1. $\int x^2\ln x\ dx \xlongequal[dv=x^2\ dx,\ v=\frac13 x^3]{u=\ln x,\ du=\frac1x\ dx} \ln x\frac13x^3-\frac13\int x^2\ dx=\frac13x^3\ln x-\frac19x^3+C$

2. $\int\theta\cos\theta\ d\theta \xlongequal[dv=\cos\theta\ d\theta,\ v=\sin\theta]{u=\theta,\ du=d\theta}\theta\sin\theta-\int\sin\theta\ d\theta=\theta\sin\theta+\cos\theta+C$

3. $\int x\cos5x\ dx \xlongequal[dv=\cos5x\ dx\Rightarrow v=\frac15\sin5x]{u=x\Rightarrow du=dx}\frac x5\sin5x-\frac15\int\sin5x\ dx=\frac x5\sin5x+\frac1{25}\cos5x+C$

4. $\int xe^{-x}\ dx\xlongequal[dv=e^{-x}\Rightarrow v=-e^{-x}]{u=x\Rightarrow du=dx}-xe^{-x}-\int e^{-x}\ dx=-xe^{-x}-e^{-x}+C$

5. $\int re^{r/2}\ dr\xlongequal[dv=e^{\frac12 r},\ v=2e^{\frac12 r}]{u=r,\ du=dv}2re^{\frac r2}-2\int e^{\frac r2}\ dr=2re^{\frac r2}-4e^{\frac r2}+C$

6. $\int t\sin2t\ dt\xlongequal[dv=\sin2t\ dt,\ v=-\frac12\cos2t]{u=t,\ du=dt}-\frac t2\cos2t+\frac12\int\cos2t\ dt=-\frac t2\cos2t+\frac14\sin2t+C$

7. $\int x^2\sin\pi x\ dx\xlongequal[dv=\sin\pi x,\ v=-\frac1\pi\cos\pi x]{u=x^2,\ du=2x\ dx}-\frac{x^2}\pi\cos\pi x+\frac2\pi\int x\cos\pi x\xlongequal[dv=\cos\pi x,\ v=\frac1\pi\sin\pi x]{u=x,\ du=dx}\frac{x^2}{\pi}\cos\pi x+\frac x\pi\sin\pi x+\frac1{\pi^2}\cos\pi x+C$

8. $\int x^2\cos mx\ dx\xlongequal[dv=\cos mx\ dx,\ v=\frac1m\sin mx]{u=x^2,\ du=2x\ dx}\frac{x^2}m\sin mx-\frac2m\int x\sin mx\ dx\xlongequal[dv=\sin mx\ dx,\ v=-\frac1m\cos mx]{u=x,\ du=dx}\frac{x^2}m\sin mx+\frac{2x}{m^2}\cos mx+\frac2{m^2}\int\cos mx\ dx=\frac{x^2}m\sin mx+\frac{2x}{m^2}\cos mx+\frac2{m^3}\sin mx+C$

9. $\int\ln(2x+1)\ dx\xlongequal[dv=dx,\ v=x]{u=\ln(2x+1),\ du=\frac2{2x+1}\ dx}x\ln(2x+1)-\int\frac{2x}{2x+1}\ dx\xlongequal[x=\frac{u-1}2]{u=2x+1,\ du=2\ dx}x\ln(2x+1)-\int\frac{\frac{u-1}2}u\ du=x\ln(2x+1)-\frac12(2x+1)-\frac12\ln(2x+1)+C$

10. $\int\arcsin x\,dx\xlongequal[dv=dx,\ v=x]{u=\arcsin x,\ du=\frac1{\sqrt{1-x^2}}\ dx}x\arcsin x+\frac12\int\frac{-2x}{\sqrt{1-x^2}}\ dx\xlongequal{u=1-x^2,\ du=-2x\ dx}x\arcsin x+\frac12\int u^{-1/2}\ du=x\arcsin x-\frac14u^{-3/2}=x\arcsin x-\frac14(1-x^2)^{-3/2}+C$

11. $\int\arctan4t\ dt\xlongequal[dv=dt,\ v=t]{u=\arctan4t,\ du=\frac4{16t^2+1}\ dt}t\arctan4t-\frac18\int\frac{32t}{16t^2+1}\ dt\xlongequal{u=16t^2,\ du=32t\ dt}t\arctan4t-\frac18\ln|16t^2+1|+C$

12. $\int p^5\ln p\ dp\xlongequal[dv=p^5\ dp,\ v=\frac16p^6]{u=\ln p,\ du=p^{-1}\ dp}\frac{p^6}6\ln p-\frac16\int p^5\ dp=\frac{p^6}6\ln p-\frac{p^6}{36}+C$

13. $\int t\sec^2 2t\ dt\xlongequal[dv=\sec^2 2t\ dt,\ v=\frac12\tan2t]{u=t,\ du=dt}\frac t2\tan2t-\frac14\int2\tan2t\ dt=\frac t2\tan2t-\ln|\sec2t|+C$

14. $\int s2^s\ ds\xlongequal[dv=2^s\ ds,\ v=\frac{2^s}{\ln2}]{u=s,\ du=ds}\frac{s2^s}{\ln2}-\frac1{\ln2}\int2^s\ ds=\frac{s2^s}{\ln2}-\frac1{(\ln2)^2}2^s+C$

15. $\int(\ln x)^2\ dx\xlongequal[dv=dx,\ v=x]{u=(\ln x)^2,\ du=\frac{2\ln x}x\ dx}x(\ln x)^2-2\int\ln x\ dx\xlongequal[dv=dx,\ v=x]{u=\ln x,\ du=\frac1x\ dx}x(\ln x)^2-2x\ln x-2x+C$

16. $\int t\sinh mt\ dt\xlongequal[dv=\sinh mt\ dt,\ v=\frac1m\cosh mt]{u=t,\ du=dt}\frac tm\cosh mt-\frac1{m^2}\sinh mt+C$

17. $\int e^{2\theta}\sin3\theta\ d\theta\xlongequal[dv=e^{2\theta}\ d\theta,\ v=\frac12e^{2\theta}]{u=\sin3\theta,\ du=3\cos3\theta\ d\theta}\frac12e^{2\theta}\sin3\theta-\frac32\int e^{2\theta}\cos3\theta\ d\theta\xlongequal[dv=e^{2\theta}\ d\theta,\ v=\frac12e^{2\theta}]{u=\cos3\theta,\ du=-3\sin3\theta\ d\theta}\frac12e^{2\theta}\sin3\theta-\frac34e^{2\theta}\cos3\theta+\frac94\int e^{2\theta}\sin3\theta\ d\theta\Rightarrow\int e^{2\theta}\sin3\theta\ d\theta=-\frac25e^{2\theta}\sin3\theta+\frac35e^{2\theta}\cos3\theta+C$

18. $\int e^{-\theta}\cos2\theta\ d\theta\xlongequal[dv=\cos2\theta\ d\theta,\ v=\frac12\sin2\theta]{u=e^{-\theta},\ du=-e^{-\theta}\ d\theta}\frac12e^{-\theta}\sin2\theta+\frac12\int e^{-\theta}\sin2\theta\ d\theta\xlongequal[dv=\sin2\theta\ d\theta,\ v=-\frac12\cos2\theta]{u=e^{-\theta},\ du=-e^{-\theta}\ d\theta}\frac12e^{-\theta}\sin2\theta-\frac14e^{-\theta}\cos2\theta-\frac12\int e^{-\theta}\cos2\theta=\frac23e^{-\theta}\sin2\theta+C$

19. $\int_0^\pi t\sin3t\ dt\xlongequal[dv=\sin3t\ dt,\ v=-\frac13\cos3t]{u=t,\ du=dt}-\frac13t\cos3t|_0^\pi+\frac13\int_0^\pi\cos3t\ dt=\frac\pi3+\frac19\sin3t|_0^\pi=\frac{pi}3$

20. $\int_0^1(x^2+1)e^{-x}\ dx\xlongequal[dv=e^{-x}\ dx,\ v=-e^{-x}]{u=x^2+1,\ du=2x\ dx}-(x^2+1)e^{-x}|_0^1+2\int xe^{-x}\ dx\xlongequal[dv=e^{-x}\ dx,\ v=-e^{-x}]{u=x,\ du=dx}-(x^2+1)e^{-x}|_0^1-2xe^{-x}-2e^{-x}|_0^1=-(x^2+x+3)e^{-x}|_0^1=-\frac5e+3$

21. $\int_0^1t\cosh t\ dt\xlongequal[dv=\cosh t\ dt,\ v=\sinh t]{u=t,\ du=dt}t\sinh t|_0^1-\int_0^1\sinh t\ dt=t\sinh t-\cosh t|_0^1=\frac{e-e^{-1}}2-(\frac{e+e^{-1}}2-1)=-\frac1e+1$

22.

$$\int_4^9 \frac{\ln y}{\sqrt{y}}\ dy \xlongequal[dv=y^{-\frac{1}{2}},\ v=2\sqrt{y}]{u=\ln y,\ du=\frac{1}{y}\ dy} \ln y\cdot 2\sqrt{y}|_4^9 - \int_4^9 2\sqrt{y}\cdot\frac{1}{y}\ dy$$
$$= 6\ln 9 - 4\ln 4 - 2\int_4^9 y^{-1/2}\ dy$$
$$= 6\ln 9 - 4\ln 4 - 4\sqrt{y}|_4^9$$
$$= 6\ln 9 - 4\ln 4 - 4$$

23.

$$\int_1^2 \frac{\ln x}{x^2}\ dx \xlongequal[dv=\frac{1}{x^2}\ dx,\ v=-\frac{1}{x}]{u=\ln x,\ du=\frac{1}{x}\ dx} -\frac{1}{x}\ln x\Big|_1^2 + \int \frac{1}{x^2}\ dx$$
$$= -\frac{1}{x}(\ln x + 1)\Big|_1^2$$
$$= -\frac{1}{2}(\ln 2 + 1) + 1$$

24.

$$\int_0^\pi x^3\cos x\ dx \xlongequal[dv=\cos x\ dx,\ v=\sin x]{u=x^3,\ du=3x^2\ dx} x^3\sin x\Big|_0^\pi - \int_0^\pi 3x^2\sin x\ dx$$
$$\xlongequal[dv=\sin x\ dx,\ v=-\cos x]{u=x^2,\ du=2x\ dx} x^3\sin x\Big|_0^\pi - 3\left[-x^2\cos x\Big|_0^\pi + 2\int_0^\pi x\cos x\ dx\right]$$
$$\xlongequal[dv=\cos x,\ v=\sin x]{u=x,\ du=dx} x^3\sin x\Big|_0^\pi - 3\left[-x^2\cos x\Big|_0^\pi + 2\left[x\sin x\Big|_0^\pi - \int_0^\pi \sin x\ dx\right]\right]$$
$$= x^3\sin x\Big|_0^\pi - 3\left[-x^2\cos x\Big|_0^\pi + 2\left[x\sin x\Big|_0^\pi + \cos x\Big|_0^\pi\right]\right]$$
$$= -3\pi^2 + 12$$

25.

$$\int_0^1 \frac{y}{e^{2y}}\ dy \xlongequal[dv=e^{-2y},\ v=-\frac{1}{2}e^{-2y}]{u=y,\ du=\ dy} \frac{-y}{2}e^{-2y}\Big|_0^1 + \frac{1}{2}\int_0^1 e^{-2y}\ dy$$
$$= \frac{-2y-1}{4}e^{-2y}\Big|_0^1$$
$$= [(\frac{-3}{4}e^{-2}) - (\frac{-1}{4})]$$

26.

$$\int_1^{\sqrt{3}} \arctan(1/x)\ dx \xlongequal[dv=1,\ v=x]{u=\arctan(1/x),\ du=\frac{-1}{x^2+1}}$$