3.2 Let $\sigma_z = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $|u\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $|d\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Then

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\Rightarrow b = c = 0$, $a = 1$, and $d = -1$.

$\therefore \sigma_n = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ ✓