$\boxed{16.6}$ Charlie prepares a 2-spin system in state $|sing\rangle = \frac{1}{\sqrt{2}}(|ud\rangle - |du\rangle)$.
Bob measures τ_y and Alice measures σ_x. What is $\langle \sigma_x \tau_y \rangle$? What is
$corr(\sigma_x, \tau_y) \equiv \langle \sigma_x \tau_y \rangle - \langle \sigma_x \rangle \langle \tau_y \rangle$?

Solution. Recall that $|T_1\rangle = |ud\rangle + |du\rangle$. Also,

$$\langle sing | T_1 \rangle = \frac{1}{2}(\langle ud| - \langle du|)(|ud\rangle + |du\rangle) = \frac{1}{2}(\overset{1}{\cancel{\langle ud|ud\rangle}} + \overset{0}{\cancel{\langle ud|du\rangle}} - \overset{0}{\cancel{\langle du|ud\rangle}} - \overset{1}{\cancel{\langle du|du\rangle}})$$

$$\langle sing | T_1 \rangle = 0$$

$$\sigma_x \tau_y |sing\rangle \overset{\text{Table 1}}{=} \sigma_x \frac{1}{\sqrt{2}}(-i|uu\rangle - i|dd\rangle) \overset{\text{Table 1}}{=} -\frac{i}{\sqrt{2}}(|du\rangle + |ud\rangle) = -i|T_1\rangle$$

$$\therefore \langle \sigma_x \tau_y \rangle = \langle sing | \sigma_x \tau_y | sing \rangle = -i \langle sing | T_1 \rangle = 0$$

$corr(\sigma_x, \tau_y) = 0 - (0)(0) = 0$. $\boxed{\sigma_x \;\&\; \tau_y \text{ are completely uncorrelated}}$