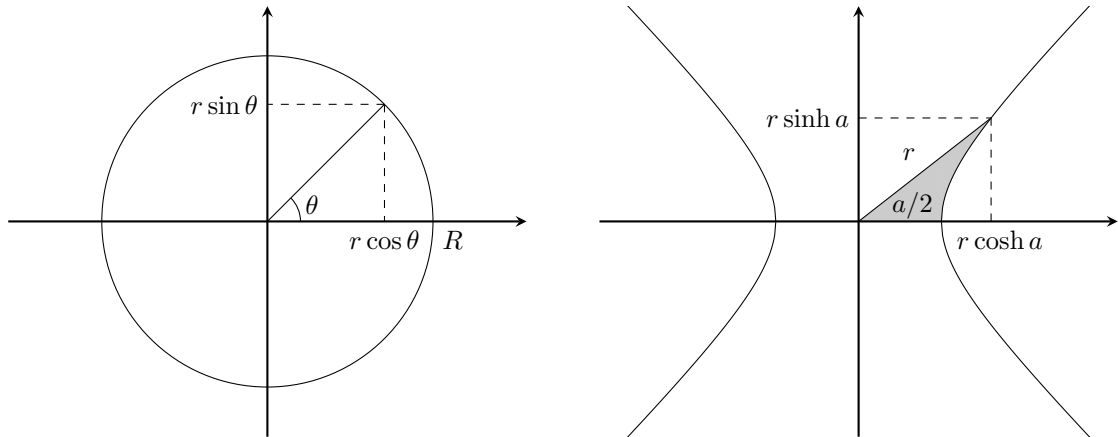


Hyperbolic functions are just like trigonometric functions, but for hyperbolas; that is, the sine hyperbolic and cosine hyperbolic functions output the vertical and horizontal projections respectively of the radial line which intersects the hyperbola - the area of the region enclosed by this line, the hyperbola, and the coordinate axis is half the parameter. Hyperbolas are defined as this: $x^2 - y^2 = a^2$.



And these are the relevant definitions, though they will not be on the test.

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad (1)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad (2)$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad (3)$$

$$\coth x = \frac{\cosh x}{\sinh x} \quad (4)$$

Then we derived a useful formula for hyperbolic tangent;

$$\begin{aligned} \tanh x &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ &= \frac{\frac{e^x}{e^{-x}} - 1}{\frac{e^x}{e^{-x}} + 1} = \frac{e^{2x} - 1}{e^{2x} + 1} \end{aligned}$$

And these hyperbolic functions have identities similar to the trigonometrics; for instance,

$$\cosh^2 x - \sinh^2 x = 1$$

Which can be proved by arithmetically working it out from the definition.