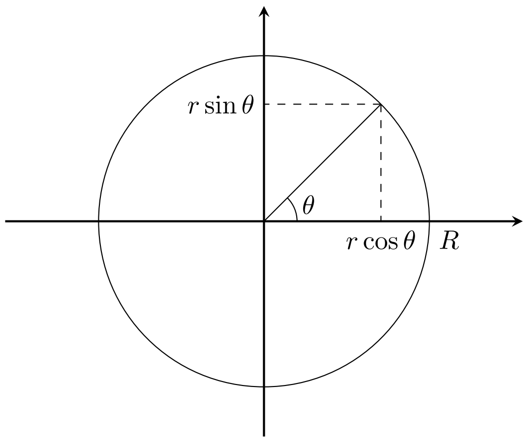
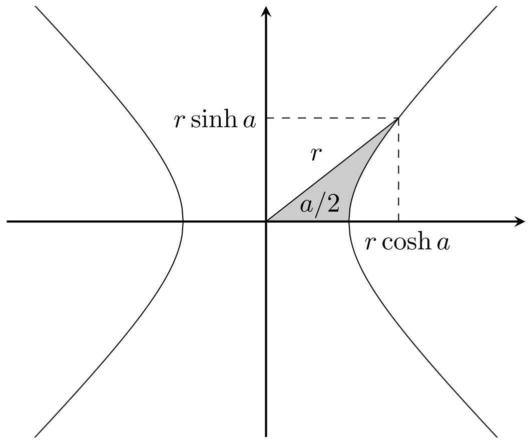
Hyperbolic functions are just like trigonometric functions, but for hyperbolas; that is, the sine hyperbolic and cosine hyperbolic functions output the vertical and horizontal projections respectively of the radial line which intersects the hyperbola - the area of the region enclosed by this line, the hyperbola, and the coordinate axis is half the paramater. Hyperbolas are defined as this: $x^2 - y^2 = a^2$.





And these are the relevant definitions, though they will not be on the test.

$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{1}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \tag{3}$$

$$\coth x = \frac{\cosh x}{\sinh x} \tag{4}$$

Then we derived a useful formula for hyperbolic tangent;

$$\begin{aligned}\tanh x &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ &\overset{\div e^{-x}}{\underset{\frac{e^x}{e^{-x}}=e^{2x}}{=\!=\!=}} \frac{e^{2x} - 1}{e^{2x} + 1}\end{aligned}$$

And these hyperbolic functions have identities similar to the trigonometrics; for instance,

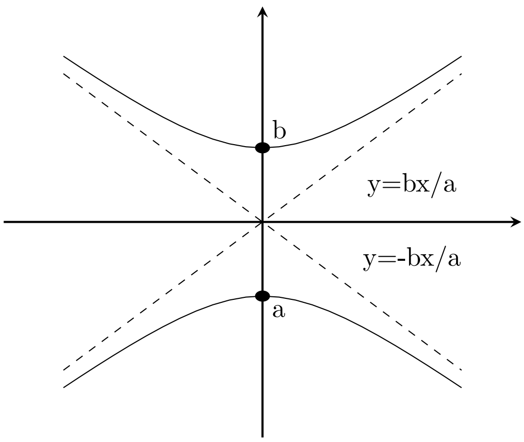
$$\cosh^2 x - \sinh^2 x = 1$$

Which can be proves by arithmetically working it out from the definition.

EXAMPLE Find the area between the branches of the hyperbola $y^2 - x^2 = 4$ for $0 \leq x \leq 1$.

$$\Rightarrow y^2 = 4 + x^2 = \begin{cases} \sqrt{x^2+4} & for \quad y < 0 \\ -\sqrt{x^2+4} & for \quad y \geq 0 \end{cases} = \begin{cases} x = a \sinh t \\ y = a \cosh t \end{cases}$$

General hyperbola: $y^2/b^2 - x^2/a^2 = 1$



And we could solve this by paramaterizing the equation or using a substitution. Both hyperbolic.

We are definitely going to be expected to know hyperbolic properties, such as $\sinh 2t = 2 \sinh t \cosh t$. These are similar to their trigonometric counterparts.