Exercise 4.2. If *M* and *L* are Hermitian, then *i* [*M*, *L*] is Hermitian but not necessarily [*M*, *L*].

Proof. Given $M^\dagger = M$ and $L^\dagger = L$, we wish to show that $\left\{i\left[M,L\right]\right\}^\dagger = i\left[M,L\right]$. Let $M = \left[m_{ij}\right]$. Then $iM = \left[i\,m_{ij}\right]$ and

$$\left(iM\right)^\dagger = \left[\left(i\,m_{ji}\right)^*\right] = \left[i^*\,m_{ji}{}^*\right] = \left[-i\,m_{ji}{}^*\right] = -i\left[m_{ji}{}^*\right] = -iM^\dagger .$$

So

$$\left\{i\left[M,L\right]\right\}^\dagger = \left\{i\left(ML - LM\right)\right\}^\dagger = \left\{\left(IM\right)L - L\left(IM\right)\right\}^\dagger = L^\dagger\left(iM\right)^\dagger - \left(iM\right)^\dagger L^\dagger$$

$$= -i\,L^\dagger M^\dagger + i\,M^\dagger L^\dagger = i\left(M^\dagger L^\dagger - L^\dagger M^\dagger\right) = i\left[M^\dagger, L^\dagger\right] = i\left[M,L\right] \quad ✔$$

However,

$$\left[M,L\right]^\dagger = \left(ML - LM\right)^\dagger = L^\dagger M^\dagger - M^\dagger L^\dagger = LM - ML = \left[L,M\right] = -\left[M,L\right],$$

not equal to $\left[M,L\right]$ unless *M* and *L* commute; i.e., unless $\left[M,L\right] = 0$. ■