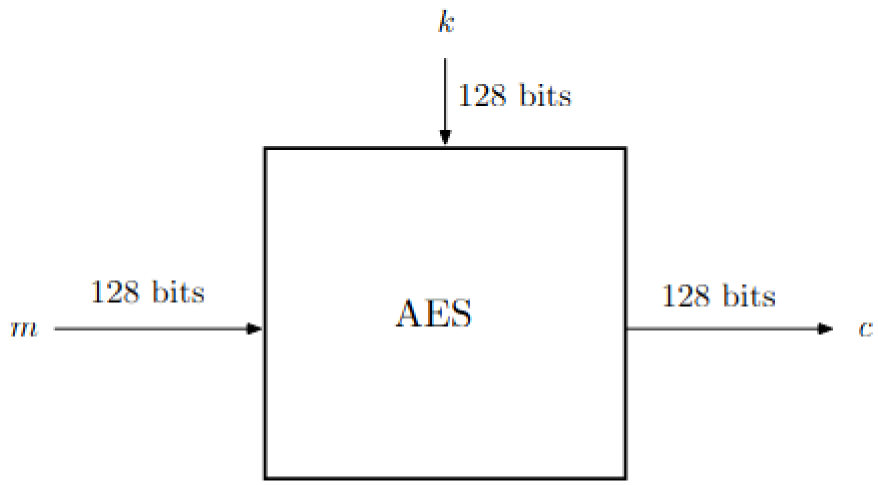
# Block ciphers

## Block ciphers

### Intro

*Intuition*:

- A block cipher is an encryption method that applies a **deterministic algorithm** along with a **symmetric key** to encrypt a **block of text**, rather than encrypting one bit at a time as in stream ciphers




**Block cipher - Definition**

Functionally, a block cipher is a deterministic cipher $(E, D)$ whose message space and ciphertext space are the same (finite) set $\mathcal{X}$.
If the key space of $(E, D)$ is $\mathcal{K}$, we say that $(E, D)$ is a block cipher defined over $(\mathcal{K}, \mathcal{X})$.
We call an element $x \in \mathcal{X}$ a data block, and refer to $\mathcal{X}$ as the datablock space of $(E, D)$

Encryption: $\forall k \in \mathcal{K}$ we define $E(k, \cdot) = f_k : \mathcal{X} \longrightarrow \mathcal{X}$

- We want the function to be one-to-one => $f_k$ is a permutation on $\mathcal{X}$

Decryption: $D(k, \cdot) = f_k^{-1}$

**Security - black box test**

- An adversary can give the challenger a value $x \in \mathcal{X}$ and receive $y = f(x)$
- The challenger will respond by applying one of the functions
  - $f_k = E(k, \cdot)$
  - $f$ = truly random function chose uniformly from all permutations on $\mathcal{X}$
- The adverary mustn't be able to distinguish which function was used => **Computationaly indistinguishable**
- The block cipher is secure if any efficient adversary have negligible advantages

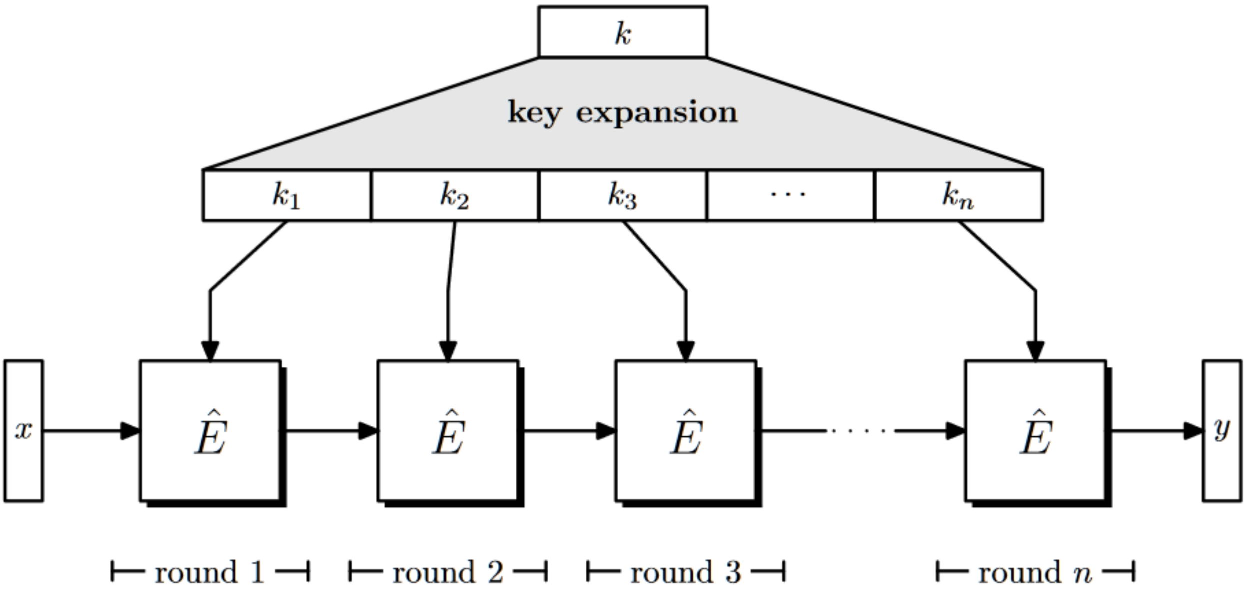
**Proprieties**:

- A secure block cipher is **unpredictable**. This means that an adversary can submit adaptive queries $(x_0, ...x_n)$ and gets their encryption mustn't be able to compute the encryption of a an extra query $x_{n+1}$.
- If a block cipher is unpredictable then it's **secure against key recovery** (finding the key $k$ used for encryptions). If we have an adversary $A$ that can recover the key, another adversary $B$ can use $A$'s attack to recover the key. This means that $B$ can compute the encryption of an extra message $E(x_{n+1,}, k)$. This makes $B$ an adversary that breaks unpredictability.

### Constructing block ciphers

- Pick a block cipher $(E, D)$ - **round cipher**
- Pick a PRG to expand the key $k$ into more keys - **key expansion function**
  - $(k_1, ..., k_d) \longleftarrow G(k)$

- Apply iteratively
    - $c = E(k_d, E(k_{d-1}, ...E(k_2, E(k_1, x))...))$
- Decrypt by applying the round keys in reverse order




**Remark**

- Linear functions never lead to secure block ciphers.
- non-linear functions *appear* to give a secure block after a few iterations. We want a *fast round cipher* that converges to a secure block cipher within a few rounds.

## Pseudo-random functions

**Pseudo random function**

A pseudo-random function (PRF) $F : \mathcal{K} \times \mathcal{X} \longrightarrow \mathcal{Y}$ is a deterministic algorithm that has two inputs:

- a key $k \in \mathcal{K}$
- an input data block $x \in \mathcal{X}$

and its output $y := F(k, x)$

*Idea*: for a randomly chosen key $k$ $F$ must look like a random function from $\mathcal{X}$ to $\mathcal{Y}$

**PRF Security**

A PRF $F$ is secure if it's indistinguishable from a random function (The advantage for all efficient adversaries is negligible)

**PRF Weak security**

A PRF $F$ is secure if it's indistinguishable from a random function when the queries are limited(The advantage for all efficient adversaries is negligible)

**When is a secure block cipher a PRF?**

Let

- $(E, D)$ be a block cipher defined over $(\mathcal{K}, \mathcal{X}))$
- $N = |\mathcal{X}|$
- $E$ be a PRF over $(\mathcal{K}, \mathcal{X}, \mathcal{X})$

> If $N$ is super-poly then $(E, D)$ is secure $\iff$ $E$ is a secure PRF

### Resources

- [Computerphile feister ciphers]
- [A graduate course in applied cryptography]