1. $\int \sin^3 x \cos^2 x\ dx = \int \sin x(1-\cos^2 x)\cos^2 x\ dx = -\int -\sin x(\cos^2 x - \cos^4 x)\ dx \xlongequal[du=-\sin x\ dx]{u=\cos x} \int u^2 - u^4\ du = \frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

2. $\int \sin^6 x \cos^3 x\ dx = \int \sin^6 x(1-\sin^2 x)\cos x\ dx = \int(\sin^6 x - \sin^8 x)\cos x\ dx \xlongequal[du=\cos x\ dx]{u=\sin x} \int u^6 - u^8\ du = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x\ dx = \int_{\pi/2}^{3\pi/4} \sin^5 x(1-\sin^2 x)\cos x\ dx = \int_{\pi/2}^{3\pi/4}(\sin^5 x - \sin^7 x)\cos x\ dx \xlongequal[du=\cos x\ dx]{u=\sin x} \int_1^{1/\sqrt{2}} u^5 - u^7\ du = \frac{1}{6}u^6 - \frac{1}{8}u^8\Big|_1^{1/\sqrt{2}} = [\frac{1}{6}(\frac{1}{\sqrt{2}})^6 - \frac{1}{8}(\frac{1}{\sqrt{2}})^8] - [\frac{1}{6}(1)^6 - \frac{1}{8}(1)^8] = [\frac{1}{48} - \frac{1}{128}] - [\frac{1}{6} - \frac{1}{8}] = \frac{10}{782} - \frac{1}{24} = \frac{-271}{48\cdot 128\cdot 12}$

4. $\int_0^{\pi/2} \cos^5 x\ dx = \int_0^{\pi/2}(1-\sin^2 x)^2\cos x\ dx = \int_0^{\pi/2} \cos x - 2\sin^2 x\cos x + \sin^4 x\cos x\ dx \xlongequal[du=\cos x\ dx]{u=\sin x} \int_0^1\ du - 2\int_0^1 u^2\ du + \int_0^1 u^4\ du = u|_0^1 - \frac{2}{3}u^3|_0^1 + \frac{1}{5}u^5|_0^1 = 1 - \frac{2}{3} + \frac{1}{5} = \frac{8}{15}$

5. $\int \sin^2(\pi x)\cos^5 x\ dx \xlongequal[dv=\cos^5 x\ dx,\ v=\frac{1}{6}\sin x]{u=\sin^2(\pi x),\ du=\frac{2}{\pi}\cos x\ dx} \frac{1}{6}\sin x\sin^2(\pi x) - \frac{2}{6\pi}\int \sin x\cos x\ dx \xlongequal[du=\cos x\ dx]{u=\sin x} \frac{1}{6}\sin x\sin^2(\pi x) - \frac{2}{12\pi}\sin^2 x$

6.

$$\begin{aligned}
\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\ dx &\xlongequal[2\sqrt{x}\ du=\ dx]{u=\sqrt{x}} 2\int \sin^3(u)\ du \\
&= 2\int \sin(u)\sin^2(u)\ du \\
&= 2\int \sin(u)(1-\cos^2(u))\ du \\
&= 2\int \sin(u)\ du - 2\int \sin(u)\cos^2(u)\ du \\
&\xlongequal[dv=\sin(u)\ du]{v=cos(u)} -2\cos(u) - \int v^2\ dv \\
&= -2\cos(u) - \frac{v^3}{3} + C|_{u=\sqrt{x},\ v=\cos(u)} \\
&= -2\cos(\sqrt{x}) - \frac{\cos^3(\sqrt{x})}{3} + C
\end{aligned}$$

7.

$$\begin{aligned}
\int_0^{\pi/2} \cos^2\theta\ d\theta &= \int_0^{\pi/2} \frac{1}{2}(1+\cos(2\theta))\ d\theta \\
&= \frac{\theta}{2} + \frac{1}{4}\sin(2\theta)|_0^{\pi/2} \\
&= \frac{\pi}{4}
\end{aligned}$$

8.

$$\begin{aligned}
\int_0^{\pi/2} \sin^2(2\theta)\ d\theta &= \int_0^{\pi/2} \frac{1}{2}(1+\cos(4\theta))\ d\theta \\
&= \frac{\theta}{2} + \frac{1}{8}\sin(4\theta)\Big|_0^{\pi/2} \\
&= \frac{\pi}{4}
\end{aligned}$$

9.

$$\begin{aligned}
\int_0^{\pi} \sin^4(3t)\ dt &= \int_0^{\pi} (1-\cos(6t))^2\ dt \\
&= \int_0^{\pi} 1 - 2\cos(6t) + \cos^2(6t)\ dt \\
&= \int_0^{\pi} 1 - 2\cos(6t) + (1+\cos(12t))\ dt \\
&= 2t - \frac{1}{3}\sin(6t) + \frac{1}{12}\sin(12t)\Big|_0^{\pi} \\
&= 2\pi
\end{aligned}$$

10.

$$\begin{aligned}
\int_0^\pi \cos^6\theta\ d\theta &= \int_0^\pi (1+\cos(2\theta))^3\ d\theta \\
&= \int_0^\pi 1+3\cos(2\theta)+3\cos^2(2\theta)+\cos^3(2\theta)\ d\theta \\
&= \int_0^\pi 1+2\cos(2\theta)+3(1+\cos(4\theta))+\cos(2\theta)(1-\sin^2(2\theta))\ d\theta \\
&= \int_0^\pi 4+3\cos(2\theta)+\cos(4\theta)-\cos(2\theta)\sin^2(2\theta)\ d\theta \\
&\xlongequal[du=2\cos(2\theta)\ d\theta]{u=\sin(2\theta)} 4\theta+\frac{3}{2}\sin(2\theta)+\frac{1}{4}\sin(4\theta)\Big|_0^\pi-\frac{1}{2}\int_0^0 u^2\ du \\
&= 4\pi
\end{aligned}$$

11.

$$\begin{aligned}
\int (1+\cos\theta)^2\ d\theta &= \int 1+2\cos\theta+\cos^2\theta\ d\theta \\
&= \int 2+2\cos\theta+\cos(2\theta)\ d\theta \\
&= 2\theta+2\sin\theta+\frac{1}{2}\sin(2\theta)+C
\end{aligned}$$

12.

$$\begin{aligned}
\int x\cos^2 x\ dx &\xlongequal[dv=\cos^2 x\ dx,\ v=\frac{x}{2}+\frac{1}{4}\sin(2x)]{u=x,\ du=\ dx} \frac{x^2}{2}+\frac{x}{4}\sin(2x)-\int \frac{x}{2}+\frac{1}{4}\sin(2x)\ dx \\
&= \frac{x^2}{2}+\frac{x}{4}\sin(2x)-\frac{x^2}{4}-\frac{1}{8}\cos(2x)+C
\end{aligned}$$

$\boxed{13.}$

$$\begin{aligned}
\int_0^{\pi/2} \sin^2 x\cos^2 x\ dx &= \int_0^{\pi/2} \frac{1}{4}(1-\cos(2x))(1+\cos(2x))\ dx \\
&= \int_0^{\pi/2} \frac{1}{4}-\frac{1}{4}\cos^2(2x)\ dx \\
&= \int_0^{\pi/2} \frac{1}{4}\cos(4x)\ dx \\
&= \frac{1}{16}\sin(4x)\Big|_0^{\pi/2} \\
&= 0
\end{aligned}$$

14.

$$\begin{aligned}
\int_0^\pi \sin^2 t\cos^4 t\ dt &= \int_0^\pi (1-\cos(2t))(1+\cos(2t))^2\ dt \\
&= \int_0^\pi (1-\cos^2(2t))(1+\cos(2t))\ dt \\
&= \int_0^\pi 1+\cos(2t)-\cos^2(2t)-\cos^3(2t)\ dt \\
&= \int_0^\pi -\cos(4t)+\cos(2t)\sin^2(2t)\ dt \\
&\xlongequal[du=2\cos(2t)\ dt]{u=\sin(2t)} -\frac{1}{4}\sin(4t)\Big|_0^\pi+\frac{1}{2}\int_0^0 u^2\ du \\
&= 0
\end{aligned}$$

15.

$$\begin{aligned}
\int \frac{\cos^5 \alpha}{\sqrt{\sin \alpha}} \ d\alpha &= \int \frac{(1-\sin^2 \alpha)^2 \cos \alpha}{\sqrt{\sin \alpha}} \ d\alpha \\
&= \int \frac{(1-2\sin^2 \alpha + \sin^4 \alpha)\cos \alpha}{\sqrt{\sin \alpha}} \ d\alpha \\
&\overset{u=\sin \alpha}{\underset{du=\cos \alpha \ d\alpha}{=\!=\!=\!=}} \int \frac{1-2u^2+u^4}{\sqrt{u}} \ du \\
&= \int u^{-1/2} - 2u^{3/2} + u^{7/2} \ du \\
&= \frac{u^{1/2}}{\frac{1}{2}} - 2\frac{u^{5/2}}{\frac{5}{2}} + \frac{u^{9/2}}{\frac{9}{2}} + C \\
&= 2\sqrt{\sin \alpha} - \frac{4}{5}\sqrt{\sin^5 \alpha} + \frac{2}{9}\sqrt{\sin^9 \alpha} + C
\end{aligned}$$

16.

$$\begin{aligned}
\int \cos\theta \cos^5(\sin\theta) \ d\theta &\overset{u=\sin \theta}{\underset{du=\cos \theta \ d\theta}{=\!=\!=\!=}} \int \cos^5 u \ du \\
&= \int (1-\sin^2 u)^2 \cos u \ du \\
&= \int (1-2\sin^2 u + \sin^4 u)\cos u \ du \\
&\overset{w=\sin u}{\underset{dw=\cos u \ du}{=\!=\!=\!=}} \int 1 - 2w^2 + w^4 \ dw \\
&= w - \frac{2}{3}w^3 + \frac{1}{5}w^5 + C \\
&= \sin u - \frac{2}{3}\sin^3 u + \frac{1}{5}\sin^5 u + C \\
&= \sin(\sin\theta) - \frac{2}{3}\sin^3(\sin\theta) + \frac{1}{5}\sin^5(\sin\theta) + C
\end{aligned}$$

17.

$$\begin{aligned}
\int \cos^2 x \tan^3 x \ dx &= \int \frac{\sin^3 x}{\cos x} \ dx \\
&=
\end{aligned}$$