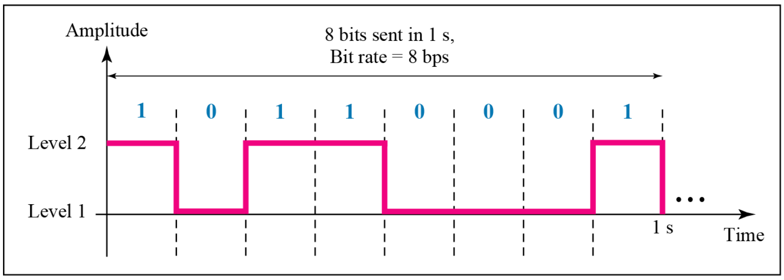
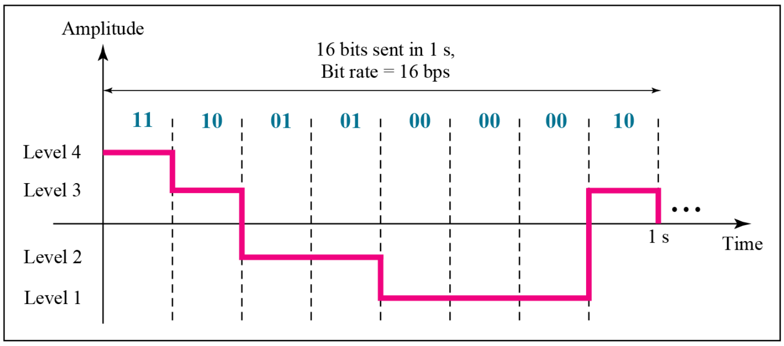
03 Data Transmission (Problems)

Figure of Two digital signals: one with two signal levels and the other with four signal levels



a. A digital signal with two levels



b. A digital signal with four levels

We send 1 bit per level in part a of the figure and 2 bits per level in part b of the figure. In general, if a signal has *M* levels, each level needs log_2M bits.

1. A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

   Solution

   Number of bits per level $= \log_2 8 = 3$

   Each signal level is represented by 3 bits.

2. The power of a signal is 10 mW and the power of the noise is 1 μW; what are the values of *SNR* and SNR_{dB}?

   Solution

   The values of *SNR* and SNR_{dB} can be calculated as follows:

$$SNR = \frac{10000\ \mu W}{1\ mW} = 10000$$

$$SNR_{dB} = 10\log_{10} 10000 = 10\log_{10} 10^4 = 40$$

- **Nyquist Formula : C = 2B $\log_2$M**
- **Shannon Capacity: C=B $\log_2$(1+SNR)**

3. Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

   Solution

   $$\text{BitRate} = 2 \text{ x } 3000 \text{ x } \log_2 2 = 6000 \text{ bps}$$

4. Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

   Solution

   $$\text{BitRate} = 2 \text{ x } 3000 \text{ x } \log_2 4 = 12{,}000 \text{ bps}$$

5. We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

   Solution

   We can use the Nyquist formula as shown:

   $$265{,}000 = 2 \text{ x } 20{,}000 \text{ x } \log_2 M$$

   $$\log_2 M = 6.625 \qquad M = 2^{6.625} = 98.7 \text{ levels}$$

   Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

6. We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000 Hz (300 to 3300 Hz) assigned for data communications. The signal- to-noise ratio is usually 3162. For this channel the capacity is calculated as

   $$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \text{ x } 11.62 = 34{,}860 \text{ bps}$$

   This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

7. The signal-to-noise ratio is often given in decibels. Assume that $\text{SNR}_{\text{dB}} = 36$ and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as

   $$SNR_{dB} = 10 \log_{10} SNR \rightarrow SNR = 10^{SNR_{dB}/10} \rightarrow SNR = 10^{3.6} = 3981$$

   $$\text{C} = B \log_2 (1 + \text{SNR}) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

8. We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

   Solution

   First, we use the Shannon formula to find the upper limit.

   $$\text{C} = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 (64) = 6 \text{ Mbps}$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$4\text{Mbps} = 2 \text{ x } 1 \text{ MHz x } log_2 L \rightarrow L=4$$

The Shannon capacity gives us the upper limit; the Nyquist formula tells us how many signal levels we need.

9. Let us consider an example that relates the Nyquist and Shannon formulations. Suppose that the spectrum of a channel is between 3 MHz and 4 MHz and SNR_{dB} = 24 dB. Then

$$B = 4 \text{ MHz} - 3 \text{ MHz} = 1 \text{ MHz}$$
$$SNR_{dB} = 24 \text{ dB} = 10 \log_{10}(SNR)$$
$$SNR = 251$$

Using Shannon's formula,

$$C = 10^6 \times \log_2 (1 + 251) \approx 10^6 \times 8 = 8 \text{ Mbps}$$

This is a theoretical limit and, as we have said, is unlikely to be reached. But assume we can achieve the limit. Based on Nyquist's formula, how many signal-ing levels are required?

We have

$$C = 2B \log_2 M$$
$$8 \times 10^6 = 2 \times (10^6) \times \log_2 M$$
$$4 = \log_2 M$$
$$M=16$$

10. What is the channel capacity for a teleprinter channel with a 300-Hz bandwidth and a signal-to-noise ratio of 3 dB, where the noise is white thermal noise?

Solution

Using Shannon's equation: $C = B \log_2 (1 + SNR)$
We have $B = 300$ Hz $(SNR)_{dB} = 3$
Therefore, $SNR = 10^{0.3}$
$C = 300 \log_2 (1 + 10^{0.3}) = 300 \log_2 (2.995) = 474$ bps

11. A digital signaling system is required to operate at 9600 bps.
    a. If a signal element encodes a 4-bit word, what is the minimum required bandwidth of the channel?
    b. Repeat part (a) for the case of 8-bit words.

Solution

Using Nyquist's equation: $C = 2B \log_2 M$
We have C = 9600 bps
a. $\log_2 M = 4$, because a signal element encodes a 4-bit word
Therefore, $C = 9600 = 2B \times 4$, and $B = 1200$ Hz

b. $9600 = 2B \times 8$, and $B = 600$ Hz

12. Given a channel with an intended capacity of 20 Mbps, the bandwidth of the channel is 3 MHz. Assuming white thermal noise, what signal-to-noise ratio is required to achieve this capacity

Solution

$C = B \log_2 (1 + SNR)$
$20 \times 10^6 = 3 \times 10^6 \times \log_2(1 + SNR)$
$\log_2(1 + SNR) = 6.67$
$1 + SNR = 102$
$SNR = 101$