ANGLES

Angles can be measured in radians or degrees. A complete revolution is $360°$ or 2πrad.

$$\boxed{\pi\text{rad} = 180°}$$

For a circular sector with central angle θ and radius r subtending an arc with lenth a, $\boxed{\theta = \frac{a}{r}}$

THE TRIGONOMETRIC FUNCTIONS

The opposite and adjacent sidelengths of a $\frac{1}{4}\pi$ right triangle are of equal value.

The opposite and adjacent sidelengths of a $\frac{1}{3}\pi$ or $\frac{1}{6}\pi$ right triangle can be found by completing the equilateral triangle to solve for one and then using the pythagorean theorem to solve for the other.

TRIGONOMETRIC IDENTITIES

It follows from the pythagorean theorem that $\boxed{\sin^2\theta + \cos^2\theta = 1}$

We can multiple and divide both sides by squared trig functions to obtain more identities.
The sine function is odd and the cosine function is even.
The **addition** formulas are proven in Ex.85-87.

$$\boxed{\begin{aligned} \sin(x+y) &= \sin x\cos y + \cos x\sin y \\ \cos(x+y) &= \cos x\cos y - \sin x\sin y \end{aligned}}$$

By substituting $-y$ for y, we obtain the subtraction formulas.

$$\boxed{\begin{aligned} \sin(x-y) &= \sin x\cos y - \cos x\sin y \\ \cos(x-y) &= \cos x\cos y + \sin x\sin y \end{aligned}}$$

Then, by dividing the addition and subtraction formulas, we obtain the corresponding formulas for tangent.

$$\boxed{\begin{aligned} \tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x\tan y} \\ \tan(x-y) &= \frac{\tan x - \tan y}{1 + \tan x\tan y} \end{aligned}}$$

If we put $y = x$ in the addition formulas, we get the **double-angle formulas**.

$$\boxed{\begin{aligned} \cos\sin 2x &= 2\sin x\cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \end{aligned}}$$

Then by using the pythagorean identity, we can obtain alternate forms of $\cos 2x$.

$$\boxed{\begin{aligned} \cos 2x &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}}$$

If we now solve these equations for $\sin^2 x$ and $\cos^2 x$, we get the **half-angle formulas**.

$$\boxed{\begin{aligned} \cos^2 x &= \frac{1+\cos 2x}{2} \\ \sin^2 x &= \frac{1-\cos 2x}{2} \end{aligned}}$$

The **product rules** can be deduced from the addition and subtraction formulas.

$$\boxed{\begin{aligned} \sin x \cos y &= \frac{1}{2}[\sin(x+y)+\sin(x-y)] \\ \cos x \cos y &= \frac{1}{2}[\cos(x+y)+\cos(x-y)] \\ \sin x \sin y &= \frac{1}{2}[\cos(x-y)-\cos(x+y)] \end{aligned}}$$

EXAMPLE Find all values of x in the interval $[0, 2\pi]$ such that $\sin x = \sin 2x$.

Using the double angle formula, we rewrite the given equation as

$$\sin x = 2\sin x \cos x \qquad \text{or} \qquad \sin x(1-2\cos x) = 0$$

Therefore there are two possibilities:

$$\begin{aligned} \sin x &= 0 & \text{or} \qquad 1-2\cos x &= 0 \\ x &= 0, \pi, 2\pi & \cos x &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

The given equation has five solutions: $0,\ \pi/3,\ \pi,\ 5\pi/3,$ and 2π