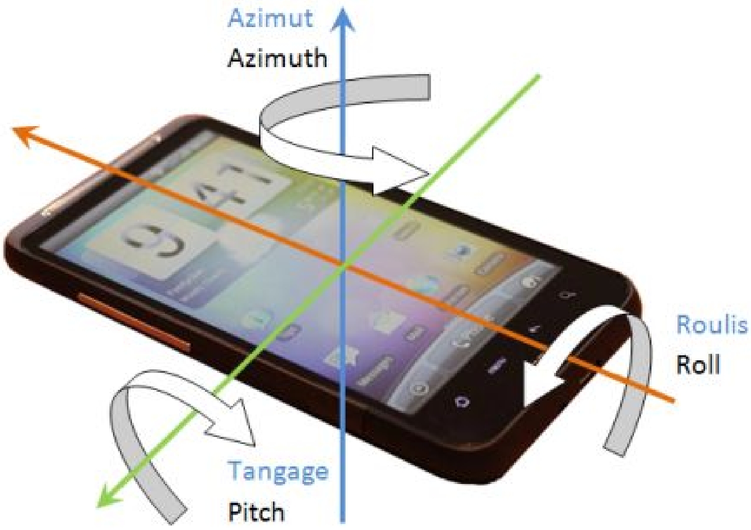


Figure 1.1: Possibilities of rotating a Smartphone[1]

1 Mathematical Basics of Gesture Recognition

The recognition of gestures is based on measured acceleration values. These values depend on the orientation of the smartphone. The mathematical relation of the measured acceleration values and the smartphone's orientation will be derived in this chapter. At first, the impact the smartphone being rotated along one of its axes is investigated in isolation. Afterwards, the results are combined and the final equation for each of the acceleration values is set up.

1.1 Terminology of the possible Rotations

There exist three different possible rotations that are measured by the magnetic field sensor:

- **Pitch**

  The angle of a rotation around the x-axis is called pitch. In the fol-

[1] Taken from: http://objnux.1s.fr/index.php?post/2012/04/29/App-Inventor-%3A-Premier-programme

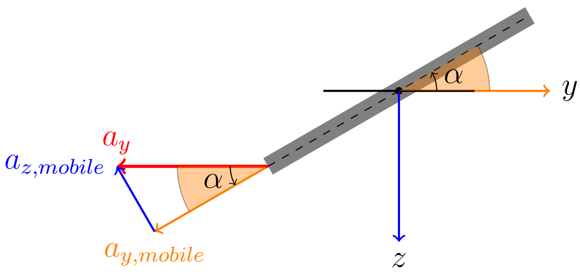
lowing equations, α will be used to describe the value of pitch that is retrieved from the magnetic field sensor.

- **Roll**
  The angle of a rotation around the y-axis is called roll. In the following equations, β will be used to describe the value of roll that is retrieved from the magnetic field sensor.

- **Azimuth**
  The angle of a rotation around the z-axis is called azimuth. In the following equations, γ will be used to describe the value of azimuth that is retrieved from the magnetic field sensor.

1.2 Acceleration depending on Pitch

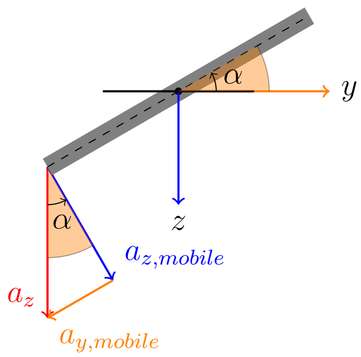
The current section investigates the effect of a rotation around the smartphone's x-axis on the measured acceleration values. Figures 1.2 and 1.3 show how accelerations in the direction of x and of z respectively might be decomposed into the different acceleration vectors that are parallel to the smartphone's axes. The magnitudes of these vectors are measured by the smartphone's acceleration sensor.



$$a_{z,mobile} = \tan(\alpha) \cdot a_{y,mobile} \tag{1}$$

$$a_y = \frac{a_{y,mobile}}{\cos(\alpha)} \tag{2}$$

Figure 1.2: a_y depending on Pitch



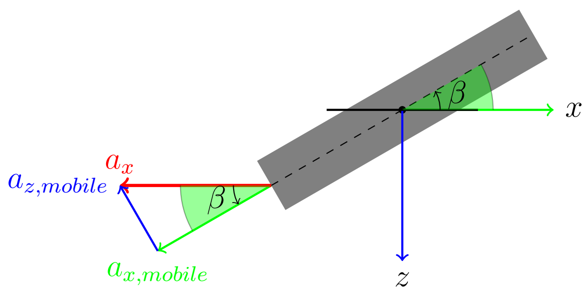
$$a_{y,mobile} = \tan(\alpha) \cdot a_{z,mobile} \quad (3)$$

$$a_z = \frac{a_{z,mobile}}{\cos(\alpha)} \quad (4)$$

Figure 1.3: a_z depending on Pitch

1.3 Acceleration depending on Roll

A rotation around a certain axis affects the measured acceleration values of the axes that are parallel to axis of rotation. Therefore, if the smartphone is rotated around its y-axis, the accelerations in the direction of x and z have to be investigated. The composition of the acceleration vectors can be retrieved from the figures 1.4 and 1.5.



$$a_{z,mobile} = \tan(\beta) \cdot a_{x,mobile} \quad (5)$$

$$a_x = \frac{a_{x,mobile}}{\cos(\beta)} \quad (6)$$

Figure 1.4: a_x depending on Roll

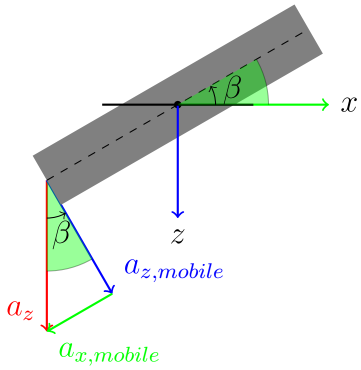


Figure 1.5: a_z depending on Roll

$$a_{x,mobile} = \tan(\beta) \cdot a_{z,mobile} \quad (7)$$

$$a_z = \frac{a_{z,mobile}}{\cos(\beta)} \quad (8)$$

1.4 Acceleration depending on Azimuth

The last rotation that is examined is the one around the z-axis. This rotation affects the measured acceleration values in the direction of x and y. The vector-decomposition is depicted in the figures 1.6 and 1.7.

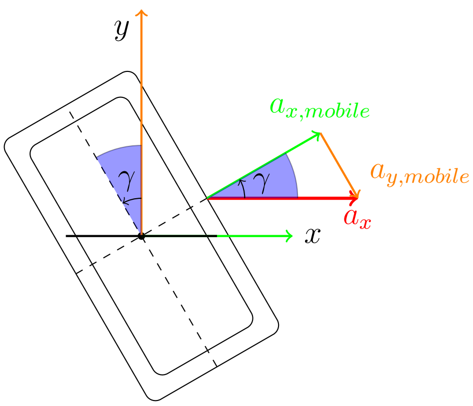


Figure 1.6: a_x depending on Azimuth

$$a_{y,mobile} = \tan(\gamma) \cdot a_{x,mobile} \quad (9)$$

$$a_x = \frac{a_{x,mobile}}{\cos(\gamma)} \quad (10)$$

1.5 Final Equations for the Acceleration Values

After the acceleration values have been examined depending on each rotation value in isolation, the results are combined together in this section.

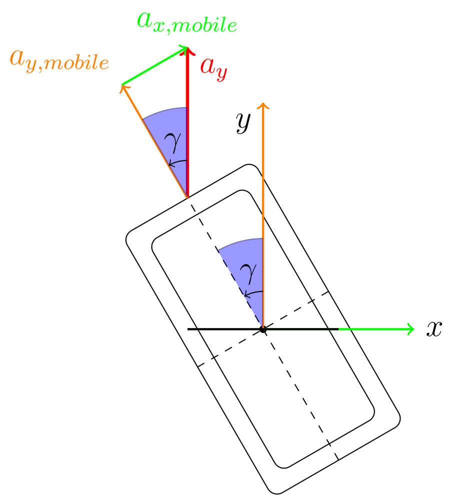


Figure 1.7: a_y depending on Azimuth

$$a_{x,mobile} = \tan(\gamma) \cdot a_{y,mobile} \tag{11}$$

$$a_y = \frac{a_{y,mobile}}{\cos(\gamma)} \tag{12}$$

1.5.1 Final Equation: X-Acceleration

As it has been shown in the equations (1.6) and (1.10), the acceleration in the direction of x depends on the values of roll and azimuth. Combining these two equations leads to the following equation:

$$a_x = \frac{a_{x,mobile}}{\cos(\beta) \cdot \cos(\gamma)} \tag{13}$$

The measured value of $a_{x,mobile}$ is affected in two cases:

- if the mobile is accelerated in the direction of z and the value of pitch is not equal to 0
- if the mobile is accelerated in the direction of y and the value of azimuth is not equal to 0

These affectations are defined by the equations (1.7) and (1.11). Taking these equations into account leads to the following, final equation:

$$a_x = \frac{a_{x,mobile} \cdot (1 - \tan(\beta) \cdot a_{z,mobile} - \tan(\gamma) \cdot a_{y,mobile})}{\cos(\beta) \cdot \cos(\gamma)} \tag{14}$$

1.5.2 Final Equation: Y-Acceleration

The equations (1.2) and (1.12) show the dependency of an acceleration in the direction of y depending on the values of pitch and azimuth. If these equations are put together, the result is given by:

$$a_y = \frac{a_{y,mobile}}{\cos(\alpha) \cdot \cos(\gamma)} \tag{15}$$

The measured value of $a_{y,mobile}$ is affected in two cases:

- if the mobile is accelerated in the direction of z and the value of pitch is not equal to 0
- if the mobile is accelerated in the direction of x and the value of azimuth is not equal to 0

Therefore, the equations (1.3) and (1.9) have to be included in the calculations. This leads to the following, final equation:

$$a_y = \frac{a_{y,mobile} \cdot (1 - \tan(\alpha) \cdot a_{z,mobile} - \tan(\gamma) \cdot a_{x,mobile})}{\cos(\alpha) \cdot \cos(\gamma)} \tag{16}$$

1.5.3 Final Equation: Z-Acceleration

The basic equation for the acceleration in the direction of z is retrieved by assembling the equations (1.4) and (1.8) that define the accelerations dependency on the values of roll and pitch respectively.

$$a_z = \frac{a_{z,mobile}}{\cos(\alpha) \cdot \cos(\beta)} \tag{17}$$

The measured value of $a_{z,mobile}$ is affected in two cases:

- if the mobile is accelerated in the direction of y and the value of pitch is not equal to 0
- if the mobile is accelerated in the direction of x and the value of roll is not equal to 0

If these cases (see equations (1.1) and (1.5)) are considered the final is given by:

$$a_z = \frac{a_{z,mobile} \cdot (1 - \tan(\alpha) \cdot a_{y,mobile} - \tan(\beta) \cdot a_{x,mobile})}{\cos(\alpha) \cdot \cos(\beta)} \tag{18}$$