$$\boxed{\int_0^a \frac{1}{x^P}\ dx \begin{array}{r}\text{convg. } p<1\\ \text{divg. } p\geq 1\end{array}} \tag{1}$$

$$\boxed{\int_1^\infty \frac{1}{x^P}\ dx \begin{array}{r}\text{convg. } p>1\\ \text{divg. } p\leq 1\end{array}} \tag{2}$$

For $f(x) \leq g(x)$, where $0 < x < m$, if $I_f = \int_0^1 f(x)\ dx \quad I_g = \int_0^1 g(x)$, then; (a) if I_g convg., then I_f is too, and (b) if I_g digv., then I_f is too.

EXAMPLE $\int_0^1 \frac{4+\sin x}{x\sqrt{x}}\ dx \leq \int_0^1 \frac{2}{x^{3/2}}\ dx$

EXAMPLE $\int_0^1 \frac{\sin x}{x\sqrt{x}}\ dx$

$$\sin x \sim x \implies \frac{\sin x}{x\sqrt{x}} \sim \frac{1}{\sqrt{x}}$$

$$\lim_{x\to 0^+}\left(\frac{\sin x}{x\sqrt{x}} \div \frac{1}{\sqrt{x}}\right) = 1$$

EXAMPLE $\int_a^\infty \frac{x\arctan x}{(1+x^2)^2}\ dx$

$\arctan x \leq \pi/2$ as $x \to \infty$.

$$\begin{aligned}\int_a^\infty \frac{x\arctan x}{(1+x^2)^2}\ dx &\leq \frac{\pi}{2}\int_a^\infty \frac{x}{(1+x^2)^2}\ dx\\ &= \frac{\pi}{2}\int_0^\infty \frac{0.5\ d(1+x^2)}{(1+x^2)^2}\\ &\Rightarrow \text{ convg.}\end{aligned}$$

Knowing that it is convergent, we can evaluate it with integration by parts; $u = \arctan x$

EXAMPLE $\int_{\pi/2}^\pi \frac{1}{\sin x}\ dx$

$$\begin{aligned}\int_{\pi/2}^\pi \frac{1}{\sin x}\ dx &= \begin{pmatrix} x=\pi-t & x=\pi \implies t=0\\ dx=-dt & x=\dfrac{\pi}{2} \implies t=\dfrac{\pi}{2}\end{pmatrix}\\ &= -\int_{\frac{\pi}{2}}^\pi \frac{1}{\sin t}\ dt = \int_0^{\frac{\pi}{2}} \frac{1}{\sin t}\ dt\\ \frac{1}{\sin t} &\sim \frac{1}{t} \text{ when } t \to 0^+\\ &\therefore \text{ divergent by comparison.}\end{aligned}$$