knn

April 13, 2020

[ ]:
```
from google.colab import drive

drive.mount('/content/drive', force_remount=True)

# enter the foldername in your Drive where you have saved the unzipped
# 'cs231n' folder containing the '.py', 'classifiers' and 'datasets'
# folders.
# e.g. 'cs231n/assignments/assignment1/cs231n/'
FOLDERNAME = None

assert FOLDERNAME is not None, "[!] Enter the foldername."

%cd drive/My\ Drive
%cp -r $FOLDERNAME ../../
%cd ../../
%cd cs231n/datasets/
!bash get_datasets.sh
%cd ../../
```

1 k-Nearest Neighbor (kNN) exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the assignments page on the course website.

The kNN classifier consists of two stages:

- During training, the classifier takes the training data and simply remembers it
- During testing, kNN classifies every test image by comparing to all training images and transfering the labels of the k most similar training examples
- The value of k is cross-validated

In this exercise you will implement these steps and understand the basic Image Classification pipeline, cross-validation, and gain proficiency in writing efficient, vectorized code.

[1]:
```
# Run some setup code for this notebook.

import random
```

```
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the␣
 ↪notebook
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/
 ↪autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

[2]:
```
# Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may cause␣
 ↪memory issue)
try:
   del X_train, y_train
   del X_test, y_test
   print('Clear previously loaded data.')
except:
   pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

[3]:
```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
 ↪'ship', 'truck']
```

```
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

[4]:
```
# Subsample the data for more efficient code execution in this exercise
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
```

```
# Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)
```

```
(5000, 3072) (500, 3072)
```

[5]:
```
from cs231n.classifiers import KNearestNeighbor

# Create a kNN classifier instance.
# Remember that training a kNN classifier is a noop:
# the Classifier simply remembers the data and does no further processing
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
```

We would now like to classify the test data with the kNN classifier. Recall that we can break down this process into two steps:

1. First we must compute the distances between all test examples and all train examples.
2. Given these distances, for each test example we find the k nearest examples and have them vote for the label

Lets begin with computing the distance matrix between all training and test examples. For example, if there are **Ntr** training examples and **Nte** test examples, this stage should result in a **Nte x Ntr** matrix where each element (i,j) is the distance between the i-th test and j-th train example.

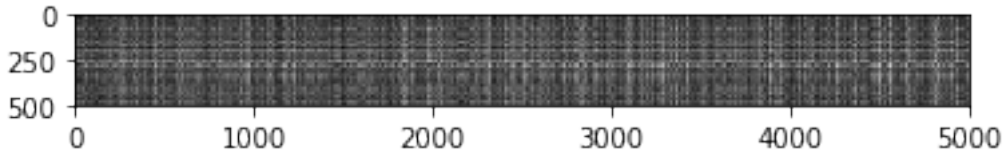
Note: For the three distance computations that we require you to implement in this notebook, you may not use the np.linalg.norm() function that numpy provides.

First, open `cs231n/classifiers/k_nearest_neighbor.py` and implement the function `compute_distances_two_loops` that uses a (very inefficient) double loop over all pairs of (test, train) examples and computes the distance matrix one element at a time.

[6]:
```
# Open cs231n/classifiers/k_nearest_neighbor.py and implement
# compute_distances_two_loops.
# Done.

# Test your implementation:
dists = classifier.compute_distances_two_loops(X_test)
print(dists.shape)
```

```
(500, 5000)
```

[7]:
```
# We can visualize the distance matrix: each row is a single test example and
# its distances to training examples
plt.imshow(dists, interpolation='none')
plt.show()
```



Inline Question 1

Notice the structured patterns in the distance matrix, where some rows or columns are visible brighter. (Note that with the default color scheme black indicates low distances while white indicates high distances.)

- What in the data is the cause behind the distinctly bright rows?
- What causes the columns?

YourAnswer : 1. The distinctively bright rows suggest that some pixel values in those datapoints are very different from that of the training example. Hence, larger Euclidiean distance and more different.

2. The difference in the pixle values of the training example to that of test data causes these columns.

[8]:
```
# Now implement the function predict_labels and run the code below:
# We use k = 1 (which is Nearest Neighbor).
y_test_pred = classifier.predict_labels(dists, k=1)

# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

```
Got 137 / 500 correct => accuracy: 0.274000
```

You should expect to see approximately 27% accuracy. Now lets try out a larger `k`, say `k = 5`:

[9]:
```
y_test_pred = classifier.predict_labels(dists, k=5)
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

```
Got 139 / 500 correct => accuracy: 0.278000
```

You should expect to see a slightly better performance than with `k = 1`.

Inline Question 2

We can also use other distance metrics such as L1 distance. For pixel values $p_{ij}^{(k)}$ at location (i, j) of some image I_k,

the mean μ across all pixels over all images is

$$\mu = \frac{1}{nhw}\sum_{k=1}^{n}\sum_{i=1}^{h}\sum_{j=1}^{w} p_{ij}^{(k)}$$

And the pixel-wise mean μ_{ij} across all images is

$$\mu_{ij} = \frac{1}{n}\sum_{k=1}^{n} p_{ij}^{(k)}.$$

The general standard deviation σ and pixel-wise standard deviation σ_{ij} is defined similarly.

Which of the following preprocessing steps will not change the performance of a Nearest Neighbor classifier that uses L1 distance? Select all that apply. 1. Subtracting the mean μ ($\tilde{p}_{ij}^{(k)} = p_{ij}^{(k)} - \mu$.) 2. Subtracting the per pixel mean μ_{ij} ($\tilde{p}_{ij}^{(k)} = p_{ij}^{(k)} - \mu_{ij}$.) 3. Subtracting the mean μ and dividing by the standard deviation σ. 4. Subtracting the pixel-wise mean μ_{ij} and dividing by the pixel-wise standard deviation σ_{ij}. 5. Rotating the coordinate axes of the data.

YourAnswer : **1 and 3**.

YourExplanation : Pixel values hold true even after the given mathematical modifiers.

[10]:
```
# Now lets speed up distance matrix computation by using partial vectorization
# with one loop. Implement the function compute_distances_one_loop and run the
# code below:
# Done
dists_one = classifier.compute_distances_one_loop(X_test)

# To ensure that our vectorized implementation is correct, we make sure that it
# agrees with the naive implementation. There are many ways to decide whether
# two matrices are similar; one of the simplest is the Frobenius norm. In case
# you haven't seen it before, the Frobenius norm of two matrices is the square
# root of the squared sum of differences of all elements; in other words, reshape
# the matrices into vectors and compute the Euclidean distance between them.
difference = np.linalg.norm(dists - dists_one, ord='fro')
print('One loop difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

```
One loop difference was: 0.000000
Good! The distance matrices are the same
```

[12]:
```
# Now implement the fully vectorized version inside compute_distances_no_loops
# and run the code
# Done
dists_two = classifier.compute_distances_no_loops(X_test)
```

```
# check that the distance matrix agrees with the one we computed before:
difference = np.linalg.norm(dists - dists_two, ord='fro')
print('No loop difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

```
No loop difference was: 0.000000
Good! The distance matrices are the same
```

[13]:
```
# Let's compare how fast the implementations are
def time_function(f, *args):
    """
    Call a function f with args and return the time (in seconds) that it took␣
 ↪to execute.
    """
    import time
    tic = time.time()
    f(*args)
    toc = time.time()
    return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops, X_test)
print('Two loop version took %f seconds' % two_loop_time)

one_loop_time = time_function(classifier.compute_distances_one_loop, X_test)
print('One loop version took %f seconds' % one_loop_time)

no_loop_time = time_function(classifier.compute_distances_no_loops, X_test)
print('No loop version took %f seconds' % no_loop_time)

# You should see significantly faster performance with the fully vectorized␣
 ↪implementation!

# NOTE: depending on what machine you're using,
# you might not see a speedup when you go from two loops to one loop,
# and might even see a slow-down.
```

```
Two loop version took 32.679075 seconds
One loop version took 40.930478 seconds
No loop version took 0.726208 seconds
```

1.0.1 Cross-validation

We have implemented the k-Nearest Neighbor classifier but we set the value k = 5 arbitrarily. We will now determine the best value of this hyperparameter with cross-validation.

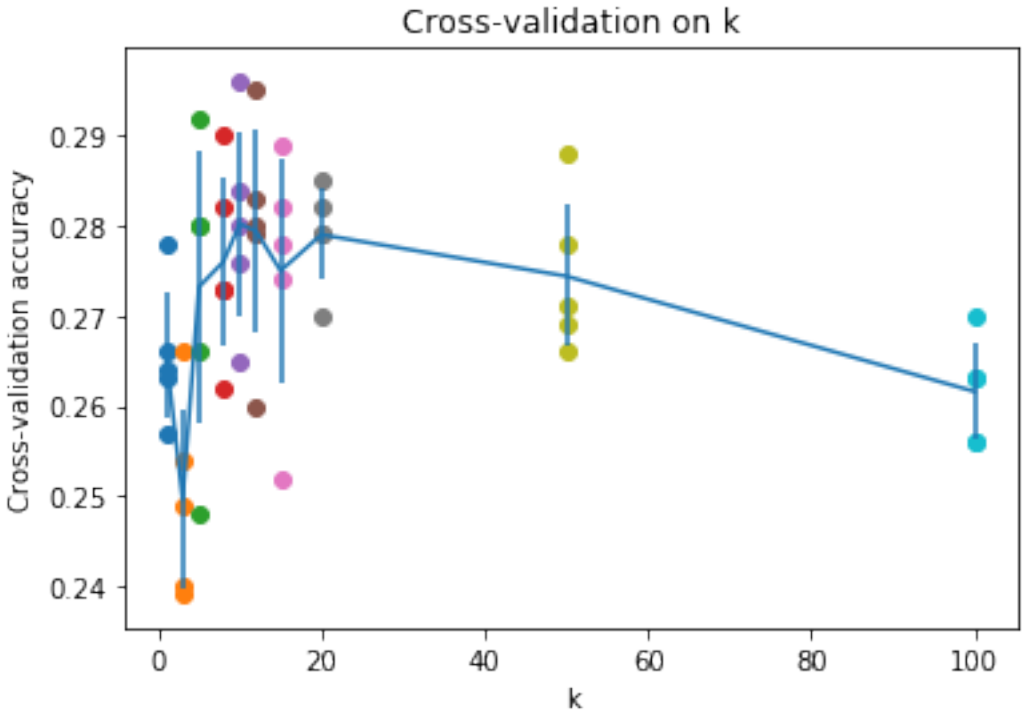
[30]:
```
num_folds = 5
k_choices = [1, 3, 5, 8, 10, 12, 15, 20, 50, 100]

X_train_folds = []
y_train_folds = []
################################################################################
# TODO:                                                                        #
# Split up the training data into folds. After splitting, X_train_folds and    #
# y_train_folds should each be lists of length num_folds, where                #
# y_train_folds[i] is the label vector for the points in X_train_folds[i].     #
# Hint: Look up the numpy array_split function.                                #
################################################################################
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
X_train_folds = np.array_split(X_train, 5)
y_train_folds = np.array_split(y_train, 5)
# print(y_train_folds[1].shape, X_train_folds[1].shape)
pass
# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# A dictionary holding the accuracies for different values of k that we find
# when running cross-validation. After running cross-validation,
# k_to_accuracies[k] should be a list of length num_folds giving the different
# accuracy values that we found when using that value of k.
k_to_accuracies = {}


################################################################################
# TODO:                                                                        #
# Perform k-fold cross validation to find the best value of k. For each        #
# possible value of k, run the k-nearest-neighbor algorithm num_folds times,   #
# where in each case you use all but one of the folds as training data and the #
# last fold as a validation set. Store the accuracies for all fold and all     #
# values of k in the k_to_accuracies dictionary.                               #
################################################################################
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
for k in k_choices:
    k_to_accuracies[k] = []
    for n in range(num_folds):
        # read through all training and test data
        classifier.train(np.vstack(X_train_folds[:n] + X_train_folds[(n+1):]),
                         np.hstack(y_train_folds[:n] + y_train_folds[(n+1):]))
        dists = classifier.compute_distances_no_loops(X_train_folds[n])
        y_pred_valid = classifier.predict_labels(dists, k=k)
        num_correct = np.sum(y_pred_valid == y_train_folds[n])
        k_to_accuracies[k].append(float(num_correct)/len(y_train_folds[n]))

pass
```

```
# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out the computed accuracies
for k in sorted(k_to_accuracies):
    for accuracy in k_to_accuracies[k]:
        print('k = %d, accuracy = %f' % (k, accuracy))
```

```
k = 1, accuracy = 0.263000
k = 1, accuracy = 0.257000
k = 1, accuracy = 0.264000
k = 1, accuracy = 0.278000
k = 1, accuracy = 0.266000
k = 3, accuracy = 0.239000
k = 3, accuracy = 0.249000
k = 3, accuracy = 0.240000
k = 3, accuracy = 0.266000
k = 3, accuracy = 0.254000
k = 5, accuracy = 0.248000
k = 5, accuracy = 0.266000
k = 5, accuracy = 0.280000
k = 5, accuracy = 0.292000
k = 5, accuracy = 0.280000
k = 8, accuracy = 0.262000
k = 8, accuracy = 0.282000
k = 8, accuracy = 0.273000
k = 8, accuracy = 0.290000
k = 8, accuracy = 0.273000
k = 10, accuracy = 0.265000
k = 10, accuracy = 0.296000
k = 10, accuracy = 0.276000
k = 10, accuracy = 0.284000
k = 10, accuracy = 0.280000
k = 12, accuracy = 0.260000
k = 12, accuracy = 0.295000
k = 12, accuracy = 0.279000
k = 12, accuracy = 0.283000
k = 12, accuracy = 0.280000
k = 15, accuracy = 0.252000
k = 15, accuracy = 0.289000
k = 15, accuracy = 0.278000
k = 15, accuracy = 0.282000
k = 15, accuracy = 0.274000
k = 20, accuracy = 0.270000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.282000
k = 20, accuracy = 0.285000
```

```
k = 50, accuracy = 0.271000
k = 50, accuracy = 0.288000
k = 50, accuracy = 0.278000
k = 50, accuracy = 0.269000
k = 50, accuracy = 0.266000
k = 100, accuracy = 0.256000
k = 100, accuracy = 0.270000
k = 100, accuracy = 0.263000
k = 100, accuracy = 0.256000
k = 100, accuracy = 0.263000
```

[31]:
```
# plot the raw observations
for k in k_choices:
    accuracies = k_to_accuracies[k]
    plt.scatter([k] * len(accuracies), accuracies)

# plot the trend line with error bars that correspond to standard deviation
accuracies_mean = np.array([np.mean(v) for k,v in sorted(k_to_accuracies.
 ↪items())])
accuracies_std = np.array([np.std(v) for k,v in sorted(k_to_accuracies.
 ↪items())])
plt.errorbar(k_choices, accuracies_mean, yerr=accuracies_std)
plt.title('Cross-validation on k')
plt.xlabel('k')
plt.ylabel('Cross-validation accuracy')
plt.show()
```



[36]:
```
# Based on the cross-validation results above, choose the best value for k,
# retrain the classifier using all the training data, and test it on the test
# data. You should be able to get above 28% accuracy on the test data.
best_k = k_choices[accuracies_mean.argmax()]

classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
y_test_pred = classifier.predict(X_test, k=best_k)

# Compute and display the accuracy
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))
```

```
Got 141 / 500 correct => accuracy: 0.282000
```

2 Inline Question 3

Which of the following statements about k-Nearest Neighbor (k-NN) are true in a classification setting, and for all k? Select all that apply. 1. The decision boundary of the k-NN classifier is linear. 2. The training error of a 1-NN will always be lower than that of 5-NN. 3. The test error of a 1-NN will always be lower than that of a 5-NN. 4. The time needed to classify a test example with the k-NN classifier grows with the size of the training set. 5. None of the above.

YourAnswer : **2 and 4**

YourExplanation : **2.** If we consider the training dataset as the test set too, then NN will be the same point thus reducing error to be 0. However, this wouldn't be the case with a 5-NN classifier under same conditions.

4. Since, we have to go through the entire training set to find nearest neighbors, as size of training set increases, time taken to classifiy on test set also increases.

---

3 IMPORTANT

This is the end of this question. Please do the following:

1. Click `File -> Save` to make sure the latest checkpoint of this notebook is saved to your Drive.
2. Execute the cell below to download the modified `.py` files back to your drive.

[ ]:
```
import os

FOLDER_TO_SAVE = os.path.join('drive/My Drive/', FOLDERNAME)
FILES_TO_SAVE = ['cs231n/classifiers/k_nearest_neighbor.py']
```

```
for files in FILES_TO_SAVE:
  with open(os.path.join(FOLDER_TO_SAVE, '/'.join(files.split('/')[1:])), 'w')␣
 ↪as f:
    f.write(''.join(open(files).readlines()))
```

```
from builtins import range
from builtins import object
import numpy as np
from past.builtins import xrange


class KNearestNeighbor(object):
    """ a kNN classifier with L2 distance """

    def __init__(self):
        pass

    def train(self, X, y):
        """
        Train the classifier. For k-nearest neighbors this is just
        memorizing the training data.

        Inputs:
        - X: A numpy array of shape (num_train, D) containing the training data
          consisting of num_train samples each of dimension D.
        - y: A numpy array of shape (N,) containing the training labels, where
             y[i] is the label for X[i].
        """
        self.X_train = X
        self.y_train = y

    def predict(self, X, k=1, num_loops=0):
        """
        Predict labels for test data using this classifier.

        Inputs:
        - X: A numpy array of shape (num_test, D) containing test data consisting
             of num_test samples each of dimension D.
        - k: The number of nearest neighbors that vote for the predicted labels.
        - num_loops: Determines which implementation to use to compute distances
          between training points and testing points.

        Returns:
        - y: A numpy array of shape (num_test,) containing predicted labels for the
          test data, where y[i] is the predicted label for the test point X[i].
        """
        if num_loops == 0:
            dists = self.compute_distances_no_loops(X)
        elif num_loops == 1:
            dists = self.compute_distances_one_loop(X)
        elif num_loops == 2:
            dists = self.compute_distances_two_loops(X)
        else:
            raise ValueError('Invalid value %d for num_loops' % num_loops)

        return self.predict_labels(dists, k=k)

    def compute_distances_two_loops(self, X):
        """
        Compute the distance between each test point in X and each training point
        in self.X_train using a nested loop over both the training data and the
        test data.

        Inputs:
        - X: A numpy array of shape (num_test, D) containing test data.

        Returns:
        - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
          is the Euclidean distance between the ith test point and the jth training
          point.
        """
        num_test = X.shape[0]
        num_train = self.X_train.shape[0]
```

```
        dists = np.zeros((num_test, num_train))
        for i in range(num_test):
            for j in range(num_train):
                #####################################################################
                # TODO:                                                             #
                # Compute the l2 distance between the ith test point and the jth    #
                # training point, and store the result in dists[i, j]. You should   #
                # not use a loop over dimension, nor use np.linalg.norm().          #
                #####################################################################
                # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
                dists[i,j] = np.sqrt(np.sum(np.square(self.X_train[j,:] - X[i,:])))
                pass
                # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
        return dists

    def compute_distances_one_loop(self, X):
        """
        Compute the distance between each test point in X and each training point
        in self.X_train using a single loop over the test data.

        Input / Output: Same as compute_distances_two_loops
        """
        num_test = X.shape[0]
        num_train = self.X_train.shape[0]
        dists = np.zeros((num_test, num_train))
        for i in range(num_test):
            #######################################################################
            # TODO:                                                               #
            # Compute the l2 distance between the ith test point and all training #
            # points, and store the result in dists[i, :].                        #
            # Do not use np.linalg.norm().                                        #
            #######################################################################
            # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
            dists[i,:] = np.sqrt(np.sum((self.X_train - X[i, :])**2, axis=1))
            pass
            # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
        return dists

    def compute_distances_no_loops(self, X):
        """
        Compute the distance between each test point in X and each training point
        in self.X_train using no explicit loops.

        Input / Output: Same as compute_distances_two_loops
        """
        num_test = X.shape[0]
        num_train = self.X_train.shape[0]
        dists = np.zeros((num_test, num_train))
        #########################################################################
        # TODO:                                                                 #
        # Compute the l2 distance between all test points and all training      #
        # points without using any explicit loops, and store the result in      #
        # dists.                                                                #
        #                                                                       #
        # You should implement this function using only basic array operations; #
        # in particular you should not use functions from scipy,                #
        # nor use np.linalg.norm().                                             #
        #                                                                       #
        # HINT: Try to formulate the l2 distance using matrix multiplication    #
        #       and two broadcast sums.                                         #
        #########################################################################
        # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
        # (x-y)^2 = x^2 + y^2 - 2xy (expand fo
        dists = np.sqrt(np.sum(self.X_train**2, axis=1)+np.sum(X**2, axis=1)[:,np.newaxis]-
2*np.dot(X, self.X_train.T))
        pass
        # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
        return dists
```

```
    def predict_labels(self, dists, k=1):
        """
        Given a matrix of distances between test points and training points,
        predict a label for each test point.

        Inputs:
        - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
          gives the distance betwen the ith test point and the jth training point.

        Returns:
        - y: A numpy array of shape (num_test,) containing predicted labels for the
          test data, where y[i] is the predicted label for the test point X[i].
        """
        num_test = dists.shape[0]
        y_pred = np.zeros(num_test)
        for i in range(num_test):
            # A list of length k storing the labels of the k nearest neighbors to
            # the ith test point.
            closest_y = []
            ###########################################################################
            # TODO:                                                                   #
            # Use the distance matrix to find the k nearest neighbors of the ith    #
            # testing point, and use self.y_train to find the labels of these       #
            # neighbors. Store these labels in closest_y.                           #
            # Hint: Look up the function numpy.argsort.                             #
            ###########################################################################
            # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

            # getting k points with lowest distance of the i'th training examples
            closest_y = self.y_train[np.argsort(dists[i])][0:k]
            pass

            # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
            ###########################################################################
            # TODO:                                                                   #
            # Now that you have found the labels of the k nearest neighbors, you    #
            # need to find the most common label in the list closest_y of labels.   #
            # Store this label in y_pred[i]. Break ties by choosing the smaller     #
            # label.                                                                  #
            ###########################################################################
            # *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

            y_pred[i] = np.bincount(closest_y).argmax()
            pass

            # *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

        return y_pred
```