A **Graphical Model** Perspective on **Multi-Task** and **Meta-RL**

CS 330

The Plan

Variational inference review

Control as inference

Control as variational inference

Meta-RL as variational inference

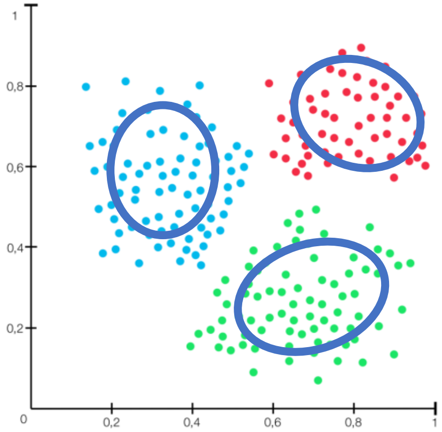
Disclaimer

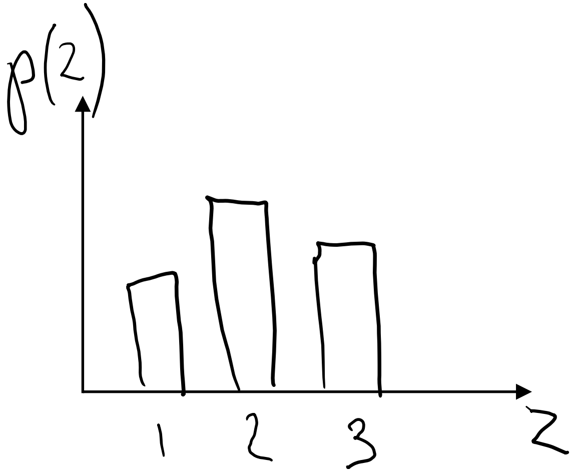
There will be quite a bit of math and derivations

This is an active research topic

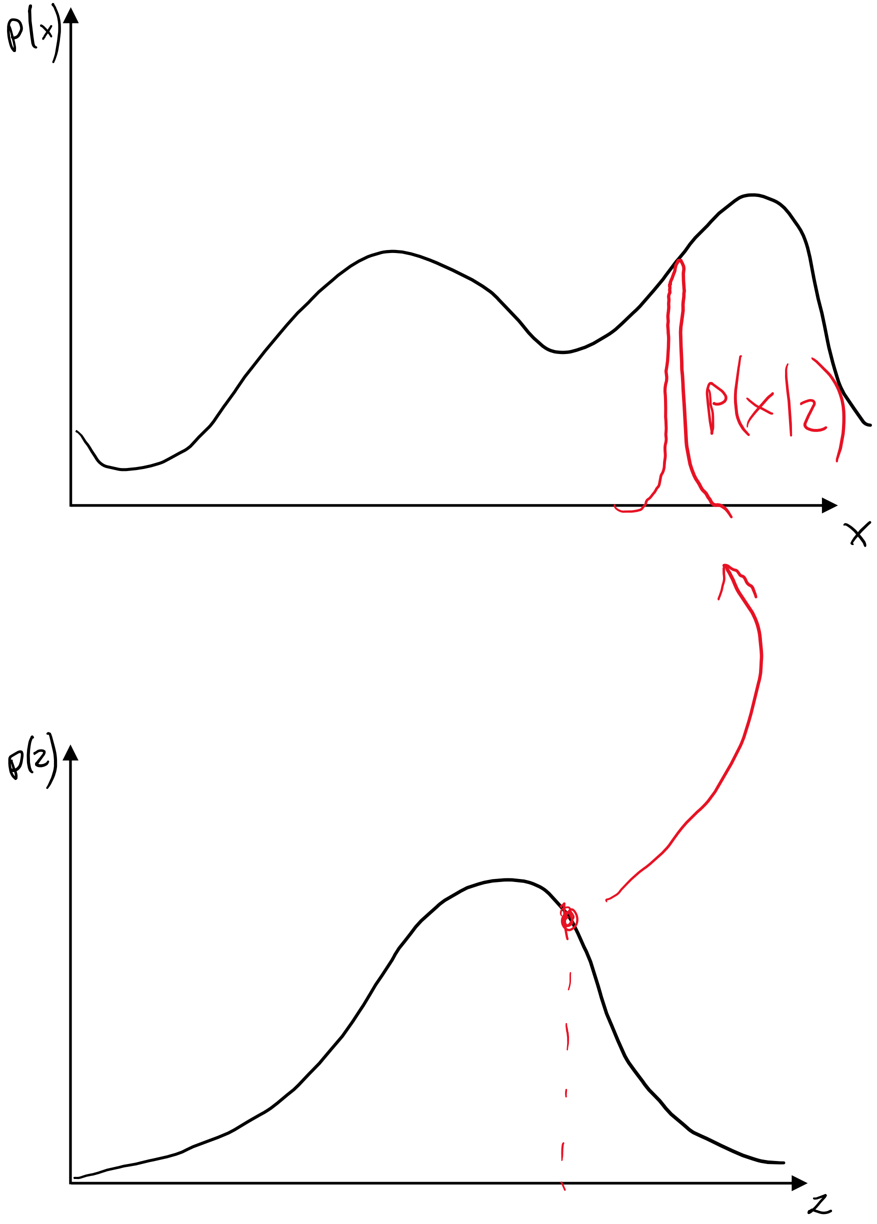
Why Variational Inference?

Main tool to cope with latent variable models

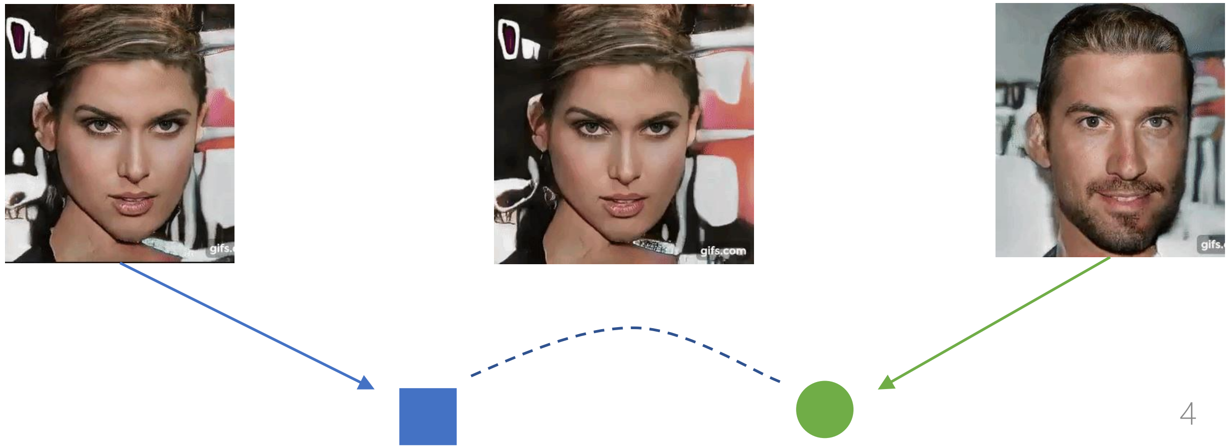




$$p(x) = \sum_z p(x|z)p(z)$$

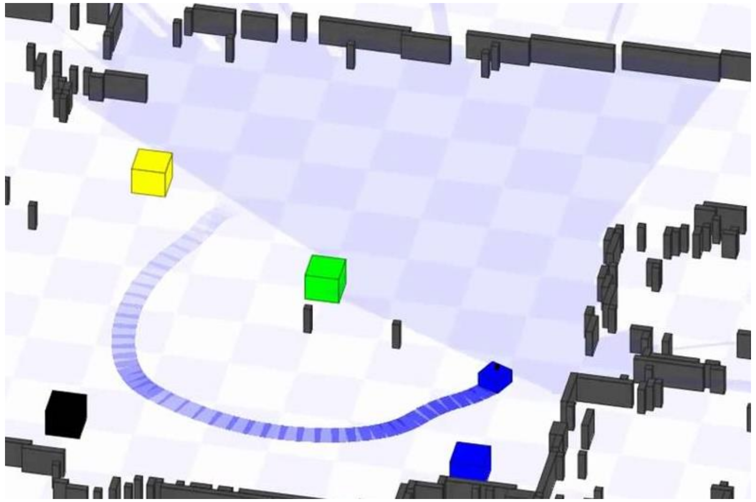


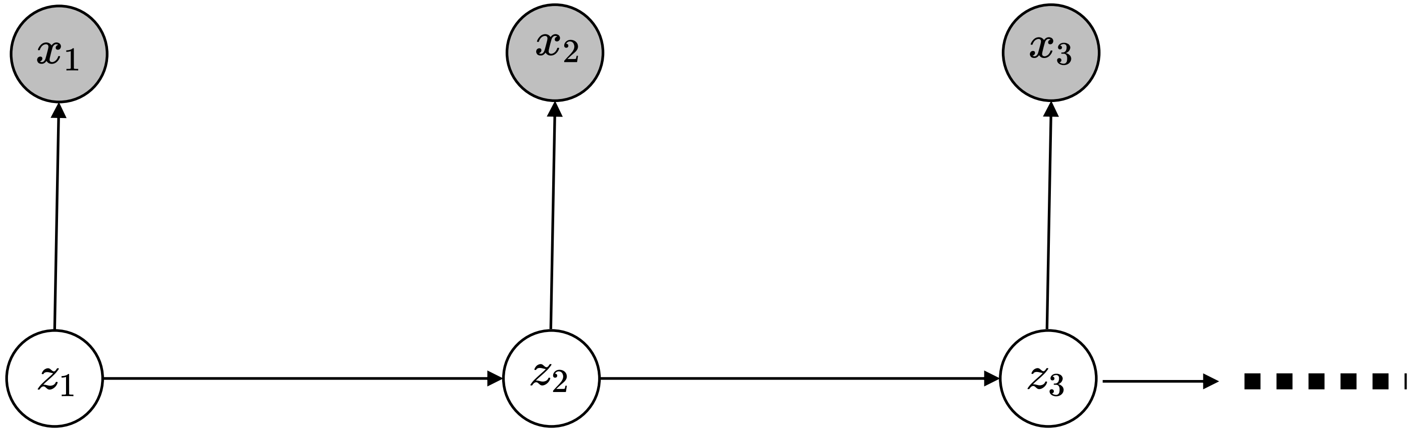
$$p(x) = \int p(x|z)p(z)dz$$



Latent Variable Models in Control

Kalman Filter

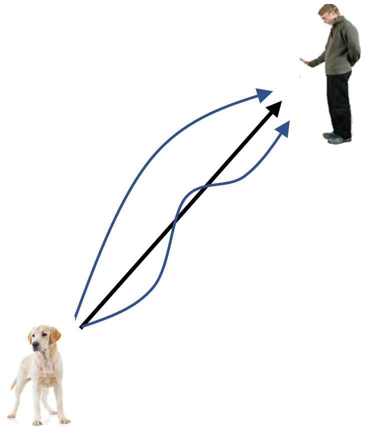




Kalman, A New Approach to Linear Filtering, 1960

Why Control as Inference?

A framework to describe stochastic optimal behavior



Knowledge from Probabilistic Graphical Models for control

- Better exploration
- Hierarchical RL
- Skill discovery

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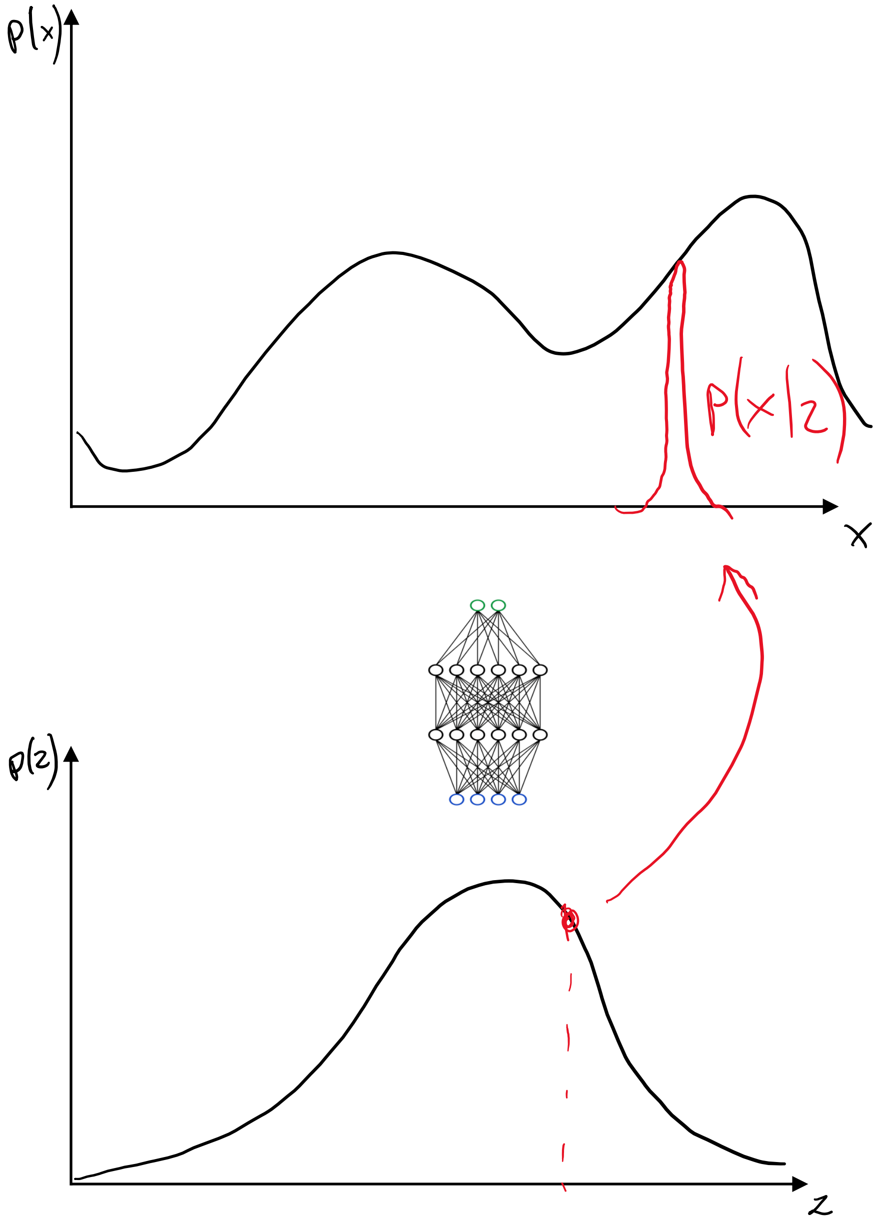
Variational Inference

$$p(x) = \int p(x|z)p(z)dz$$

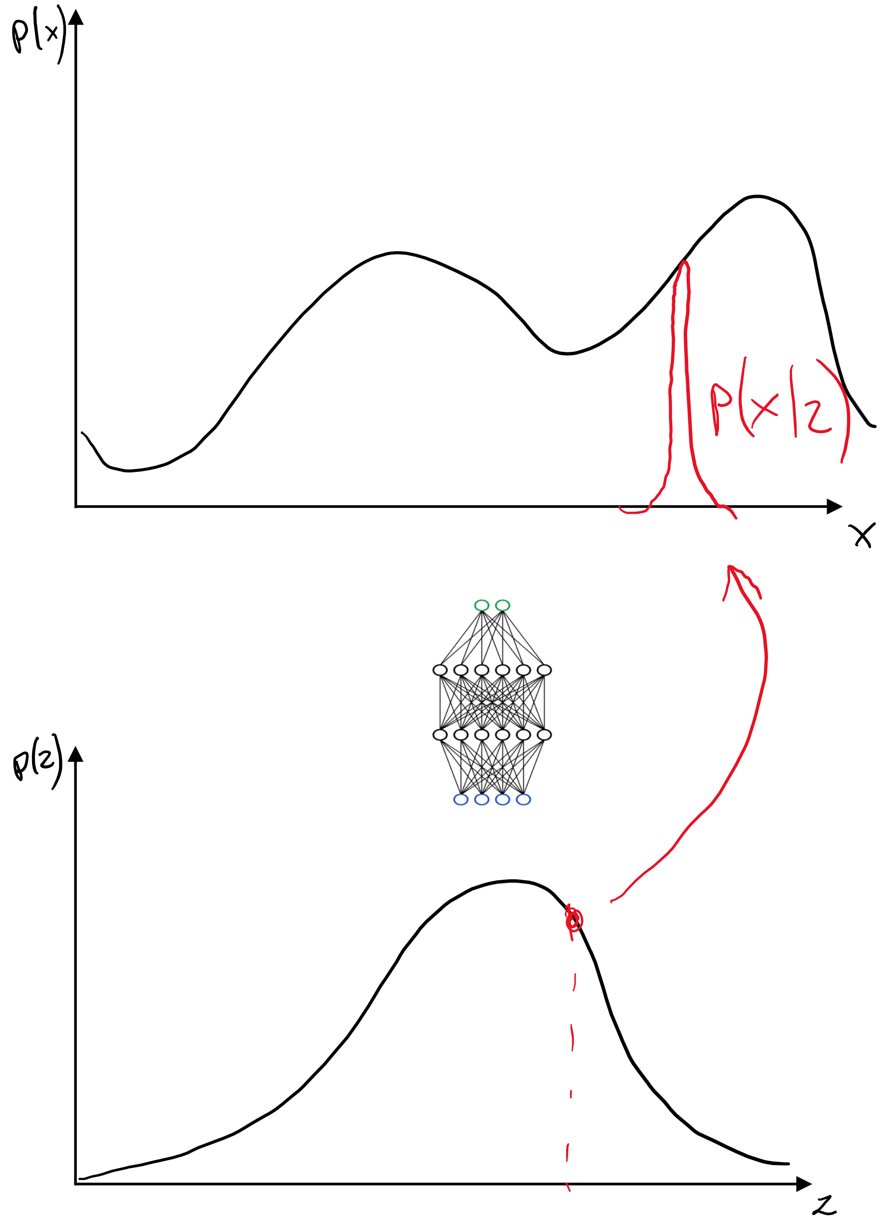


Variational Inference

$$p(x) = \int p(x|z)p(z)dz$$

$$\arg\max_{\theta} \prod_{x_i} p_\theta(x_i) = \arg\max_{\theta} \sum_{x_i} \log p_\theta(x_i)$$

$$\log p_\theta(x_i) = \log \int p(x_i|z)p(z)dz$$



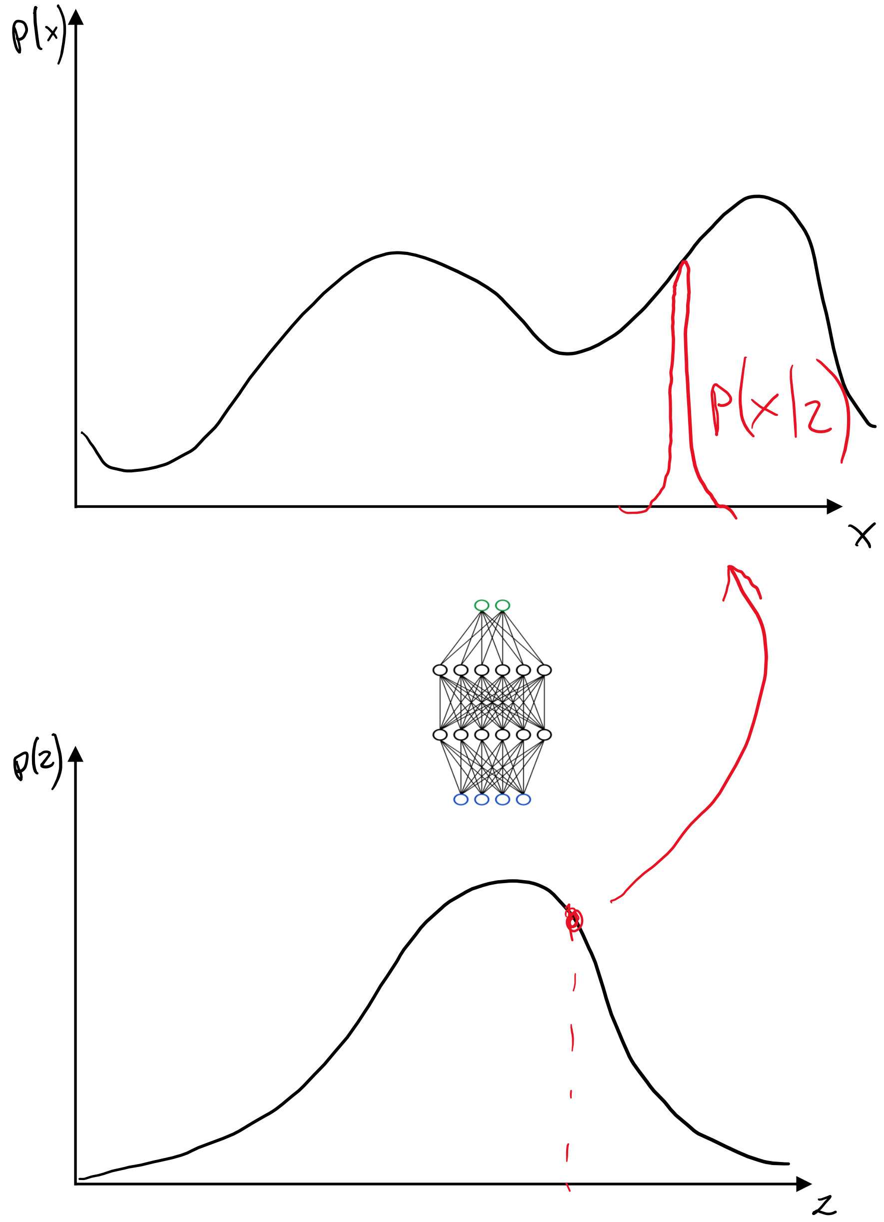
Variational Inference

$$p(x) = \int p(x|z)p(z)dz$$

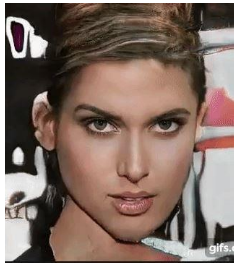
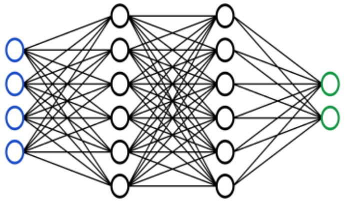
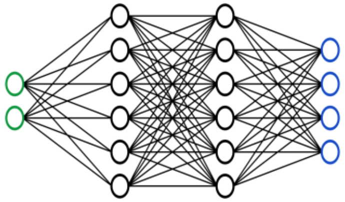
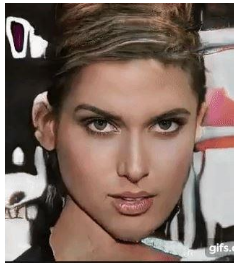
$$\log p_\theta(x_i) \geq \mathbb{E}_{z\sim q_i(z)} \log p(x_i|z) - \mathbb{D}_{KL}(q_i(z)||p(z))$$

Amortized VI

$$\log p_\theta(x_i) \geq \mathbb{E}_{z\sim q_{\phi}(z|x_i)} \log p_{\theta}(x_i|z) - \mathbb{D}_{KL}(q_{\phi}(z|x_i)||p(z))$$

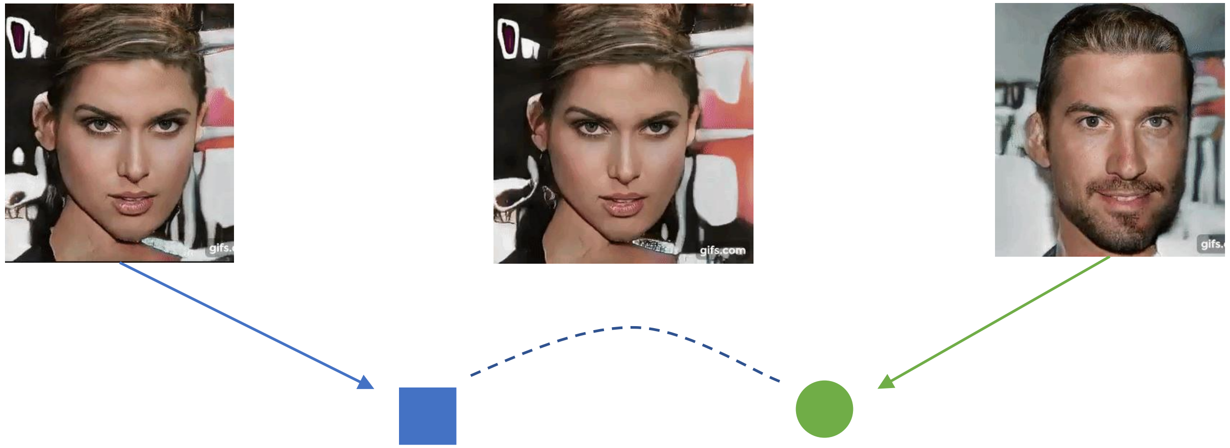


Variational Autoencoder (VAE)

x_i   $q_\phi(z|x_i)$   $p_\theta(x_i|z)$

$$\max_{\phi,\theta} \frac{1}{N}\sum_i \mathbb{E}_{z\sim q_\phi(z|x_i)} \log p_\theta(x_i|z) - \mathbb{D}_{KL}(q_\phi(z|x_i)||p(z))$$

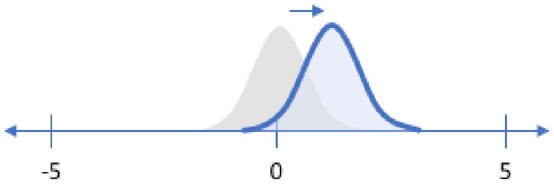
$\mathcal{N}(0, I)$



Variational Autoencoder (VAE) – what did we just do?

$$\max_{\phi,\theta} \frac{1}{N} \sum_i \mathbb{E}_{z \sim q_\phi(z|x_i)} \log p_\theta(x_i|z) - \mathbb{D}_{KL}(q_\phi(z|x_i)||p(z))$$

Penalizing reconstruction loss encourages the distribution to describe the input



Our distribution deviates from the prior to describe some characteristic of the data

Image credit: Jeremy Jordan

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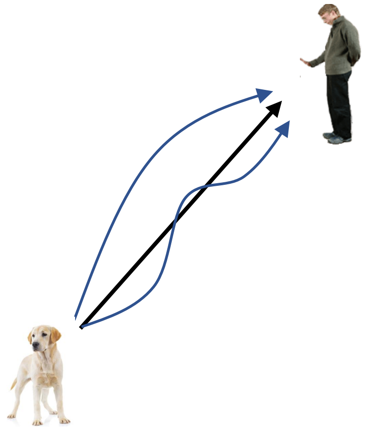
Control as Inference

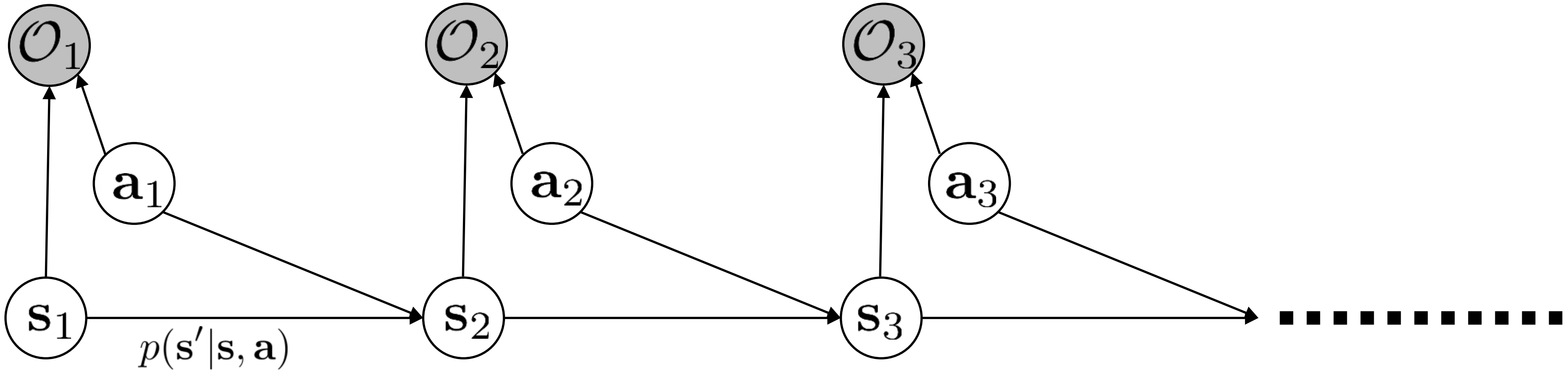
A framework to describe stochastic optimal behavior



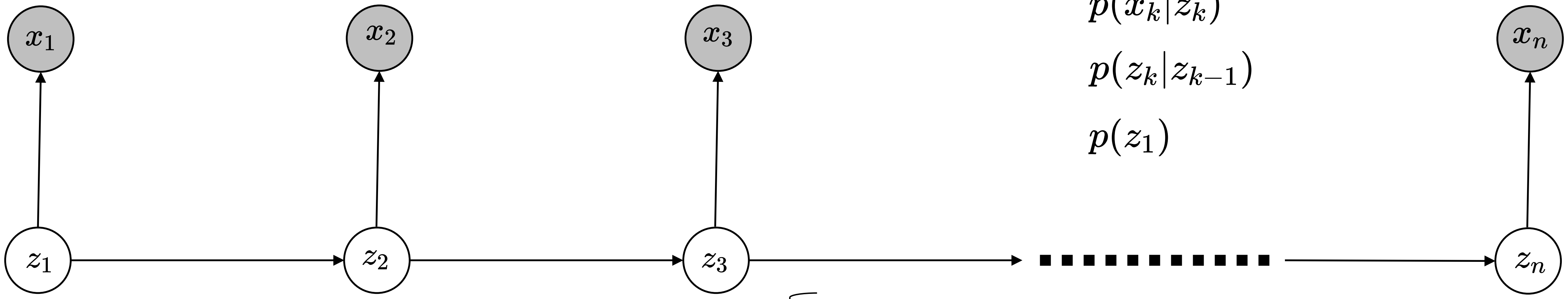
$$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau})$$

$$p(\tau | \mathcal{O}_{1:T})$$

$$p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$



Hidden Markov Models



$p(x_k|z_k)$

$p(z_k|z_{k-1})$

$p(z_1)$

Forward-backward algorithm: compute $p(z_k|x_{1:n})$

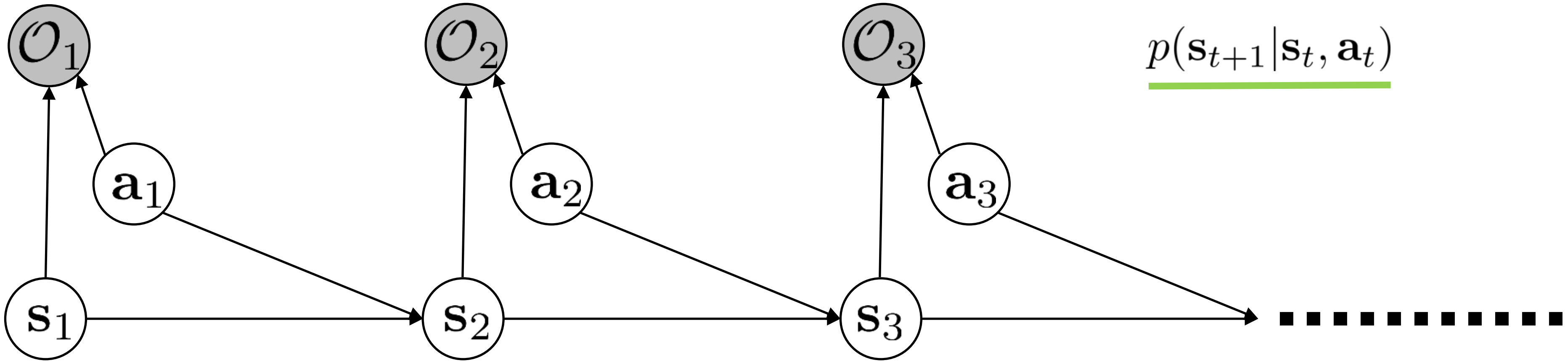
Forward algorithm: compute $p(z_k|x_{1:k-1})$

Backward algorithm: compute $p(x_{k:n}|z_k)$

$$p(z_k|x_{1:n}) = \frac{p(z_k, x_{1:n})}{p(x_{1:k-1})p(x_{k:n})} = \frac{p(x_{k:n}|z_k, \cancel{x_{1:k-1}})p(z_k|x_{1:k-1})\cancel{p(x_{1:k-1})}}{\cancel{p(x_{1:k-1})}p(x_{k:n})} = \frac{p(x_{k:n}|z_k)p(z_k|x_{1:k-1})}{p(x_{k:n})}$$

$$\propto p(x_{k:n}|z_k)p(z_k|x_{1:k-1})$$

Backward messages



$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t, \mathbf{a}_t)$ probability that we can be optimal at steps t through T given that we take action $\mathbf{a}_t$ in state $\mathbf{s}_t$

$$= \int p(\mathcal{O}_{t:T}, \mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_{t+1}$$

$$= \int p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}) p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_{t+1}$$

$$p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}) = \int \underbrace{p(\mathcal{O}_{t+1:T}|\mathbf{s}_{t+1}, \mathbf{a}_{t+1})}_{\beta_t(\mathbf{s}_{t+1}, \mathbf{a}_{t+1})} \cancel{p(\mathbf{a}_{t+1}|\mathbf{s}_{t+1})} d\mathbf{a}_{t+1}$$

which actions are likely *a priori*
(assume uniform for now)

for $t = T-1$ to 1:

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[\beta_{t+1}(\mathbf{s}_{t+1})]$$

$$\beta_t(\mathbf{s}_t) = E_{\mathbf{a}_t \sim p(\mathbf{a}_t|\mathbf{s}_t)}[\beta_t(\mathbf{s}_t, \mathbf{a}_t)]$$

Slide adapted from Sergey Levine

A closer look at the backward pass

for $t = T-1$ to 1:

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[\beta_{t+1}(\mathbf{s}_{t+1})]$$

$$\beta_t(\mathbf{s}_t) = E_{\mathbf{a}_t \sim p(\mathbf{a}_t|\mathbf{s}_t)}[\beta_t(\mathbf{s}_t, \mathbf{a}_t)]$$

let $V_t(\mathbf{s}_t) = \log \beta_t(\mathbf{s}_t)$

let $Q_t(\mathbf{s}_t, \mathbf{a}_t) = \log \beta_t(\mathbf{s}_t, \mathbf{a}_t)$

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t)) d\mathbf{a}_t$$

$V_t(\mathbf{s}_t) \to \max_{\mathbf{a}_t} Q_t(\mathbf{s}_t, \mathbf{a}_t)$ as $Q_t(\mathbf{s}_t, \mathbf{a}_t)$ gets bigger!

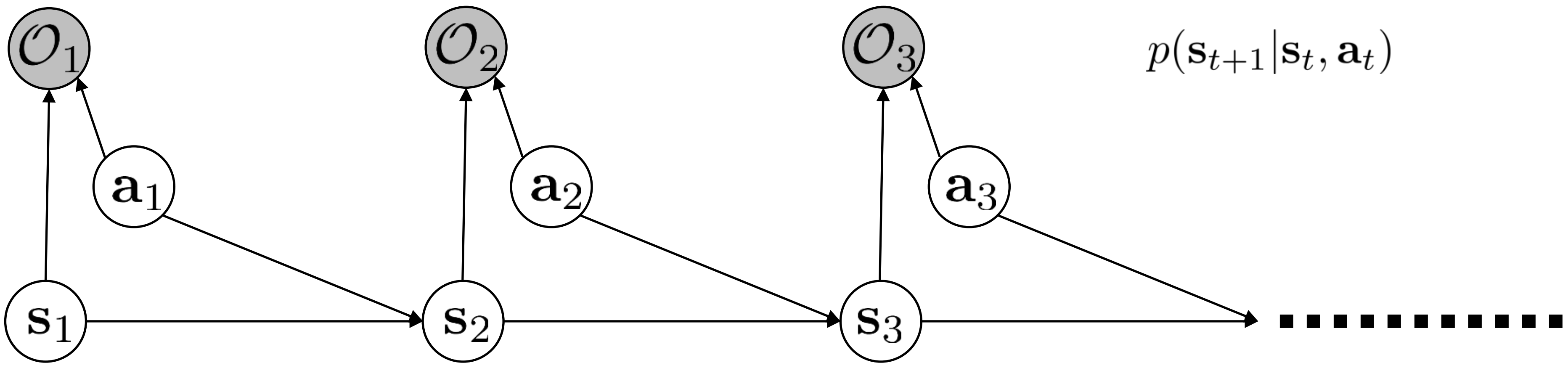
$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \overbrace{\log E[\exp(V_{t+1}(\mathbf{s}_{t+1}))]}^{\text{“optimistic” transition (something is a little off)}}$$

deterministic transition: $Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + V_{t+1}(\mathbf{s}_{t+1})$

we'll come back to the stochastic case later!

Slide adapted from Sergey Levine

Policy computation



$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) \propto \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

2. compute policy $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T})$

$$\beta_t(\mathbf{s}_t, \mathbf{a}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\beta_t(\mathbf{s}_t) = p(\mathcal{O}_{t:T}|\mathbf{s}_t)$$

$$\begin{aligned}
p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}) &= \pi(\mathbf{a}_t|\mathbf{s}_t) \\
&= p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{t:T}) \\
&= \frac{p(\mathbf{a}_t, \mathbf{s}_t|\mathcal{O}_{t:T})}{p(\mathbf{s}_t|\mathcal{O}_{t:T})} \\
&= \frac{p(\mathcal{O}_{t:T}|\mathbf{a}_t, \mathbf{s}_t)p(\mathbf{a}_t, \mathbf{s}_t)/\cancel{p(\mathcal{O}_{t:T})}}{p(\mathcal{O}_{t:T}|\mathbf{s}_t)p(\mathbf{s}_t)/\cancel{p(\mathcal{O}_{t:T})}} \\
&= \frac{p(\mathcal{O}_{t:T}|\mathbf{a}_t, \mathbf{s}_t)}{p(\mathcal{O}_{t:T}|\mathbf{s}_t)} \frac{p(\mathbf{a}_t, \mathbf{s}_t)}{p(\mathbf{s}_t)} = \frac{\beta_t(\mathbf{s}_t, \mathbf{a}_t)}{\beta_t(\mathbf{s}_t)} \cancel{p(\mathbf{a}_t|\mathbf{s}_t)}
\end{aligned}$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \frac{\beta_t(\mathbf{s}_t, \mathbf{a}_t)}{\beta_t(\mathbf{s}_t)}$$

Slide adapted from Sergey Levine

Policy computation with value functions

for $t = T-1$ to 1:

$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \log E[\exp(V_{t+1}(\mathbf{s}_{t+1}))]$$

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t))\mathbf{a}_t$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \frac{\beta_t(\mathbf{s}_t, \mathbf{a}_t)}{\beta_t(\mathbf{s}_t)}$$

$$V_t(\mathbf{s}_t) = \log \beta_t(\mathbf{s}_t)$$

$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = \log \beta_t(\mathbf{s}_t, \mathbf{a}_t)$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t, \mathbf{a}_t))$$

Slide adapted from Sergey Levine

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Control via variational inference

let $q(\mathbf{s}_{1:T}, \mathbf{a}_{1:T}) = p(\mathbf{s}_1) \prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) q(\mathbf{a}_t|\mathbf{s}_t)$

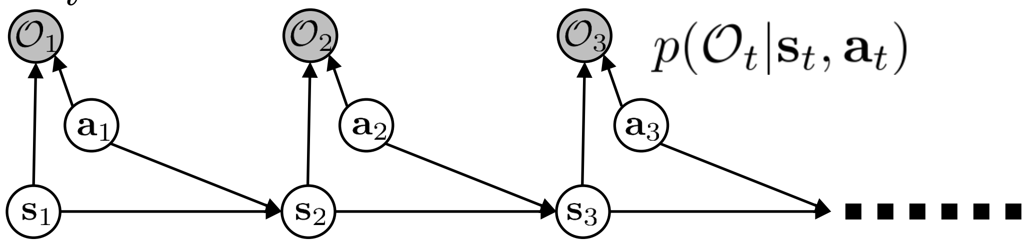
same dynamics and initial state as p

only new thing

let $\mathbf{x} = \mathcal{O}_{1:T}$ and $\mathbf{z} = (\mathbf{s}_{1:T}, \mathbf{a}_{1:T})$

$$p(\mathbf{s}_{1:T},\mathbf{a}_{1:T},\mathcal{O}_{1:T}) = p(\mathbf{s}_1) \prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t) p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t)$$

$p(\mathbf{s}_{1:T},\mathbf{a}_{1:T},\mathcal{O}_{1:T})$



$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t)$

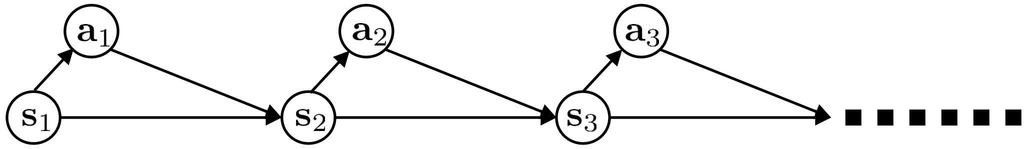
$p(\mathbf{z}, \mathbf{x})$

$p(\mathbf{s}_1)$

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

$q(\mathbf{a}_t|\mathbf{s}_t)$

$q(\mathbf{s}_{1:T}, \mathbf{a}_{1:T})$



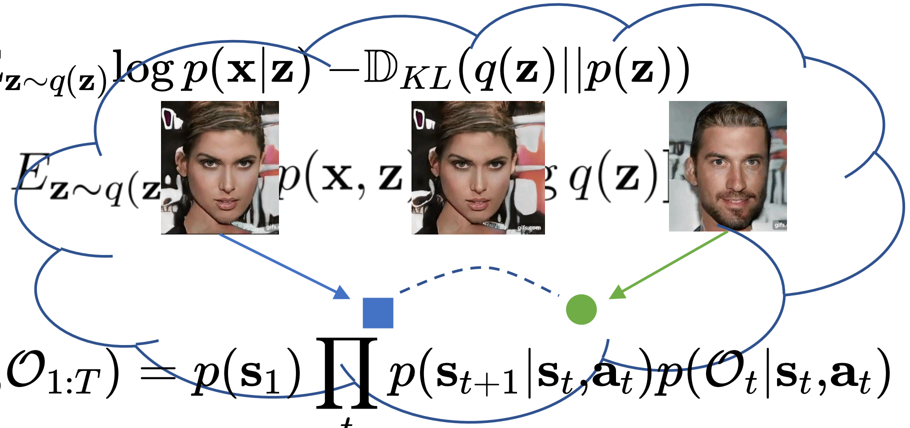
$q(\mathbf{z})$

$p(\mathbf{s}_1)$

$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

Slide adapted from Sergey Levine

The variational lower bound



$$\log p(\mathbf{x}) \geq \mathbb{E}_{\mathbf{z}\sim q(\mathbf{z})}\log p(\mathbf{x}|\mathbf{z}) - \mathbb{D}_{KL}(q(\mathbf{z})||p(\mathbf{z}))$$

$$\log p(\mathbf{x}) \geq E_{\mathbf{z}\sim q(\mathbf{z})}[\log p(\mathbf{x},\mathbf{z}) - \log q(\mathbf{z})]$$

let $\mathbf{x} = \mathcal{O}_{1:T}$ and $\mathbf{z} = (\mathbf{s}_{1:T}, \mathbf{a}_{1:T})$

$$p(\mathbf{s}_{1:T},\mathbf{a}_{1:T},\mathcal{O}_{1:T}) = p(\mathbf{s}_1)\prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t)$$

let $q(\mathbf{s}_{1:T},\mathbf{a}_{1:T}) = \underline{p(\mathbf{s}_1)}\, \underline{\prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}\, q(\mathbf{a}_t|\mathbf{s}_t)$

$$\log p(\mathcal{O}_{1:T}) \geq E_{(\mathbf{s}_{1:T},\mathbf{a}_{1:T})\sim q}\Big[\cancel{\log p(\mathbf{s}_1)} + \sum_{t=1}^{T}\cancel{\log p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)} + \sum_{t=1}^{T}\log p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t)$$

$$\underline{-\cancel{\log p(\mathbf{s}_1)}} - \underline{\sum_{t=1}^{T}\cancel{\log p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}} - \sum_{t=1}^{T}\log q(\mathbf{a}_t|\mathbf{s}_t)\Big]$$

$$= E_{(\mathbf{s}_{1:T},\mathbf{a}_{1:T})\sim q}\left[\sum_t r(\mathbf{s}_t,\mathbf{a}_t) - \log q(\mathbf{a}_t|\mathbf{s}_t)\right]$$

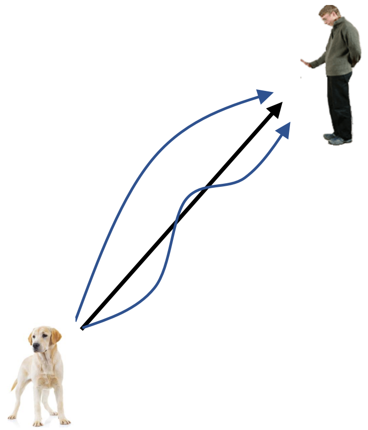
$$= \sum_t E_{(\mathbf{s}_t,\mathbf{a}_t)\sim q}[r(\mathbf{s}_t,\mathbf{a}_t) + \mathcal{H}(q(\mathbf{a}_t|\mathbf{s}_t))] \longleftarrow \text{maximize reward and maximize action entropy!}$$

Slide adapted from Sergey Levine

Summary

Objective:

$$\sum_t E_{(\mathbf{s}_t,\mathbf{a}_t)\sim q}\left[r(\mathbf{s}_t,\mathbf{a}_t) + \mathcal{H}(q(\mathbf{a}_t|\mathbf{s}_t))\right]$$



Value-, Q-functions, and the policy

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t,\mathbf{a}_t))\mathbf{a}_t$$

$$\cancel{Q_t(\mathbf{s}_t,\mathbf{a}_t) = r(\mathbf{s}_t,\mathbf{a}_t) + \log E[\exp(V_{t+1}(\mathbf{s}_{t+1}))]} \longrightarrow Q_t(\mathbf{s}_t,\mathbf{a}_t) = r(\mathbf{s}_t,\mathbf{a}_t) + E[(V_{t+1}(\mathbf{s}_{t+1})]$$

$$\pi(\mathbf{a}_t|\mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t,\mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t,\mathbf{a}_t))$$

Q-learning

1. collect dataset $\{(\mathbf{s}_i,\mathbf{a}_i,\mathbf{s}_i',r_i)\}$

$K\times$

2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i,\mathbf{a}_i) + \gamma \max_{\mathbf{a}_i'} Q_\phi(\mathbf{s}_i',\mathbf{a}_i')$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|Q_\phi(\mathbf{s}_i,\mathbf{a}_i) - \mathbf{y}_i\|^2$

$$\pi(\mathbf{a}|\mathbf{s}) = \underset{\mathbf{a}}{\mathrm{arg}max} Q_\phi(\mathbf{s},\mathbf{a})$$

Soft Q-learning

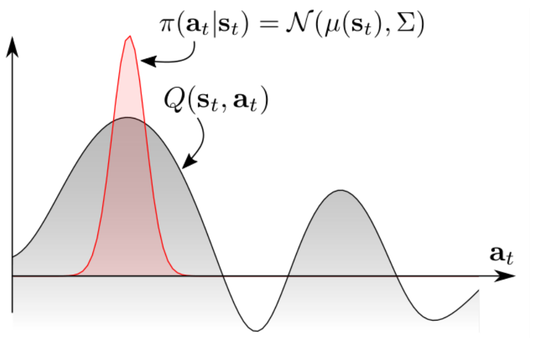
1. collect dataset $\{(\mathbf{s}_i,\mathbf{a}_i,\mathbf{s}_i',r_i)\}$

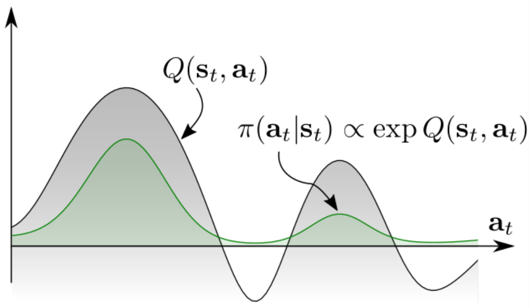
$K\times$

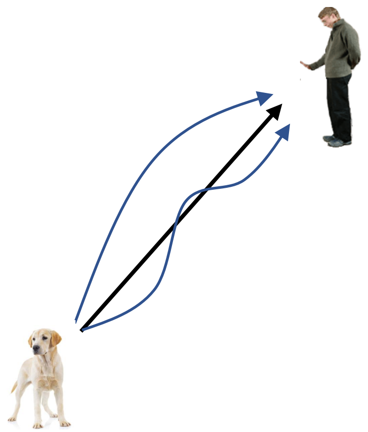
2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i,\mathbf{a}_i) + \gamma \overset{\text{softmax}}{\cancel{\max}}_{\mathbf{a}_i'} Q_\phi(\mathbf{s}_i',\mathbf{a}_i')$
3. set $\phi \leftarrow \arg\min_\phi \frac{1}{2}\sum_i \|Q_\phi(\mathbf{s}_i,\mathbf{a}_i) - \mathbf{y}_i\|^2$

$$\pi(\mathbf{a}|\mathbf{s}) = \mathrm{arg}\cancel{\underset{\mathbf{a}}{max} Q_\phi(\mathbf{s},\mathbf{a})}\ \exp(A_t(s_t, a_t))$$

Soft Q-learning



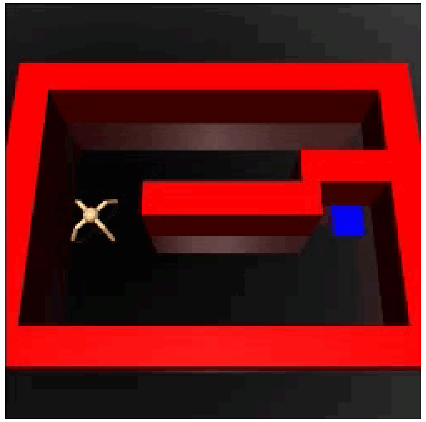


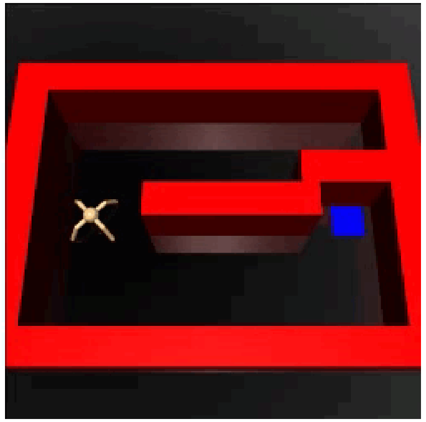


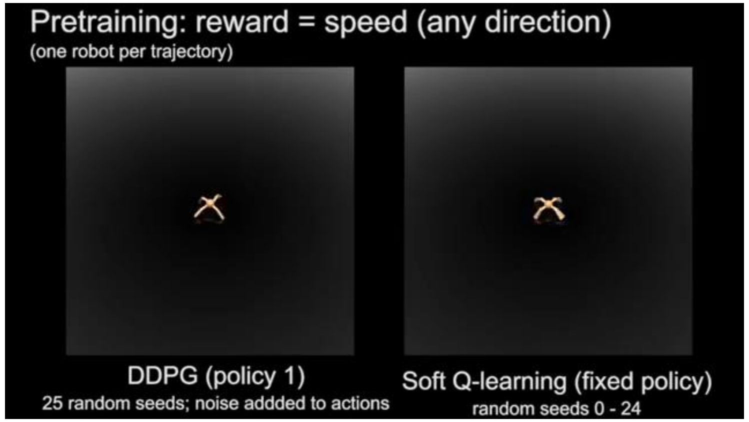
Exploration

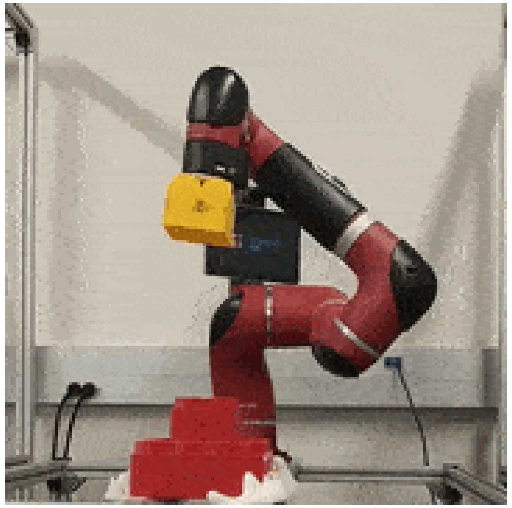
Fine-tunability

Robustness









 Haarnoja et al. RL with Deep Energy-Based Policies, 2017

The Plan

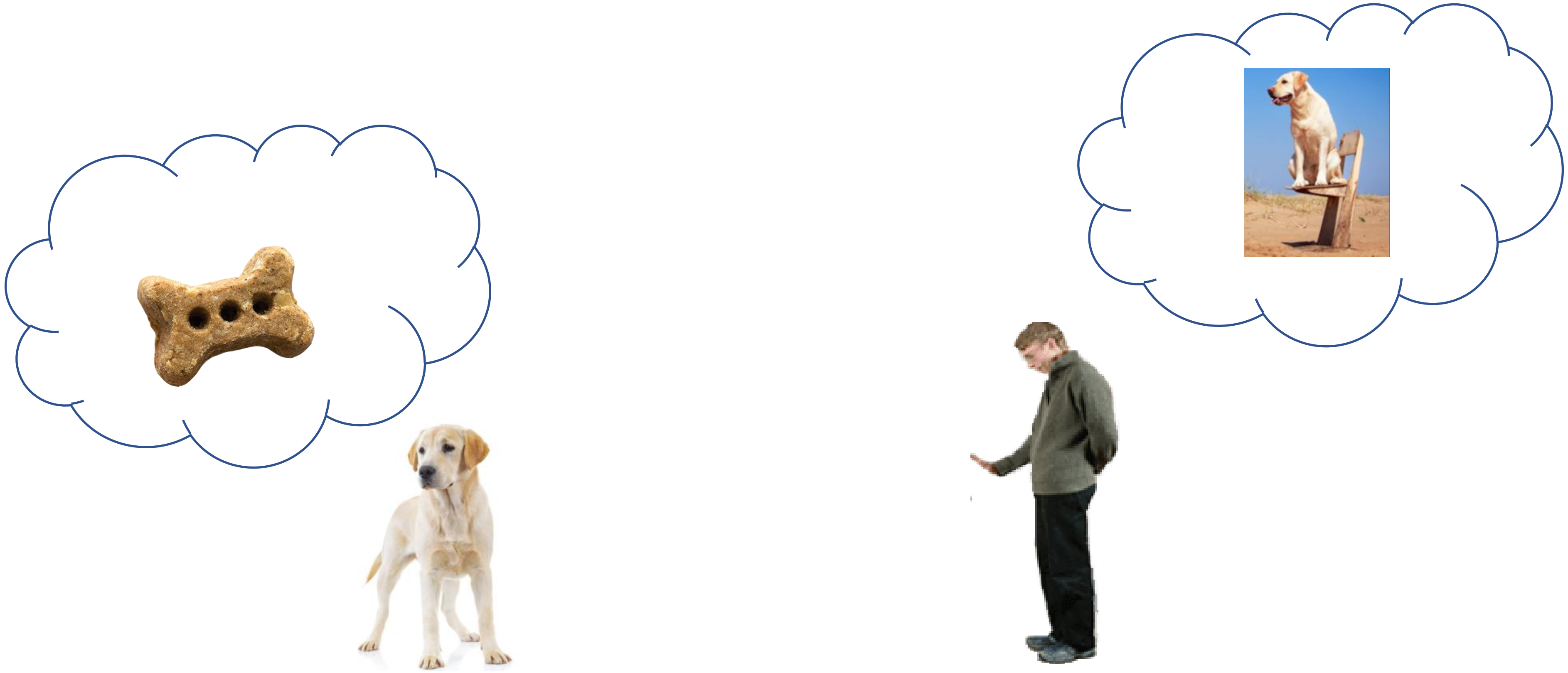
Variational inference review

Control as inference

Control as variational inference

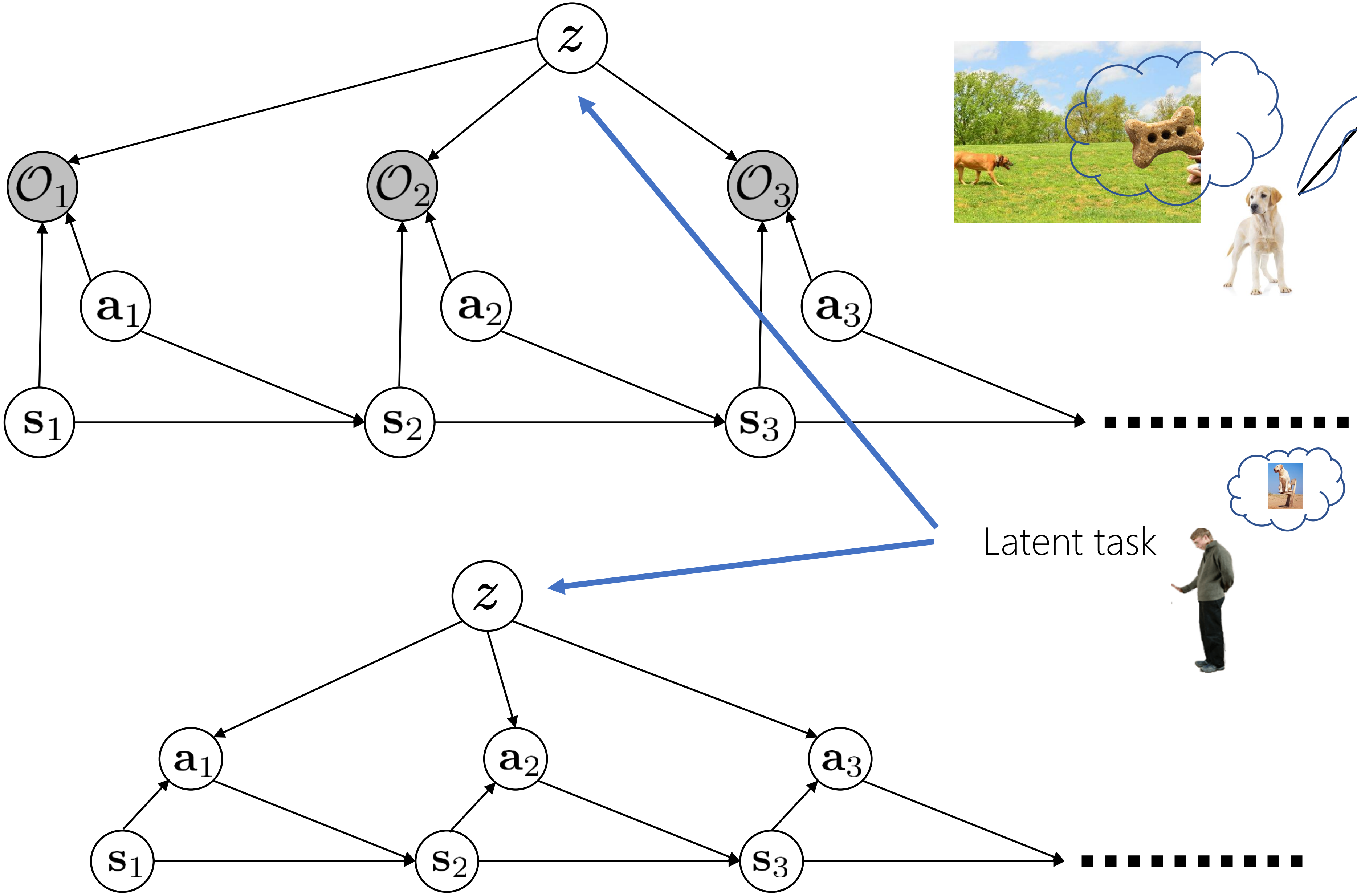
Meta-RL as variational inference

Meta-RL via Variational Inference



What's a good strategy for the dog?

$$\max_{\phi,\theta} \frac{1}{N} \sum_i \mathbb{E}_{z \sim q_\phi(z|x_i)} \log p_\theta(x_i|z) - \mathbb{D}_{KL}(q_\phi(z|x_i)||p(z))$$

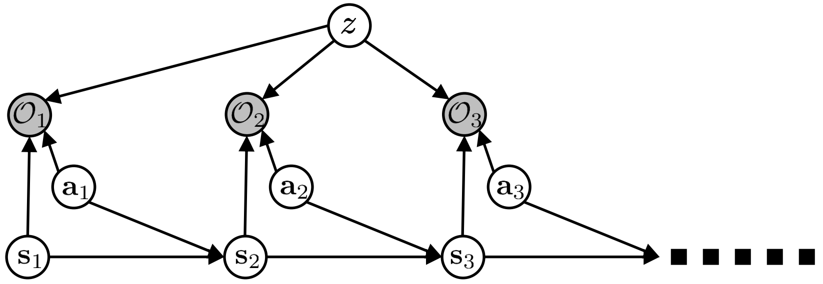


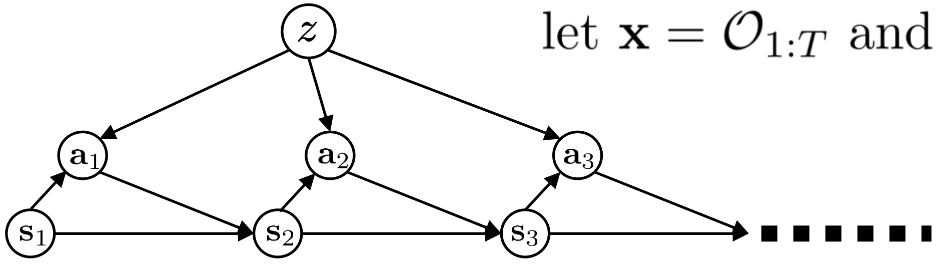
z
$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_3$
$\mathbf{a}_1$
$\mathbf{a}_2$
$\mathbf{a}_3$
$\mathbf{s}_1$
$\mathbf{s}_2$
$\mathbf{s}_3$
Latent task
z
$\mathbf{a}_1$
$\mathbf{a}_2$
$\mathbf{a}_3$
$\mathbf{s}_1$
$\mathbf{s}_2$
$\mathbf{s}_3$

Variational Inference Again!

let $\mathbf{x} = \mathcal{O}_{1:T}$ and $\cancel{\mathbf{z} = (\mathbf{s}_{1:T}, \mathbf{a}_{1:T})}$

let $\mathbf{x} = \mathcal{O}_{1:T}$ and $\mathbf{z} =$ latent task





$$\log p_\theta(x_i) \geq \mathbb{E}_{z\sim q(z|x_i)} \log p(x_i|z) - \mathbb{D}_{KL}(q(z|x_i)||p(z))$$

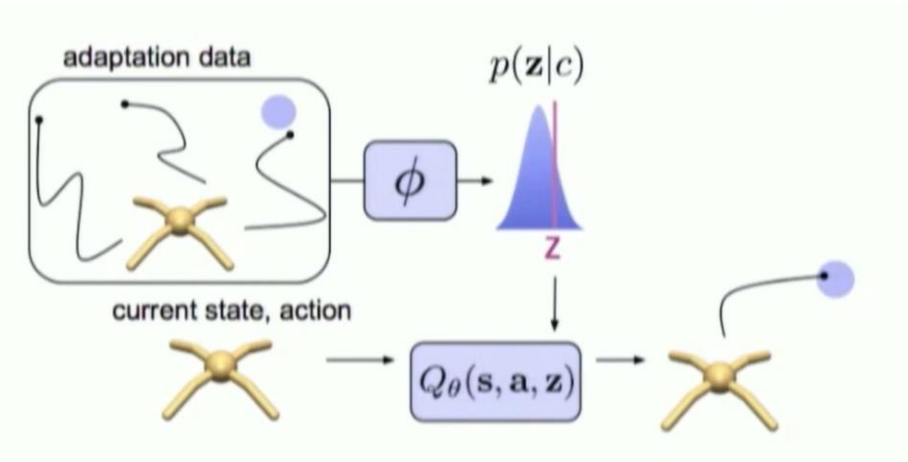
$$\log p(\mathcal{O}_{1:T}) \geq \mathbb{E}_{q(z|x_i)} \log p(\mathcal{O}_{1:T}|z) - \mathbb{D}_{KL}(q(z|x_i)||p(z))$$

$(\mathcal{O}_{1:T} | z)$ (s_t, a_t, z) $r(s_t, a_t, z)$ $q(a_t | s_t, z)$

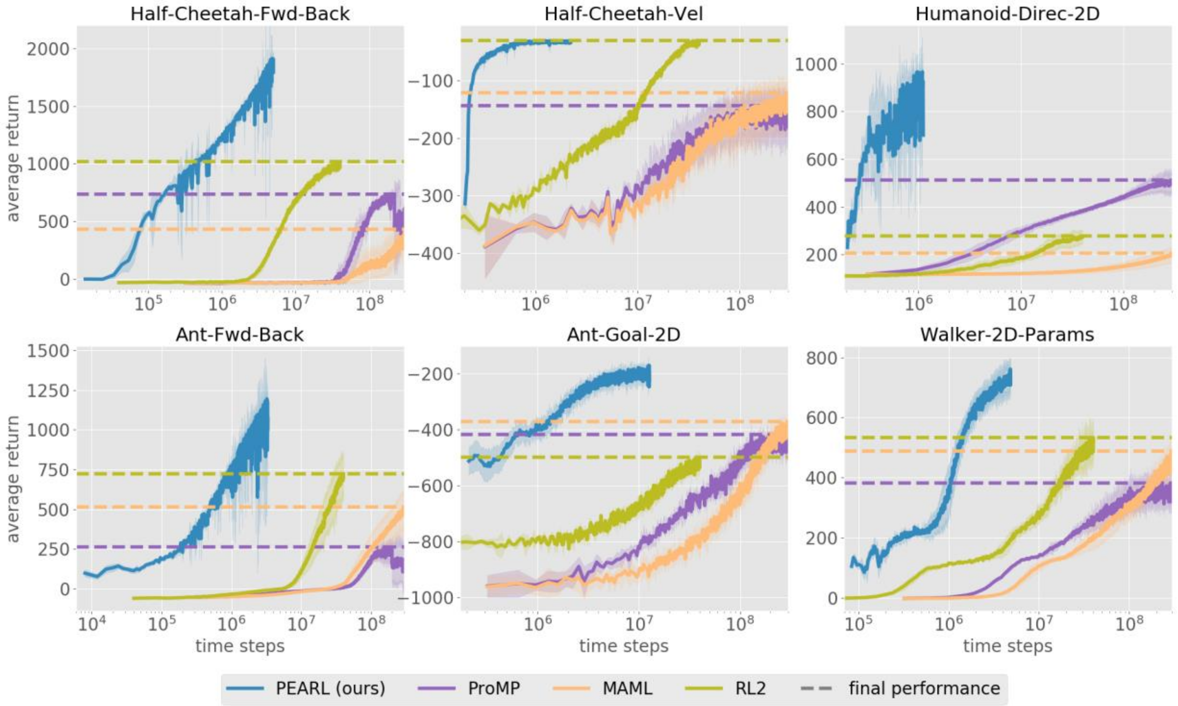
$$\log p(\mathcal{O}_{1:T}) \geq \sum_t E_{(\mathbf{s}_t, \mathbf{a}_t)\sim q}\left[r(\mathbf{s}_t, \mathbf{a}_t) + \mathcal{H}(q(\mathbf{a}_t|\mathbf{s}_t))\right]$$

What's a good x_i ? → How about all the transitions seen so far?

PEARL

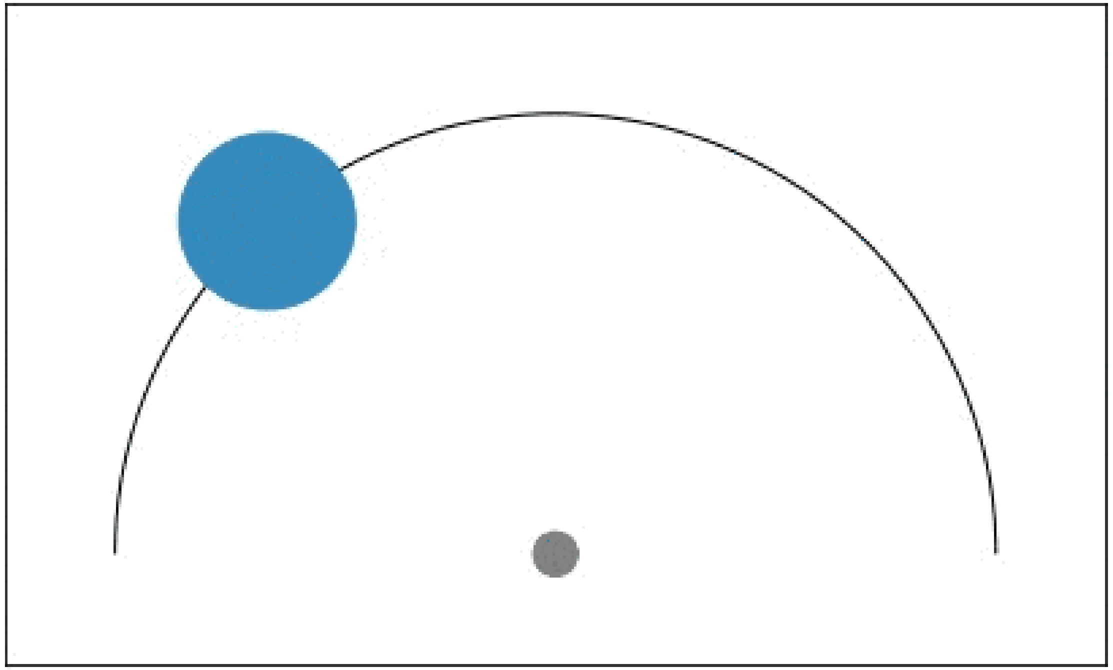


$$\mathbb{E}_{\mathcal{T}}[\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{c}^{\mathcal{T}})}[R(\mathcal{T},\mathbf{z}) + \beta D_{\mathrm{KL}}(q_\phi(\mathbf{z}|\mathbf{c}^{\mathcal{T}})||p(\mathbf{z}))]]$$



Rakelly et al. PEARL, 2019

PEARL



Rakelly et al. PEARL, 2019

The Plan

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Next Week

What if we want the agent to **come up with the tasks**?

Hierarchical RL and Skill Discovery - **Nov 2**

What about **hierarchies** of tasks?

Can the agent **learn continuously** over their life-time?

Lifelong learning – **Nov 4**

Additional Resources

RL and Control as Probabilistic Inference, Levine, 2018

Learning to Learn with Probabilistic Task Embeddings, BAIR blog post,

Berkeley CS285: Deep Reinforcement Learning