

Given

$$[2.2] \quad |i\rangle = \frac{1}{\sqrt{2}} |u\rangle + \frac{i}{\sqrt{2}} |d\rangle \quad \langle i| = \langle u| \frac{1}{\sqrt{2}} - \langle d| \frac{i}{\sqrt{2}}$$

$$|0\rangle = \frac{1}{\sqrt{2}} |u\rangle - \frac{i}{\sqrt{2}} |d\rangle \quad \langle 0| = \langle u| \frac{1}{\sqrt{2}} + \langle d| \frac{i}{\sqrt{2}}$$

$$(2.7): \langle u|0\rangle = \frac{1}{2} \langle u|u\rangle - \frac{i}{2} \langle d|d\rangle = 0 - 0 = 0 \quad \checkmark$$

$$(2.8): \langle u|u\rangle \langle u|i\rangle \stackrel{(2.8)}{=} \left(\langle u| \frac{1}{\sqrt{2}} - \langle d| \frac{i}{\sqrt{2}} \right) \left(\langle u| \left\{ \frac{1}{\sqrt{2}} |u\rangle + \frac{i}{\sqrt{2}} |d\rangle \right\} \right)$$

$$= \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) = \frac{1}{2} \quad \checkmark$$

$$(2.9): \langle i|d\rangle \langle d|i\rangle \stackrel{(2.9)}{=} \left(\langle u| \frac{1}{\sqrt{2}} - \langle d| \frac{i}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} |u\rangle - \frac{i}{\sqrt{2}} |d\rangle \right)$$

$$\left(\langle u| \frac{1}{\sqrt{2}} - \langle d| \frac{i}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} |u\rangle + \frac{i}{\sqrt{2}} |d\rangle \right)$$

$$= \left(\frac{1}{2} + \frac{i}{2} \right) \left(\frac{1}{2} - \frac{i}{2} \right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad \checkmark$$