6.9

$$\langle sing|sing\rangle = \tfrac{1}{2}\left(\langle ud| - \langle du|\right)\left(|ud\rangle - |du\rangle\right)$$
$$= \tfrac{1}{2}\left(\overset{1}{\cancel{\langle ud|ud\rangle}} - \overset{0}{\cancel{\langle ud|du\rangle}} - \overset{0}{\cancel{\langle du|ud\rangle}} + \overset{1}{\cancel{\langle du|du\rangle}}\right)$$
$$= 1$$

$$\sigma_z \tau_z |sing\rangle = \sigma_z \tfrac{1}{\sqrt{2}}\left(-|ud\rangle - |du\rangle\right) = \tfrac{1}{\sqrt{2}}\left(-|ud\rangle + |du\rangle\right) = -|sing\rangle$$

$\Rightarrow \langle \sigma_z \tau_z \rangle = -1$ ✓

$$\sigma_x \tau_x |sing\rangle = \sigma_x \tfrac{1}{\sqrt{2}}\left(|uu\rangle - |dd\rangle\right) = \tfrac{1}{\sqrt{2}}\left(|du\rangle - |ud\rangle\right) = -|sing\rangle$$

$\Rightarrow \langle \sigma_x \tau_x \rangle = -1$ ✓

$$\sigma_y \tau_y |sing\rangle = \sigma_y \tfrac{1}{\sqrt{2}}\left(-i|uu\rangle - i|dd\rangle\right) = \tfrac{-i}{\sqrt{2}}\left(i|du\rangle - i|ud\rangle\right)$$
$$= -\tfrac{1}{\sqrt{2}}\left(-|du\rangle + |ud\rangle\right) = -|sing\rangle$$

$\Rightarrow \langle \sigma_y \tau_y \rangle = -1$ ✓

$\Rightarrow \langle \vec{\sigma}\cdot\vec{\tau} \rangle = \langle \sigma_x \tau_x \rangle + \langle \sigma_y \tau_y \rangle + \langle \sigma_z \tau_z \rangle = -3$

$\Rightarrow$ $|sing\rangle$ is an eigenvector of $\vec{\sigma}\cdot\vec{\tau}$ with eigenvalue (-3)

In problem 6.7, I showed $|T_1\rangle$ is an eigenvector of $\vec{\sigma}\cdot\vec{\tau}$ w/ eigenvalue +1

In problem 6.8 " $|T_2\rangle$ & $|T_3\rangle$ are " " " " "

Summary

Eigenvalues

	$\|sing\rangle$	$\|T_1\rangle$	$\|T_2\rangle$	$\|T_3\rangle$
$\sigma_x \tau_x$	-1	1	1	-1
$\sigma_y \tau_y$	-1	1	-1	1
$\sigma_z \tau_z$	-1	-1	1	1
$\vec{\sigma}\cdot\vec{\tau}$	-3	+1	+1	+1

- Singlet has eigenvalue (-3)
- Triplets have eigenvalue (+1)