A question was asked at the beginning of; the answer was that we can ignore the exact value of, for example, $\arctan(-2)$ in situations where we are asked to take the inverse of the operation. That is, $\tan(\arctan(-2)) = -2$ and it doesn't matter what $\arctan(-2)$ equals - I will omit the specific question though as I don't want this to be confusing.

EXAMPLE Evaluate $\displaystyle\int \frac{1+x}{1+x^2}\ dx$

We must immediately recognize this as a sum of the derivative of the tangent function and another function which is integrable by substitution - and a reciprocal integration, though this is something we'd probable determine after the obvious substitution. By obvious, I mean that we could do it with a table integral.

$$\begin{aligned}
\int \frac{1+x}{1+x^2}\ dx &= \arctan x + \int \frac{x}{1+x^2}\ dx \\
&= \left(\begin{array}{c} x\ dx = \frac{1}{2}\ d(x^2+1) \\ d(x^2+1) = 2x\ dx \end{array} \right. \\
\text{Let } u = x^2+1 &\Rightarrow d\ dx = \frac{1}{2}\ du \\
&= \arctan(x) + \frac{1}{2}\ln|u| + C \\
&= \arctan(x) + \frac{1}{2}\ln(x^2+1) + C
\end{aligned}$$

EXAMPLE Evaluate $\displaystyle\int \cos x \cdot 3^{\sin x}\ dx$

Always look for derivative-antiderivative pairs when integrating functions; in this case, this substitution is also "obvious." The reason these kinds of derivative-antiderivative substitutions work is because of an advanced concept called "invariance of the form of the first differential." We are not expected to know how this works - remotely; we were just made aware of its existence.

And by letting $u = \sin x$, we get $\int \cos x \cdot 3^{\sin x}\ dx = 3^{\sin x}/\ln 3 + C$.

A question was asked regarding one of the homework problems, $\tan(\arctan \frac{1}{2} + \arccos \frac{1}{3}$. Essentially, we rewrite cosine in terms of tangent, then use the tangent sum formula.

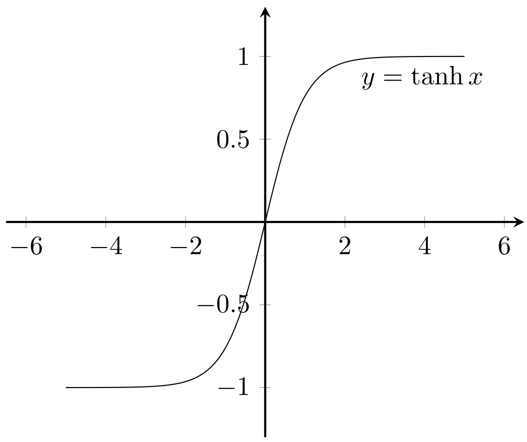
IMPORTANT Dr. Solomonovich indicated that there will be a related-rates question on the test - related to the related rates of the sidelengths of a right triangle in the context of finding out the horizontal velocity of an ascending air-plane. Essentially, we implicitly differentiate - with respect to time - a function which relates our quantities of interest, then plug in whatever values that are provided by the question.

We need to know the definitions of hyperbolic sine cosine and tangent - hyperbolic tangent has three equivalent definitions which can be transitioned between using the algebraic manipulations mentioned last class. The one that is easiest to graph is $\tanh x = (e^{2x}-1)/(e^{2x}+1)$

We can prove that the tangent of hyperbolic sine at the origin is one;

$$
\begin{aligned}
(\sinh x)' &= \cosh x \\
\text{So, } (\sinh x)'\big|_{x=0} &= 1 \\
\text{and } \cosh x\big|_{x=0} &= 1
\end{aligned}
$$

We can prove that hyperbolic cosine is horizontal when $x = 0$ using the same method. We are expected to know the graphs and long-term behavior of the hyperbolic functions. Notable is that hyperbolic sine and cosine are "curvilinear asymptotes" of eachother - in the positive direction. That is, they approach each other asymptotically. We can prove this by taking the limit of their quotient as x becomes arbitrarily large and getting 1 as our answer.



We need to be able to define the inverse hyperbolic parent functions, as they are important in certain types of integration. For example; sine hyperbolic:

$$
\begin{aligned}
y &= \text{arcsinh } x \\
x &= \sinh y \\
2x &= e^y + e^{-y} \\
e^y \cdot 2x &= e^{2y} + 1 \\
e^{2y} - 2xe^y - 1 &= 0
\end{aligned}
$$

Therefore, by the quadratic formula $e^y = x + \sqrt{x^2+1} \Rightarrow y = \ln(x + \sqrt{x^2+1})$

HOMEWORK Derive the formulas for the inverse hyperbolic tangent and cosine functions.

We will not be given trick questions (eg. find arctanh (3)) on the test. All questions will actually have an answer. This question may be rephrased as "find arctanh $\frac{1}{3}$."