[6.2] Suppose that $\left|A\right\rangle = \alpha_u\left|u\right\rangle + \alpha_d\left|d\right\rangle$ and $\left|b\right\rangle = \beta_u\left|u\right\rangle + \beta_d\left|d\right\rangle$. Show that if $\left|A\right\rangle$ and $\left|B\right\rangle$ are normalized then so is $\left|A\right\}\otimes\left|B\right\}$.

Solution. The basis for S_A is $\left\{\left|a'\right\}, \left|a\right\}\right\}$ and for S_B it is $\left\{\left|b'\right\rangle, \left|b\right\rangle\right\}$. So a basis for S_{AB} is $\left\{\ \left|a'b'\right\rangle, \left|a'b\right\rangle, \left|ab'\right\rangle, \left|ab\right\rangle\ \right\}$.

Alice's basis is orthonormal $\Leftrightarrow$ $\left\langle a'\middle|a\right\rangle = \delta_{a'a}$.
Bob's basis is orthonormal $\Leftrightarrow$ $\left\langle b'\middle|b\right\rangle = \delta_{b'b}$.

Hence

$$\left\langle a'b'\middle|ab\right\rangle \overset{(1.14)}{=} \left(\ \left\langle a'b'\right|\ \right)\left(\ \left|ab\right\rangle\ \right) = \left(\ \left\{a'\right|\otimes\left|\ b'\ \right\rangle\ \right)\left(\ \left|\ a\ \right\}\otimes\left|\ b\ \right\rangle\ \right)$$

$$= \left\langle a'\middle|a\right\rangle\left\langle b'\middle|b\right\rangle$$ since we do not combine *a*'s and *b*'s.

Note that $\left\langle a'\middle|a\right\rangle$ and $\left\langle b'\middle|b\right\rangle$ are complex numbers so we drop the tensor sign.

Therefore $\left\langle a'b'\middle|ab\right\rangle = \delta_{a'a}\,\delta_{b'b}$.