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1. Aggregate Method

The aggregate method calculates the total cost of operations and divides by the number of operations to find the average (or amortized) cost per operation.

- Suppose we have a dynamic array that starts with a capacity of 1.
- Each time we insert an element, and the array is full, the array size doubles, requiring copying all elements to the new array.

To analyze the cost:

- **Single insertions:** Inserting an element costs $O(1)$ unless resizing is required.
- **Resizing:** Each time resizing happens, all elements need to be copied to the new array, so the cost of resizing after k insertions is $O(k)$ (copying k elements).

When we insert n elements, the total cost consists of both the simple $O(1)$ cost for each insertion and the costs of each resizing operation.

The total cost of inserting n elements is:

$$\sum_{i=0}^{\log n} O(2^i) = O(n)$$

Thus, the amortized cost per insertion is $O(1)$.

2. Accounting Method

In the accounting method, we assign a "credit" to each operation, which covers not only the immediate cost but also any future costs associated with the operation.

- For each insertion, we charge a cost of 3 units:
 - 1 unit for the actual insertion
 - 2 units to "save up" for future resizing operations

When a resize, operation occurs (i.e., when the table is full), the saved units cover the copying cost:

- Doubling from k to $2k$ requires copying k elements, which will be covered by the previously saved credits.

Since each insertion is charged a constant cost of 3 units, the amortized cost per insertion remains $O(1)$.

In both methods, the amortized runtime for inserting n elements is $O(1)$ per insertion.