ME 535 - Computer-Aided Geometric Design

Homework #1
Hermite and Bézier Curves

Submit the homework (including the source code and results). Due Sep 22^{nd} class.
Late submission (< 2 days) accepted with half credit.

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1. **Hermite curve 20%**
   Demonstrate how unequal tangent vector magnitude with the symmetric slope end condition would affect the resulting Hermite curves for the following coefficients.

   $$\begin{aligned}
   &x_0 = 4,\ y_0 = 4,\ z_0 = 0;\\
   &x_1 = 24,\ y_1 = 4,\ z_1 = 0;\\
   &t_{0x} = 0.8320k,\ t_{0y} = 0.5547k,\ t_{0z} = 0;\\
   &t_{1x} = 0.8320,\ t_{1y} = -0.5547,\ t_{1z} = 0;
   \end{aligned}$$

   where $k = 0.5, 1, 1.5$ and 2. Display the results graphically.

2. **Hermite curve 20%**
   A Hermite curve is defined by two points $\boldsymbol{P}_0 = (0,0,0)$ and $\boldsymbol{P}_1 = (4,0,0)$ and two tangents $\boldsymbol{t}_0 = [1,4,0]$ and $\boldsymbol{t}_1 = [1,-4,0]$. What is the equation for the curve, i.e. the polynomial form? Plot the curve.

3. **Matrix form of Bézier curve 25%**

   (a) Derive the matrix representation for a degree $d = 4$ Bezier curve.

   (b) Based on the matrix representation, for a Bézier curve with control points $\boldsymbol{P} = [0,0;1,2;3,5;4,4;5,0]$, compute the curve points when $u = 0.2, u = 0.5, u = 0.8$.

   (c) Draw the above curve. (programming)

4. **de Casteljau Algorithm 35%**

   (a) Implement the de Casteljau algorithm.

   (b) for a Bézier curve with control points $\boldsymbol{P} = [0,0;1,2;3,5;4,4;5,0]$, compute the curve points when $u = 0.2, u = 0.5, u = 0.8$.

   (c) Compare the results with the same curve points in Problem 3.

   (d) How can you add one more control point to $\boldsymbol{P}$ in b) to make a closed curve? Draw the curve.

   (e) How can you add two more control points to $\boldsymbol{P}$ in b) to make a closed and smooth curve with C^1 continuity. Draw the curve.

5. **Quintic Hermite curve** (50%)

(a) Quintic Hermite curves have been found useful in applications such as robot motion planning and animation where velocity and acceleration continuity in the path is desired to avoid *jerky* motion. Given the geometric coefficients of a quintic (degree 5) Hermite curve, including $\boldsymbol{c}(0) = \boldsymbol{p}_0$, $\boldsymbol{c}'(0) = \boldsymbol{v}_0$, $\boldsymbol{c}''(0) = \boldsymbol{a}_0$, $\boldsymbol{c}(1) = \boldsymbol{p}_1$, $\boldsymbol{c}'(1) = \boldsymbol{v}_1$, and $\boldsymbol{c}''(1) = \boldsymbol{a}_1$, (they correspond to position, velocity and acceleration at two end points), find its algebraic coefficients $\boldsymbol{b}_i, i = 0, \cdots, 5$ of the quintic Hermite curve

$$\boldsymbol{c}(t) = \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \boldsymbol{b}_2 t^2 + \cdots + \boldsymbol{b}_5 t^5.$$

(b) Find the Hermite blending functions $F_i(t), i = 0, \cdots, 5$ for the quintic curve such that

$$\boldsymbol{c}(t) = F_0(t)\boldsymbol{p}_0 + \boldsymbol{F}_1(t)\boldsymbol{v}_0 + \boldsymbol{F}_2(t)\boldsymbol{a}_0 + F_3(t)\boldsymbol{p}_1 + \boldsymbol{F}_4(t)\boldsymbol{v}_1 + \boldsymbol{F}_5(t)\boldsymbol{a}_1.$$

(c) Find the acceleration $\boldsymbol{a}_0$ that minimizes the distance travelled by a particle following the quintic Hermite curve $\boldsymbol{c}(t)$ with $\boldsymbol{c}(0) = [1, 1], \boldsymbol{c}'(0) = [1, 1], \boldsymbol{c}''(0) = \boldsymbol{a}_0, \boldsymbol{c}(1) = [4, 2], \boldsymbol{c}'(1) = [1, -1], \boldsymbol{c}''(1) = [0, 0]$.