[6.2] Suppose that $|A\rangle = \alpha_u |u\rangle + \alpha_d |d\rangle$ and $|b\rangle = \beta_u |u\rangle + \beta_d |d\rangle$. Show that if $|A\rangle$ and $|B\rangle$ are normalized then so is $|A\} \otimes |B\}$.

Solution. The basis for S_A is $\{ |a'\}, |a\} \}$ and for S_B it is $\{ |b'\}, |b\} \}$. So a basis for S_{AB} is $\{ |a'b'\}, |a'b\}, |ab'\}, |ab\} \}$.

Alice's basis is orthonormal $\Leftrightarrow \langle a'|a\rangle = \delta_{a'a}$.

Bob's basis is orthonormal $\Leftrightarrow \langle b'|b\rangle = \delta_{b'b}$.

Hence

$$\langle a'b'|ab\rangle \overset{(1.14)}{=} \left(\langle a'b'| \right)\left(|ab\rangle \right) = \left(\{a'| \otimes \langle b'| \right)\left(|a\} \otimes |b\rangle \right)$$

$$= \langle a'|a\rangle \langle b'|b\rangle$$ since we do not combine *a*'s and *b*'s.

Note that $\langle a'|a\rangle$ and $\langle b'|b\rangle$ are complex numbers so we drop the tensor sign.

Therefore $\langle a'b'|ab\rangle = \delta_{a'a}\delta_{b'b}$.