[1, 2] Let $|A\rangle = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$ and $|B\rangle = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$. Then $\langle A| = [\alpha_1^* \ \alpha_2^*]$ and $\langle B| = [\beta_1^* \ \beta_2^*]$

Let $|C\rangle = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$. Then $\langle C| = [\gamma_1^* \ \gamma_2^*]$.

$$\langle C| \{ |A\rangle + |B\rangle \} = [\gamma_1^* \ \gamma_2^*] \begin{bmatrix} \alpha_1+\beta_1 \\ \alpha_2+\beta_2 \end{bmatrix} = \gamma_1^*(\alpha_1+\beta_1) + \gamma_2^*(\alpha_2+\beta_2)$$

$$\langle C|A\rangle + \langle C|B\rangle = [\gamma_1^* \ \gamma_2^*] \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + [\gamma_1^* \ \gamma_2^*] \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$= \gamma_1^*\alpha_1 + \gamma_2^*\alpha_2 + \gamma_1^*\beta_1 + \gamma_2^*\beta_2 \ \checkmark$$

$$\langle B|A\rangle = [\beta_1^* \ \beta_2^*] \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \alpha_1\beta_1^* + \alpha_2\beta_2^*$$

$$\langle A|B\rangle^* = \left([\alpha_1^* \ \alpha_2^*] \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \right)^* = \left(\alpha_1^*\beta_1 + \alpha_2^*\beta_2\right)^* \overset{\star}{=} (\alpha_1^*\beta_1)^* + (\alpha_2^*\beta_2)^*$$

$$= \alpha_1\beta_1^* + \alpha_2\beta_2^* \ \checkmark$$

$$\star \quad \underset{\overset{\parallel}{\alpha_1^*\beta_1}}{\left[(a_1 - a_2 i)(b_1 + b_2 i)\right]^*} = \left[(a_1b_1 + a_2b_2) + (a_1b_2 - a_2b_1)i\right]^* = (a_1b_1 + a_2b_2) + (a_1b_2 - a_2b_1)i$$

$$\alpha_1\beta_1^* = (a_1 + a_2 i)(b_1 - b_2 i) = (a_1b_1 + a_2b_2) + (a_2b_1 - a_1b_2)i \ \checkmark$$