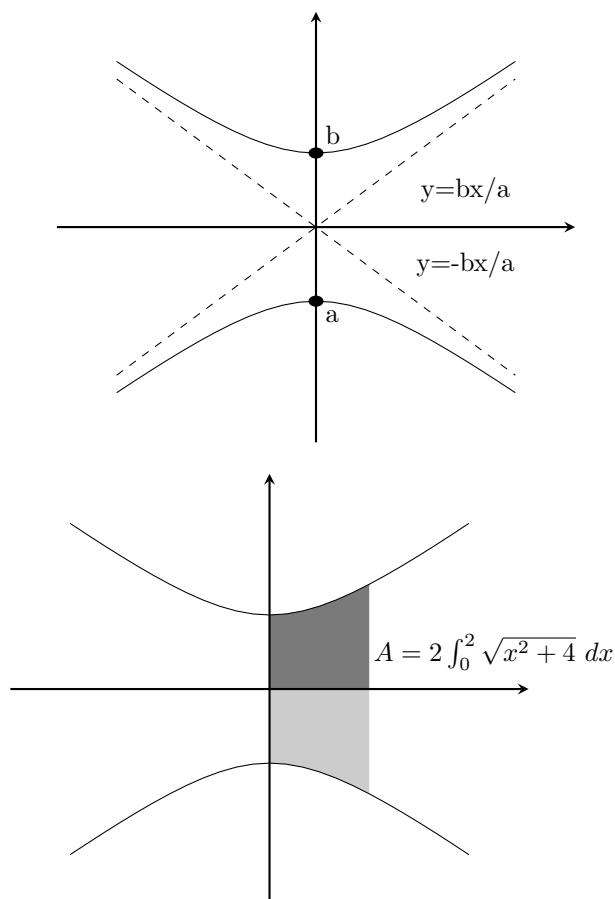We will need to know the limit test for the comparison of improper integrals.

EXAMPLE Find the area between the branches of the hyperbola $y^2 - x^2 = 4$ for $0 \le x \le 1$.

$$\Rightarrow y^2 = 4 + x^2 = \begin{cases} \sqrt{x^2 + 4} & for \quad y < 0 \\ -\sqrt{x^2 + 4} & for \quad y \ge 0 \end{cases} = \begin{cases} x = a \sinh t \\ y = a \cosh t \end{cases}$$

General hyperbola: $y^2/b^2 - x^2/a^2 = 1$

$$A = 2 \int_0^2 \sqrt{x^2 + 4} \; dx$$

And we could solve this by paramaterizing the equation or using a substitution. Both hyperbolic.

We are definitely going to be expected to know hyperbolic properties, such as $\sinh 2t = 2 \sinh t \cosh t$. These are similar to their trigonometric counterparts.

In this class, out problems involved a lot of previous material, such as solving for inverse hyperbolic functions.

We ended class by studying some volumes.

1
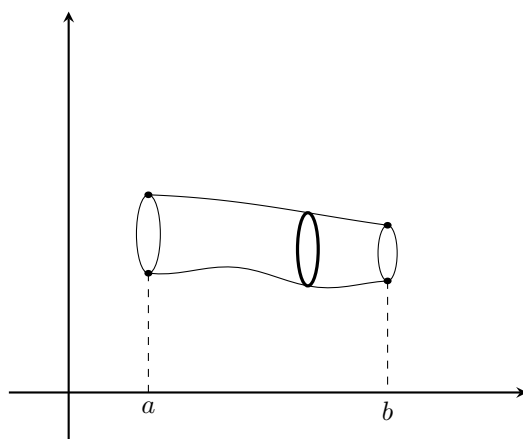
$$dV = A(x)\ dx; \qquad \int dV = V = \int_a^b A(x)\ dx$$

EXAMPLE Volume of conic frustum: