$$\frac{dN}{dt} = rn\left(1 - \frac{N}{k}\right)$$

$$\frac{1}{N\left(1 - \frac{N}{k}\right)}\frac{dN}{dt} = r$$

partial fraction decomposition of denominator

$$\left(\frac{1}{N} + \frac{\frac{1}{k}}{1 - \frac{N}{k}}\right)\frac{dN}{dt} = r$$

$$\frac{1}{N}\frac{dN}{dt} - \frac{-\frac{1}{k}}{1 - \frac{N}{k}}\frac{dN}{dt} = r$$

$$\ln(N) - \ln(1 - \frac{N}{k}) = rt + C$$

$$\ln\left(\frac{N}{1 - \frac{N}{k}}\right) = rt + C$$

$$\frac{N}{1 - \frac{N}{k}} = Ce^{rt}$$

$$\frac{\frac{1 - \frac{N}{k}}{N}}{N} = C$$

$$\frac{1}{N} = Ce^{-rt} + \frac{1}{k}$$

$$n(t) = \frac{1}{Ce^{-rt} + \frac{1}{k}}$$