[2.3] $|i\rangle = \alpha|u\rangle + \beta|d\rangle$ so $\langle i| = \langle u|\alpha^* + \langle d|\beta^*$

$|o\rangle = \gamma|u\rangle + \delta|d\rangle$ so $\langle o| = \langle u|\gamma^* + \langle d|\delta^*$

$$\frac{1}{2} \overset{(2.8)}{=} \langle i|u\rangle\langle u|i\rangle = \{\langle u|\alpha^* + \langle d|\beta^*\}|u\rangle\langle u|\{\alpha|u\rangle + \beta|d\rangle\}$$

$$= (\alpha^* + 0)(\alpha + 0) = \alpha^*\alpha \checkmark$$

$$\frac{1}{2} \overset{(2.9)}{=} \langle i|r\rangle\langle r|i\rangle = \{\langle u|\alpha^* + \langle d|\beta^*\}|r\rangle\langle r|\{\alpha|u\rangle + \beta|d\rangle\}$$

$$= (\alpha^*\langle u|r\rangle + \beta^*\langle d|r\rangle)(\alpha\langle r|u\rangle + \beta\langle r|d\rangle)$$

$$= \left(\alpha^*\langle u|\{\tfrac{1}{\sqrt2}|u\rangle + \tfrac{1}{\sqrt2}|d\rangle\} + \beta^*\langle d|\{\tfrac{1}{\sqrt2}|u\rangle + \tfrac{1}{\sqrt2}|d\rangle\}\right)$$

$$\left(\alpha\{\langle u|\tfrac{1}{\sqrt2} + \langle d|\tfrac{1}{\sqrt2}\}|u + \beta\{\langle u|\tfrac{1}{\sqrt2} + \langle d|\tfrac{1}{\sqrt2}\}|d\rangle\}\right)$$

$$= \frac{1}{\sqrt2}(\alpha^* + \beta^*)\frac{1}{\sqrt2}(\alpha + \beta) = \frac12(\alpha^*\alpha + \beta^*\beta + \alpha^*\beta + \alpha\beta^*)$$

$$= \frac12\left(\frac12 + \frac12 + \alpha^*\beta + \alpha\beta^*\right) \Rightarrow 1 = 1 + \alpha^*\beta + \alpha\beta^*$$

$\Rightarrow \alpha^*\beta + \alpha\beta^* = 0 \checkmark$ Let $\alpha = a_1 + a_2 i$ $\beta = b_1 + b_2 i$

$\|$

$$(a_1 - a_2 i)(b_1 + b_2 i) + (a_1 + a_2 i)(b_1 - b_2 i)$$

$$= (a_1b_1 + a_2b_2) + \cancel{(a_1b_2 - a_2b_1)i} + (a_1b_1 + a_2b_2) + \cancel{(a_2b_1 - a_1b_2)i}$$

$\Rightarrow a_1b_1 = -a_2b_2$ $\therefore \alpha^*\beta = (a_1b_2 - a_2b_1)i$ pure imaginary ✓

$\alpha\beta^* = (a_2b_1 - a_1b_2)i$ ✓