

$$\begin{aligned}
 \langle \text{sing} | \text{sing} \rangle &= \frac{1}{2} (\langle ud | - \langle du |) (|ud\rangle - |du\rangle) \\
 &= \frac{1}{2} \langle ud | ud \rangle - \langle ud | du \rangle - \langle du | ud \rangle + \langle du | du \rangle \\
 &= 1
 \end{aligned}$$

$$\sigma_z \tau_z | \text{sing} \rangle = \sigma_z \frac{1}{\sqrt{2}} (-|ud\rangle - |du\rangle) = \frac{1}{\sqrt{2}} (-|ud\rangle + |du\rangle) = -| \text{sing} \rangle$$

$$\Rightarrow \langle \sigma_z \tau_z \rangle = -1 \checkmark$$

$$\sigma_x \tau_x | \text{sing} \rangle = \sigma_x \frac{1}{\sqrt{2}} (|uu\rangle - |dd\rangle) = \frac{1}{\sqrt{2}} (|du\rangle - |ud\rangle) = -| \text{sing} \rangle$$

$$\Rightarrow \langle \sigma_x \tau_x \rangle = -1 \checkmark$$

$$\begin{aligned}
 \sigma_y \tau_y | \text{sing} \rangle &= \sigma_y \frac{1}{\sqrt{2}} (-i|uu\rangle - i|dd\rangle) = \frac{-i}{\sqrt{2}} (i|du\rangle - i|ud\rangle) \\
 &= -\frac{1}{\sqrt{2}} (-|du\rangle + |ud\rangle) = -| \text{sing} \rangle
 \end{aligned}$$

$$\Rightarrow \langle \sigma_y \tau_y \rangle = -1 \checkmark$$

$$\Rightarrow \langle \vec{\sigma} \cdot \vec{\tau} \rangle = \langle \sigma_x \tau_x \rangle + \langle \sigma_y \tau_y \rangle + \langle \sigma_z \tau_z \rangle = -3$$

$\Rightarrow | \text{sing} \rangle$ is an eigenvector of $\vec{\sigma} \cdot \vec{\tau}$ with eigenvalue -3

Since $\sigma_x \tau_x | \text{sing} \rangle = -| \text{sing} \rangle$ eigenvector w/ eig. value of -1
 $\sigma_y \tau_y | \text{sing} \rangle = -| \text{sing} \rangle$ " " " " " "
 $\sigma_z \tau_z | \text{sing} \rangle = -| \text{sing} \rangle$ " " " " " "

Then $| \text{sing} \rangle$ is an eigenvector of $\vec{\sigma} \cdot \vec{\tau}$ with eigenvalue -3

In problem 6.7, I showed $|T_1\rangle$ is an eigenvector of $\vec{\sigma} \cdot \vec{\tau}$ w/ eigenvalue $+1$

In problem 6.8 " $|T_2\rangle$ & $|T_3\rangle$ are " " " " " "

Summary

	Eigenvalues			
	$ \text{sing} \rangle$	$ T_1\rangle$	$ T_2\rangle$	$ T_3\rangle$
$\sigma_x \tau_x$	-1	1	1	-1
$\sigma_y \tau_y$	-1	1	-1	1
$\sigma_z \tau_z$	-1	-1	1	1
$\vec{\sigma} \cdot \vec{\tau}$	-3	+1	+1	+1

• Singlet has eigenvalue (-3)
 • Triplets have eigenvalue $(+1)$
 of multiplicity 3