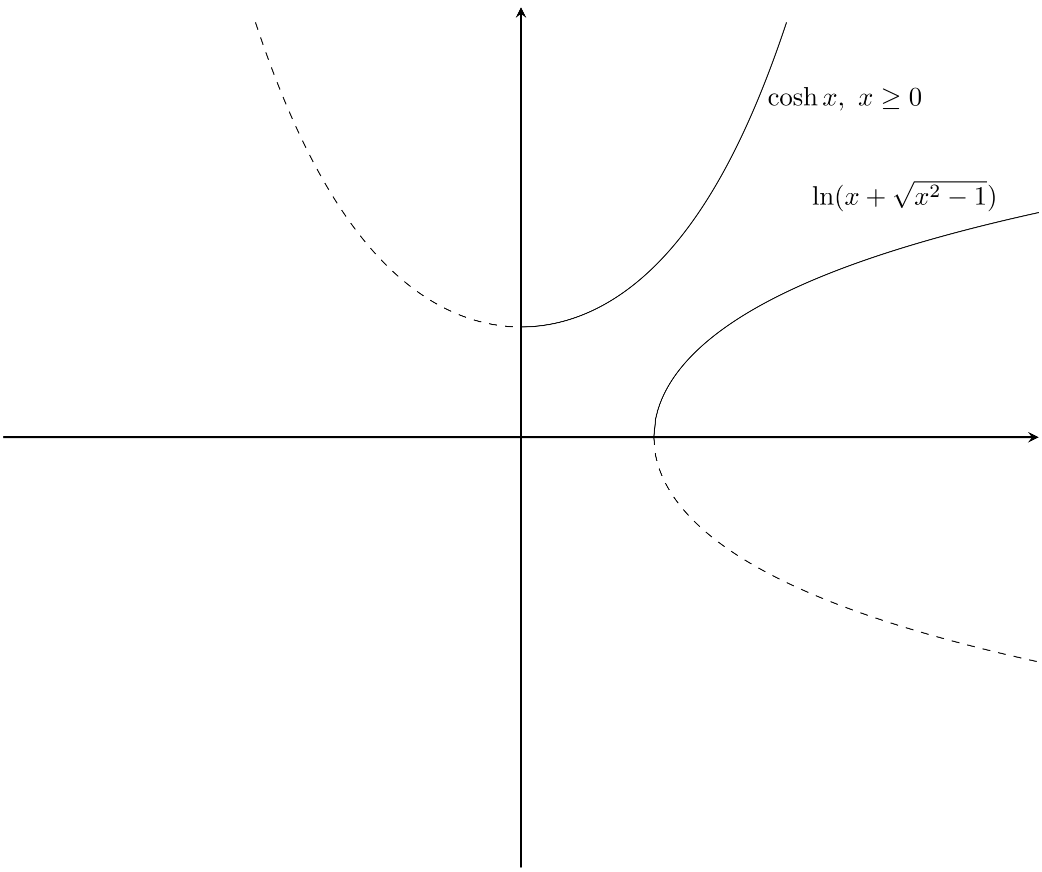
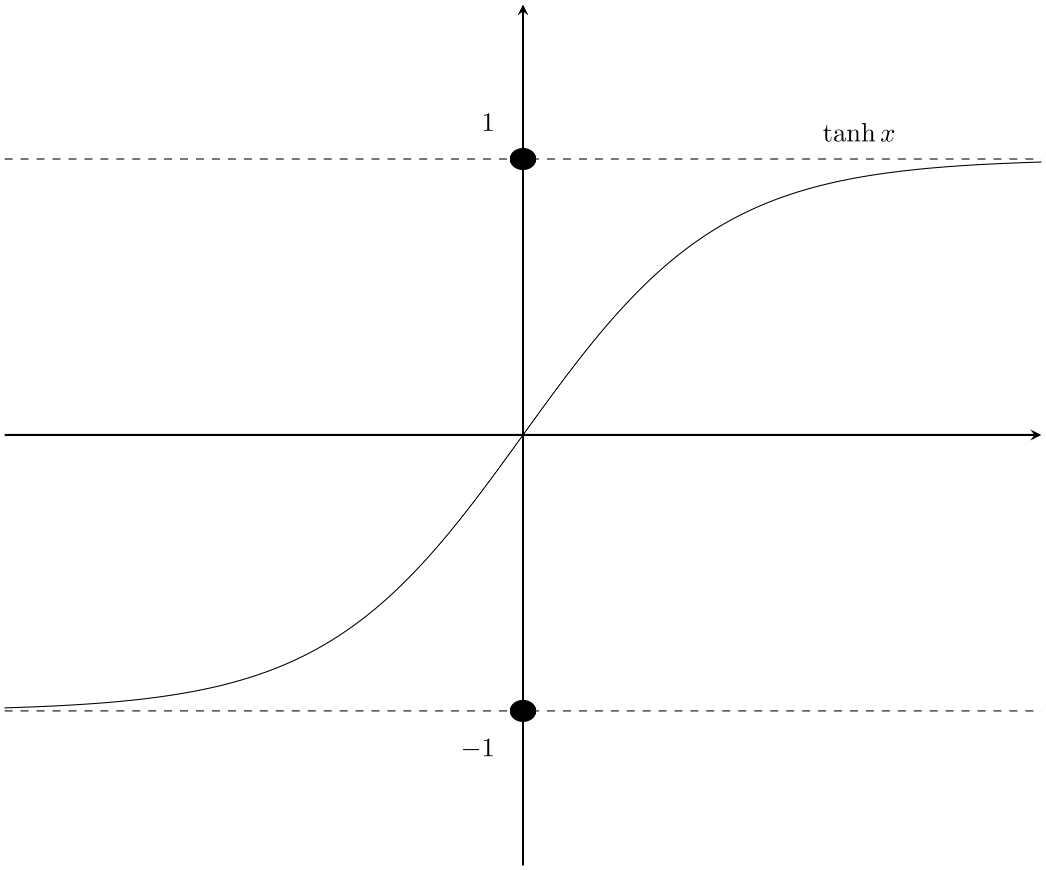
Our Math Professor says we need to be able to derive the inverse hypewrbolic cosine and tangent functions.



EXAMPLE Evaluate $\operatorname{arctanh}\frac{1}{3}$



$$\begin{aligned}
\tanh x &= \frac{\sinh x}{\cosh x} \\
&= \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{3} \\
&= \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1}{3} \\
\Rightarrow 0 < t &= e^{2x} \\
\frac{t-1}{t+1} &= \frac{1}{3} \\
3t - 3 &= t + 1 \\
t &= 2 \\
\therefore x &= \frac{1}{2}\ln 2 = \ln\sqrt{2}
\end{aligned}$$

EXAMPLE $\displaystyle\int \frac{\sinh x}{4+\cosh^2 x}\,dx$

$$\begin{aligned}
\int \frac{\sinh x}{4+\cosh^2 x} &= \left(\begin{array}{l} \sinh x\ dx = d\ \cosh x \\ \cosh x = t \end{array}\right) \\
&= \int \frac{1}{4+t^2}\ dt \\
&= \frac{1}{2}\arctan\frac{t}{2} + C \\
&= \frac{1}{2}\arctan(0.5\cosh x) + C
\end{aligned}$$

Our Math Professor recommends we study chapter 6.7 1-22, 23, 30-47.

We then moved onto the topic of limits which approach indeterminant forms of the types "$\frac{0}{0}$" and "$\frac{\infty}{\infty}$"

These limits are solved using $L'Hospital's$ Rule. This rule says that if you take a limit and it approaches an "indeterminant form," that you can differentiate both the numerator and the denominator and take the same limit.

EXAMPLE Now we have a more powerful tool to solve the problem in Calc 1 where we used geometry to prove $\displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 0$

And having identified the removable discontinuity, we can write $f(x) = \begin{cases} \frac{\sin x}{x};\ x \neq 0 \\ 1;\ x = 0 \end{cases}$

EXAMPLE Evaluate $\displaystyle\lim_{x\to\infty}\frac{x^2-x+1}{x^3+2}$

$$\lim_{x\to\infty}\frac{x^2-x+1}{x^3+2} = \lim_{x\to\infty}\frac{\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}}{1+\frac{2}{x^3}} = \frac{0}{1} = 0$$

EXAMPLE $\displaystyle\lim_{x\to\infty}(\sqrt{x}-\sqrt{x-1}) \Rightarrow \infty - \infty$

$$\lim_{x\to\infty}(\sqrt{x}-\sqrt{x-1})\cdot\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}} = \frac{1}{\sqrt{x}+\sqrt{x-1}} = \frac{1}{\infty} = 0$$

EXAMPLE Evaluate $\displaystyle\lim_{x\to 0} xe^{1/x} = 0\cdot\infty$

$$\begin{aligned}
\lim_{x\to 0}\frac{e^{1/x}}{1/x} &= \left(\begin{array}{c} \frac{1}{x} = t \\ t\to\infty \end{array}\right) \\
&= \lim_{t\to+\infty}\frac{e^t}{t} \\
&\overset{L'H}{=\!=} \lim_{t\to+\infty}\frac{e^t}{1} = \infty
\end{aligned}$$

THEOREM

If $f(x)$ and $g(x)$ are differentiable "near" (which is actually a rigorously defined term) a, and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$, and $\lim_{x\to a} \left(\frac{f'}{g'}\right)(x) = L$ exists,

Then: $\lim_{x\to a} \left(\frac{f}{g}\right)(x) = \lim_{x\to a} \left(\frac{f'}{g'}\right)(x)$ where L can be $\pm\infty$

EXAMPLE Evaluate $\displaystyle\lim_{x\to 0} \frac{\sinh 3x}{\tanh 2x}$

$$\lim_{x\to 0} \frac{\sinh 3x}{\tanh 2x} = \lim_{x\to 0} \frac{3\cosh 3x}{\frac{2}{\cosh^2 2x}} = \frac{3\cdot 1}{2\cdot 1} = \frac{3}{2}$$

Recall that we d/dx tanh with quotient rule