ME 535 Computer-Aided Geometric Design

Homework #2
Blossoms and Bézier/B-spline Curves

Submit the homework (including the source code and results). Due October 13.
Late submission (< 2 days) accepted with half credit.

Note: in all B-spline examples, no superfluous knots are given.

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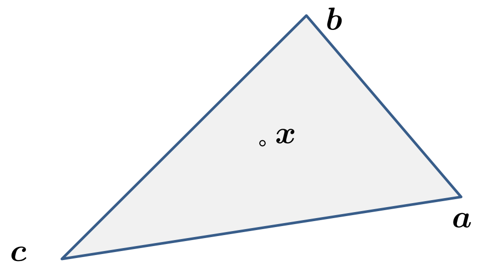
1. **Shear transform of Bézier curves** (15%)
   - (a) Use cubic Bézier curves to model the 1st letter of your last name in the Times New Roman font with size 16.
   - (b) Construct a matrix that maps the unit square with points $(0,0),(1,0),(1,1),(0,1)$ to the parallelogram with image points $(0,0,),(1,0),(2,1),(1,1)$.
   - (c) Apply the matrix to the Bézier curves in (a).

2. **Barycentric coordinates**(10%)
   Using geometric means to solve the following problems:
   - Determine $\boldsymbol{p}_1 = 0.25\boldsymbol{a} + 0.25\boldsymbol{b} + 0.5\boldsymbol{c}$ in the triangle $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ below.
   - Determine $\boldsymbol{p}_2 = -0.25\boldsymbol{a} - 0.25\boldsymbol{b} + 1.5\boldsymbol{c}$ in the triangle $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ below.
   - Determine the point $\boldsymbol{x}$'s barycentric coordinates wrt to the triangle $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ below.



3. **Bézier curves** (10%)
   For a Bézier curve with control points $\boldsymbol{P} = [0,0;1,2;3,5;4,4]$.
   - (a) Compute the derivative at $t = 0.5$.
   - (b) Compute the integral of the x and y coordinates of the curve using the Bézier basis from t=0 to t=1.
   - (c) Raise the degree of the Bézier curve by 1 and give the control points for this curve.
   - (d) Subdivide the curve at $t = 0.75$ and provide the control points for the two subdivided Bézier curves

4. **Blossom for polynomials** (10%)
   Given the polynomial $f(t) = 5t^3 - 4t^2 - 4t + 5$

(a) Blossom this polynomial in four variables.

(b) Use the blossom in part (a) to write down the control points for a quartic Bézier curve over the parameter interval [0,1] that represents this function.

(c) Blossom this polynomial in three variables.

(d) Use the blossom above to write down the control points of a cubic B-spline curve using the knots -2, -1, 0, 2, 4, 5, 6, 6.

5. **Blossom for B-splines** (10%)
Given a cubic B-spline curve with the knots -2, -1, 0, 2, 4, 5, 6, 6 and associated control points (0,0), (1,0), (1,1), (0,1), (0,2), (2,2)

   (a) Use the deBoor (pyramid) algorithm to evaluate the curve at t=3.

   (b) Calculate the derivative at t=3.

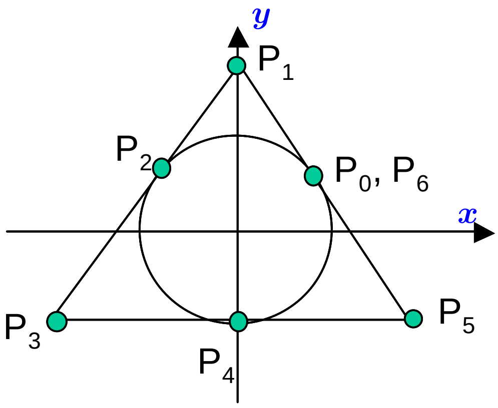
   (c) Obtain the polynomial for knot interval $[2, 4]$

   (d) Obtain the piecewise polynomial form of the B-spline basis function corresponding to control point $(0, 2)$

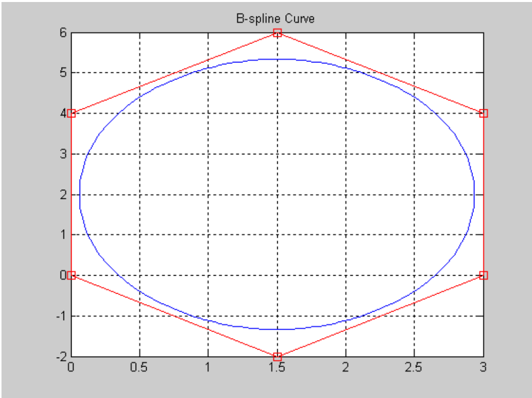
   (e) Use Boehm's algorithm to insert the knots 3, 3 and give the modified control points.

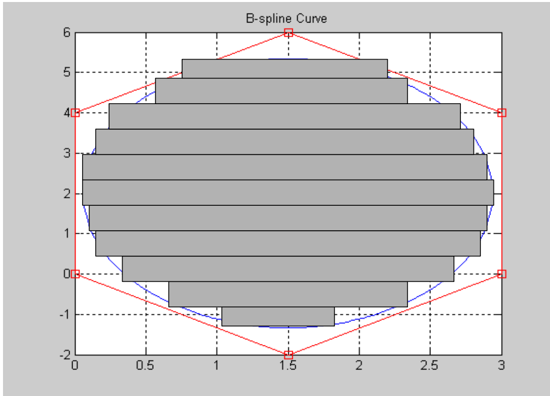
   (f) With the original control points, use the Oslo algorithm to insert the knots 3, 3.5 and give the modified control points.

6. **NURBS curve** (10%)
Represent a unit circle centered at (0,0) by the NURBS curve defined by 7 control points, as shown in the accompanying figure. In other words, derive the degree, knot values, and coordinates of the control points and the corresponding weights.



7. **Slicing B-splines** (20%)
Given a closed uniform cubic B-spline curve, described by control points as P=[0 0; 0 4; 1.5 6; 3 4; 3 0; 1.5 -2; 0 0; 0 4; 1.5 6]. In order to manufacture an object of this shape using additive manufacturing, i.e. by successive layer stacking, there is a need to compute the number of layers and the two end points of each layer. Assume the layer thickness is 0.1. What are the number of layers required to make the part and what are the two end points for each layer?





8. **Programming de Boor Algorithm** (20%)

   (a) Implement the de Boor Algorithm.

   (b) With the implemented algorithm, create a cubic B-spline curve with the following control points. Assuming it is a non-periodic uniform B-spline curve. Assuming the knot vector is 0 0 0 1 2 3 4 5 6 6 6 . Evaluate the point at u=1.5.