors of the economy and evaluate portfolio investment strategies that can be derived from these assets. Throughout this paper, we will explore the analyses and conclusions arising from our investigation of the empirical properties of these stocks in the U.S. financial market.

2. The importance of the volatility and the influence of the correlation

Volatility plays a crucial role in understanding the risk and variability of financial asset returns. It can influence investment decisions and portfolio strategies, as assets with higher volatility may represent greater risk to investors.

To visualize the volatility of the selected assets, we present below a graph that displays daily, weekly, and monthly log return volatilities. As highlighted in the graph, assets with monthly volatilities exceeding 8% include META, BA, and AMZN, indicating a greater price fluctuation over a month. Conversely, assets with lower monthly volatility are JNJ and PG, suggesting that these stocks are relatively more stable in terms of price variation throughout the month. This information is critical for investors as it helps assess the risk associated with each asset and can impact their portfolio allocation decisions.



The correlation between assets plays a crucial role in building a diversified portfolio, as it can impact the effectiveness of risk mitigation and the pursuit of investment opportunities. To visualize the degree of correlation among these assets, we present below a heatmap-style graph that illustrates the correlation relationships among them.

We have observed that all identified correlations are positive, indicating that the performance of the returns of these assets tends to move in the same direction. Notably, we have identified positive correlations, exceeding 50%, among the following pairs of assets:

- ECL and JPM;
- XOM and JPM;
- PG and JNJ;
- BA and JPM;
- AMT and NEE.

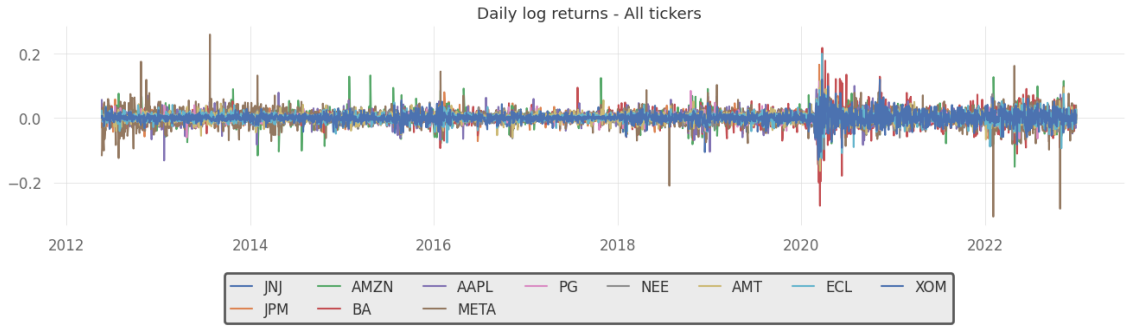
Strong correlations between assets carry significant implications for portfolio diversification. Investors can leverage this knowledge to strike a balance between risk exposure and the identification of investment opportunities rooted in these positive interconnections.

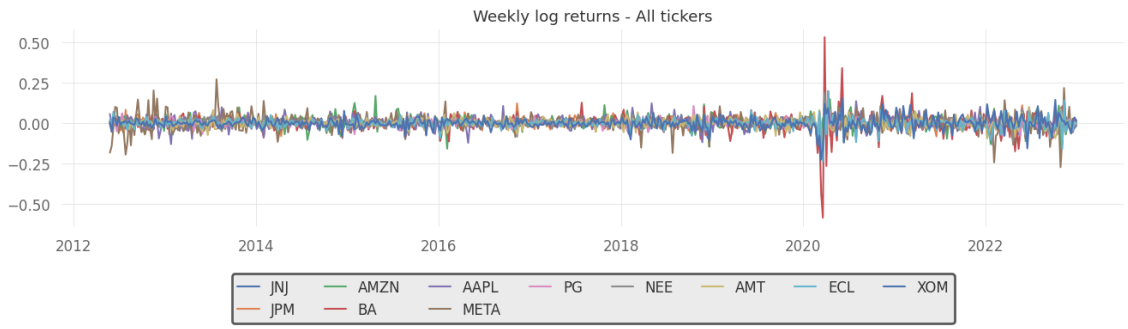
Understanding these correlations helps in crafting well-informed diversification strategies, allowing for risk minimization and the potential enhancement of returns, especially when spanning different sectors or industries. This insight empowers investors to refine their portfolios, ensuring they remain agile and responsive to market conditions, facilitating more effective investment decisions and a better risk-reward profile.

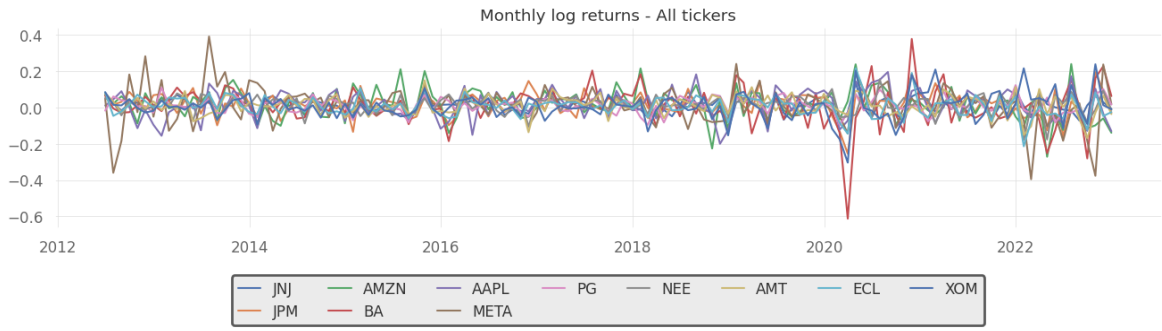
3. Empirically investigate the financial market returns

To assess the performance of any stock, we calculate daily, weekly, and monthly log returns, which are essential metrics in financial analysis for understanding the price variation of stocks over time.

$$Log\ Returns = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$$







The charts above depict the returns of selected companies from 2012 to 2022, based on different time frequencies (daily, weekly, and monthly). Overall, it can be observed that asset returns follow a trend of stability between 2014 and 2020. However, the periods from 2012 to 2014 and from 2020 to 2022 were characterized by higher volatility, with a focus on the following companies:

Meta exhibited higher volatility between 2012 and 2014, with losses of approximately -40% in 2012 and gains of up to approximately +30% in 2013. This period coincided with the company's initial public offering (IPO), followed by the acquisition of Instagram.

Boeing experienced increased volatility between 2020 and 2022, with losses of approximately -60% in 2020 and gains of approximately +40% in early 2021. The global aviation industry was severely impacted by the COVID-19 pandemic, leading to sharp declines in demand for commercial aircraft. The ongoing Boeing 737 MAX crisis continued to affect the company, despite efforts to address safety issues and resume operations, resulting in significant impact.

These observations provide insights into the performance of these companies during these periods, with noteworthy events and market dynamics influencing their returns.

3.1 Absence of autocorrelation

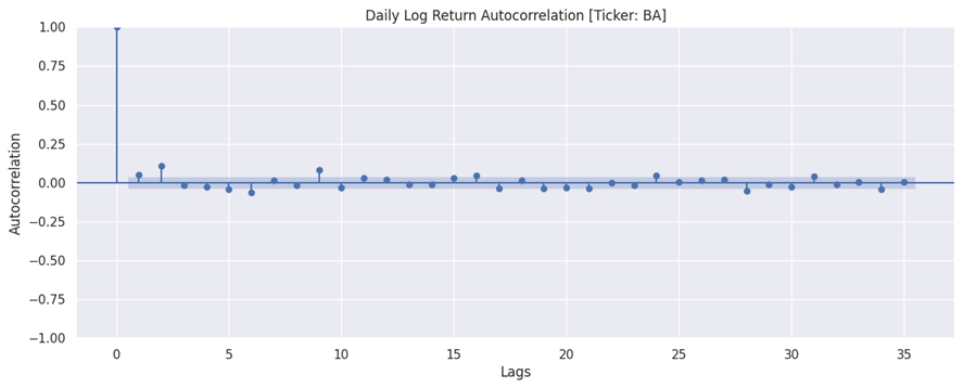
Autocorrelation is a fundamental statistical concept that measures the relationship between the values of a time series and its previous values. In other words, autocorrelation assesses the presence of temporal patterns or dependencies in the data. This means that autocorrelation analyzes whether the values at a specific point in time are related to the values that occurred in previous periods.

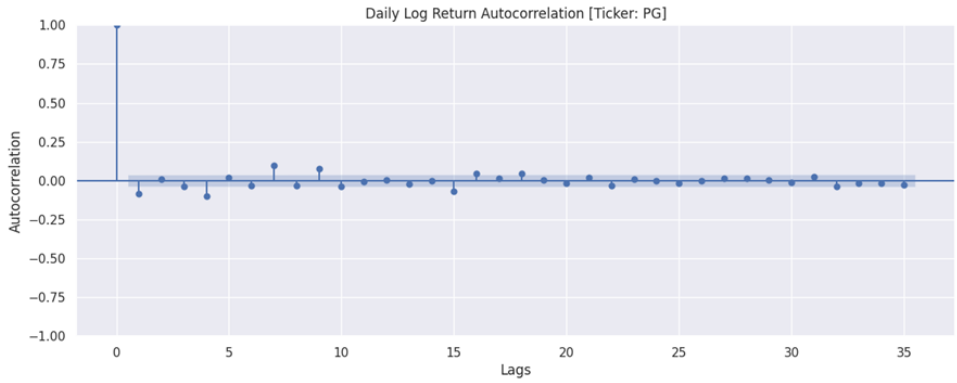
$$Autocorrelation = \left(\frac{\sum_{t=1}^{N}(X_t - \bar{X})(X_{t-k} - \bar{X})}{\sum_{t=1}^{N}(X_t - \bar{X})^2}\right)$$

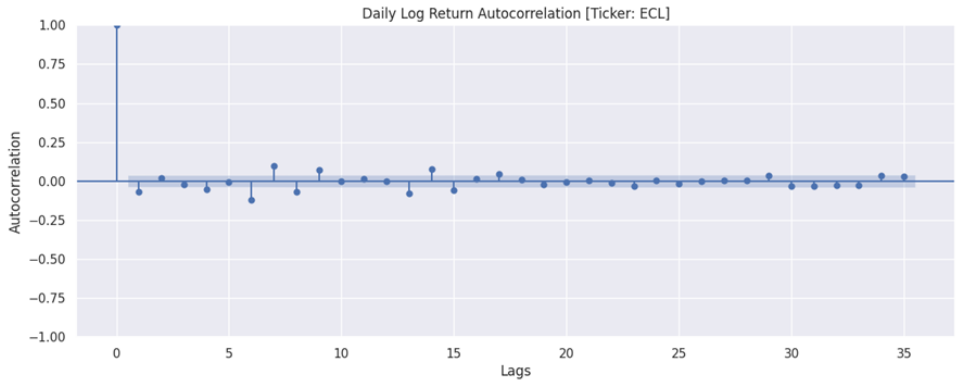
- AC represents autocorrelation.
- X_t denotes a value in the time series at time t.
- N is the total number of observations in the series.
- *k is* the lag interval being compared (e.g., first-order autocorrelation, k =1)
- $\bar{X}$ is the mean of the values in the time series

Observing the daily, weekly, and monthly autocorrelation plots for all companies reveals interesting patterns.

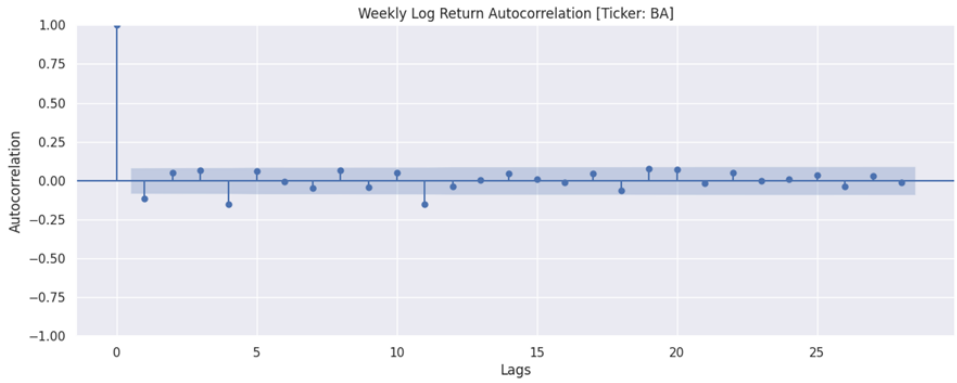
Regarding daily autocorrelation, over 70% of the stocks exhibit significant levels of dependence between daily returns and their previous values within one day. This suggests that events or patterns occurring in a single day may influence the returns of the following companies: JNJ, JPM, BA, AAPL, PG, NEE, AMT, and ECL. We provide some examples below:

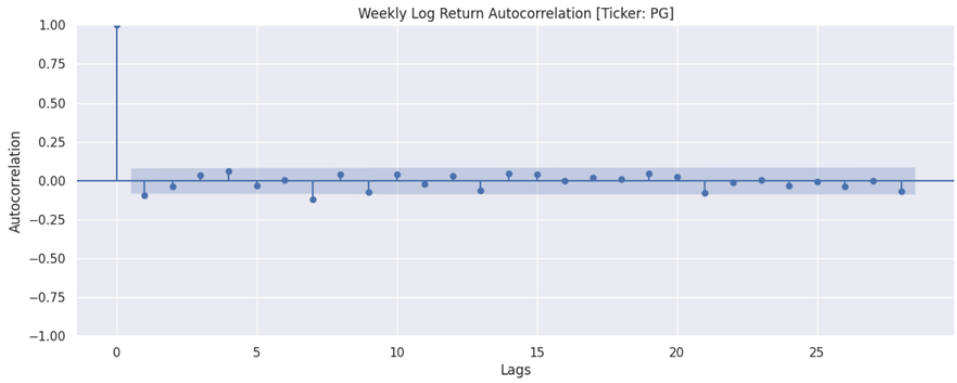


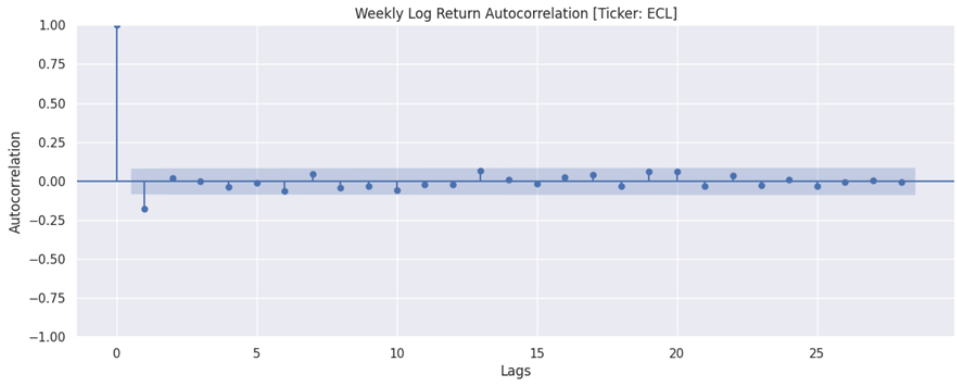




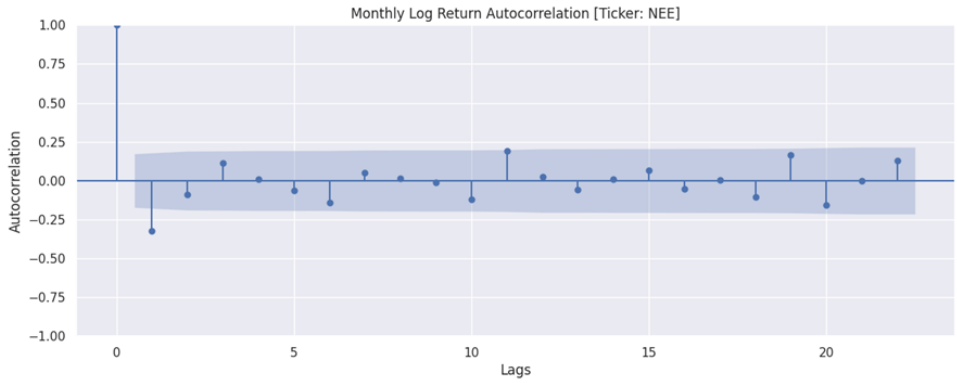
As for weekly autocorrelation, approximately 27.3% of the stocks exhibit significant dependence between weekly returns and their past values within a one-week period. This suggests that trends or events unfolding over a week can influence the following stocks: BA, PG, and ECL. The charts are presented below:







Finally, the company NEE is the only one to exhibit monthly autocorrelation, indicating that monthly events or patterns influence its returns. This finding may be particularly relevant for long-term investment strategies, where investors adjust their positions monthly based on monthly trends.



It is important to note that when analyzing autocorrelations with a lag of one day, all companies exhibited negative autocorrelations, except for company BA, when considering daily log returns.

This observation is significant as it suggests that, in general, daily returns tend to move in the opposite direction of the previous day. This phenomenon indicates a daily

reversal pattern, where negative fluctuations on one day tend to be followed by positive fluctuations the next day, and vice versa.

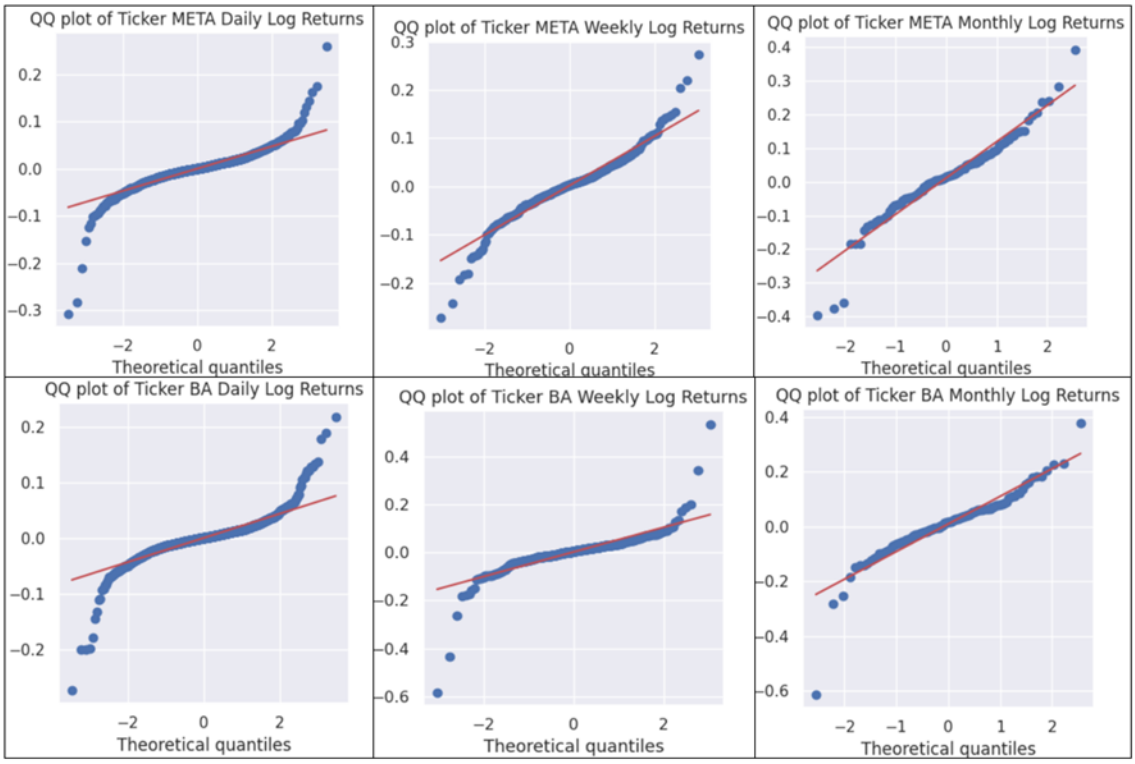
On the other hand, company BA stands out by displaying positive autocorrelation with a lag of one. This means that its daily returns tend to move in the same direction as the previous day. This observation may be of interest to investors looking to identify daily continuity trends in the stock price movements of company BA.

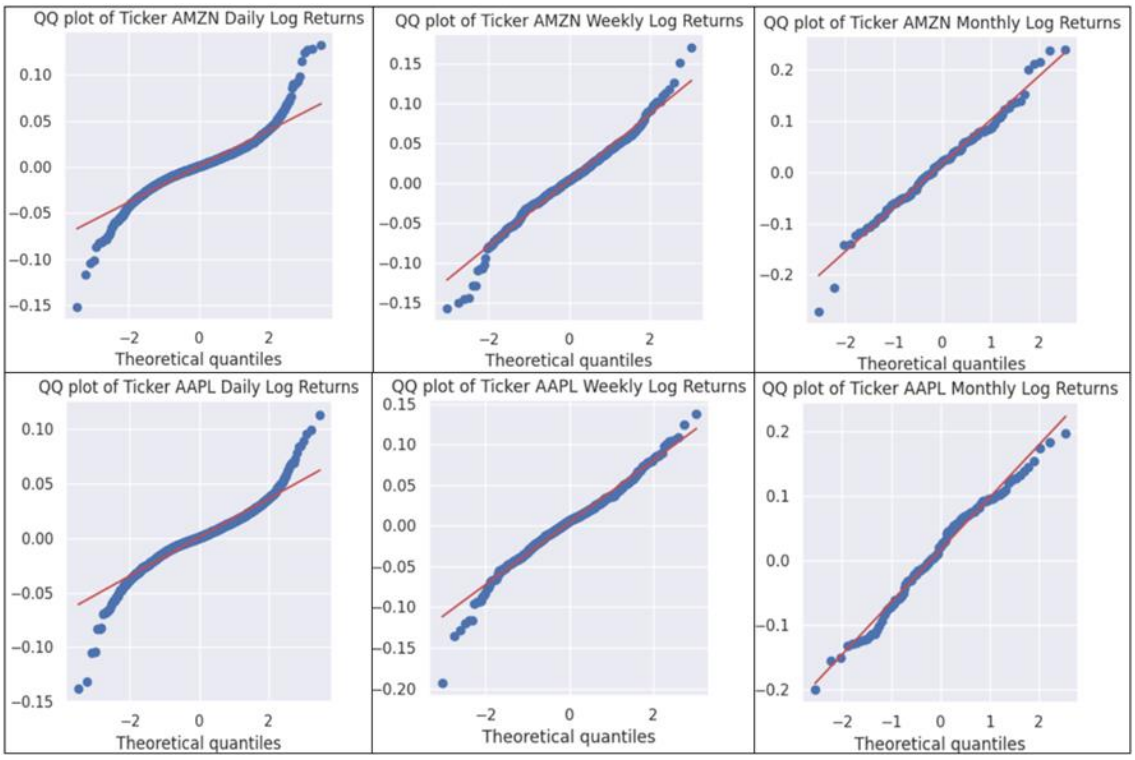
In summary, the analysis of autocorrelations with a lag of one day reveals a prevailing daily reversal pattern among the stocks, except for company BA, which demonstrates a continuity trend in its price movements. It is worth noting that the presence of autocorrelation does not guarantee predictability but suggests that certain stocks may be influenced by events on different time horizons.

3.2 The unconditional distribution of daily returns does not follow the normal distribution / Fat Tails

To analyze the unconditional distribution of monthly returns for the selected stocks, we employ normal probability plots, commonly known as QQ plots. QQ plots are a vital visual tool used to compare the distribution of a data sample to a theoretical normal distribution. They enable us to assess whether the data adheres to a normal distribution or exhibits deviations.

In a QQ plot, the points represent the quantiles of the data sample and the expected quantiles for a normal distribution. When the points in the plot closely follow a straight line, it suggests that the data may conform to a normal distribution. Conversely, deviations from the straight line indicate that the data exhibit non-normal characteristics. If the points are above the line, it suggests a heavy tail, indicating that extreme values are higher than expected in a normal distribution. On the other hand, if the points are below the line, it suggests a light tail, indicating that extreme values are lower than expected in a normal distribution.





The charts presented above indicate that the distribution of returns closely approaches the straight line (normal distribution) when returns are observed monthly. On the other hand, deviations from the straight line suggest that the data exhibit non-normal characteristics, as observed in the daily observations. It is important to note that if the points are above the line, it suggests a fat tail, indicating that extreme values are higher than expected in a normal distribution.

To verify if the data adheres to a normal distribution, we will apply the Shapiro-Wilk normality test to the daily, weekly, and monthly returns of the selected stocks, considering a significance level of 5%.

The Shapiro-Wilk test is a statistical tool used to assess whether a dataset follows a normal distribution. This test calculates a test statistic based on the discrepancies between the observed values and the expected values of a normal distribution. The null hypothesis (H0) of the test asserts that the data follows a normal distribution, while the alternative hypothesis (H1) suggests that the data does not follow a normal distribution.

The test statistic follows a specific distribution under the null hypothesis. Subsequently, the calculated test statistic is compared to critical values from the distribution under H0. If the test statistic is significantly lower than the critical value

(based on a predetermined level of significance), the null hypothesis is rejected, indicating that the data does not follow a normal distribution.

When subjecting the data to the Shapiro-Wilk test, valuable insights were gained regarding the distribution of returns over different time intervals.

Applying the Shapiro-Wilk test to daily logarithmic returns revealed that the distributions of daily returns significantly deviate from the bell-shaped curve of a normal distribution. These findings underscore the inherent complexity and unpredictability of daily market fluctuations.

Stock	Shapiro-Wilk Statistic (Dialy)	P-Value	Follows Normal Distribution
JNJ	0.903200	3.448363e-38	False
JPM	0.900483	1.369443e-38	False
AMZN	0.925458	1.642593e-34	False
BA	0.821065	0.000000e+00	False
AAPL	0.938656	6.787308e-32	False
META	0.859347	8.688050e-44	False
PG	0.888272	2.729435e-40	False
NEE	0.889029	3.443691e-40	False
AMT	0.917225	5.810653e-36	False
ECL	0.866070	4.974610e-43	False
XOM	0.915576	3.072283e-36	False

The analysis extended to weekly logarithmic returns, where the results highlighted the non-normality observed in daily returns. Once again, the data exhibited characteristics that diverged from the Gaussian ideal. Weekly returns, while offering a broader perspective, continued to reveal deviations from normality.

Stock	Shapiro-Wilk Statistic Weekly	P-Value	Follows Normal Distribution
JNJ	0.969432	2.439284e-09	False
JPM	0.951323	1.531676e-12	False
AMZN	0.979671	5.688481e-07	False
BA	0.728719	4.824931e-29	False
AAPL	0.978768	3.327512e-07	False
META	0.940575	4.441235e-14	False
PG	0.957037	1.251281e-11	False
NEE	0.902128	2.328996e-18	False
AMT	0.956274	9.357715e-12	False
ECL	0.884791	6.809719e-20	False
XOM	0.916840	6.806092e-17	False

However, a notable shift was observed when applying the test to monthly logarithmic returns. Surprisingly, the returns of five distinct companies exhibited a closer alignment with a normal distribution. This alignment implied a degree of predictability in the broader monthly market performance of these selected entities, contrasting with the daily and weekly data that challenged such predictability.

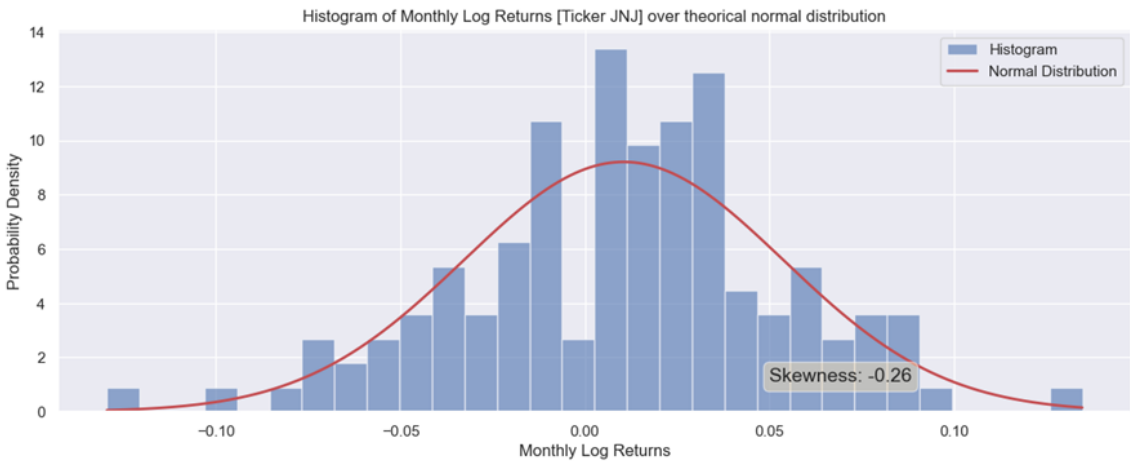
Stock	Shapiro-Wilk Statistic Monthly	P-Value	Follows Normal Distribution
JNJ	0.991386	6.231308e-01	True
JPM	0.974906	1.835312e-02	False
AMZN	0.985256	1.856677e-01	True
BA	0.885173	1.829593e-08	False
AAPL	0.983595	1.284284e-01	True
META	0.939655	2.445272e-05	False
PG	0.990090	4.995379e-01	True
NEE	0.955005	3.354752e-04	False
AMT	0.985625	2.013161e-01	True
ECL	0.964420	2.028902e-03	False
XOM	0.942957	4.170826e-05	False

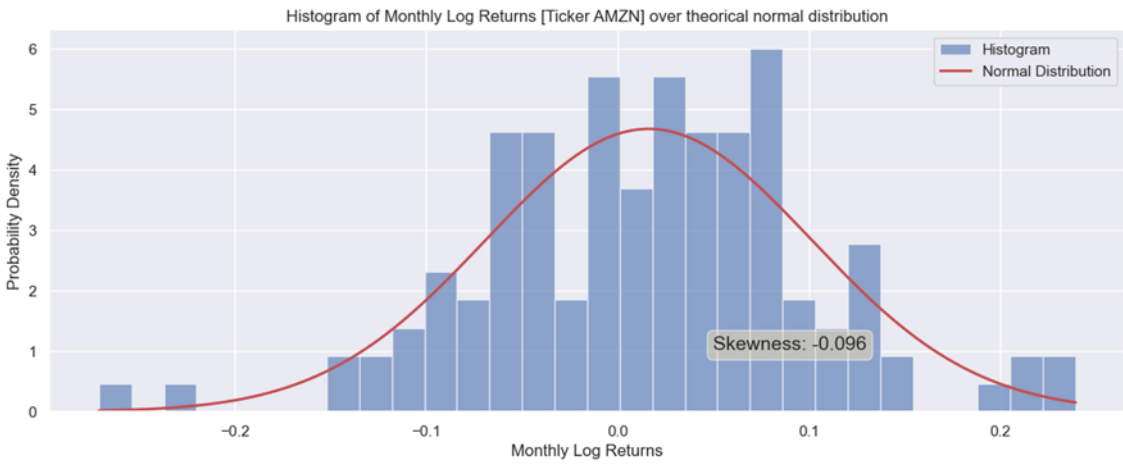
These results emphasize the significance of choosing the appropriate time frame when analyzing financial data. While daily and weekly returns may display non-normal behavior, monthly returns, encompassing a longer period, suggest a semblance of normality.

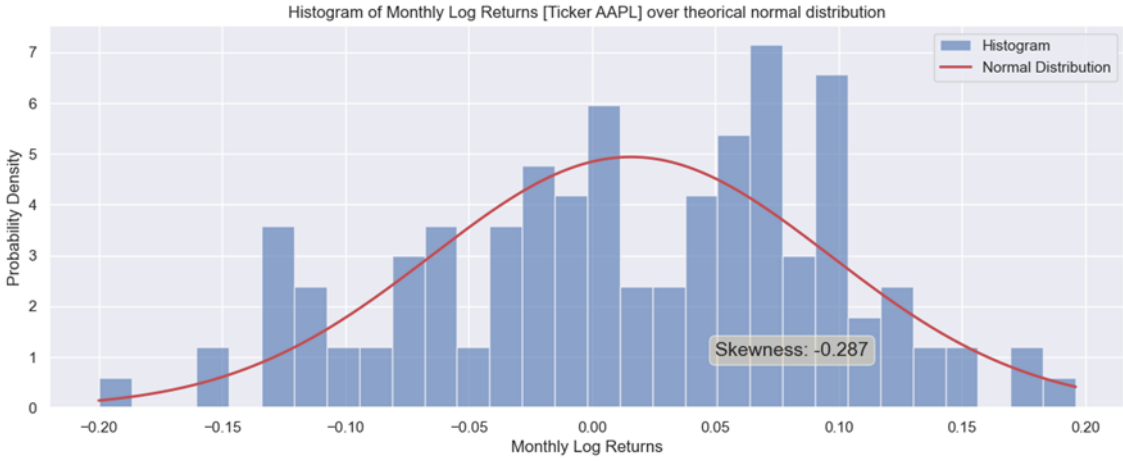
3.4 The return distribution is asymmetric or negatively skewed

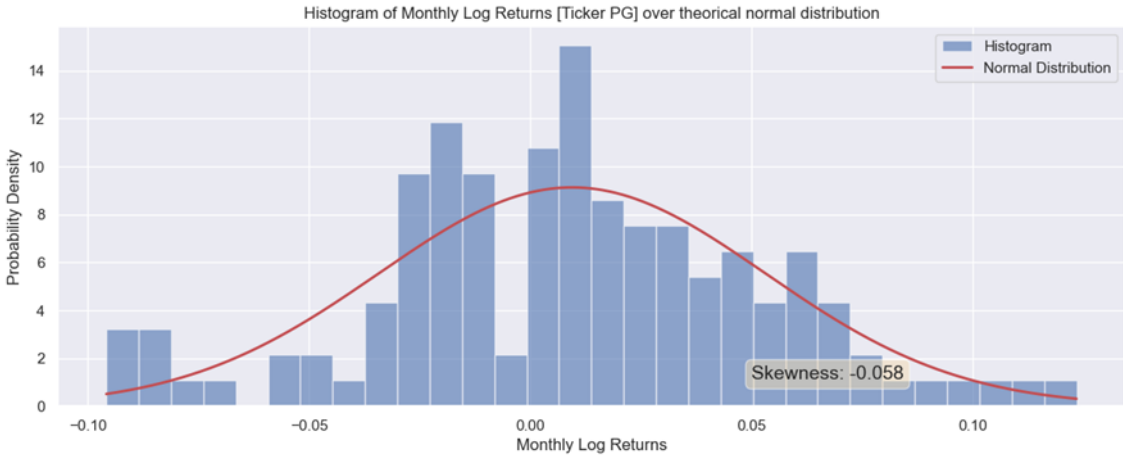
The Shapiro-Wilk test suggests that the tickers JNJ, AMZN, AAPL, PG, and AMT exhibit a normal distribution. To verify if these stocks follow a normal distribution, it is essential to consider additional aspects, such as:

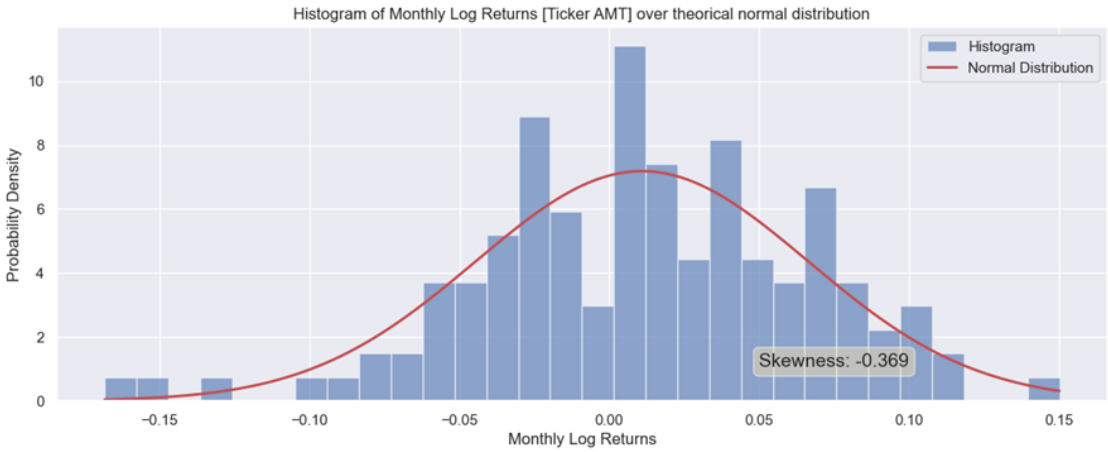
1. Bell-shaped curve: The curve is symmetrical and takes the shape of a bell, with a peak in the center of the distribution.
2. Equal mean and median: The mean (expected value) and the median of a normal distribution are equal and located at the peak of the curve.
3. Standard deviation: The standard deviation controls the data's dispersion. A higher standard deviation results in a wider curve, while a lower standard deviation leads to a sharper curve.
4. Symmetry: The tails of the curve extend infinitely in both directions and never touch the horizontal axis.











	Monthly mean	Monthly median	Monthly std. deviation	Monthly variance
JNJ	0.010474	0.012542	0.043312	0.001876
AMZN	0.016265	0.021446	0.085300	0.007276
AAPL	0.015746	0.022172	0.080812	0.006531
PG	0.009443	0.009633	0.043720	0.001911
AMT	0.010913	0.010107	0.055528	0.003083

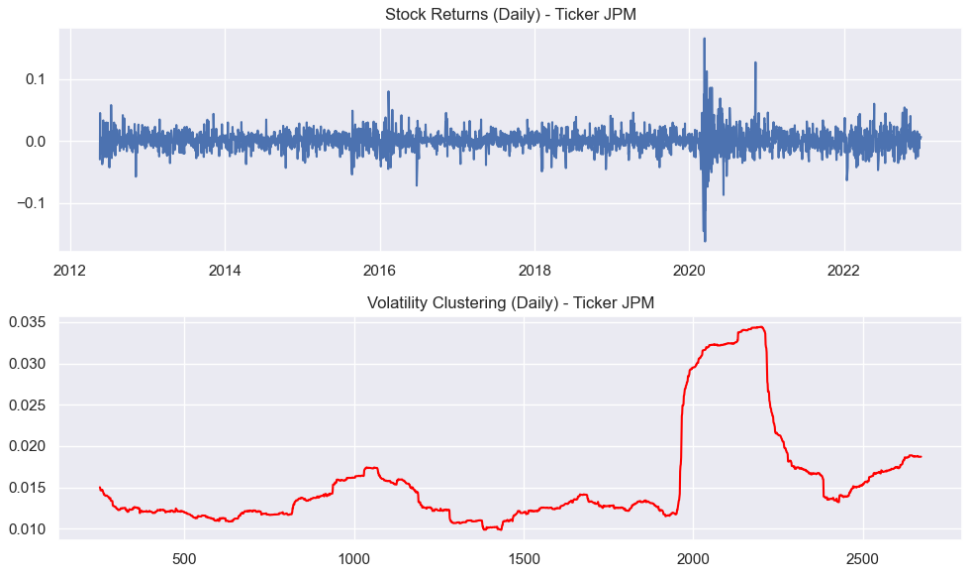
The analysis of the histogram charts provided valuable insights that question the results of the Shapiro-Wilk test applied to the tickers following a normal distribution based on monthly returns. Based on these charts, we can observe the following:

1. The curve does not display asymmetry, showing that all of them have negative skewness. This skewness suggests that the distribution is not symmetric, which is an important characteristic of a normal distribution.
2. The difference between the mean and median of the returns is an additional indication of non-conformity with a normal distribution. In normal distributions, the mean and median are equal and located at the peak of the curve. When these values differ, it suggests that the distribution is asymmetric.
3. The high standard deviation to the mean of the returns is also a sign that the data has significant variability. In a normal distribution, the standard deviation controls the data's dispersion, and a high standard deviation to the mean indicates that the values are more dispersed.

Therefore, based on these observations, it is reasonable to conclude that, despite passing the Shapiro-Wilk test, the monthly returns of these stocks do not follow a normal distribution. This highlights the importance of considering not only the results of statistical tests but also visual analysis and other measures to assess the distribution of financial data.

3.4 Volatility Clustering

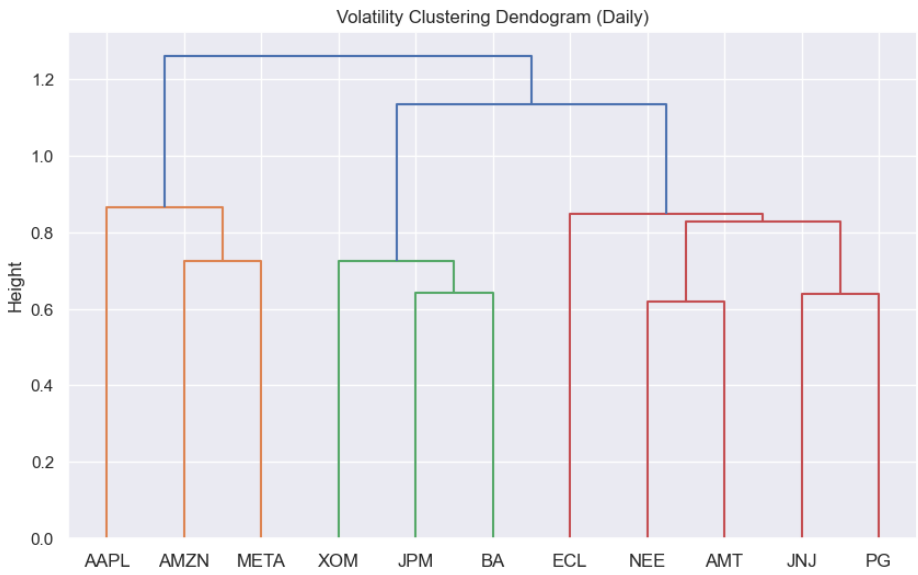
Volatility Clustering is a commonly occurring circumstance in which periods of high volatility carry over their "momentum" and are thus followed by more periods of high volatility, tending to cluster over time.



Observing the graphic above, we can spot some volatility clusters, especially around the 2020 period (COVID-19 pandemic). This is problematic since it diminishes the usefulness of the Gaussian curve to model stock returns.

Because of the effect of volatility clustering that "extends" the high volatility periods, it causes the distribution of returns to have higher extremities (fat tails) and thus not approximate a normal distribution. It then becomes necessary to correct it to allow Gaussian distributions to approximate reality.

An alternative interesting point of view is also viewing the clustering of volatility not across a single asset but for multiple assets. For that, we can use a dendrogram:



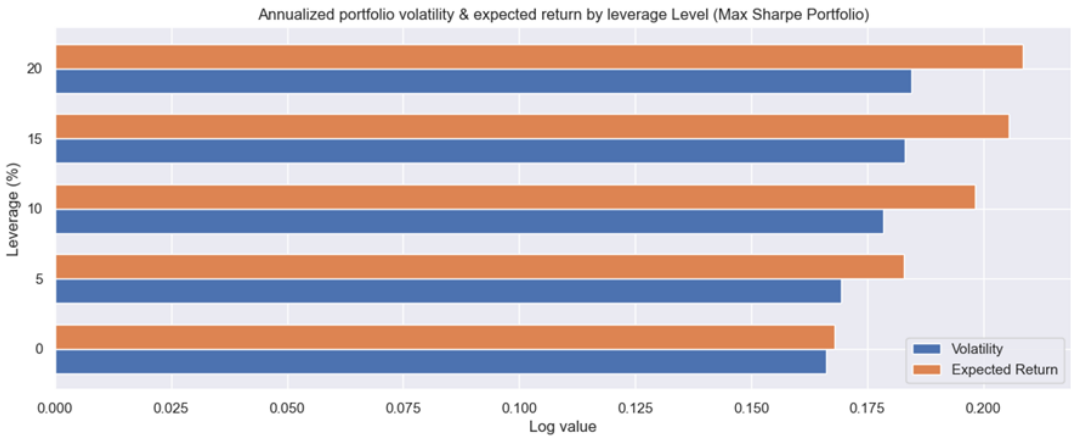
We can see that through the dendrogram certain assets are "linked" together regarding volatility. For example, Apple, Amazon, and Meta assets are all technology stocks, and when clustering for volatility we can spot their close ties in this regard, giving credence to the idea that the periods of volatility from one carry over to the other.

3.5 Leverage Effects

For testing the leverage effects on a portfolio, a simple strategy was devised: using the entire data frame of the returns data, the weights were calibrated, consecutively applying various levels of leverage to determine the expected portfolio returns and volatility.

The levels selected were from 0% (no leverage) up to 20%. The portfolio strategy employed was the Maximum Sharpe Ratio Portfolio (MSRP) for its ease of understanding, use, and implementation with leverage.

The results obtained can be seen in the figure below:

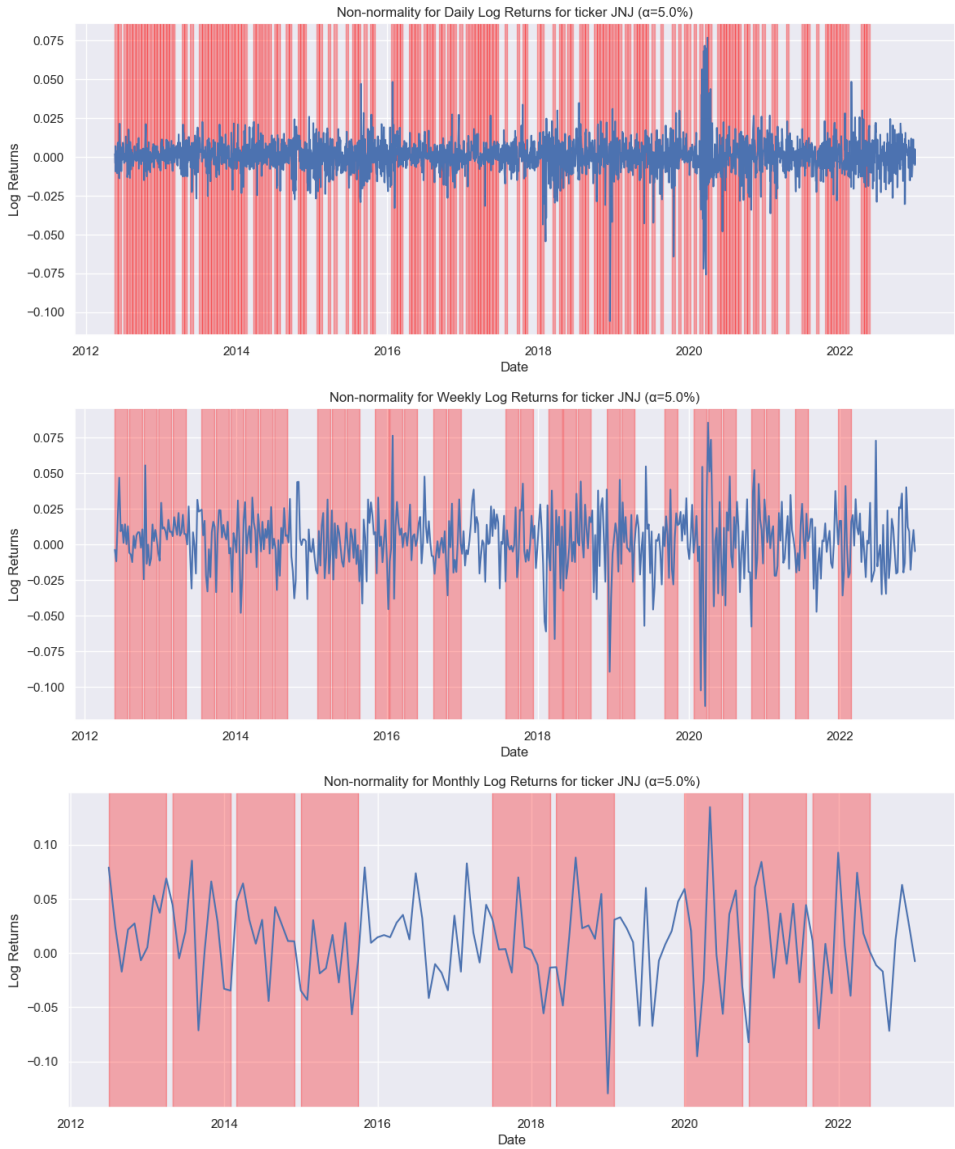


Observing the figure several key observations can be made that, all together, help summarize the main effects of leverage on any given portfolio:

- As the leverage increases, so does the volatility. This happens because, with the increase of leverage, the position is amplified negatively (losses), and positively (gains). For any movement in the expected returns, there is an additional increment/decrement proportional to our leverage level.
- As the leverage increases, the expected returns do as well. The main reason is the same as the previous point since the portfolio is working with extra capital.
- The expected volatility and expected returns increase are not linear nor proportional to one another. In other words, a 5% increase in leverage does not mean that the volatility and expected returns will increase 5% nor does it mean that volatility will increase the same as the expected returns.

3.6 Conditional Non-Normality

In this section we are testing our stock log returns for conditional non-normality, to test for this we split our stocks into samples of one-year periods and put the samples through the Shapiro-Wilk Test with a confidence level equal to 0.05 and obtained the following results.



For all the tickers most of the periods were filled with non-normal periods which supports the results obtained in the previous point a). However, all the tickers also experienced a bigger period of normality between the end of 2015 and the latter parts of 2017, when checked on a monthly frequency. When we go to the opposite end of the spectrum and see the daily charts, we can see that there was not a big period where there

was a streak of normal periods as expected, since we tend to smooth data and lose some information when going from daily to monthly frequency.

Regarding the impact of frequencies on this test, we can see that as we decrease the frequency of the data obtained, we tend to reduce the number of non-normal periods in exchange for having periods that cover much higher periods, which is expected.

Some interesting facts are observable across the different frequencies, for example, in most tickers there are periods with a lot of non-normal periods when checked on the daily frequency (for example JPM in 2017) that when checked in the weekly frequency disappear, which means that these periods with fluctuation in prices when averaged out in weeks disappear and become normal periods.

4. Empirically investigate the performance of the following investment strategies

4.1 Buy & Hold

The Buy and Hold (BnH) strategy is a long-term investment approach in which investors purchase stocks and hold them for an extended period, regardless of short-term market fluctuations. This strategy is based on the belief that over time, assets tend to appreciate and provide positive returns.

In our empirical study, the BnH strategy assumes the allocation of 100% of the investment budget to a single company. This means that each portfolio consists of a single company. Below, we present the weight matrix that reflects the BnH strategy.

	JNJ	JPM	AMZN	BA	AAPL	META	PG	NEE	AMT	ECL	XOM
JNJ	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
JPM	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AMZN	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
BA	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AAPL	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
META	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
PG	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
NEE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
AMT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
ECL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
XOM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

Based on the allocation matrix, it is possible to test the model's performance by simply multiplying the weight of each company by its cumulative return growth over the test period. It is worth mentioning that for the BnH strategy, the division between training and test sets may not have the same impact as in more active trading strategies. This is because the BnH strategy involves purchasing assets and maintaining positions for an

extended period, usually without the need for frequent adjustments based on short-term predictions.

Below, we present the performance chart of the 11 portfolios for the training and test periods.

During the training period, we can observe that several companies performed positively, with cumulative returns exceeding 100%. Among them, companies like AMZN, BA, JPM, and META stand out, showing significant growth in their value over the period.

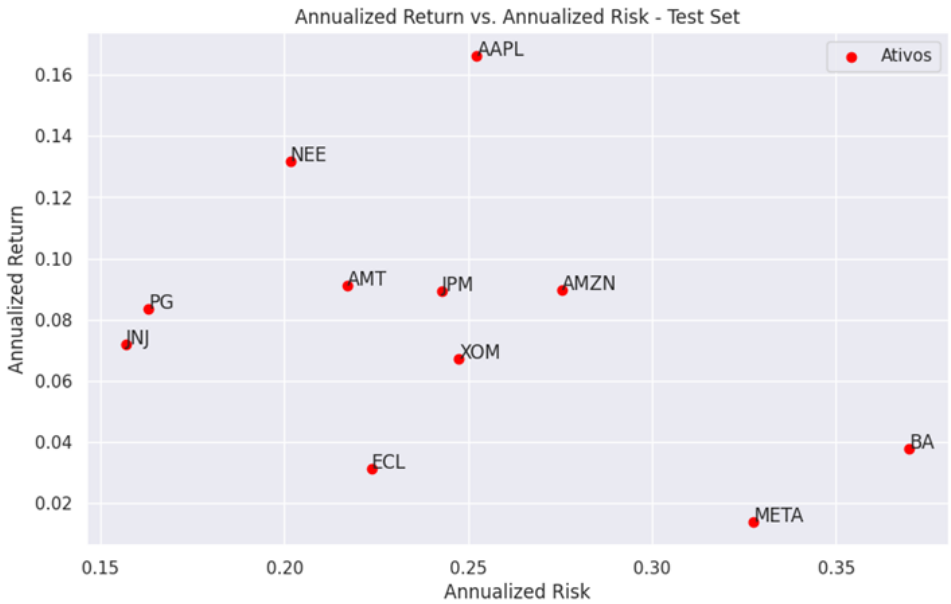
It is interesting to note that company XOM maintained stable returns, without significant growth, but also without losses. In this context, its stability can be considered a positive feature, providing consistent performance.

Furthermore, it is essential to highlight that none of the companies evaluated during the training period exhibited negative returns. This suggests that in this long-term scenario, the Buy and Hold strategy demonstrated resilience, as all companies maintained or increased their value over time.

During the test period, we observed that some companies delivered notable performance, with cumulative returns exceeding 100%. Among these companies, names like AAPL, PG, NEE, and AMT stood out, achieving substantial returns over the period, and reflecting their potential for continuous growth.

However, it is essential to note that META and BA had a challenging performance during this period, recording negative returns. While these two companies faced challenges in the test period, it is crucial to remember that the BnH strategy is designed for the long term and may not necessarily reflect short-term performance.

Next, we present a scatter plot of each company in the BnH strategy:



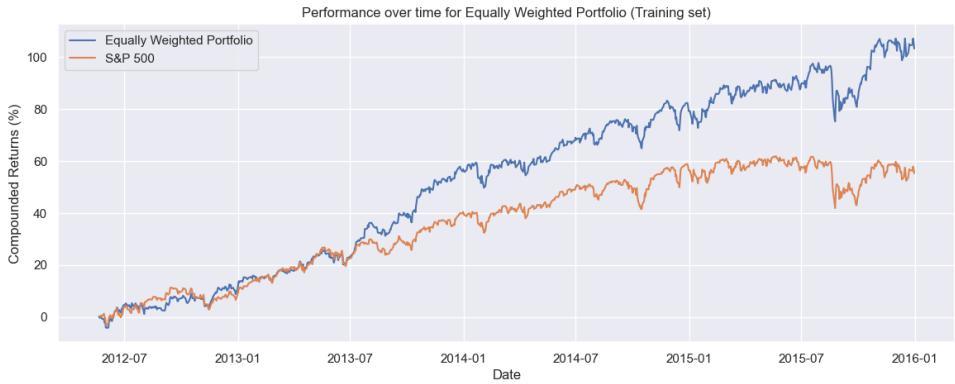
The scatter plot compares the annualized return with the annualized risk of specific assets used in the Buy and Hold strategy. Each point on the graph represents an asset, and the point's position is determined by its annualized risk and annualized return. Additionally, while observing the data, some notable facts were identified:

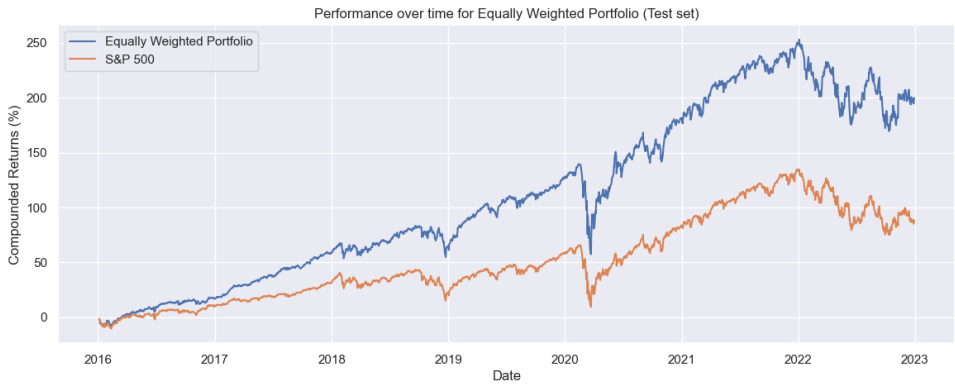
- During the training period, the META company stood out as the riskiest, presenting a significantly high annualized risk. However, this elevated risk was offset by the highest annualized return among all companies in the Buy and Hold portfolio. META achieved the highest return during the training period. In the test period, although META's risk remained similar, its return approached zero, indicating a significantly different performance in the second period.
- The JNJ company exhibited remarkable performance during the training period, with an annualized return of 12% and an annualized risk of only 5%. This indicates that JNJ delivered a solid return with a low risk. However, in the test period, JNJ's annualized return decreased to 7%, while the annualized risk increased to 16%. This change in performance may indicate challenges faced by JNJ in the test period.

AAPL, as depicted in the scatter plot, maintained a consistent annualized risk, ranging between 20% and 25% in both periods. However, its annualized return increased from 7.5% in the training period to 17% in the test period. This suggests that while the risk remained within a stable range, AAPL managed to significantly improve its performance in the second period.

4.2 Equally weighted portfolio

The equally weighted portfolio is one of the most basic strategies for portfolio investment which consists of giving equal weights to all the assets chosen for this project, which averages out to each asset consisting of 9.1% of the portfolio.



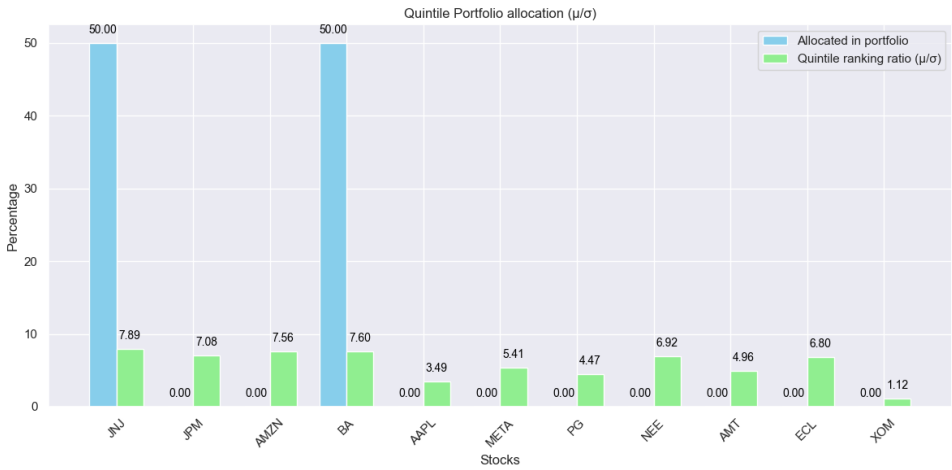


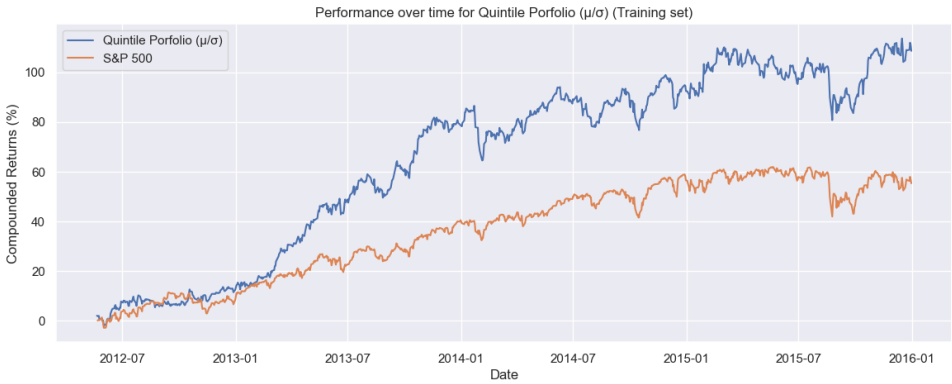
Regarding the performance of the said strategy, we can see that when compared with the S&P 500 it outperformed it by a considerable margin for the duration of the training set, which is only attributable to the stocks chosen being among the best performing for this period and as so a portfolio which is constituted only by some of the best portfolios of the S&P but with higher weights on these portfolios will naturally outperform the index by a decent margin, and we can even see that the graphs closely mirror each other except the EWP being on a higher return level, with the difference that the portfolio is more susceptible to a higher volatility which can be seen for example in the period after July of 2015 when the EWP suffers a higher decrease on returns in comparison to the S&P 500, this is simply attributable to a smaller number of assets that constitute the EWP in comparison to the S&P.

The same can be verified when applied to the test set since this strategy doesn't imply calculating weights using the training set for the testing set, since the weights need to be equal for all stocks and that fact doesn't change from set to set, so the conclusion taken from the training set also apply to the test set.

4.3 Quintile portfolio

The quintile portfolio consists of a portfolio where you invest in a group of assets that give you the highest returns. To get to the assets in which we invested, firstly we ordered the 11 stocks by returns, then divided the 11 stocks into 5 groups, and invested in the group with the highest returns. Through the process described, we got to the 2 assets where we would invest equally, JNJ and BA.





For the training set, we can see that this strategy produces great returns since we are 40% higher in compounded returns when compared to the S&P 500, which is no surprise since the 2 stocks chosen not only to constitute the S&P 500 but are among the ones with the higher returns for the period comprehended in the test set. The 2 graphs are identical only changing in the degree of returns and the volatility, since the quintile portfolio only has 2 assets will always be more volatile than an index which has 500 assets, this can be seen in the much smoother curve that the S&P 500 has compared to the QP.

For the test set, we can see that the results aren't the same as in the test set, with the 2 assets that we got from our process we can see that until 2020 we were averaging returns that were much higher than the S&P 500, at a point in 2019 the portfolio was almost 100% higher than the S&P 500, but around 2020, the covid period, the higher volatility that a portfolio with only 2 stocks is subject too made our quintile portfolio reach the same levels seen in the S&P and since then both of them have been comparable and average around the same returns with the QP edging out the S&P in 2023 in 20% of extra returns. With this set it is possible to see all the benefits and consequences of this strategy, with the extremely high returns but also the extreme volatility of investing in only 2 assets.

4.4 Markowitz's mean-variance portfolio (MVP) with no short-selling

The previous portfolios were all portfolios based on heuristics of some kind. The Markowitz Mean-Variance Portfolio (MVP) is a fresh approach, being purely based on quantitative factors using two quantitative measures as the foundation.

The two measures it uses are the mean return given by $w^T\mu$ and volatility $\sqrt{w^T\Sigma w}$. Since higher returns are associated with higher risks (volatility), the MVP tries to optimize for both, maximizing returns for a given level of risk.

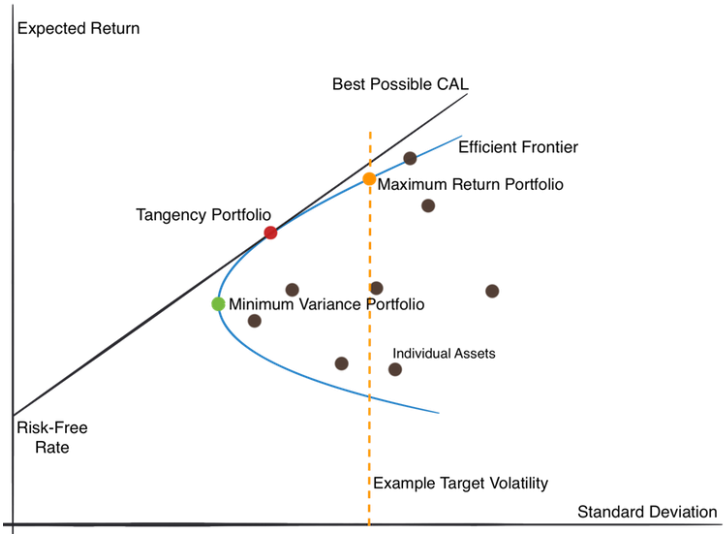
In short, it can be summarized by the problem:

$$\begin{array}{ll} \underset{\mathbf{w}}{\text{maximize}} & \mathbf{w}^T\mu - \lambda\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} & \mathbf{1}^T\mathbf{w} = 1 \end{array}$$

The variable λ is the risk aversion parameter, which can be tweaked according to the risk profile of the investor. If the investor is less risk-averse, preferring to maximize returns, then $0< \lambda<1$; otherwise, we will have that $\lambda>1$.

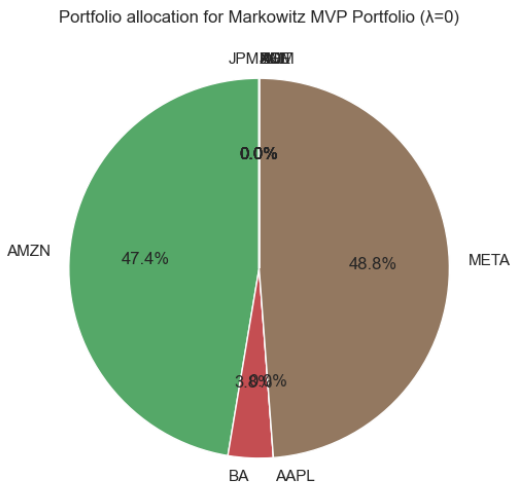
Another interpretation is that the λ parameter is a percentage of how much of the generated expected volatility we want to account for. If $\lambda = 0$, then we are accounting for no volatility; if $\lambda = 0.5$, then only 50% of it; if $\lambda > 1$, then we are amplifying generated expected volatility for an overall less volatile portfolio.

A simpler way to look at this is through the Efficient Frontier, which is a curve defining the highest-yielding portfolios for any given level of volatility. The MVP will be along the curve as the maximum return portfolio for target volatility, moving on the volatility axis according to the λ parameter.

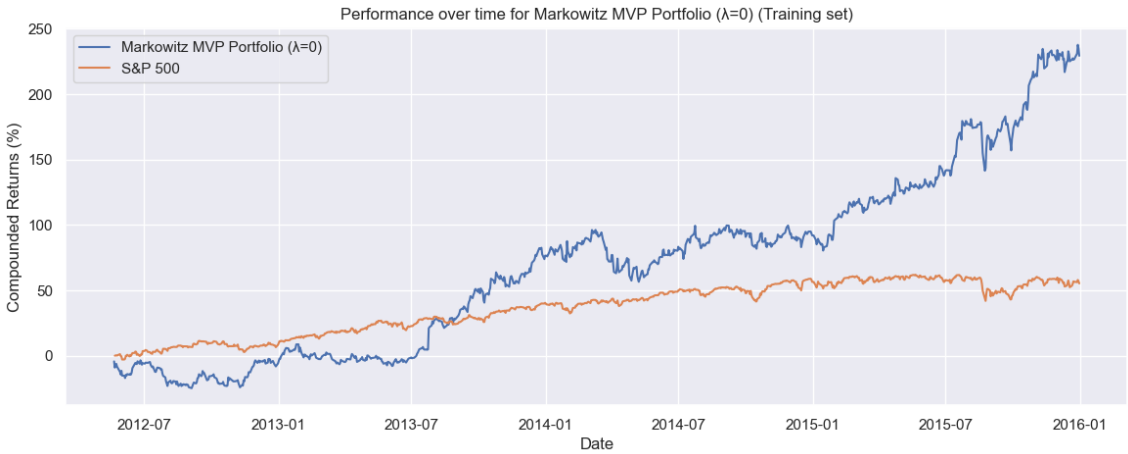


For our portfolio, we computed the various MVP for different risk-aversion parameters., from a non-risk-averse profile to a more cautious one.

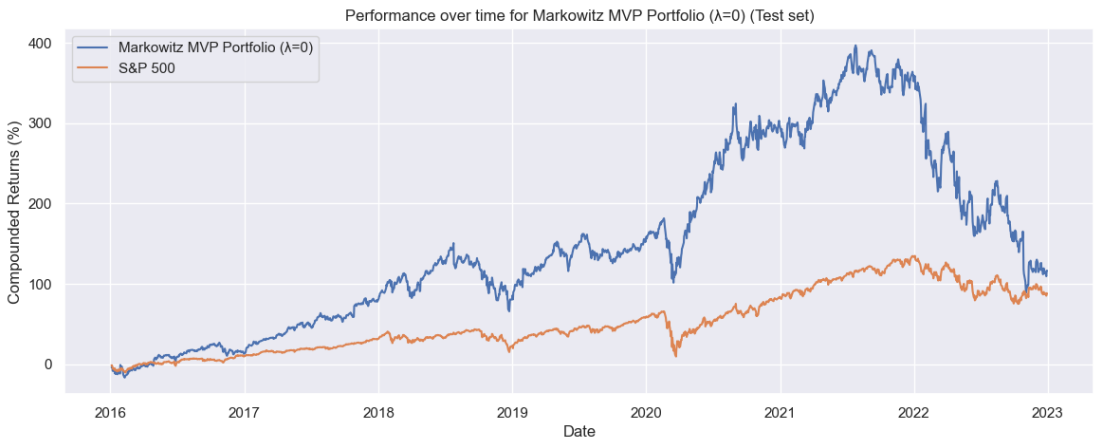
For a risk-seeking profile (λ=0), we obtained a very concentrated allocation on the 3 most yield assets:



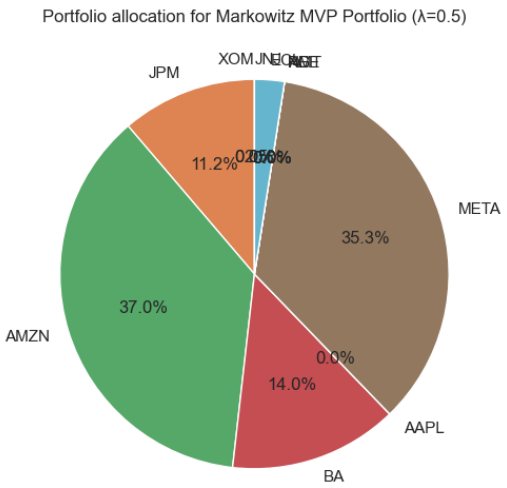
Observing the returns over time for the training portion of the data, we observe that for a little over a year, it returned less than the S&P 500, however, it beat the benchmark eventually:



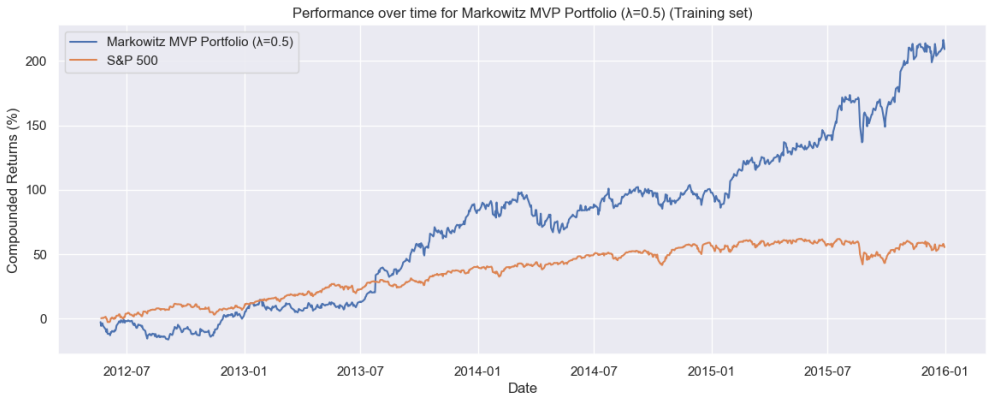
For the test portion of the data, it achieved a very worthy performance. However, it needs to be noted that this performance is atypical and mostly due to the portfolio composition which includes mostly tech stocks that have seen good performance during the COVID pandemic.

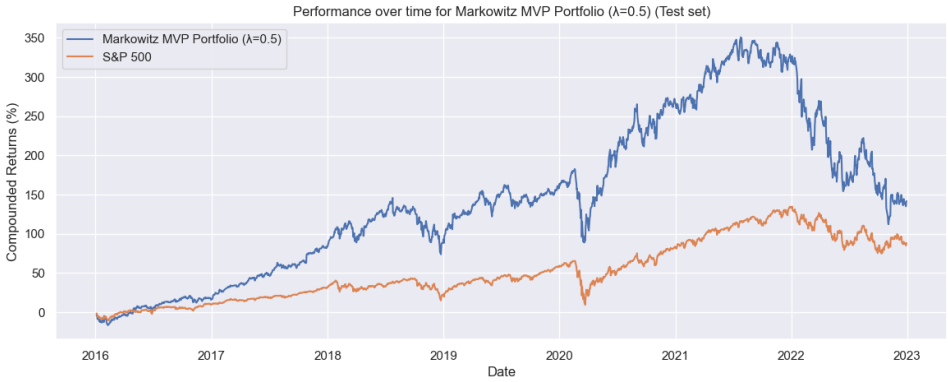


For a more risk-averse - but still preferring returns over risk - profile, the composition is more balanced but still heavily concentrated on the highest-yielding stocks with little regard for volatility (50% of expected volatility is ignored since λ=0.5):

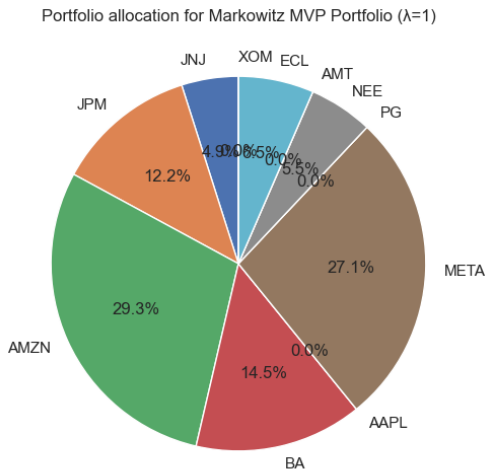


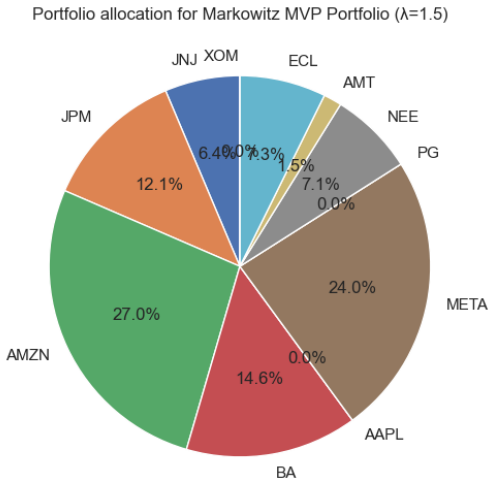
We now observe that the performance still beat the market, however, it did not yield as the previous one, especially when comparing the peaks of the test portion for both profiles.

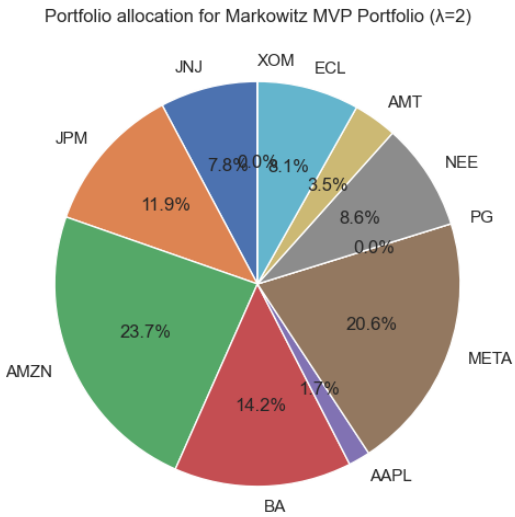


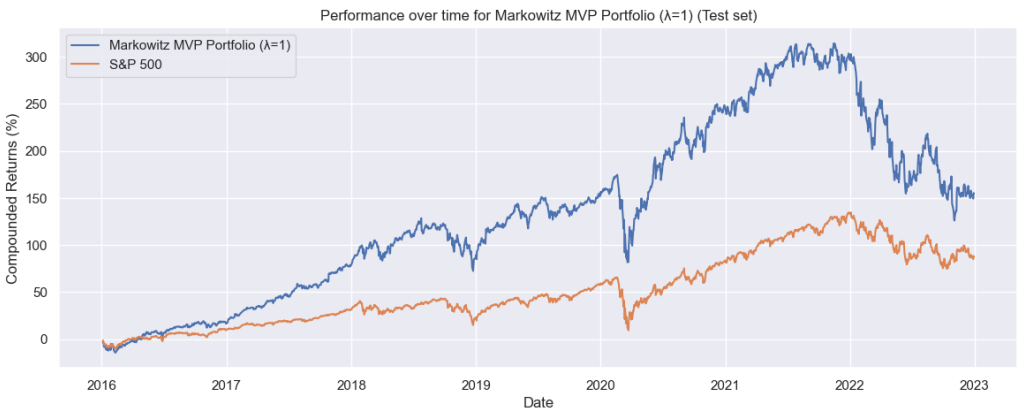


As expected, for higher risk-aversion parameter values, the portfolio allocations tend to be more diversified, albeit with fewer returns over time. In this case, since the allocations do not differ much, the performance will not either.









4.5 Global Minimum Variance Portfolio (GMVP) with no short selling

The Global Minimum Variance Portfolio (GMVP) is a simplification of the MVP, only accounting for volatility and attempting to minimize it, generating the most volatility-resilient portfolio regardless of expected returns.

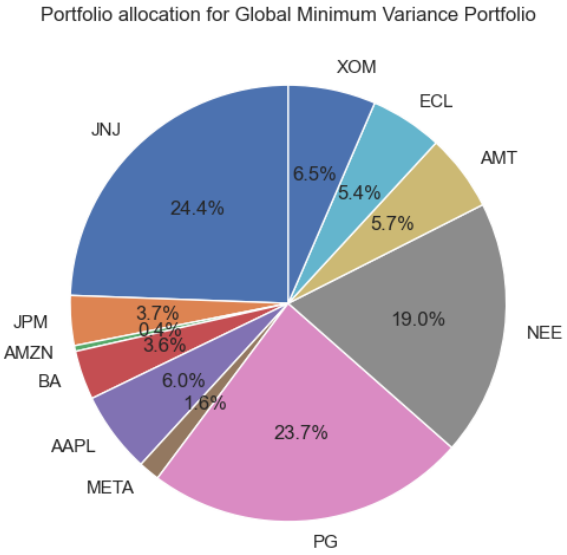
$$\begin{array}{ll} \underset{\mathbf{w}}{\text{minimize}} & \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} & \mathbf{1}^T\mathbf{w} = 1. \end{array}$$

Or the closed-form solution:

$$\mathbf{w}_{\text{GMVP}} = \frac{1}{\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}}\boldsymbol{\Sigma}^{-1}\mathbf{1}.$$

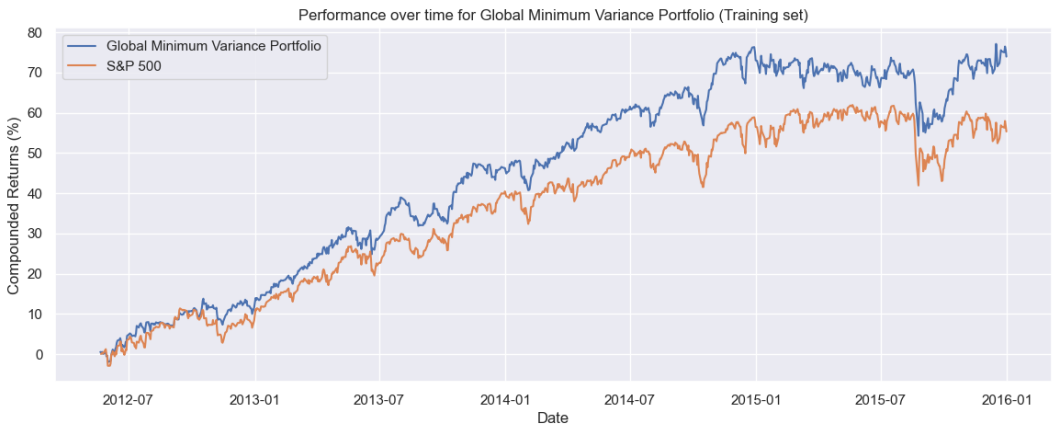
In the efficient frontier, the GMVP will be the most backward-facing portfolio on the curve, serving as an inflection point between worthwhile portfolios, and portfolios that should be discarded (same risk as other portfolios but fewer returns).

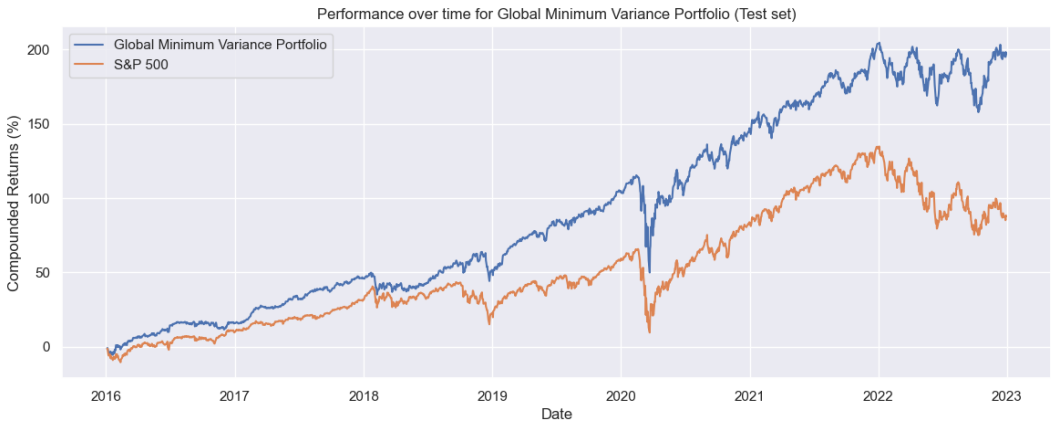
In our portfolio, the allocation reflected the objective of minimizing risk, allocating more to the assets with less expected volatility and little to the highest yield ones (regardless of their returns).



As expected, the performance over time did not yield as many returns as the previous ones. In this case, it barely beat the market for the training portion and half of the test portion.

However, the performance visibly beat the market during high volatility periods such as the COVID pandemic period, which can be seen by the change of "pace" starting in 2020:





4.6 Estimate the Maximum Sharpe ratio portfolio (MSRP)

The Maximum Sharpe Ratio is a concept from Modern Portfolio Theory, developed by William F. Sharpe. It is a metric that helps investors assess the performance of a portfolio relative to the risk taken. The goal is to find the optimal balance between return and risk, summarizing The MSRP is a measure of the relationship between the return of an investment portfolio and its volatility. It is calculated as the portfolio's return divided by volatility.

$$SR = \frac{R_p - R_f}{\sigma_p}$$

Where: SR represents the Sharpe Ratio.

Rp is the portfolio mean returns.

Rf is the risk-free rate.

σ_p is the standard deviation (or volatility) of the portfolio.

The MSRP occurs when the portfolio provides the highest risk-adjusted return (measured by standard deviation). In other words, it is the asset allocation that offers the greatest excess return over the risk-free rate per additional unit of risk.

The Maximum Sharpe Ratio is a valuable tool for investors looking to maximize the return on their investments while minimizing risk.

Below are some highlights points about the Maximum Sharpe Ratio:

- It is a measure of risk-return.
- It is calculated as the portfolio's return divided by volatility.
- A portfolio with a Maximum Sharpe Ratio offers the highest possible return per unit of risk.
- It is a useful tool for investors looking to maximize the return on their investments while minimizing risk.

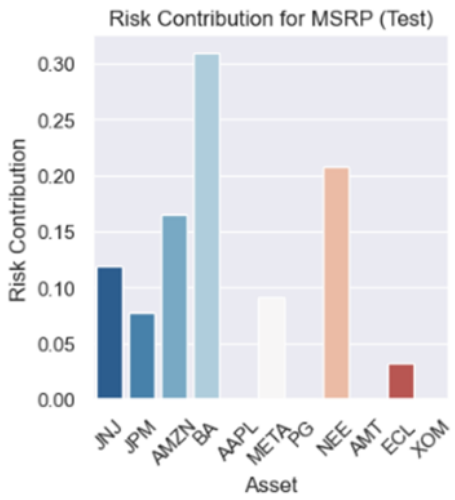
The Maximum Sharpe Ratio is a useful measure for comparing different investment portfolios. Here are some examples of how the Maximum Sharpe Ratio can be used:

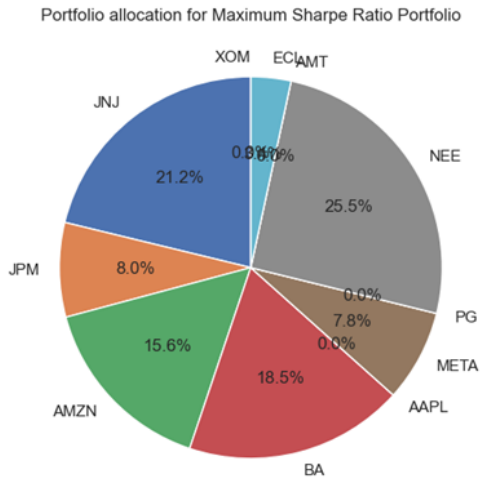
- ✓ An investor can use the MSRP to compare different mutual funds.
- ✓ A company can use the MSRP to compare different stock portfolios.
- ✓ A financial advisor can use the MSRP to recommend an investment portfolio to a client.

To find the Maximum Sharpe Ratio, investors can use portfolio optimization techniques, which involve combining different assets in specific proportions to maximize this metric.

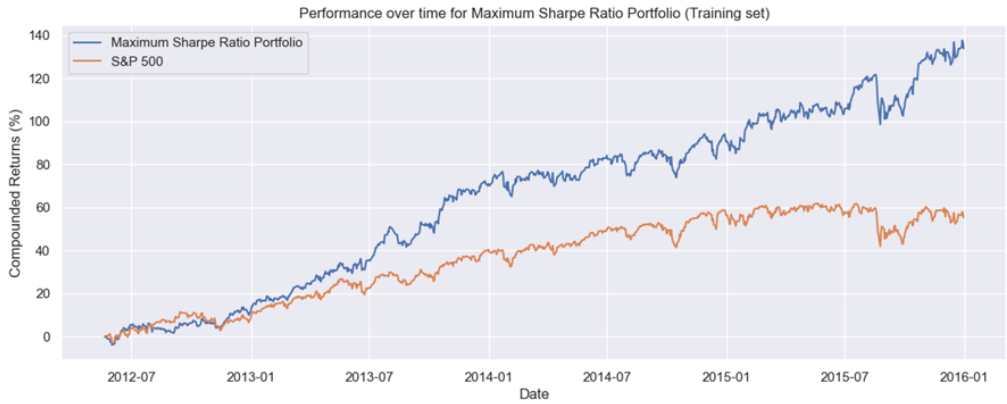
In the context of the study of the eleven selected assets in the research project, when using the Maximum Sharpe Ratio, we can observe certain variations over each asset.

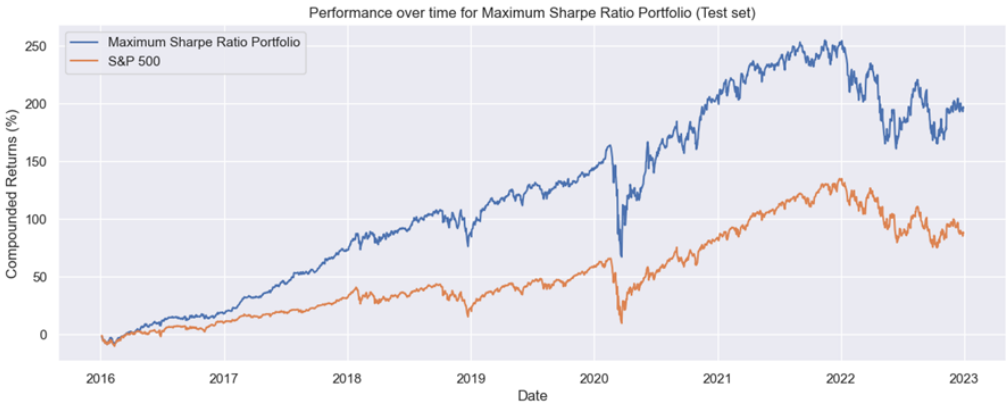
As we can observe below, there is evidence for the optimal allocation (%) of investments within the concept whose objective is to maximize returns while minimizing risk, the allocations of investment would be split up:





In this model of allocations, the major concentration of investments would be focused on four assets (NEE, JNJ, BA, AMZN) which are responsible for around 75% of the total portfolio investment.





4.7 Inverse Volatility Portfolio (IVP)

The Inverse Volatility Portfolio is an asset allocation strategy that focuses on assigning weights to assets based on their inverse volatilities rather than their expected returns. In other words, more volatile assets receive less weight, while less volatile ones receive more weight in the composition of the portfolio.

The main idea behind this approach is that by giving more weight to less volatile assets, the portfolio can be more stable and resilient to significant market fluctuations. This is especially valuable for investors seeking to minimize risk and volatility in their portfolios.

Less Volatility → More Weight

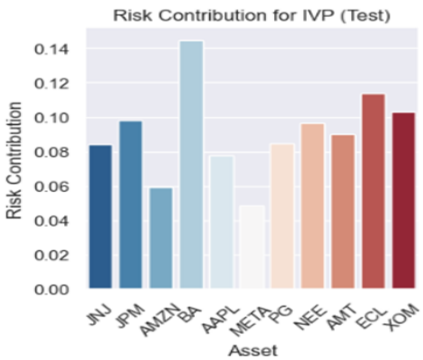
The basic formula for calculating the weights of an Inverse Volatility Portfolio is:

$$w_i = \frac{1}{\sigma_i}$$

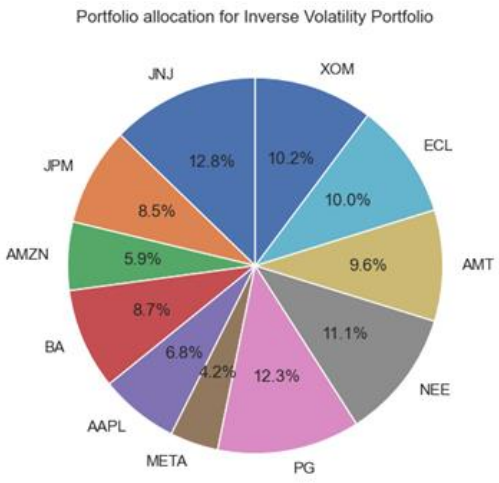
Where: w_i represents the weight of asset in the portfolio.

σ_i is the volatility of the asset.

This approach may be particularly attractive to risk-averse investors, such as pension funds and institutional investors who prefer stability and capital preservation.



Analyzing within the context of the assets selected for the project, similarly with MSRP the allocations would be focused on four assets (NEE, JNJ, BA, AMZN) being responsible for around 75% of the total portfolio investment as well.



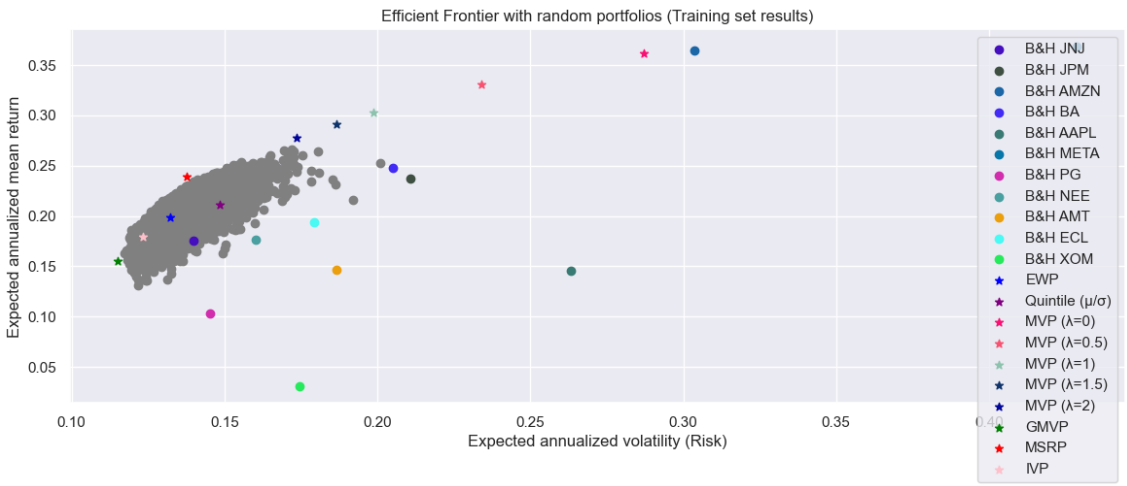
The returns over time using this strategy seem quite high compared with the benchmark (S&P 500), with more consistent returns between 2021 and 2022.

Another important highlight is the fact that despite the market in 2022 until the current day demonstrating some slight decrease, the model sustains returns above the S&P 500 over time in the period tested.

5. Critically discuss the results considering alternative risk-adjusted performance metrics (e.g., returns, volatility, Sharpe ratio, Sterling ratio, drawdown).

Training set results

This chapter will analyze all the strategies and discuss the results obtained through different metrics.



When observing the efficient frontier for the training set results, we can see that all the Buy and Hold Portfolios are not efficient and there are a large number of portfolios that outperform this strategy, which is expected considering the simplicity of the portfolio and the fact that the fewer stocks you have on a portfolio the more diversified it is and the more implied volatility it will have.

Going to the Equally Weighted Portfolio this is an inefficient strategy as seen in the graph, since we can see that we have a high number of random portfolios with either better returns with the same volatility or less volatility for the same level of returns.

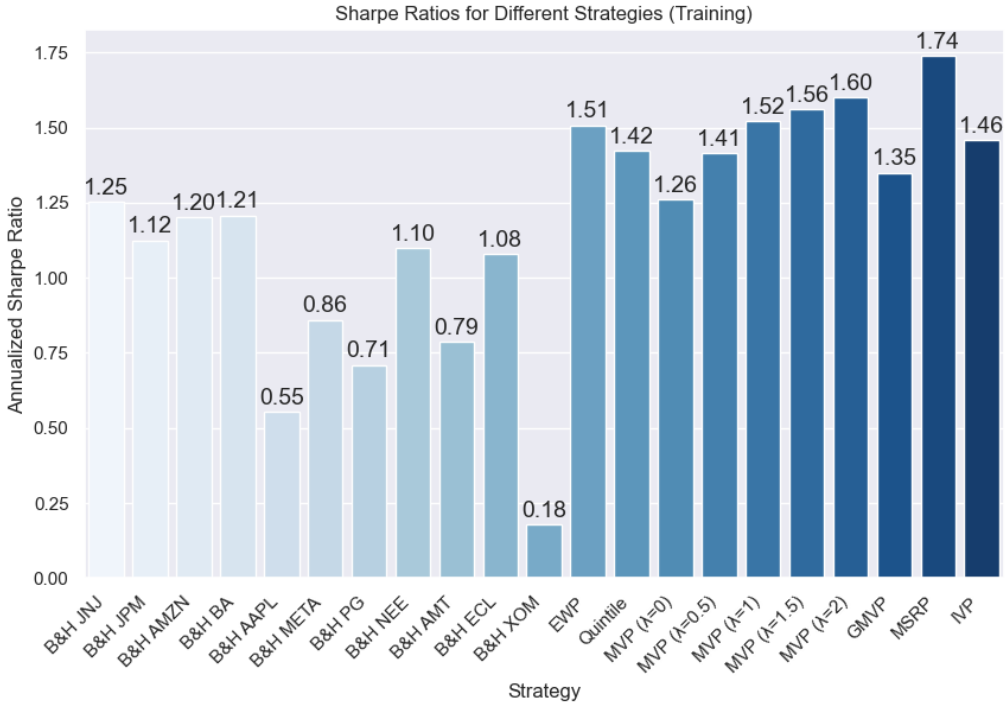
The Quintile Portfolio follows the trend set by the previous portfolios as it´s not a great performer overall for the training set it has the same problem as the B&H portfolios since investing in a reduced number of assets will lead to higher volatility for the portfolio and it won't lead to such a higher increase in returns that it would compensate for the increased volatility, in fact the expected returns for this strategy are comparable to the ones seen in the portfolio analyzed before this one, and the EWP has no strategy on how to distribute the weights, based on any mathematical concept.

As expected of all the Markowitz Mean Variance Portfolios, all of them are situated on the efficient frontier border and the only thing differentiating them is the value of lambda which dictates the level of risk aversion, making the portfolios move along the efficient frontier. Overall, for this training set all of them are efficient and there are no better options since they are all placed in the efficient line of this data set.

The Global Minimum Variance Portfolio presents the lowest possible volatility and returns while being in the efficient frontier, according to theory. Globally this is the lowest expected return portfolio strategy situated on the efficient frontier border, while also having the lowest expected volatility, thus being the inflection point of the efficient frontier and determining worthwhile efficient portfolios.

The Maximum Sharpe Ratio portfolio is all about constructing an investment portfolio that balances risk and returns effectively, aiming to maximize the Sharpe Ratio, which is a measure of the portfolio's risk-adjusted performance. In our graph we can precisely see this, the portfolio is situated on the efficient frontier, and it would be the point where the efficient frontier is tangent with the capital market line. This portfolio aims to maximize the returns for units of risk and as such, it balances the expected returns and the volatility it takes on.

Lastly, we have the Inverse Volatility Portfolio which tries to minimize the volatility of the assets that make the portfolio give higher weights to less volatile assets and do the opposite for high volatility assets. As seen in the graph this portfolio is not efficient and is not a great option to minimize risk in comparison to the GMVP that carries less risk, although also a lower return, but we can see that for its level of risk some portfolios carry more expected returns.

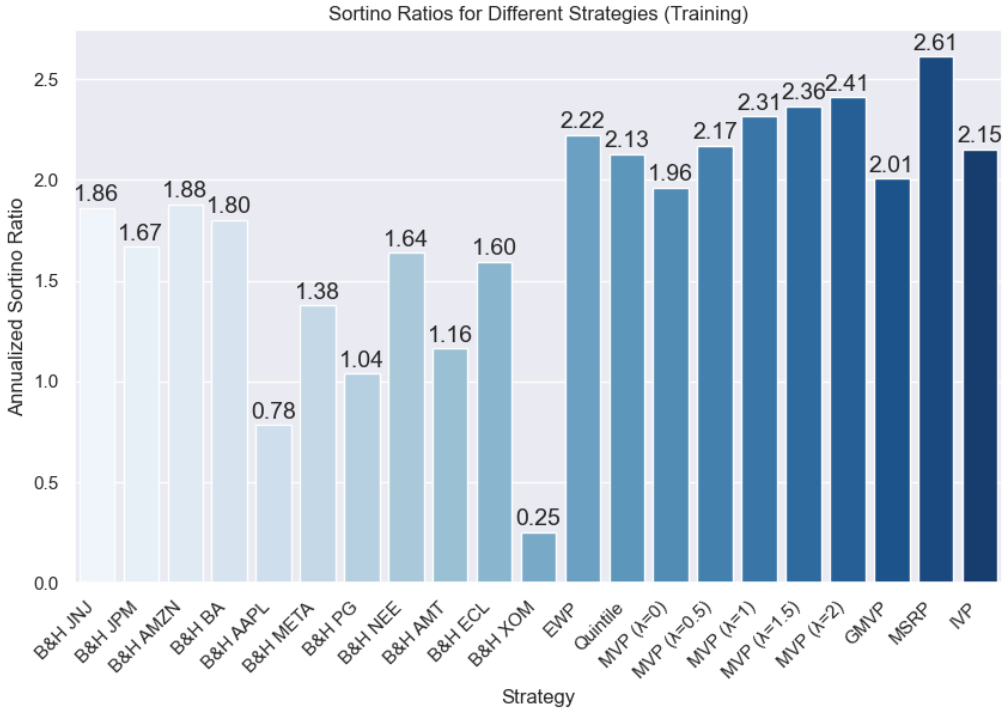


Analyzing Sharpe Ratios for the various strategies during the training phase, we can see that we have the highest Sharpe Ratio on the MSRP as this portfolio was designed to max this metric and get the most return for an additional volatility unit.

For the Buy & Hold portfolios (BnH) we can see that XOM is the stock that underperforms most in this metric, due to the high volatility seen and the lower returns on average when compared to the other stocks.

Analyzing the Mean-Variance Portfolios, we can see that when the risk aversion grows the Sharpe ratio grows as well, this is because as said before this metric isn´t only measuring returns, it takes also into consideration the volatility that the Portfolio carries. So, taking in less volatility will put us more in line with other strategies that don't look only at returns, and thus maximizing the two metrics will give us an overall better ratio, since the Sharpe ratio is the proportion of the risk premium against the portfolio volatility, the higher the portfolio volatility, the lower the Sharpe Ratio will be.

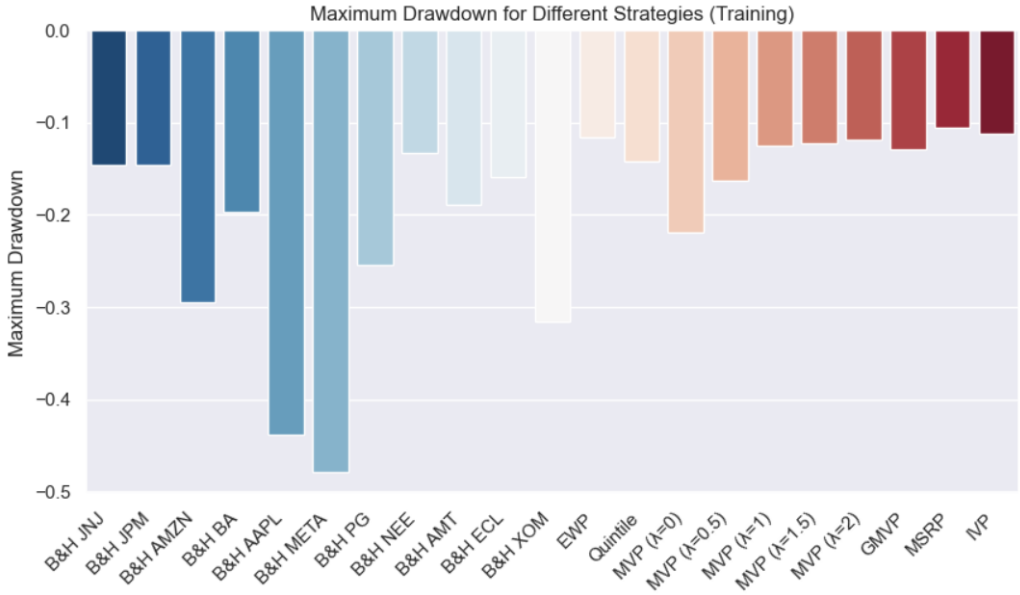
Also, an interesting case to note is the GMVP portfolio which despite having the lowest portfolio volatility, has a lower Sharpe Ratio than its risk-averse strategy counterparts because of its low returns.



On the Sortino ratio analysis, the main difference between this ratio from the Sharpe ratio is that the Sortino ratio takes into consideration the drawdown instead of the overall volatility, this means that instead of looking at the overall volatility, the Sortino ratio only looks at the negative volatility, or the risk of losing money.

Due to this, most of the facts seen in the previous ratio stay the same. The XOM Buy & Hold portfolio is still the worst overall portfolio, due to the facts stated in the previous analysis, and the MSRP is still the best performer.

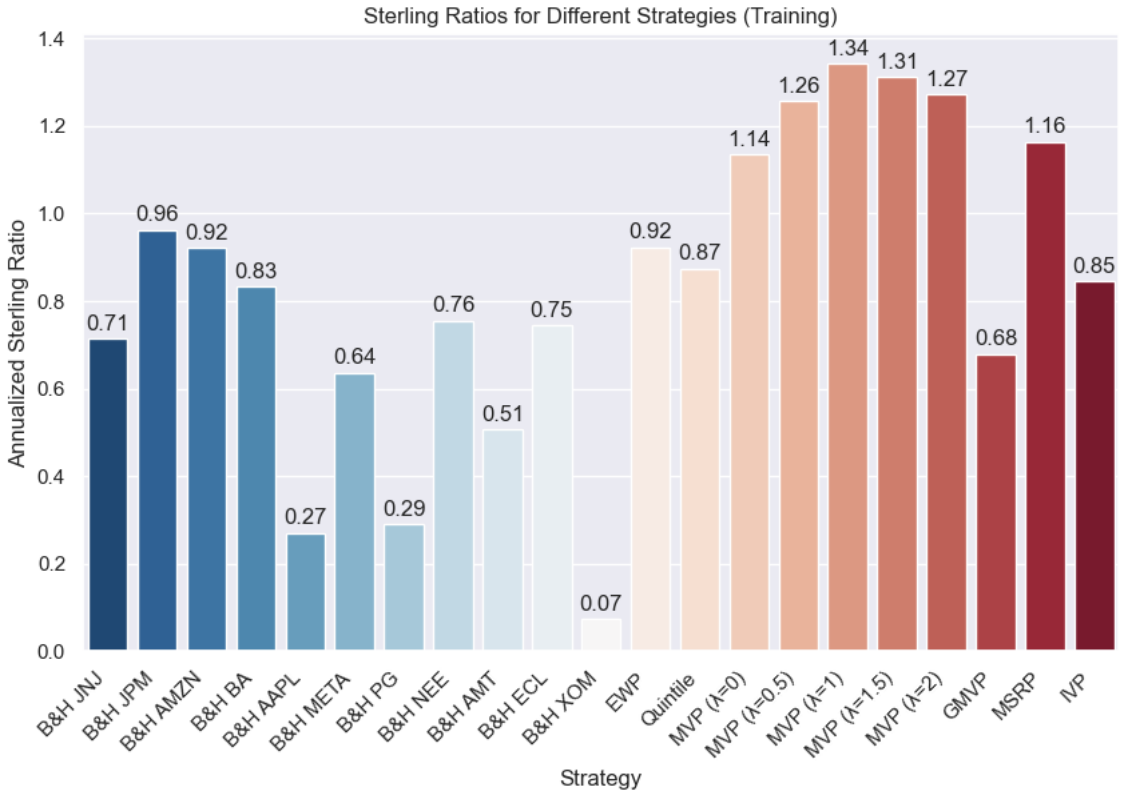
All the Mean-Variance Portfolios stay the same as the analysis done before and still stand in this ratio, nothing changed in the overall scheme of the theory.



The maximum drawdown is the difference between the maximum return that a portfolio will yield and its lowest before this portfolio has begun to recover, it represents the worst-case scenario for an investor. In other words: how much could a portfolio lose?

The portfolios that reveal the worst possible losses are META and AAPL B&H Portfolios, as they were the ones with the highest peaks, and as such, they will also have the highest possible losses.

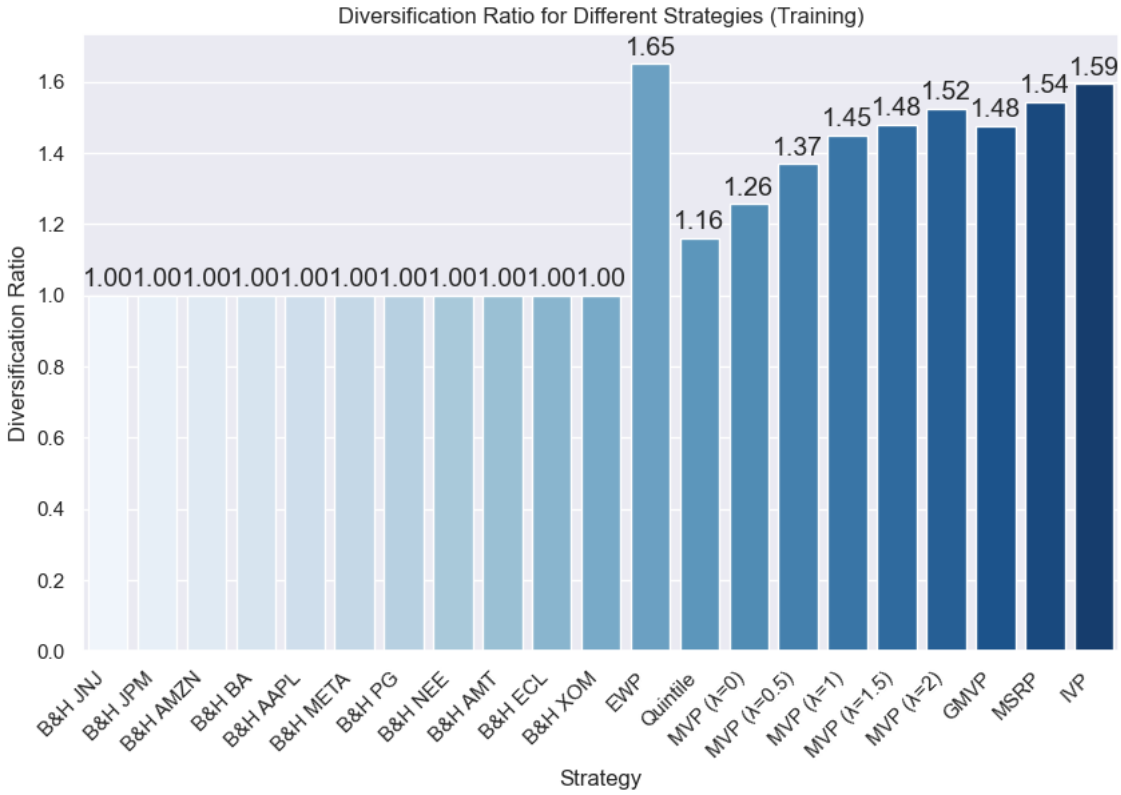
Regarding the diversified portfolios, we can see that all of them have lower drawdowns since they are more diversified, and as such they will have lower returns to balance their volatility, still, the MVP (λ=0) is the one with the highest possible drawdown for the fact that it maximizes returns only, not looking into any metric of volatility, and as such will have a higher drawdown.



Going with the Sterling Ratio, which unlike the Sortino ratio takes in the maximum drawdown as its measurement for downside risk instead of the downside deviation, implies that this ratio focuses more on capital preservation and risk mitigation.

Still in last place, we have the B&H XOM portfolio, this is due to its high maximum drawdown and low returns, so it also won´t be a good performer for this metric.

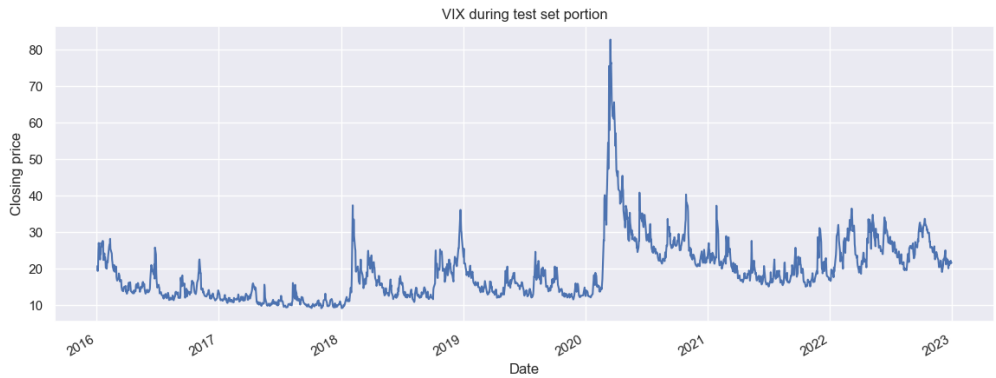
The Mean-Variance Portfolios all had great performance in this metric, having three portfolios as the best performers overall, being the MVP (λ=1) the best portfolio overall, this is probably due to this being the value of the risk-neutral Mean-Variance Portfolio, and thus just maximizing returns and volatility without giving preference on one over the other.



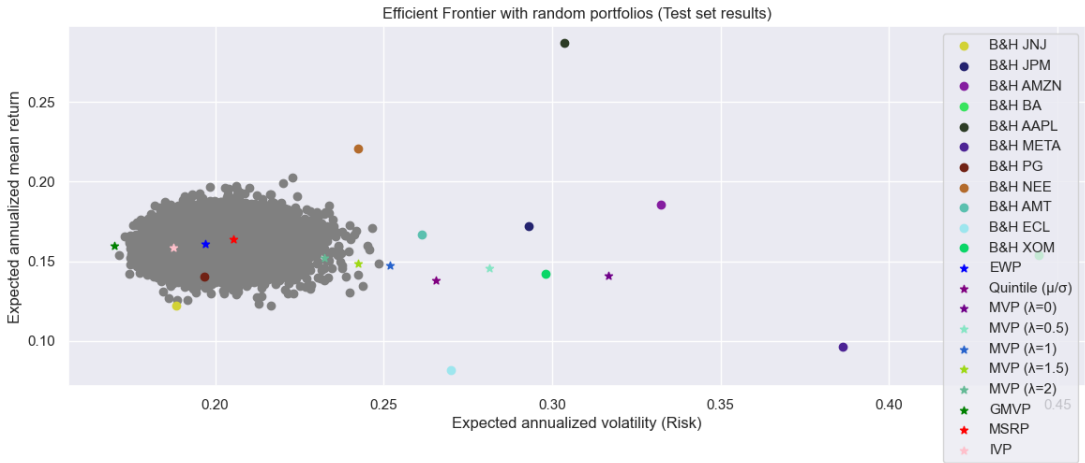
Regarding portfolio diversification, we can see that all the B&H portfolios share the lowest value, as all these portfolios only have 1 asset in them. The best performer overall is the EWP as it gives the same weight to the 11 portfolios and as such will be the most diversified portfolio that we can obtain with these 11 assets.

A fact that is also confirmed by this graph is that the most risk-averse portfolios are more diversified, as diversifying your assets will lead to a reduction in portfolio volatility.

Test set results



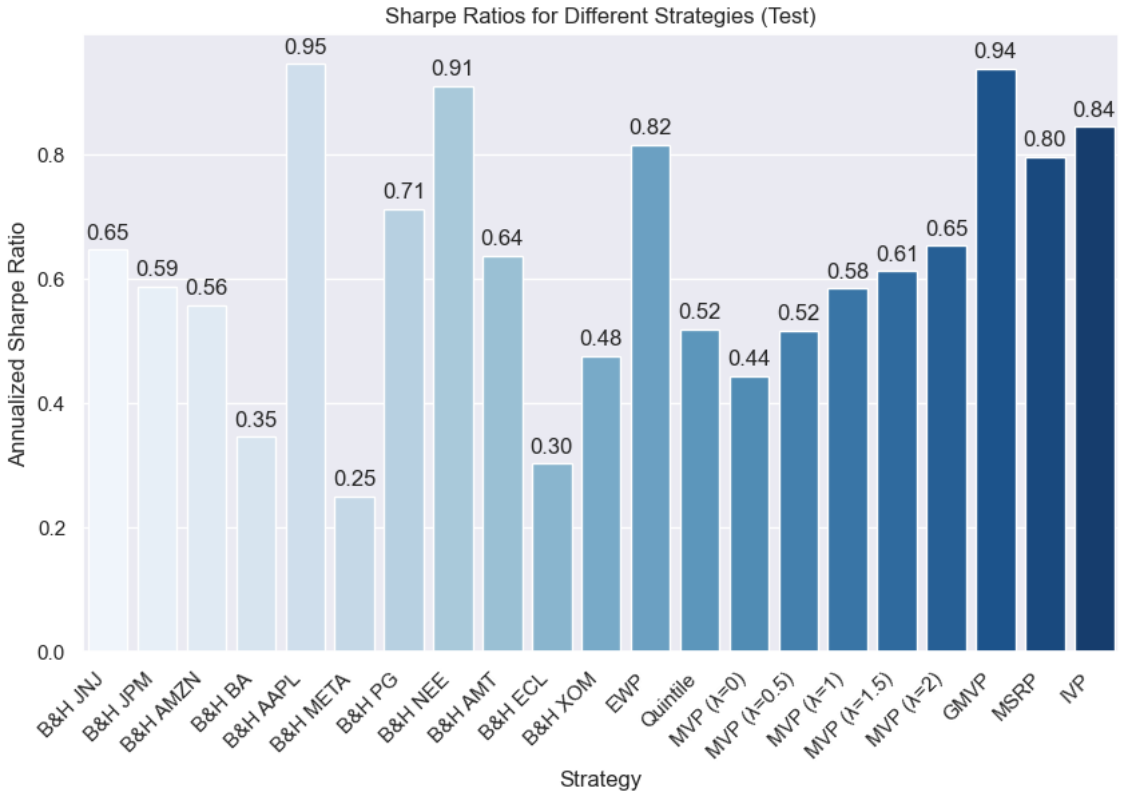
Before analyzing the results, it is necessary to create context. Observing the Volatility Index for the period contained in the test set, we can see in the graph this period is characterized by its high volatility, peaking at over 80 during the COVID pandemic, which signals a period of extremely high volatility. Values over 30 are considered tumultuous market periods.[1]



Now looking at the test set results, it is expected for low volatility portfolios to have better performances. We can see that the overall outlook changed a lot, suddenly the B&H for NEE and JPM are way better options as portfolios as they are the two portfolios that are closest to the efficient frontier, if we don't count the one that barely moved as expected which is the GMVP, since it's a very safe portfolio that minimizes volatility.

[1] Source: Fidelity Singapore - https://www.fidelity.com.sg/beginners/what-is-volatility/volatility-index

Our past top performers, in terms of efficiency, such as the MSRP, and the Mean-Variance Portfolios all became inefficient, and as we can see are quite a bit far away from what they were.

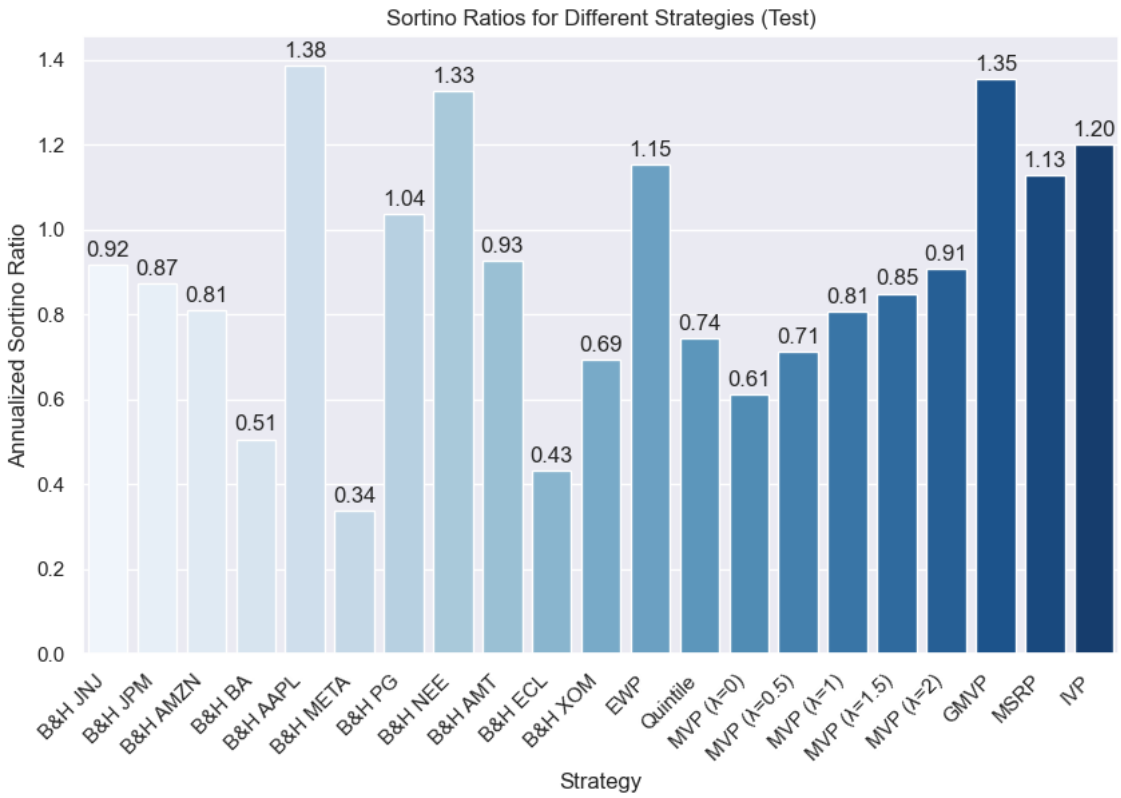


Analyzing the Sharpe Ratio for each strategy, we can see that the MSRP loses the first place that it held in the training set, and the new leader on this ratio is the GMVP, as it minimizes volatility and the period that the test set covers is characterized by its high volatility.

The top performers are not in the diversified portfolio group but reside in the B&H group, namely the AAPL and the NEE Portfolios, which due to the nature of their sectors (technology and energy, respectively) are more shielded.

Oppositely, Boeing (BA) which is in the industrial sector, mainly produces planes, which were heavily affected by the pandemic since people couldn´t travel due to lockdowns, thus orders went down.

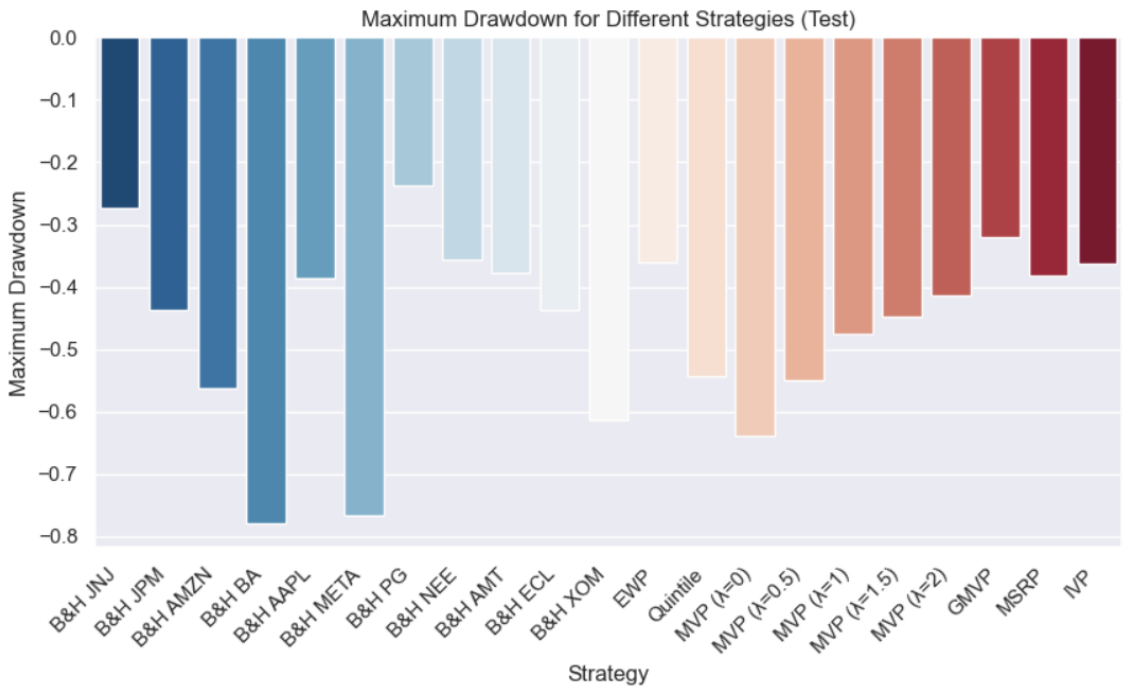
Overall, all the ratios are under 1, which means that risk premium does not justify investing in any portfolio due to the high volatility, thus making risk-free assets a better choice (e.g., Treasury bills).



In the Sortino ratio metric, we can see that the portfolios that had the best performance were the ones that minimized risk in some way, for example, the EWP minimized risk by having an equal share for every stock, and the GMVP minimized risk as its main strategy objective.

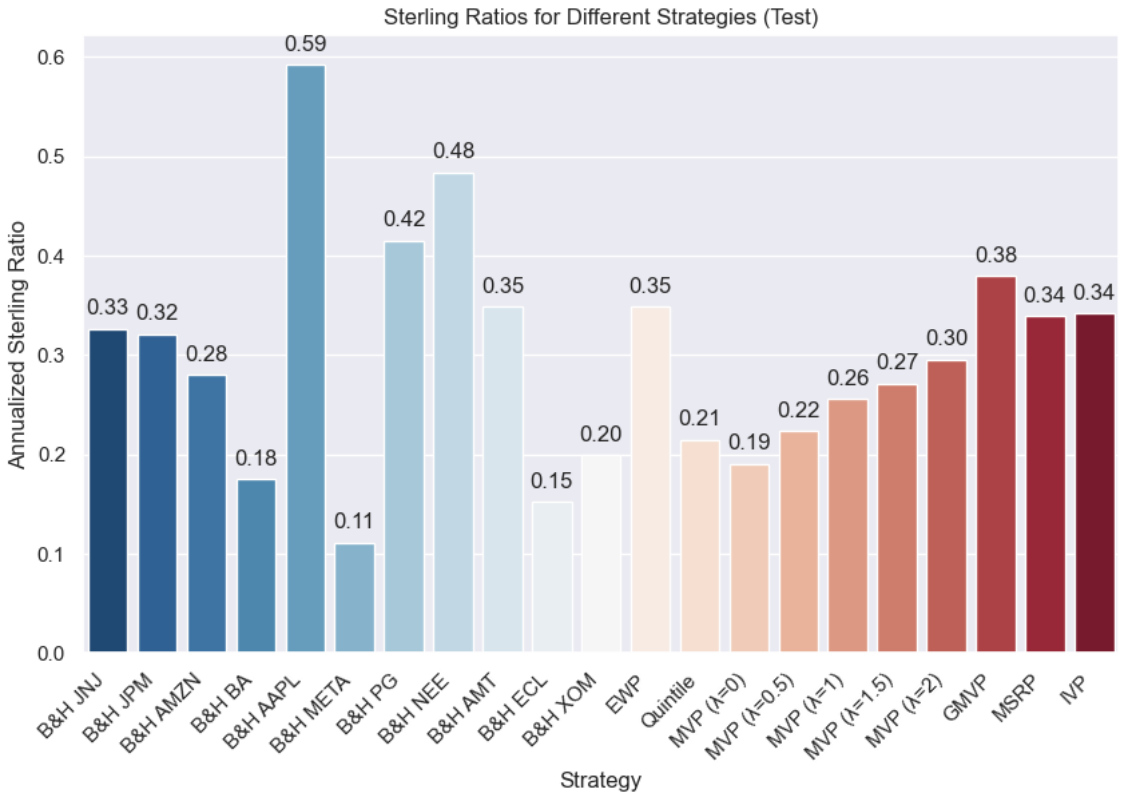
There were some exceptions to this, namely the previously referred AAPL and NEE B&H portfolios, but these will keep on performing at an elevated level due to what was explained previously.

Unlike the Sharpe ratio, we have values for the Sortino ratio above 1, which means that we have portfolios that are worth investing in.



Since the test period is characterized by its high volatility, almost all the portfolios increased their maximum drawdown.

The more diversified portfolios were better at handling this high volatility due to their diversifications, while less diversified like the B&H portfolios, increased their drawdowns. For example, Boing (BA) stock more than doubled its maximum drawdown, going over -0.7, which means that if you bought at the top and sold at the bottom you would lose over 70% of what was invested in the beginning.



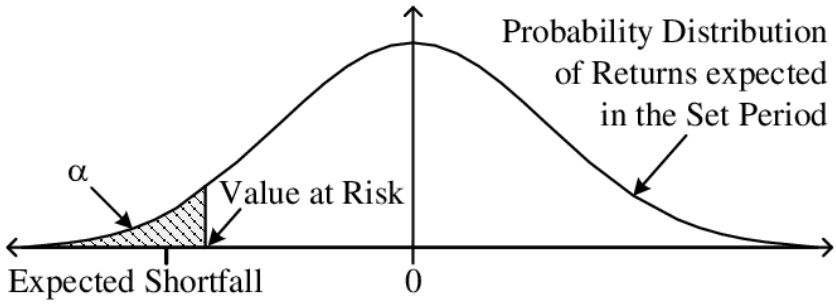
Looking at the Sterling ratio, we can see that all values are under 1, and therefore the risk of losing money is higher than the chance of making a profit.

In this metric, as seen before, all the values are lower than in the training set, and our best-performing portfolios are still the AAPL and NEE B&H. With the more diversified and risk-averse portfolios, we can see a trend that the more risk-averse and diversified a portfolio is the higher the value for the sterling ratio.

6. Extra points (VaR)

In this section, we have chosen to build a portfolio around an alternative risk metric: Value at Risk (VaR).

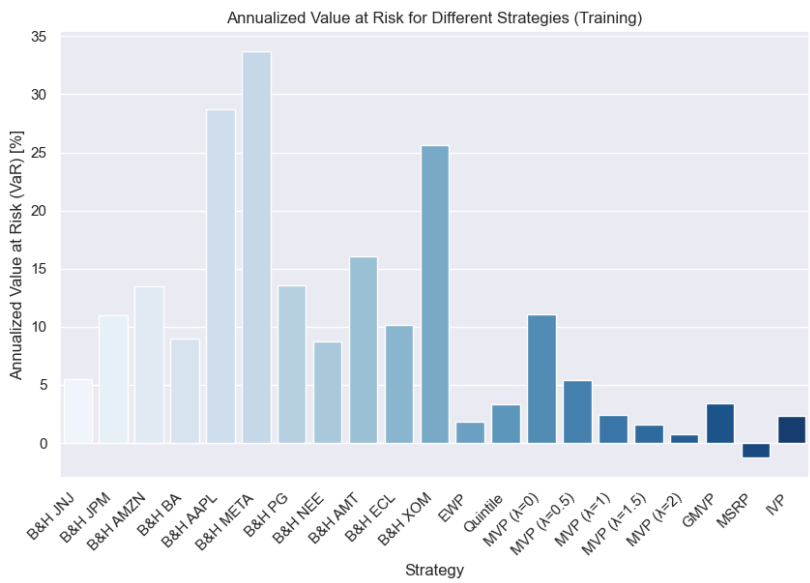
The VaR metric is a statistic that quantifies how much are the possible losses of a portfolio/asset in a given period. Using the mean returns, the standard deviation of the returns for that period, and a confidence level, then evaluating it on a Gaussian curve.



A way to interpret the VaR metric is that if the 95% confidence level VaR is €1, then it means that within that period there is a 5% chance of losing €1.

Dealing with percentage returns simplifies this even further: imagining an initial investment of €1, the losses will be between 0 and 1 (not only because the returns are percentages) so if we have a VaR of 35 cents, but we can also simplify it to 35% of the value.

Taking that approach, we can study the annualized VaR for the different strategies during the training set period:



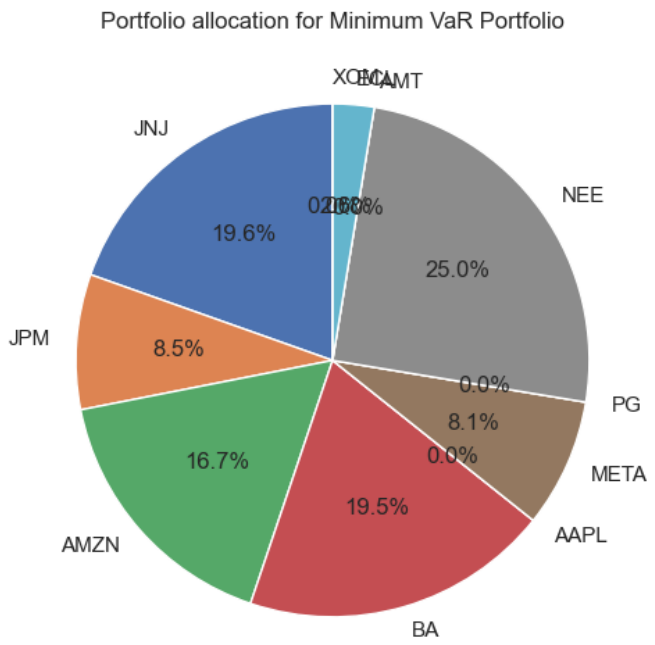
It is immediately notable that more risk-averse/diversified portfolios present a lower VaR, while more risk-seeking portfolios present higher VaR values. Curiously, the MSRP has a negative VaR value. This means that within the training set period, it was more likely to profit than lose money.

With this idea present, we can formulate a new portfolio strategy: minimize VaR. Simply, the portfolio strategy can be summarized by the problem:

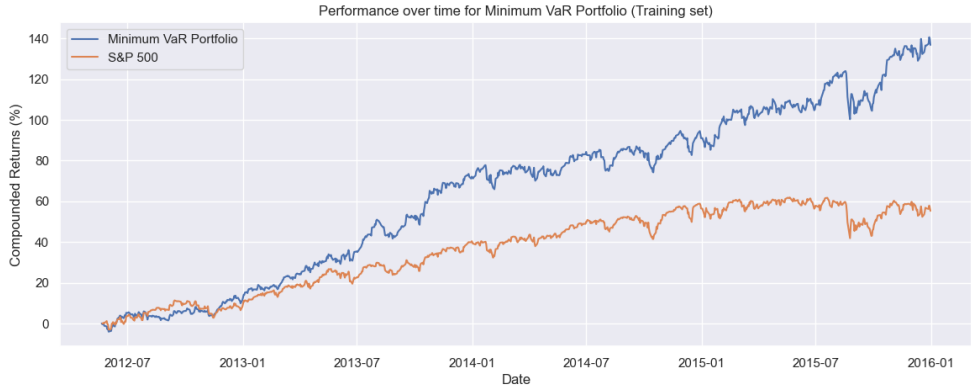
$$\min_w \quad VaR(w|\alpha = 0.05)$$
$$s.t. \quad w \geq 1 \quad and \quad 1^T w = 1$$

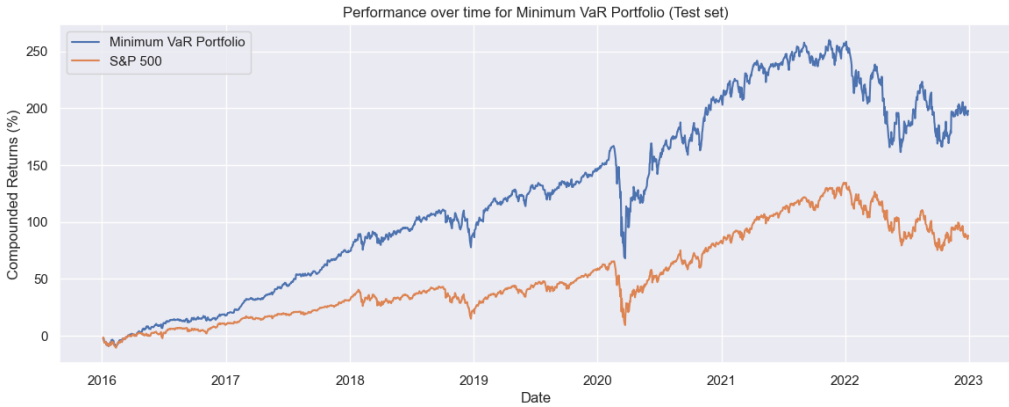
Doing so will result, when calibrating using the training dataset, in a portfolio with a negative VaR and lowest possible of -1.2% over the training dataset period, shooting up to 17.8% during the test dataset period.

The weight allocation is similar to the MSRP portfolio (also calibrated using the training dataset and tested with the test dataset):



During the training and test set period, it showed a decent performance, clearly beating the S&P 500:





Overall, its performance metrics were similar to the MSRP portfolio. This portfolio achieved the 2nd highest Sharpe Ratio of 1.737 during training, while in testing it achieved 0.789.

7. Conclusion

In conclusion, the comprehensive analysis of the diversified portfolio consisting of 11 S&P500 listed stocks across various sectors provides valuable insights into financial market behavior and investment strategies. Over the period from January 1, 2010, to December 31, 2022, we conducted a series of tasks aimed at understanding the characteristics of returns and evaluating investment strategies.

First, we collected daily, weekly, and monthly log returns for each stock, revealing that financial market returns exhibit distinct stylized facts at different frequencies. We observed the absence of autocorrelation, non-normality with fat tails, negative skewness, volatility clustering, leverage effects, and conditional non-normality, which highlighted the complex and non-linear nature of financial returns.

Subsequently, we split the dataset into a training set and a test set, allowing us to evaluate various investment strategies in the out-of-sample period. The strategies ranged from the straightforward Buy and hold and Equally Weighted Portfolio to more sophisticated approaches, such as Markowitz's Mean-Variance Portfolio (MVP), Global Minimum Variance Portfolio (GMVP), Maximum Sharpe Ratio Portfolio (MSRP), and Inverse Volatility Portfolio (IVP). We calibrated these models using the training data and assessed their performance during the test period.

To critically assess these strategies, we employed a range of risk-adjusted performance metrics, including returns, volatility, Sharpe ratio, Sterling ratio, and drawdown. This holistic evaluation provided a comprehensive perspective on each strategy's strengths and weaknesses.

In sum, our analysis demonstrates the importance of understanding the stylized facts of financial returns and the need to tailor investment strategies to the unique characteristics of the market. It is evident that there is no one-size-fits-all approach to portfolio management, and the choice of strategy should be guided by the investor's risk tolerance, objectives, and a deep understanding of market dynamics. As financial markets continue to evolve, ongoing research and adaptability in investment strategies will be crucial to achieving long-term success in an ever-changing landscape.