Block ciphers

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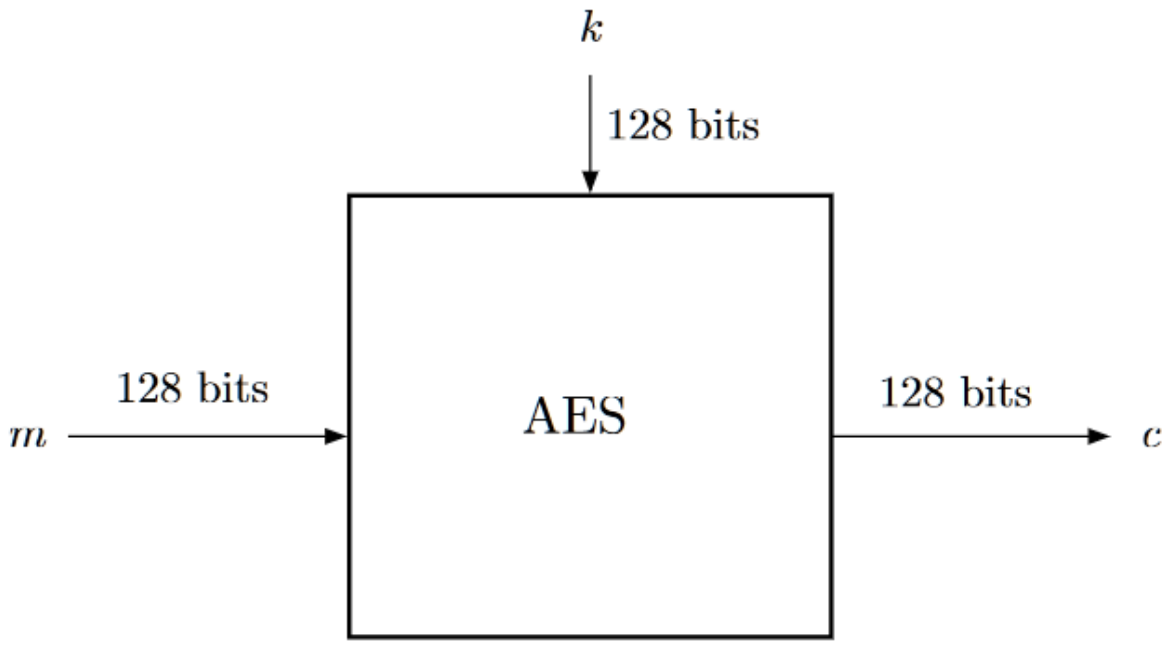
Intro

- https://www.youtube.com/watch?v=FGhj3CGxl8I

Intuition:

- A block cipher is an encryption method that applies a **deterministic algorithm** along with a **symmetric key** to encrypt a **block of text**, rather than encrypting one bit at a time as in stream ciphers



Definition - block cipher

- Functionally, a block cipher is a deterministic cipher (E, D) whose message space and ciphertext space are the same (finite) set $\mathcal{X}$.
- If the key space of (E, D) is $\mathcal{K}$, we say that $(E, D)$is a block cipher defined over $(\mathcal{K}, \mathcal{X})$.
- We call an element $x \in \mathcal{X}$ a data block, and refer to $\mathcal{X}$ as the datablock space of (E, D)

Encryption and decryption

- $\forall k \in \mathcal{K}$ we define $E(k, \cdot) = f_k : \mathcal{X} \longrightarrow \mathcal{X}$
  - We want the function to be one-to-one => f_k is a permutation on $\mathcal{X}$
- $D(k, \cdot) = f_k^{-1}$

Security - black box test

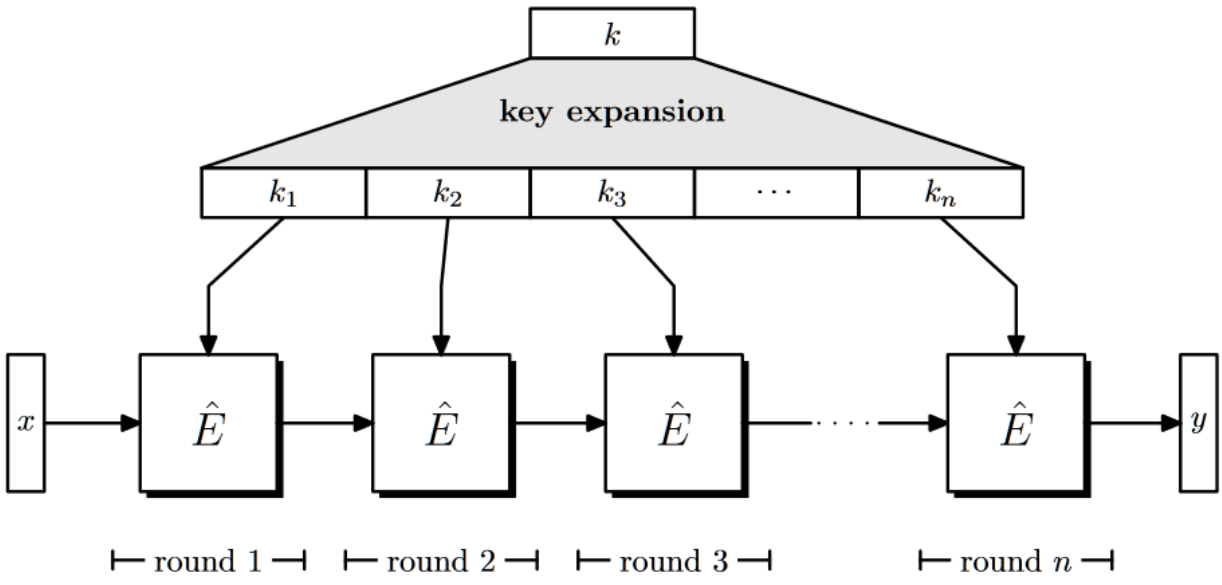
- An adversary can give the challenger a value $x \in \mathcal{X}$ and receive $y = f(x)$
- The challenger will respond by applying one of the functions
  - $f_k = E(k, \cdot)$
  - f = truly random function chose uniformly from all permutations on $\mathcal{X}$
- The adverary mustn't be able to distinguish which function was used => **Computationaly indistinguishable**
- The block cipher is secure if any efficient adversary have negligible advantages

Remarks

- A secure block cipher is unpredictable

Constructing block ciphers

- Pick a block cipher (E, D) - **round cipher**
- Pick a PRG to expand the key k into more keys - **key expansion function**
  - $(k_1, ..., k_d) \longleftarrow G(k)$
- Apply iteratively
  - $c = E(k_d, E(k_{d-1}, ...E(k_2, E(k_1, x))...))$
- Decrypt by applying the round keys in reverse order



Remark

- Linear functions never lead to secure block ciphers
- non-linear functions appear to give a secure block after a few iterations

Pseudo-random functions

A pseudo-random function (PRF) $F : \mathcal{K} \times \mathcal{X} \longrightarrow \mathcal{Y}$ is a deterministic algorithm that has two inputs:

- a key $k \in \mathcal{K}$
- an input data block $x \in \mathcal{X}$

Its output $y := F(k, x)$

Idea: for a randomly chosen key k F must look like a random function from $\mathcal{X}$ to $\mathcal{Y}$

Security

> A PRF F is secure if it's indistinguishable from a random function (The advantage for all efficient adversaries is negligible)

Weak security

> A PRF F is secure if it's indistinguishable from a random function when the queries are limited(The advantage for all efficient adversaries is negligible)

When is a secure block cipher a PRF?

Let

- (E, D) be a block cipher defined over $(\mathcal{K}, \mathcal{X}))$
- $N = |\mathcal{X}|$
- E be a PRF over $(\mathcal{K}, \mathcal{X}, \mathcal{X})$

> If N is super-poly then (E, D) is secure $\iff$ E is a secure PRF