

# Hierarchical RL and Skill Discovery

CS 330

# The Plan

Information-theoretic concepts

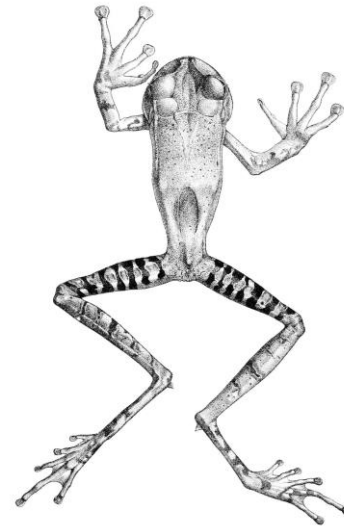
Skill discovery

Using discovered skills

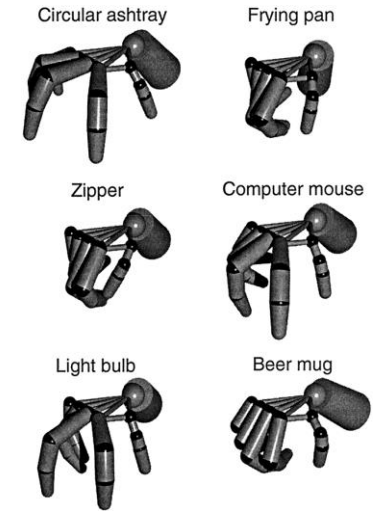
Hierarchical RL

# Why Skill Discovery?

What if we want to discover interesting behaviors?



[The construction of movement by the spinal cord, *Tresch et al.*, 1999]

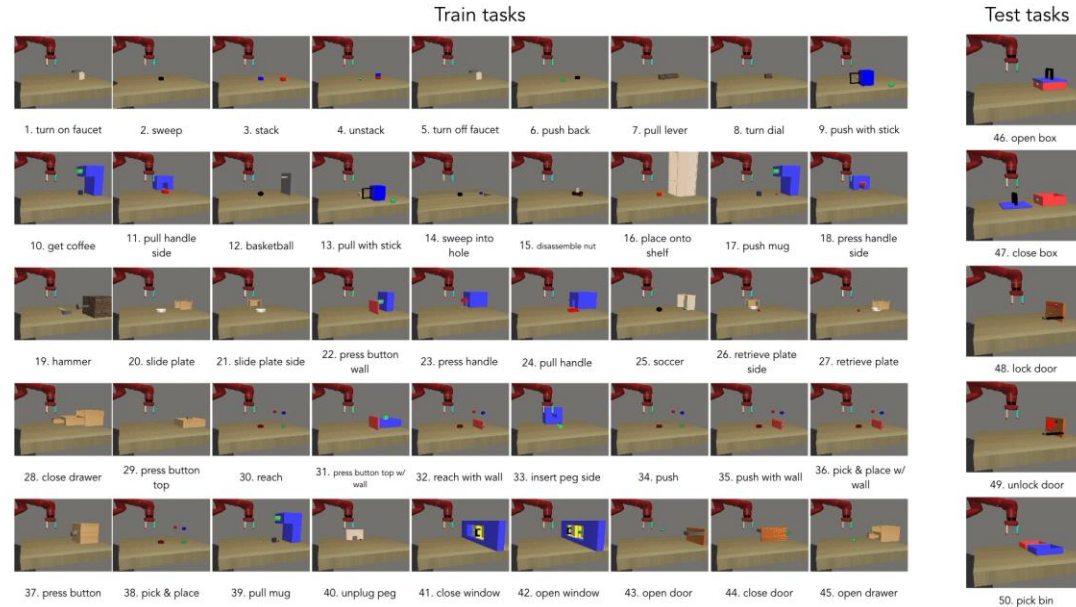


[Postural hand synergies for tool use, *Santello, et al.*, 1998]

# Why Skill Discovery? More practical version

Coming up with tasks is tricky...

Write down task ideas for a tabletop manipulation scenario



# Why Hierarchical RL?

Performing tasks at various levels of abstractions

Bake a cheesecake

Buy ingredients

Go to the store

Walk to the door

Take a step

Contract muscle X

Exploration



# The Plan

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Using discovered skills

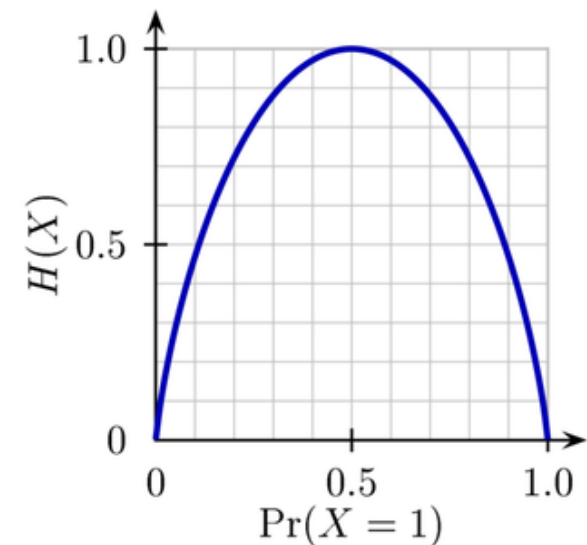
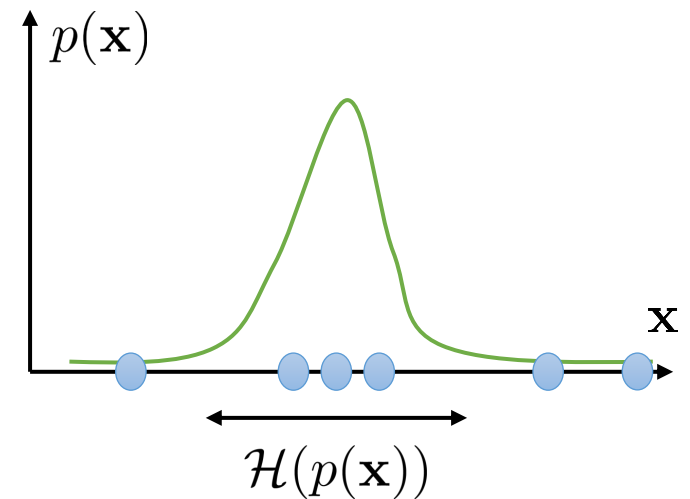
Hierarchical RL

# Entropy

$p(\mathbf{x})$  distribution (e.g., over observations  $\mathbf{x}$ )

$$\mathcal{H}(p(\mathbf{x})) = -E_{\mathbf{x} \sim p(\mathbf{x})} [\log p(\mathbf{x})]$$

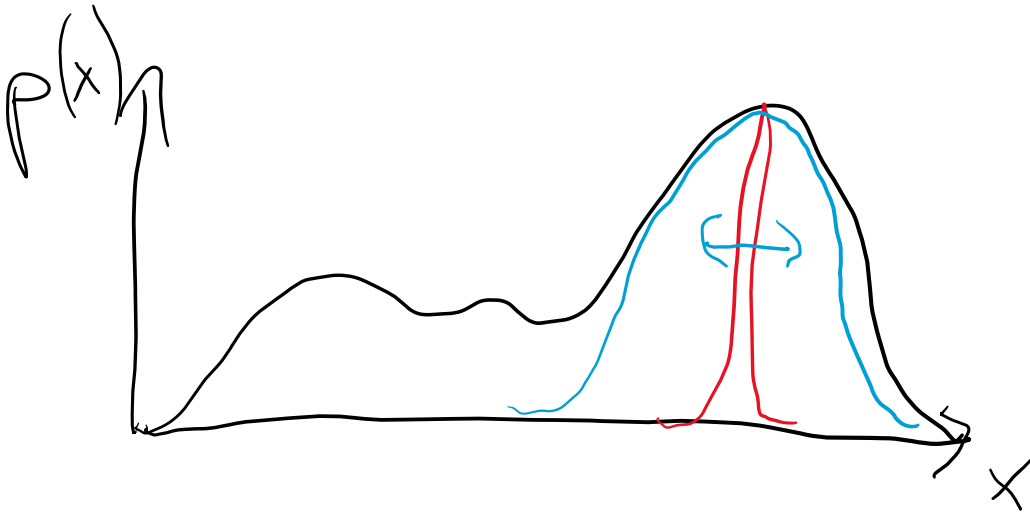
entropy – how “broad”  $p(\mathbf{x})$  is



# KL-divergence

Distance between two distributions

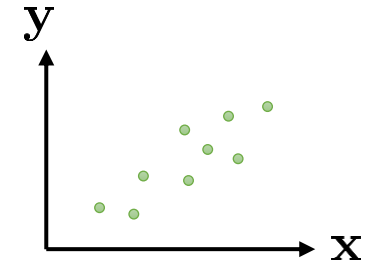
$$\mathbb{D}_{KL}(q||p) = \mathbb{E}_q \left[ \log \frac{q(x)}{p(x)} \right] = \mathbb{E}_q \log q(x) - \mathbb{E}_q \log p(x) = \underbrace{-\mathbb{E}_q \log p(x)}_{\text{cross entropy}} - \underbrace{\mathcal{H}(q(x))}_{\text{entropy}}$$



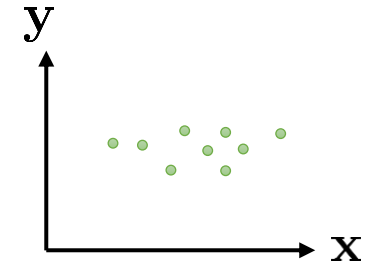


# Mutual information

$$\begin{aligned}\mathcal{I}(\mathbf{x}; \mathbf{y}) &= D_{\text{KL}}(p(\mathbf{x}, \mathbf{y}) \| p(\mathbf{x})p(\mathbf{y})) \\ &= E_{(\mathbf{x}, \mathbf{y}) \sim p(\mathbf{x}, \mathbf{y})} \left[ \log \frac{p(\mathbf{x}, \mathbf{y})}{p(\mathbf{x})p(\mathbf{y})} \right] \\ &= \mathcal{H}(p(\mathbf{y})) - \mathcal{H}(p(\mathbf{y}|\mathbf{x})) = \mathcal{H}(p(\mathbf{x})) - \mathcal{H}(p(\mathbf{x}|\mathbf{y}))\end{aligned}$$



high MI:  $\mathbf{x}$  and  $\mathbf{y}$  are *dependent*



low MI:  $\mathbf{x}$  and  $\mathbf{y}$  are *independent*

High MI?

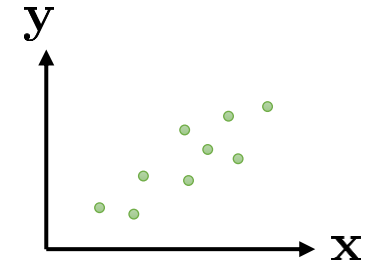
x- it rains tomorrow, y – streets are wet tomorrow

x- it rains tomorrow, y – we find life on Mars tomorrow

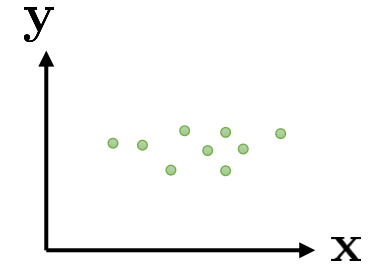
# Mutual information

$$\begin{aligned}\mathcal{I}(\mathbf{x}; \mathbf{y}) &= D_{\text{KL}}(p(\mathbf{x}, \mathbf{y}) \| p(\mathbf{x})p(\mathbf{y})) \\ &= E_{(\mathbf{x}, \mathbf{y}) \sim p(\mathbf{x}, \mathbf{y})} \left[ \log \frac{p(\mathbf{x}, \mathbf{y})}{p(\mathbf{x})p(\mathbf{y})} \right]\end{aligned}$$

$$= \mathcal{H}(p(\mathbf{y})) - \mathcal{H}(p(\mathbf{y}|\mathbf{x})) = \mathcal{H}(p(\mathbf{x})) - \mathcal{H}(p(\mathbf{x}|\mathbf{y}))$$



high MI:  $\mathbf{x}$  and  $\mathbf{y}$  are *dependent*



low MI:  $\mathbf{x}$  and  $\mathbf{y}$  are *independent*

example of mutual information: “empowerment” (Polani et al.)

$$\mathcal{I}(\mathbf{s}_{t+1}; \mathbf{a}_t) = \mathcal{H}(\mathbf{s}_{t+1}) - \mathcal{H}(\mathbf{s}_{t+1} | \mathbf{a}_t)$$

# The Plan

Information-theoretic concepts

Skill discovery

Using discovered skills

Hierarchical RL

# Soft Q-learning

Objective:

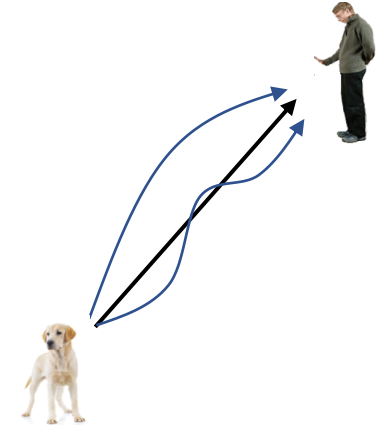
$$\sum_t E_{(\mathbf{s}_t, \mathbf{a}_t) \sim q} [r(\mathbf{s}_t, \mathbf{a}_t) + \mathcal{H}(q(\mathbf{a}_t | \mathbf{s}_t))]$$

Value-, Q-functions, and the policy

$$V_t(\mathbf{s}_t) = \log \int \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t)) \mathbf{a}_t$$

$$Q_t(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E[(V_{t+1}(\mathbf{s}_{t+1}))]$$

$$\pi(\mathbf{a}_t | \mathbf{s}_t) = \exp(Q_t(\mathbf{s}_t, \mathbf{a}_t) - V_t(\mathbf{s}_t)) = \exp(A_t(\mathbf{s}_t, \mathbf{a}_t))$$



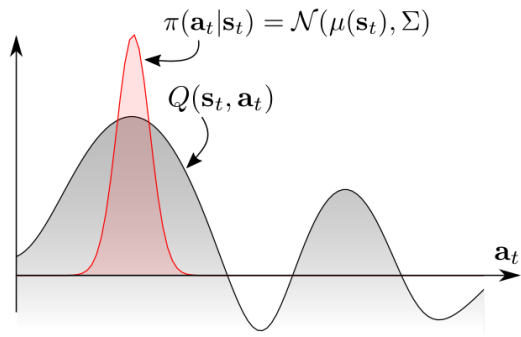
Q-learning

1. collect dataset  $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$
  2. set  $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
  3. set  $\phi \leftarrow \arg \min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$
- $\pi(\mathbf{a} | \mathbf{s}) = \arg \max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a})$

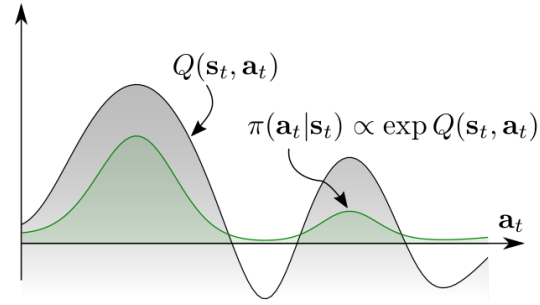
Soft Q-learning

1. collect dataset  $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$
  2. set  $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \overset{\text{softmax}}{\max_{\mathbf{a}'_i}} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
  3. set  $\phi \leftarrow \arg \min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$
- $\pi(\mathbf{a} | \mathbf{s}) = \arg \max_{\mathbf{a}} Q_\phi(\mathbf{s}, \mathbf{a}) \exp(A_t(\mathbf{s}_t, \mathbf{a}_t))$

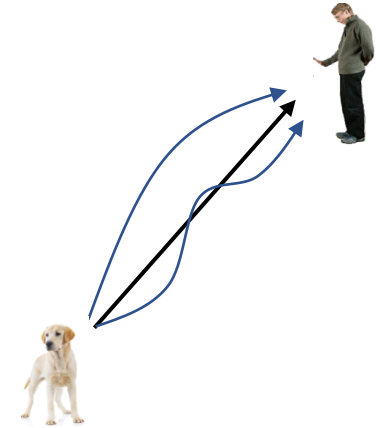
# Soft Q-learning



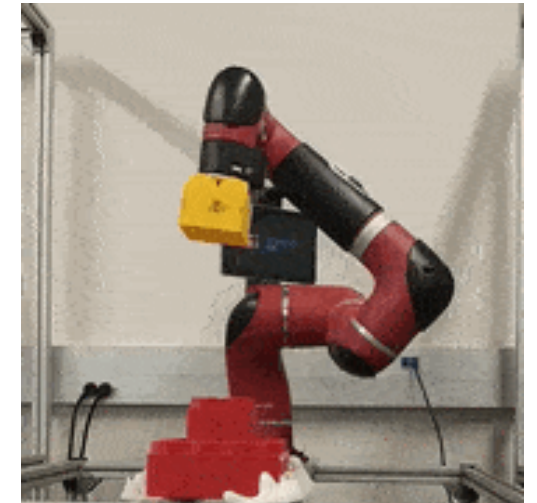
Exploration



Fine-tunability



Robustness



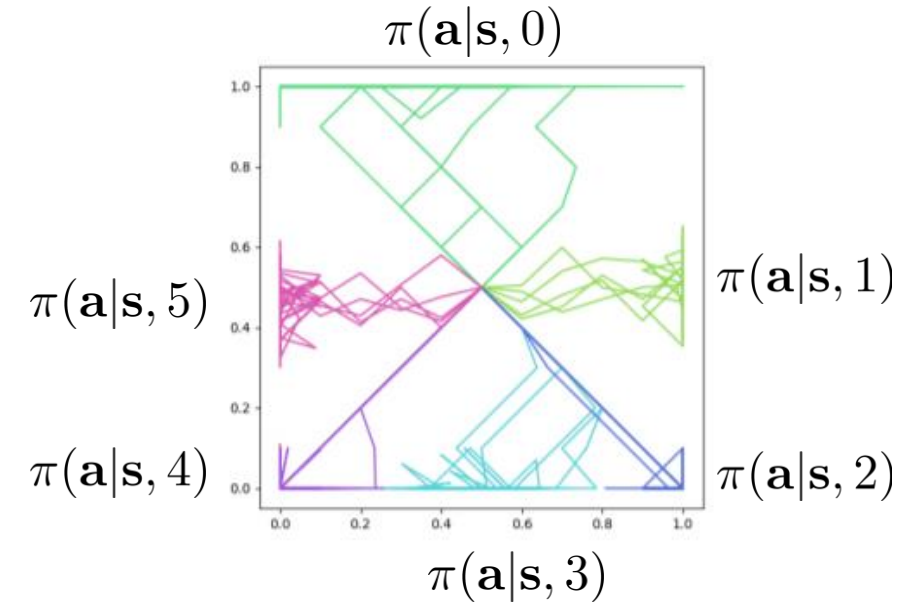
# Learning diverse skills

$$\pi(\mathbf{a}|\mathbf{s}, z)$$

↑  
task index

Why can't we just use MaxEnt RL

1. **action** entropy is not the same as **state** entropy  
agent can take very different actions, but land in similar states
2. MaxEnt policies are stochastic, but not always **controllable**  
intuitively, we want **low** diversity for a fixed  $z$ , high diversity *across*  $z$ 's



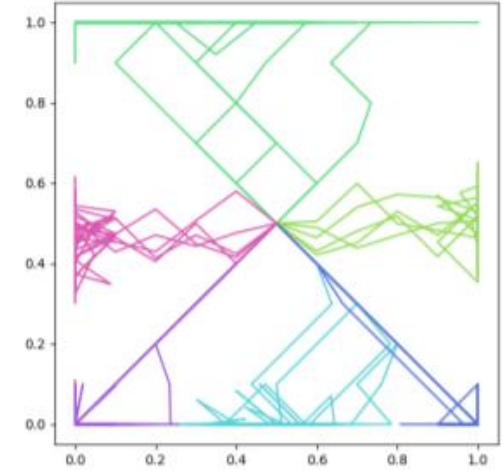
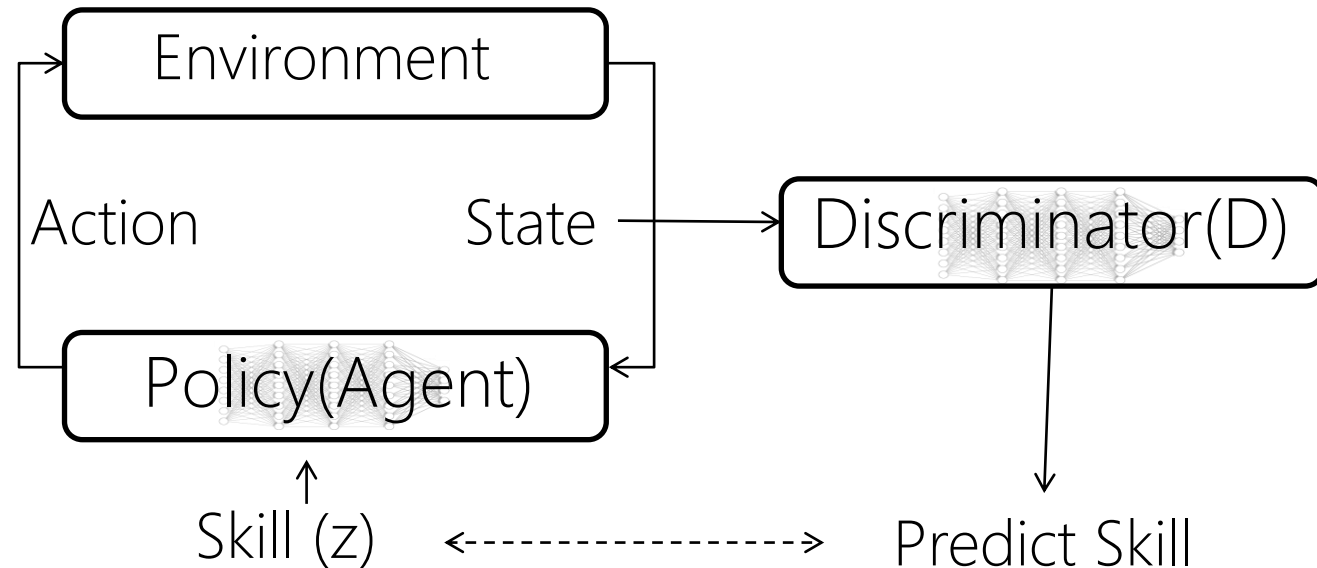
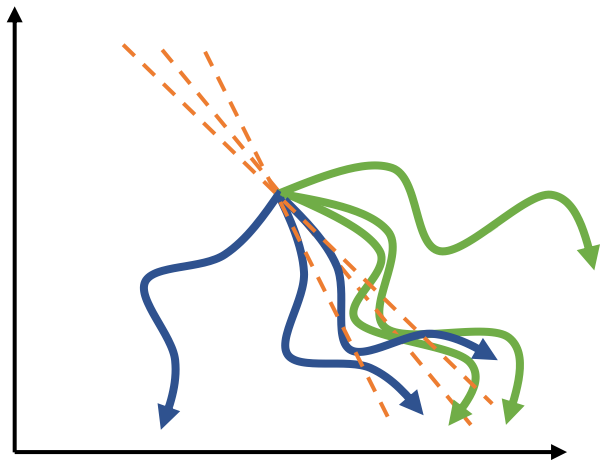
**Intuition:** different **skills** should visit different **state-space** regions

# Diversity-promoting reward function

$$\pi(\mathbf{a}|\mathbf{s}, z) = \arg \max_{\pi} \sum_z E_{\mathbf{s} \sim \pi(\mathbf{s}|z)} [r(\mathbf{s}, z)]$$

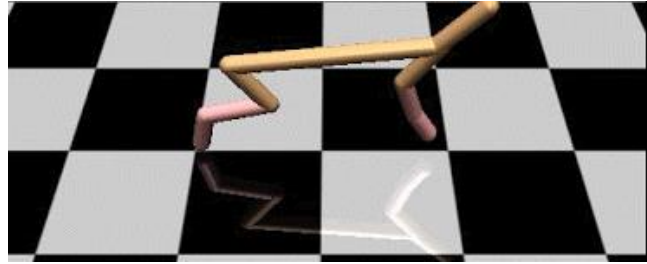
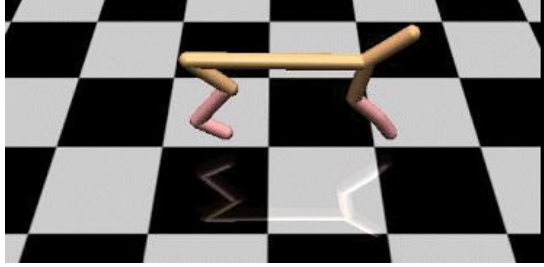
reward states that are unlikely for other  $z' \neq z$

$$r(\mathbf{s}, z) = \log p(z|\mathbf{s})$$

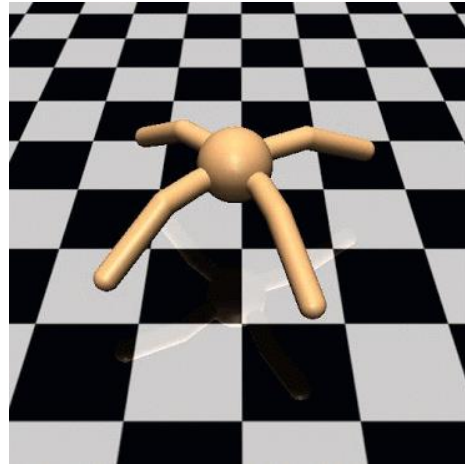
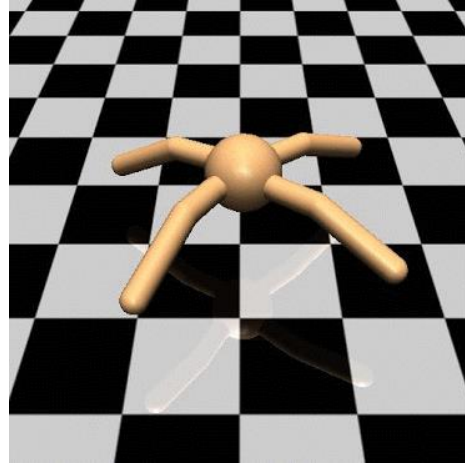




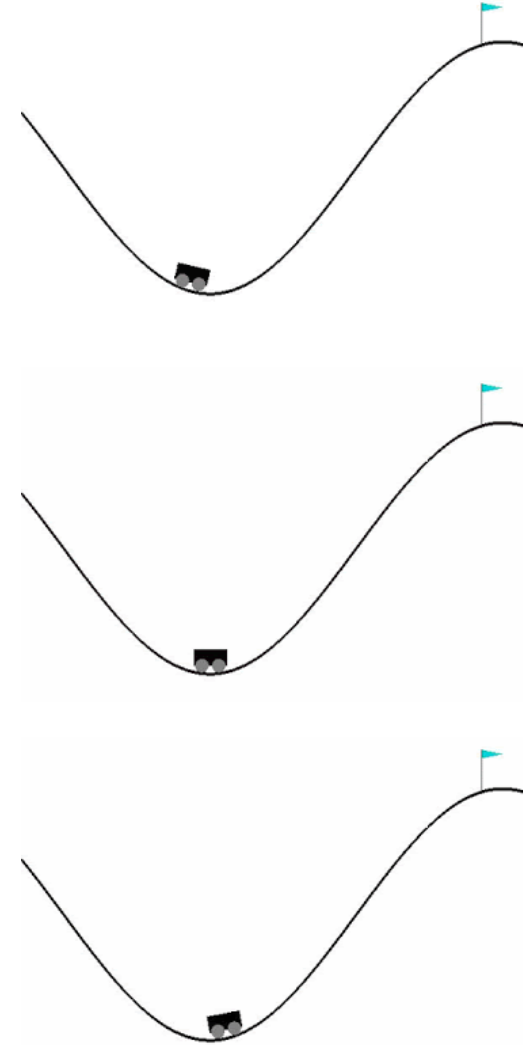
# Examples of learned tasks



Cheetah



Ant



Mountain car



# A connection to mutual information

$$\pi(\mathbf{a}|\mathbf{s}, z) = \arg \max_{\pi} \sum_z E_{\mathbf{s} \sim \pi(\mathbf{s}|z)} [r(\mathbf{s}, z)]$$

$$r(\mathbf{s}, z) = \log p(z|\mathbf{s})$$

$$I(z, \mathbf{s}) = H(z) - H(z|\mathbf{s})$$

maximized by using uniform prior  $p(z)$



minimized by maximizing  $\log p(z|\mathbf{s})$



# The Plan

Information-theoretic concepts

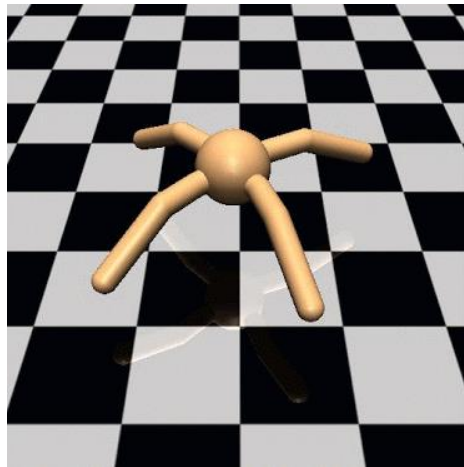
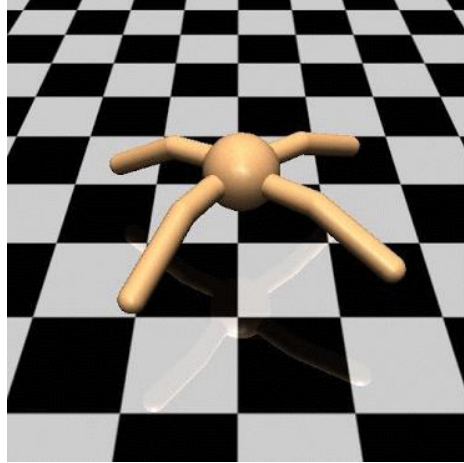
Skill discovery

Using discovered skills

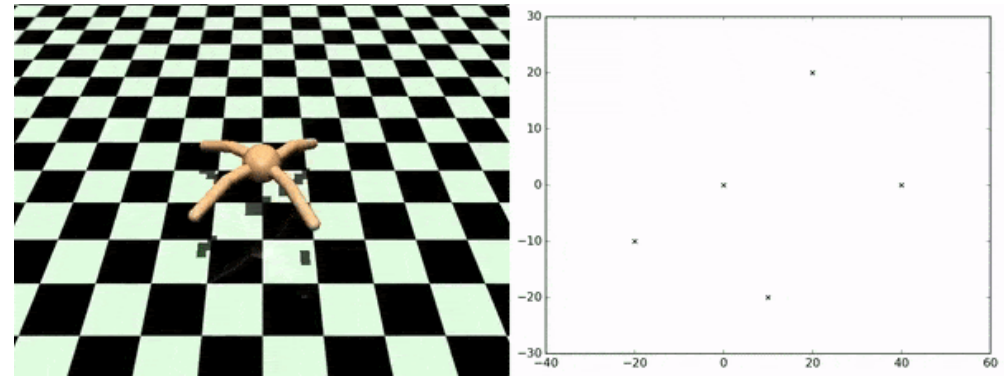
Hierarchical RL

# How to use learned skills?

$$\pi(\mathbf{a}|\mathbf{s}, z)$$

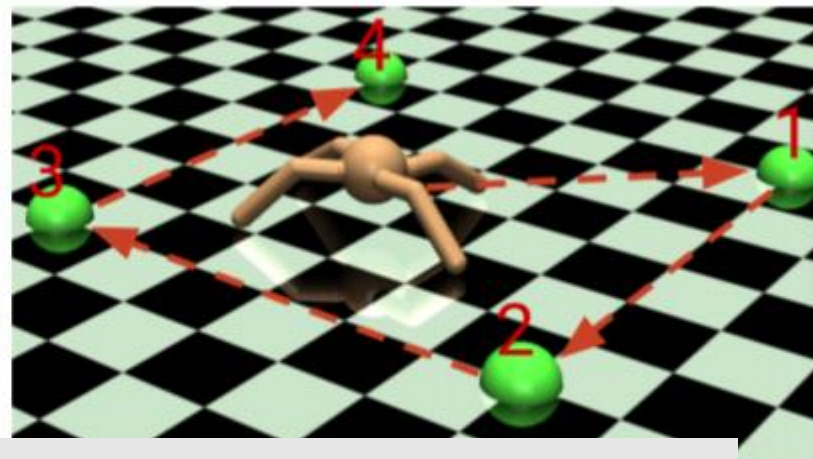
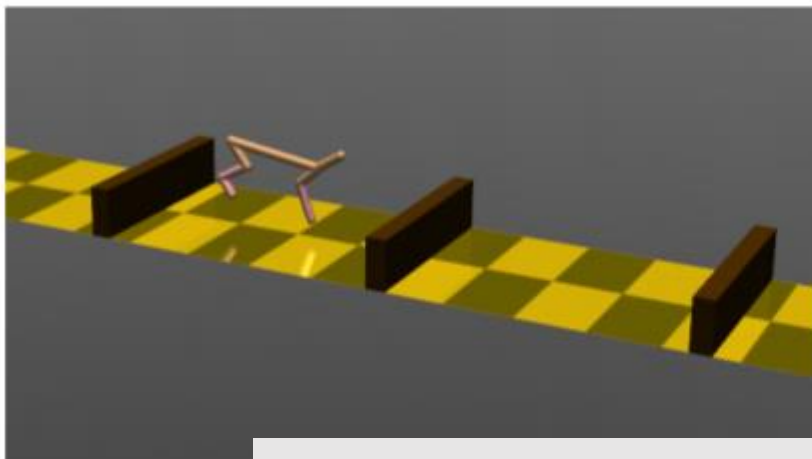


How can we use the learned skills to accomplish a task?



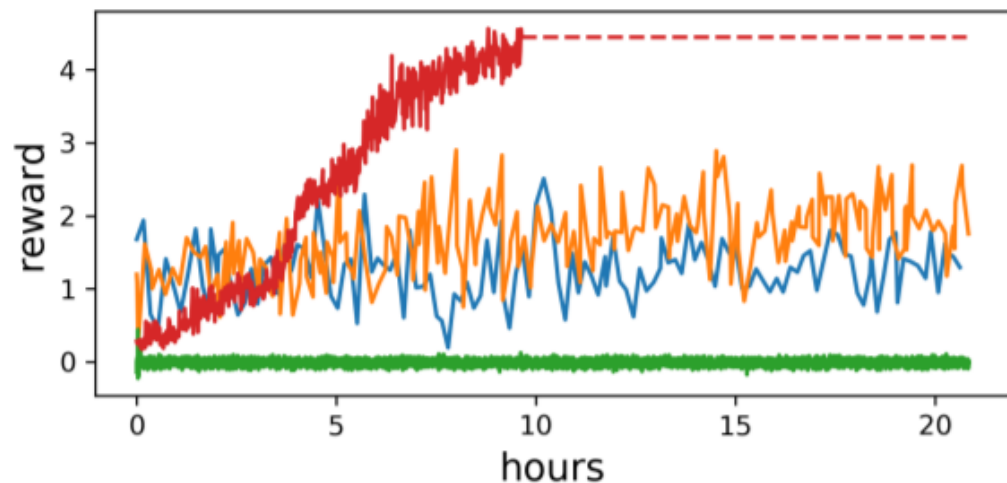
Learn a policy that operates on  $z$ 's

# Results: hierarchical RL

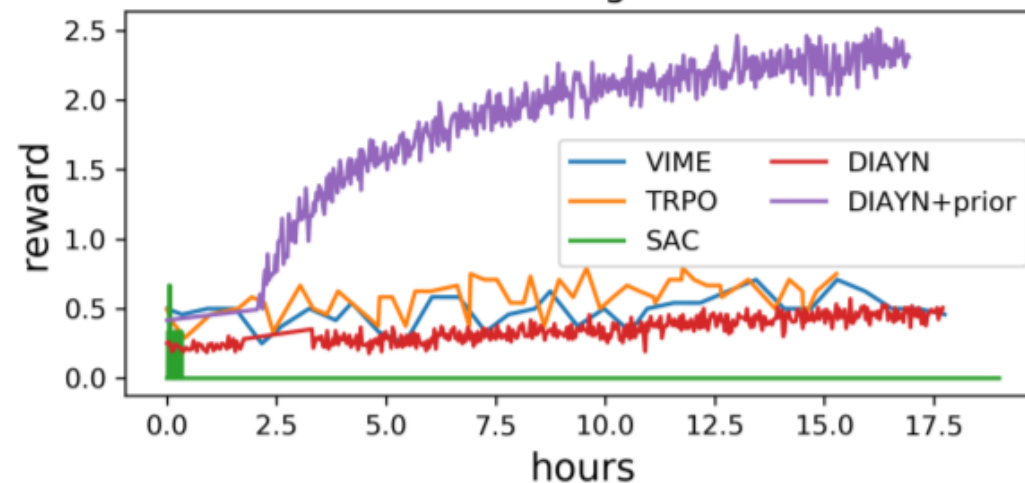


Can we do better?

Half Cheetah Hurdle

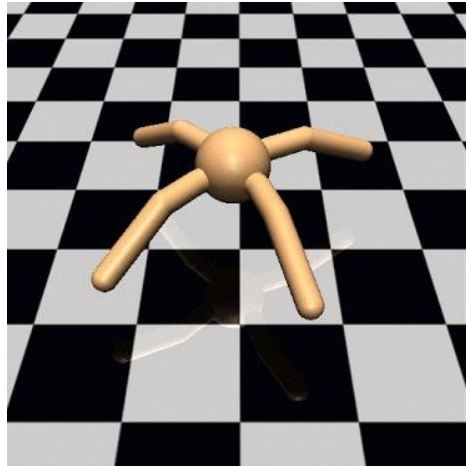
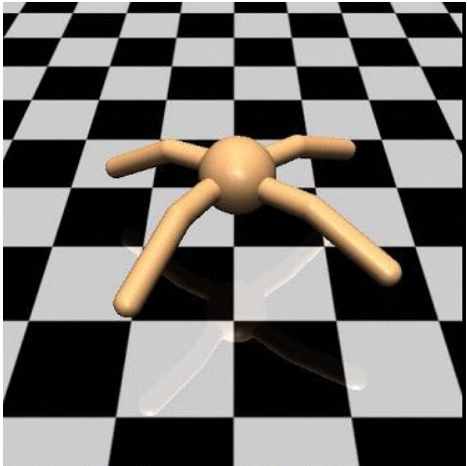


Ant Navigation

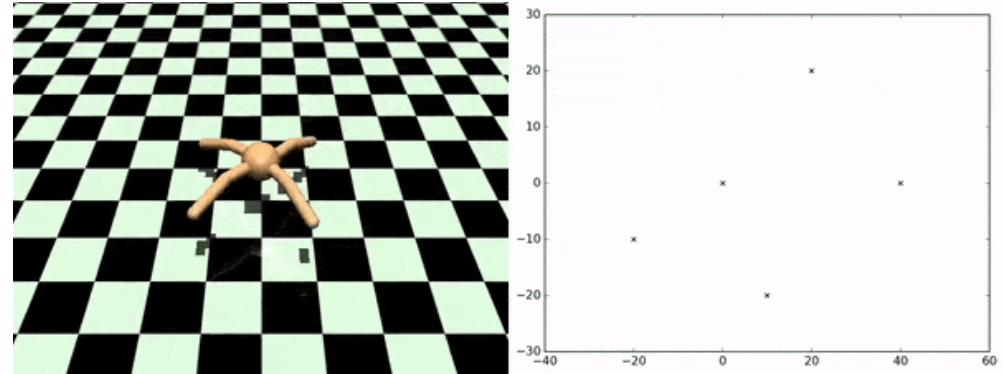


# What's the problem?

Skills might not be particularly useful



It's not very easy to use the learned skills



What makes a useful skill?

# What's the problem?

Consequences  
are **hard** to  
predict



Consequences  
are **easy** to  
predict



# Slightly different mutual information

$$I(z, \mathbf{s}) = H(z) - H(z|s)$$

$$\max \mathcal{I}(s', z | s) = \max \left( \mathcal{H}(s' | s) - \mathcal{H}(s' | s, z) \right)$$

~~$I(x, y | z)$~~   
 ~~$\mathcal{I}(\mathbf{x}; \mathbf{y}) = D_{\text{KL}}(p(\mathbf{x}, \mathbf{y}) || p(\mathbf{x})p(\mathbf{y}))$~~   
 Future hard to predict for different skills      Predictable future for a given skill

~~$p(x, y | z)$~~      ~~$p(x | z)$~~      ~~$p(y | z)$~~

$$\begin{aligned}
 I(s'; z | s) &\geq \mathbb{E}_s \mathbb{E}_z \mathbb{E}_{p(s' | s, z)} \left[ \log \frac{q_\phi(s' | s, z)}{p(s' | s)} \right] \\
 &\approx \mathbb{E}_s \mathbb{E}_z \mathbb{E}_{p(s' | s, z)} \left[ \log \frac{q_\phi(s' | s, z)}{\sum_{i=1}^L q_\phi(s' | s, z_i)} + \log L \right]
 \end{aligned}$$

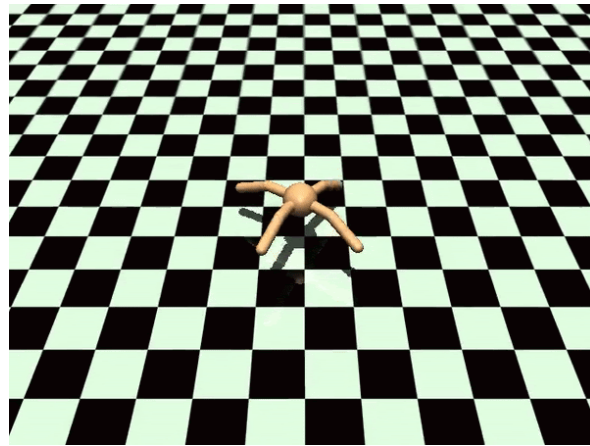
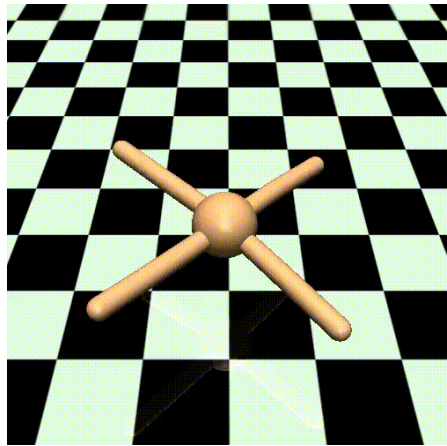


# Skill-dynamics model

We are learning a skill-dynamics model  $q(s' \mid s, z)$

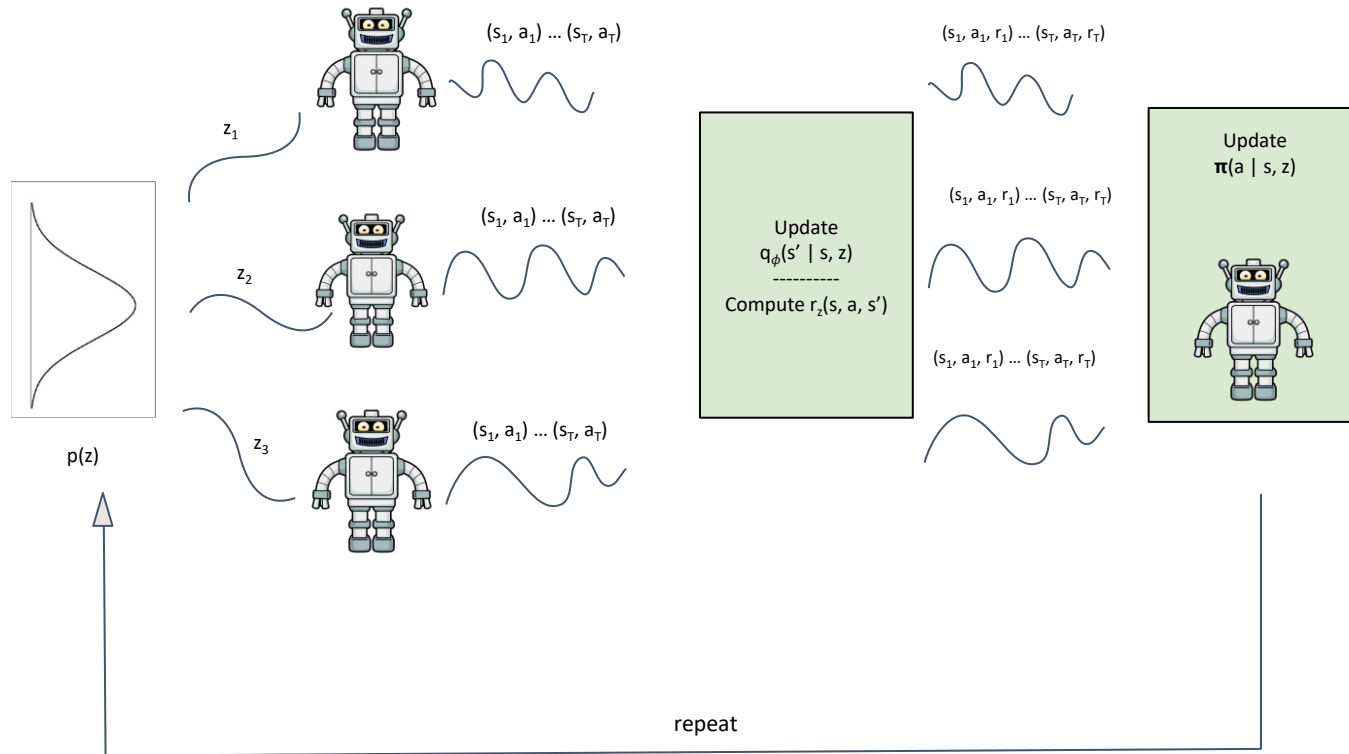
compared to conventional global dynamics  $p(s' \mid s, a)$

Skills are optimized specifically to make skill-dynamics easier to model





# DADS algorithm




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## Algorithm 1: Dynamics-Aware Discovery of Skills (DADS)

---

Initialize  $\pi, q_\phi$ ;

**while not converged do**

    Sample a skill  $z \sim p(z)$  every episode;

    Collect new  $M$  on-policy samples;

    Update  $q_\phi$  using  $K_1$  steps of gradient descent on  $M$  transitions;

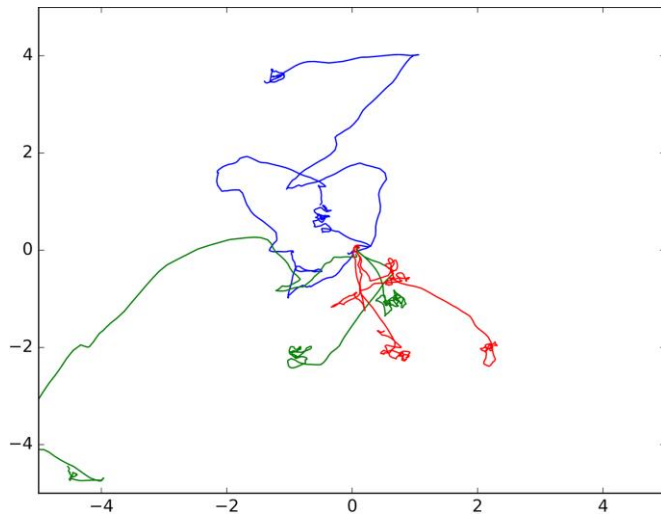
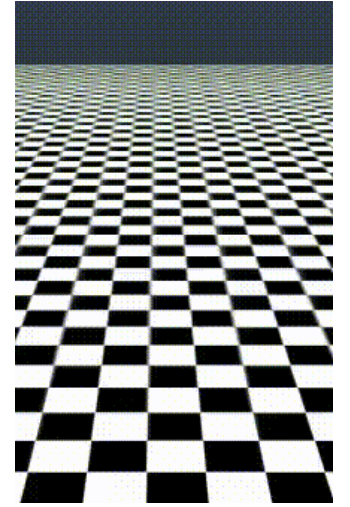
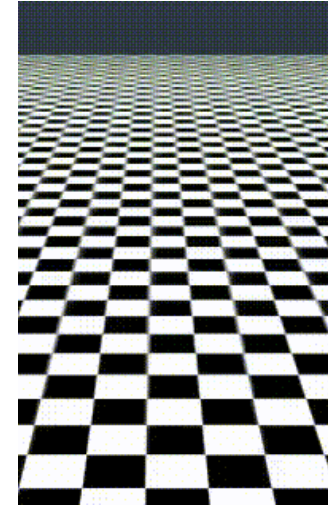
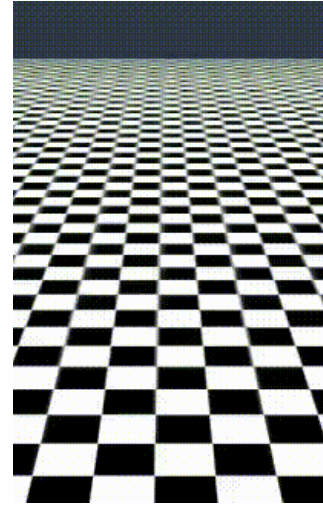
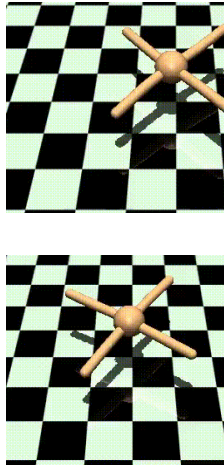
    Compute  $r_z(s, a, s')$  for  $M$  transitions;

    Update  $\pi$  using any RL algorithm;

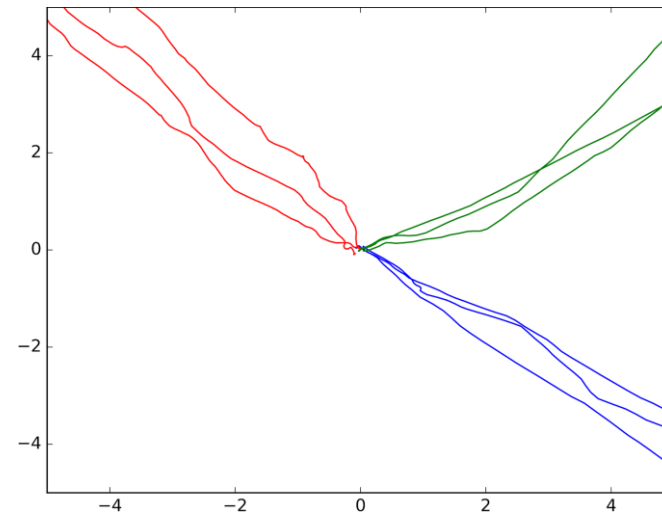
**end**

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# DADS results



DIAYN



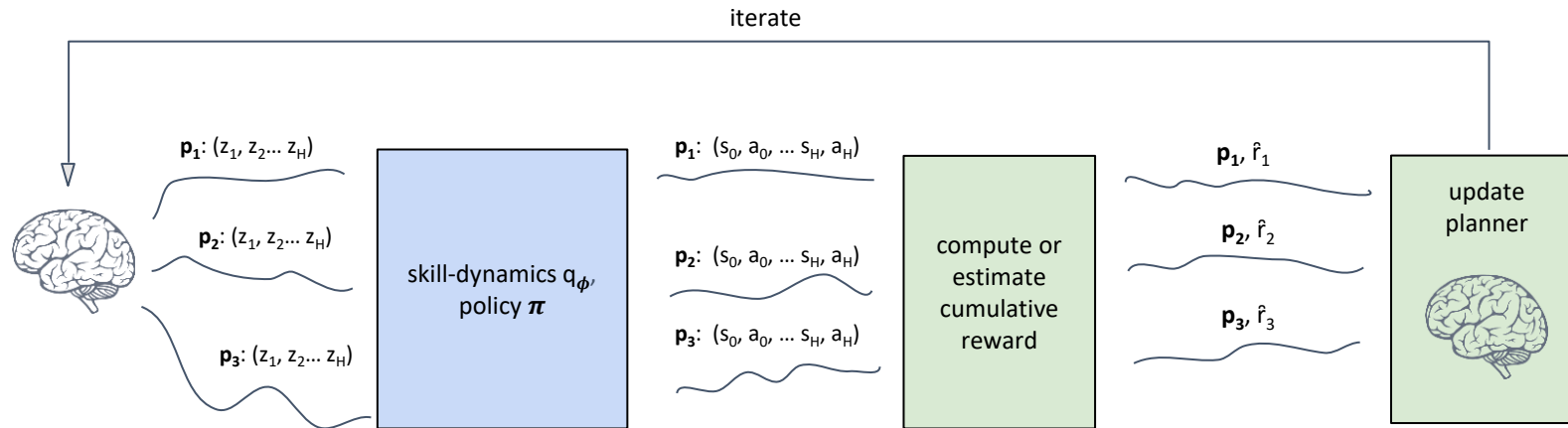
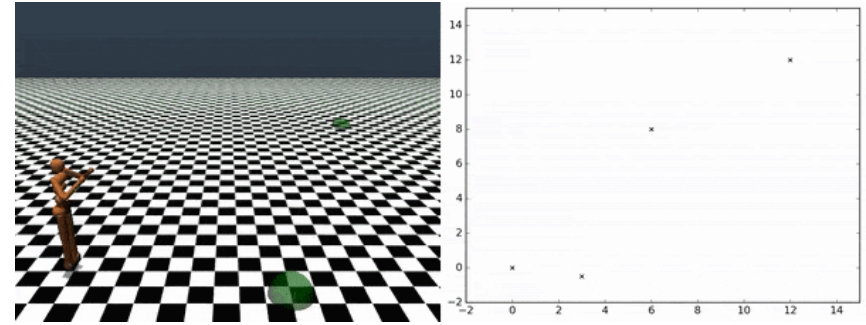
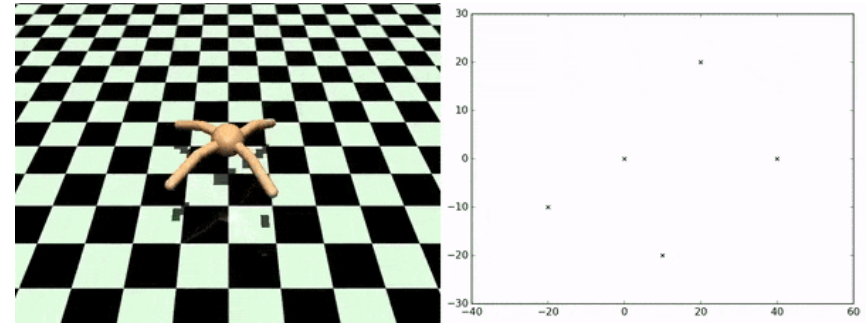
DADS

# Using learned skills

Use skill-dynamics for model-based planning

Plan for skills not actions

Tasks can be learned zero-shot



# Summary

- Two skill discovery algorithms that use mutual information
- Predictability can be used as a proxy for “usefulness”
- Method that optimizes for both, predictability and diversity
- Model-based planning in the skill space
- Opens new avenues such as unsupervised meta-RL
  - Gupta et al. *Unsupervised Meta-Learning for RL*, 2018



# The Plan

Information-theoretic concepts

Skill discovery

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Hierarchical RL

# Why Hierarchical RL?

Performing tasks at various levels of abstractions

Bake a cheesecake

Buy ingredients

Go to the store

Walk to the door

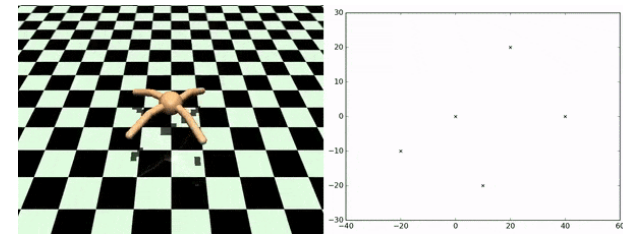
Take a step

Contract muscle X

Exploration

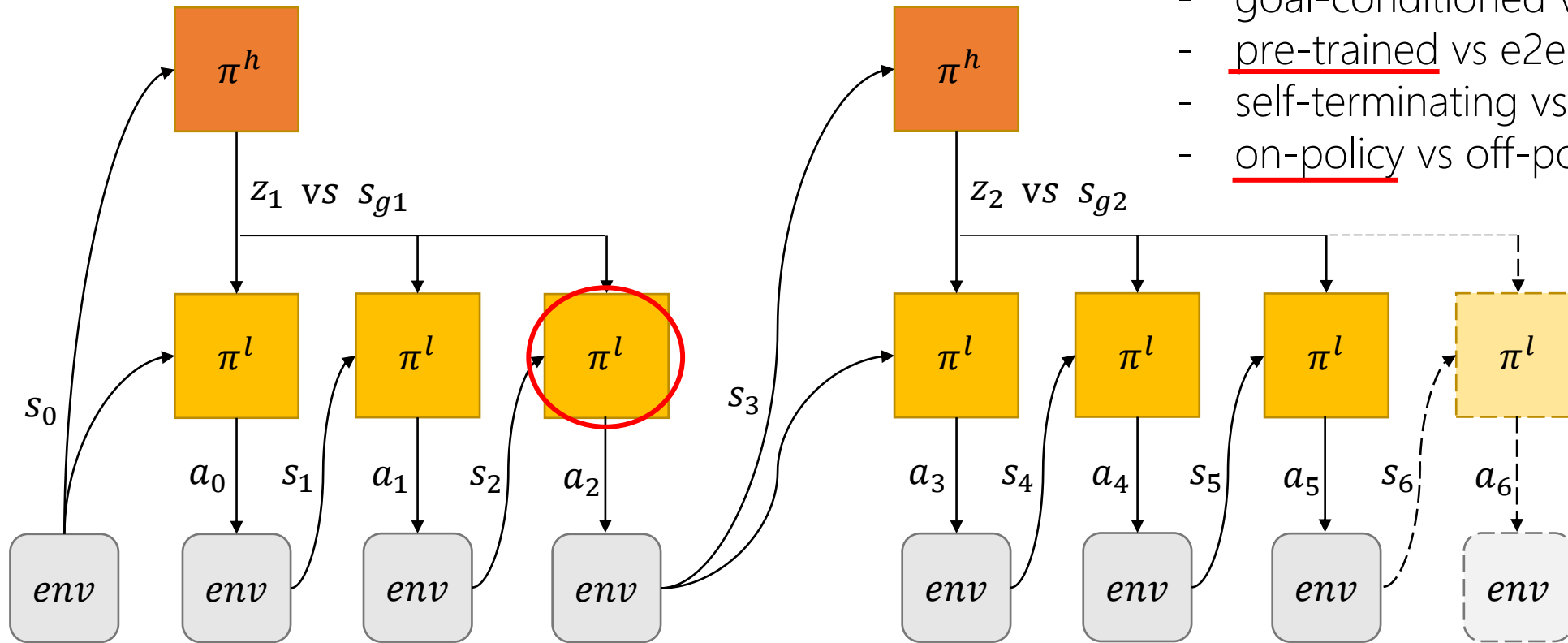


# Hierarchical RL – design choices



Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy

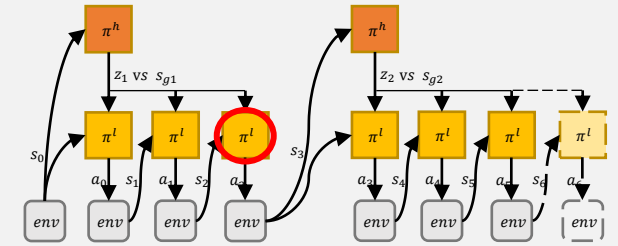


# Learning Locomotor Controllers

Command updated  
every K steps

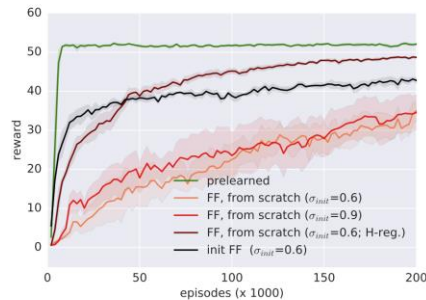
High-level  
controller

Low-level  
controller

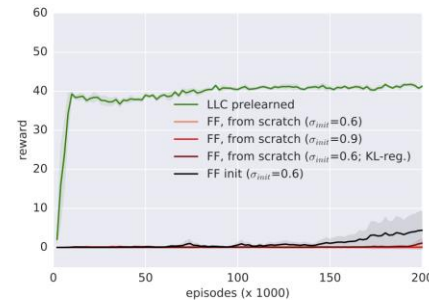


Design choices:

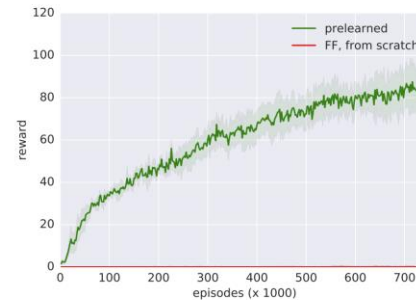
- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy



(a) target-seeking (easy)



(b) target-seeking (hard)

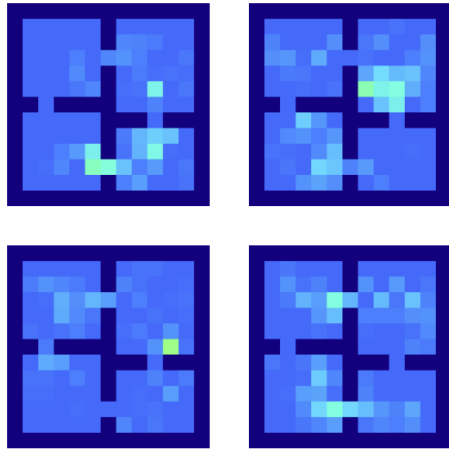


(c) soccer

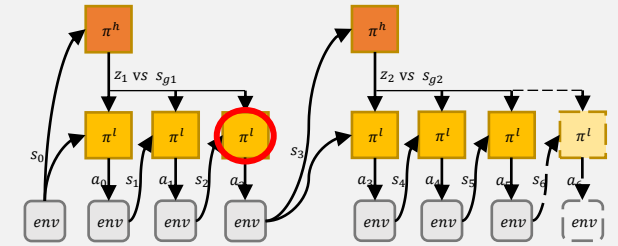
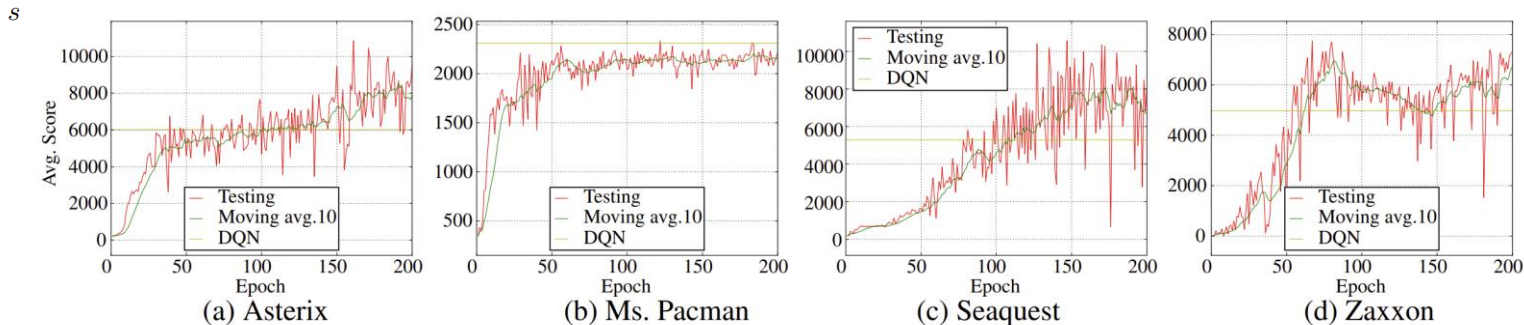


# Option Critic

A Markovian option  $\omega \in \Omega$  is a triple  $(\mathcal{I}_\omega, \pi_\omega, \beta_\omega)$  in which  $\mathcal{I}_\omega \subseteq \mathcal{S}$  is an initiation set,  $\pi_\omega$  is an *intra-option* policy, and  $\beta_\omega : \mathcal{S} \rightarrow [0, 1]$  is a termination function. We also assume that  $\forall s \in \mathcal{S}, \forall \omega \in \Omega : s \in \mathcal{I}_\omega$  (i.e., all options are available everywhere)



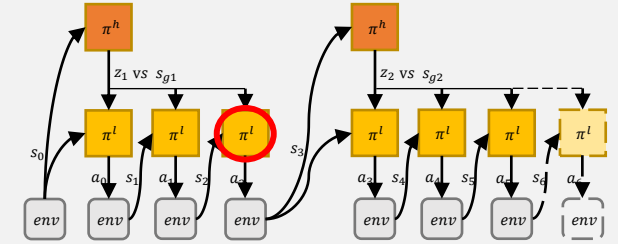
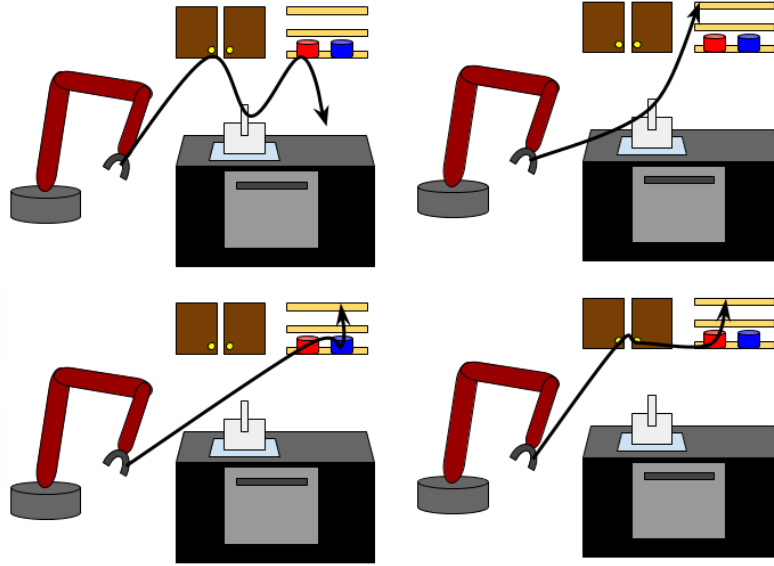
- Option is a self-terminating mini-policy
- Everything trained together with policy gradient



Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy

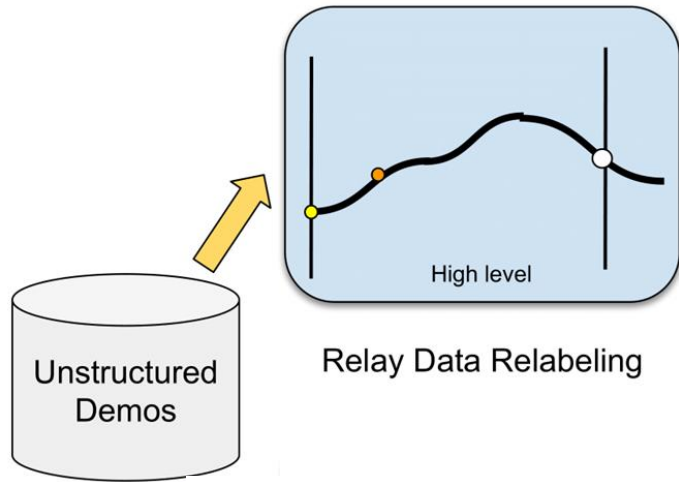
# Relay Policy Learning



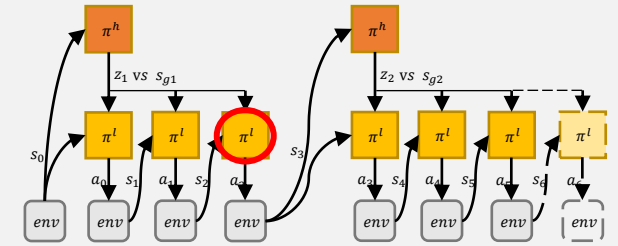
Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy

# Relay Policy Learning



- Goal-conditioned policies with relabeling
- Demonstrations to pre-train everything
- On-policy



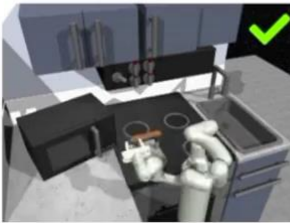
Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy

Long Horizon Goal



RPL (Ours)



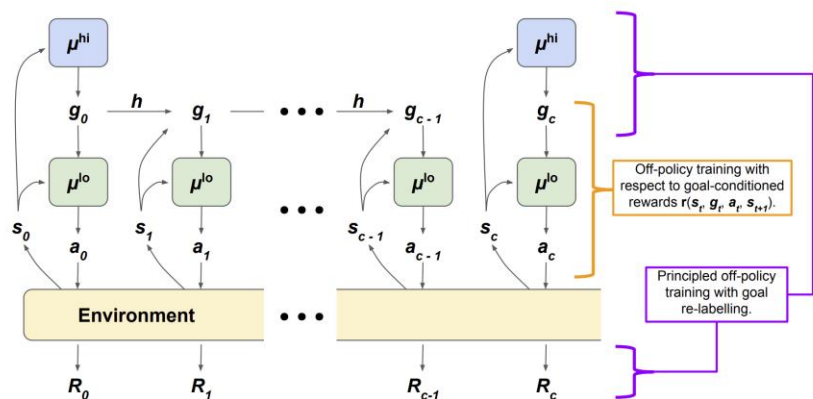
DAPG-GCBC



On-policy HIRO

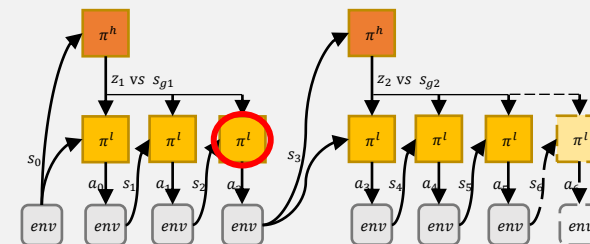


# HIRO



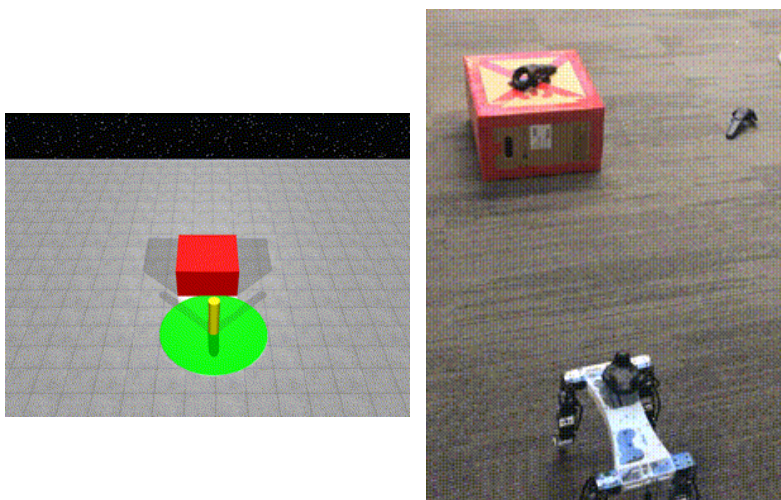
1. Collect experience  $s_t, g_t, a_t, R_t, \dots$ .
2. Train  $\mu^{lo}$  with experience transitions  $(s_t, g_t, a_t, r_t, s_{t+1}, g_{t+1})$  using  $g_t$  as additional state observation and reward given by goal-conditioned function  $r_t = -||s_t + g_t - s_{t+1}||_2$ .
3. Train  $\mu^{hi}$  on temporally-extended experience  $(s_t, \tilde{g}_t, \sum R_{t:t+c-1}, s_{t+c})$ , where  $\tilde{g}_t$  is re-labelled high-level action to maximize probability of past low-level actions  $a_{t:t+c-1}$ .
4. Repeat.

Figure 2: The design and basic training of HIRO. The lower-level policy interacts directly with the environment. The higher-level policy instructs the lower-level policy via high-level actions, or goals,  $g_t \in \mathbb{R}^{d_s}$  which it samples anew every  $c$  steps. On intermediate steps, a fixed goal transition function  $h$  determines the next step's goal. The goal simply instructs the lower-level policy to reach specific states, which allows the lower-level policy to easily learn from prior off-policy experience.

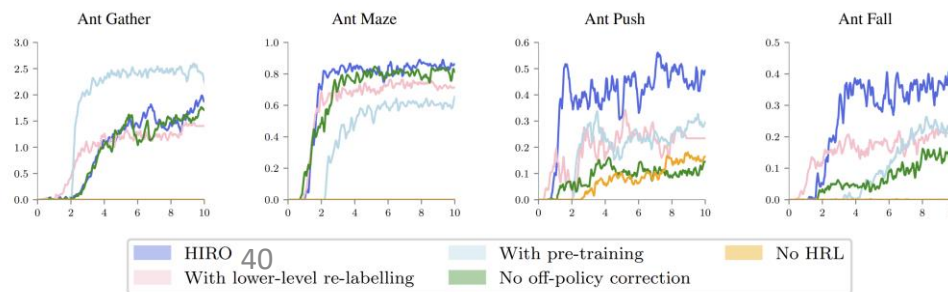


Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy



- Goal-conditioned policies with relabeling
- Off-policy training through off-policy corrections

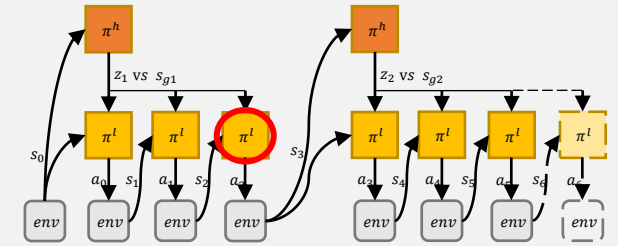


# HRL Summary

- Multiple design choices and frameworks
- Helps with exploration and temporally extended tasks
- Can be difficult to get it to work
- Seems like a natural direction for harder RL problems

Hypothesis	Experiments	Important?
(H1) Temporal training	Figures 2, 3	Yes, but only for the use of multi-step rewards ( $n$ -step returns).
(H2) Temporal exploration	Figures 2, 4	Yes, and this is important even for non-hierarchical exploration.
(H3) Semantic training	Figure 3	No.
(H4) Semantic exploration	Figure 4	Yes, and this is important even for non-hierarchical exploration.

Figure 5: A summary of our conclusions on the benefits of hierarchy.



Design choices:

- goal-conditioned vs not
- pre-trained vs e2e
- self-terminating vs fixed rate
- on-policy vs off-policy

# The Plan

Information-theoretic concepts

Skill discovery

Using discovered skills

Hierarchical RL

# Next Week

Can the agent **learn continuously** over their life-time?

Lifelong learning – **Nov 4**