2023Fall Probability & Statistics for EECS 2023/12/15

Homework 10

Professor: Ziyu Shao & Dingzhu Wen *Due: 2023/12/24 10:59pm*

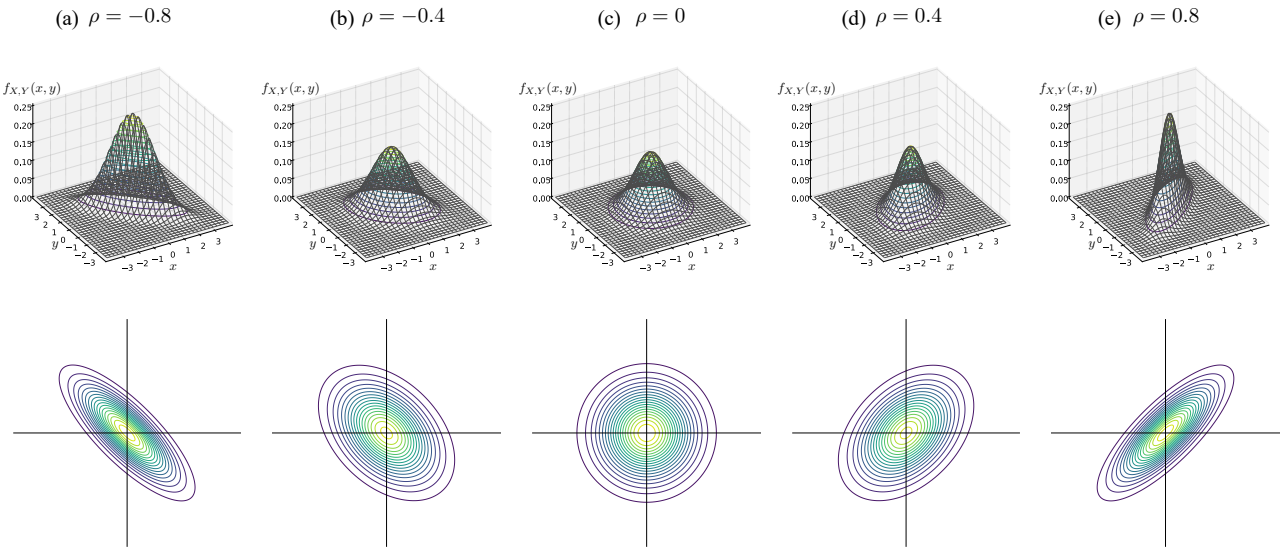
This is the programming assignment using Python. You are required to use Python for the programming part and submit all things in Jupyter Notebook format, including Python codes, simulation results, analysis, discussions, tables, figures, *etc.* Always keep the **academic integrity** in mind and remember to give credit to any source that inspires you.

1. Use the method of inverse transform sampling to obtain samples from each of the following continuous distributions:
   (a) Logistic distribution with CDF $F(x) = e^x/(1+e^x), x \in R$.
   (b) Rayleigh distribution with CDF $F(x) = 1 - e^{-x^2/2}, x > 0$.
   (c) Exponential distribution with CDF $F(x) = 1 - e^{-x}, x > 0$.
2. Use the method of inverse transform sampling to obtain samples from each of the following discrete distributions:
   (a) Bernoulli distribution Bern(0.5).
   (b) Binomial distribution Bin(20, 0.5).
   (c) Geometric distribution Geom(0.5).
   (d) Negative Binomial distribution NBin(10, 0.5).
   (e) Poisson distribution Pois(1)
3. (a) Use the Box-Muller method to obtain samples from standard Normal distribution $\mathcal{N}(0,1)$.
   (b) Use the Acceptance-Rejection method to obtain samples from standard Normal distribution $\mathcal{N}(0,1)$.
   (c) Compare the pros and cons of the above two methods (e.g. variance, sampling efficiency, running speed, etc.)
   (d) Use the following transformation to generate samples from bivariate Normal distribution with correlation coefficient ρ:

$$X = Z$$
$$Y = \rho Z + \sqrt{1-\rho^2}W$$

   where $-1 < \rho < 1$, Z and W are i.i.d. random variables satisfying $\mathcal{N}(0,1)$.

(e) The joint pdf function of bivariate Normal distribution with correlation coefficient ρ and the corresponding contour(or isocontour) are shown in the following figure for your reference:



4. (a) Use the Acceptance-Rejection Method to generate a random variable with distribution Beta(2, 4)

   (b) Use Monte Carlo methods to evaluate the integration

$$\int_0^1 \frac{4}{1+x^2} dx.$$

   (c) Use Monte Carlo methods to evaluate the integration

$$\int_0^4 \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}}\, dx.$$

   (d) Use Monte Carlo methods to estimate the value of π

   (e) Use importance sampling method to evaluate the probability of rare event $c = P(Y > 8)$, where $Y \sim N(0, 1)$.

5. (Optional Challenging Problem) Use Monte Carlo methods to compute the area of Batman Curve and compare it with the exact value. Please refer to the following webpage for more details: `https://mathworld.wolfram.com/BatmanCurve.html`