qLearn Week 2 Mathematical Intro to Quantum Algorithms

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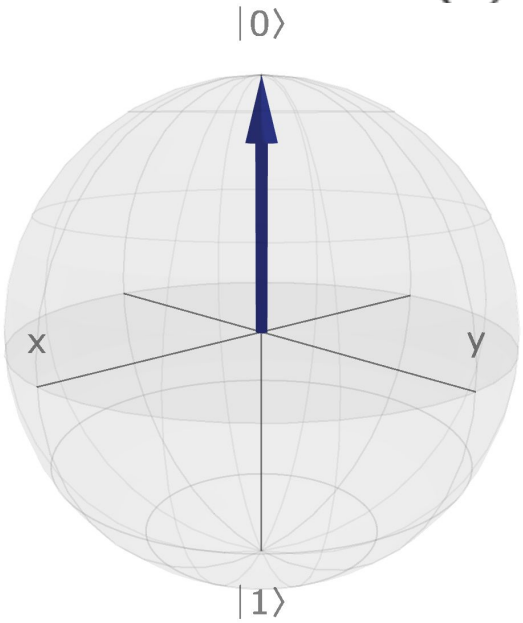
Qubits and their Mathematical Methods

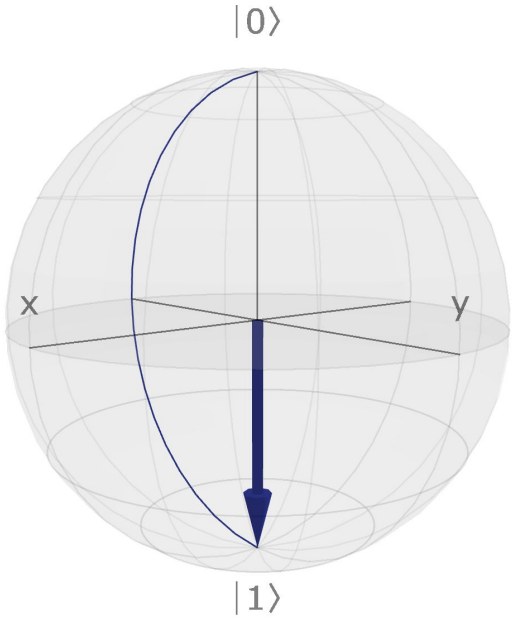
In regular computers…

In Quantum Computers…

$$\text{qubit state } 0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

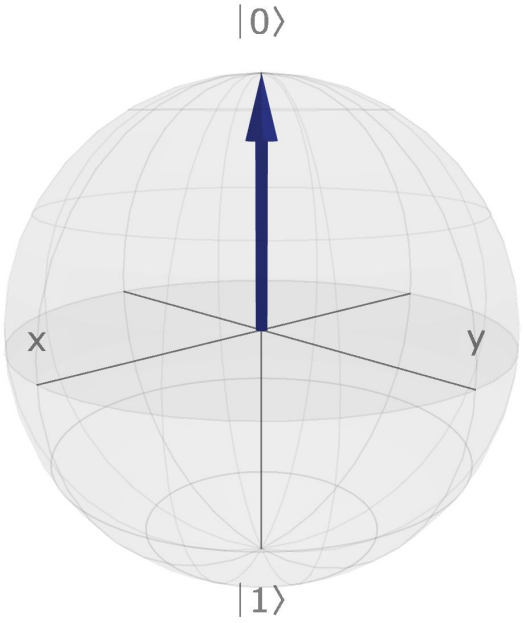


$$\text{qubit state } 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

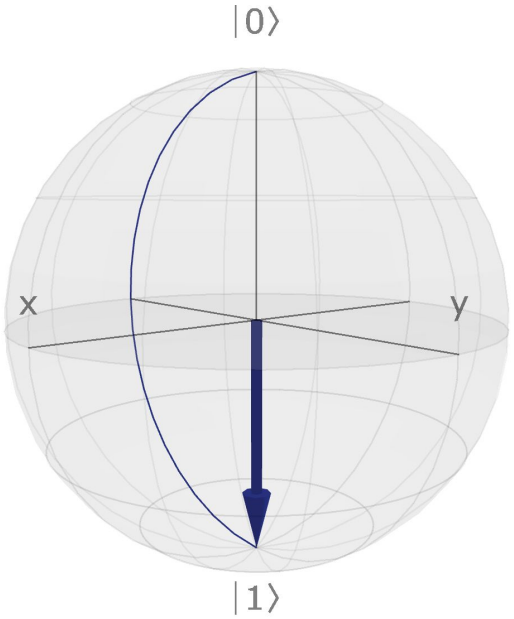


Using Simpler Terms… (Dirac Notation)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Bra-Ket Notation Details

BRA

KET

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Conjugate Transpose

$$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Two Key Identities

The Computational Basis

$$\langle 0|1\rangle = \begin{pmatrix}1 & 0\end{pmatrix} \cdot \begin{pmatrix}0\\1\end{pmatrix}$$
$$= 1 \cdot 0 + 0 \cdot 1$$
$$= 0$$

Preservation of Length

$$\langle 1|1\rangle = \sqrt{\begin{pmatrix}0 & 1\end{pmatrix} \cdot \begin{pmatrix}0\\1\end{pmatrix}}$$
$$= \sqrt{0 \cdot 0 + 1 \cdot 1} = 1$$

Quantum Superposition

Superposition Notation in QC

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\alpha\alpha^* + \beta\beta^* = 1$$

COMPLEX NUMBERS

The Idea of Superposition

Where the Advantage Comes In

A Note on Qubit Measurement

IMPORTANT: MEASURING A QUBIT COLLAPSES ITS SUPERPOSITION!!!!!!!!!

$$\text{Prob}(\text{measure and observe } |0\rangle) = |\alpha|^2 = \alpha\alpha^*$$

$$\text{Prob}(\text{measure and observe } |1\rangle) = |\beta|^2 = \beta\beta^*$$

I LOVE HOMEWORK! Example Time

$$|\psi\rangle = \frac{1}{2}|0\rangle - \frac{\sqrt{3}i}{2}|1\rangle$$

$$|\frac{1}{2}^2| + |\frac{\sqrt{3}i}{2}^2| = 1$$

State is Normalized

$$-\frac{\sqrt{3}i}{2} \cdot \frac{\sqrt{3}i}{2} = \frac{3}{4}$$

Probability of Observing 1 is $\frac{3}{4}$

All this talk... Make a Computer Already!!

How do we Transform Vectors?

MATRICES!!! Used in Quantum Circuits, come next week to find out!

See you Next Week!