The Solar Neutrino Problem

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The Solar Neutrino Problem was an area of active research and debate in modern physics from the 60s to the 2000s when it was resolved with some amount of certainty, thanks to relentless experimental and theoretical work in the area. More specifically, it concerned a deficit in the observed solar neutrino flux as compared to the calculated solar neutrino flux. In this term paper, some of the work concerning the problem, it's background, and it's resolution will be taken up and discussed.

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I. INTRODUCTION

The Solar Neutrino Problem provides a great example of ground breaking experimental and theoretical work that was carried out in modern physics. The problem actually revolved around a mismatch of the solar neutrino flux. More specifically, the observed solar neutrino flux, measured by different observatories across the globe, working on different principles, and the theoretical solar neutrino flux, calculated according to the Standard Solar Model of the sun, did not match. The observed neutrinos were far lower than the expected number, and this deficit came to be known as the Solar Neutrino Problem. Initially, the problem was considered to be within the confines of solar physics and owing to the fact that, at first, there was only one particular observatory measuring the flux continuously throughout several years (Homestake experiment, headed by J. Bahcall and Ray Davis [5]) was accorded a low importance. However, with the advent of other observatories across the globe, like the Kamiokande, the Super Kamiokande, Gallex etc, all working on different principles but all recording the same deficit, the problem garnered attention, and several ideas were pitched to resolve it, including modifications to the Standard Solar Model, neutrino oscillations and so on. A lot of approaches also relied on tackling the problem in a different way. Instead of worrying about the solar neutrinos and their processes in particular, these were aimed at increasing our understanding of neutrinos themselves. Ultimately, neutrino oscillations were confirmed by the Super Kamiokande in Japan, and this particular phenomenon was used to resolve the Solar Neutrino Problem. This term paper will start off with an introduction to the Standard Model pf particle physics, the Standard Solar Model, the detection principles and detectors, a brief idea of the mathematics associated, the problem and it's resolution.

II. THE STANDARD MODEL

The Standard Model of particle physics is regarded as one of the strongest pillars of modern physics. It unites three of the four known fundamental forces in nature, i.e., the electromagnetic force, the weak force and the strong force. It has made several predictions, and a lot of these predictions have been later experimentally verified, the most recent one being the experimental verification of the existence of the Higgs Boson in 2012 at the Large Hadron Collider, CERN, (this was followed by the Nobel Prize in Physics being awarded to the physicists who had postulated it's existence and helped to increase our

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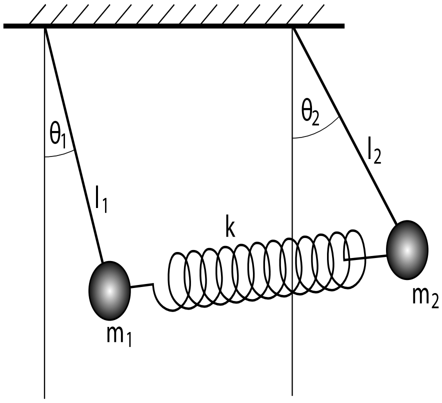

FIG. 1 A system of coupled harmonic oscillators.
(Image source: [7])

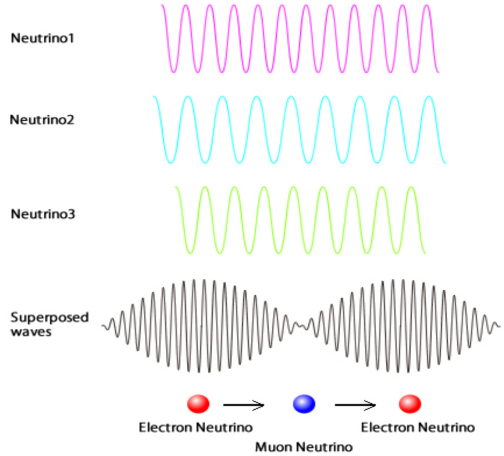

FIG. 2 Neutrino oscillations as they propagate through space. As neutrinos travel through space, the superposition of the different mass states varies, leading to a change in the flavor of the neutrino.
(Image source: [8])

understanding of the origin of the mass of subatomic particles, Peter Higgs and Francois Englert [12]) leading it to become one of the most successful theories of modern day physics.
Inspite of it's widespread success, like every scientific theory it still has it's drawbacks and failures, the most notable one being the inability to incorporate gravity within it's framework. However, as of now, the Standard Model is a towering testament to the work of thousands of scientists who have and still do work on deep questions throughout their life relentlessly.

A. Particles within the Standard Model

The Standard model incorporates several classes and types of particles based on various characteristics.
Particles can be broadly divided into 2 categories:

- Fermions (Half integer spins)
  - Leptons and antileptons
  - Quarks and antiquarks
- Bosons (Integer spins)
  - Gauge bosons
  - Scalar bosons

B. Neutrinos

Neutrinos (ν) are spin $\frac{1}{2}$ particles, that is, they are leptons (subclass of fermions). They are electrically neutral particles and interact weakly with matter through the weak interaction and gravity.
There are 3 kinds of neutrinos (3 leptonic flavors):

1. Electron neutrinos, ν_e
2. Muon neutrinos, ν_μ
3. Tau neutrinos, ν_τ

Solar neutrinos, those produced in the solar interior, are mostly electron neutrinos, ν_e. [2]
The Standard Model assumes neutrinos to be massless, but it is now known, due to the phenomenon of neutrino oscillations which has been experimentally verified, that they actually should have non zero masses.
There are three discrete and distinct neutrino masses but these do not correspond uniquely to the three different flavors of the neutrinos. Usually, neutrinos are in a quantum superposition state of all three different mass eigenstates.

1. Neutrino oscillations

Neutrinos have three distinct mass and flavor eigenstates. A neutrino created in weak processes is of any one specific flavor type, that is, ν_e, ν_μ, ν_τ. Each of these flavor states correspond to a specific superposition of the three neutrino mass states. That is, neutrinos created in a specific flavor state propagate as a superposition of distinct mass eigenstates. In other words, each neutrino flavor state corresponds to a specific superposition of the mass eigenstates.
Consider a system consisting of two pendulums attached to one another by a spring (FIG. 1). Initially, the system

is at rest, but then the first pendulum is put into oscillations. As the first pendulum oscillates, since it is connected to the second one through a spring, the second one also begins to oscillate. Gradually, the first pendulum's amplitude decreases and the second one's increases; this is due to transfer of energy from the first pendulum to the second one. This goes on until the first one comes to rest, while the second one is oscillating at it's maximum possible amplitude (depending on the energy provided initially and other factors), and then the process repeats with the first one slowly gaining energy, oscillating with higher and higher amplitude, while the second one loses energy, and slowly comes to rest. If there is no energy dissipation, like through air resistance, the process goes on in this fashion forever.

Neutrino oscillations are analogous to this classical system of coupled harmonic oscillators [14]. As the neutrinos propagate through vacuum, the quantum mechanical phases of the different mass eigenstates (due to the different masses), advance at different rates. Therefore, the superposition of the different mass eigenstates changes over time.. This means that the superposition of the mass eigenstates at any given time will be different from the initial superposition state. However, each of the neutrino flavor states corresponds to differing superpositions of the mass eigenstates, implying that a neutrino that started off in one particular flavor state is now, due to change in the superposition of the mass states, in a different flavor state. These sorts of oscillations result in one type of neutrino changing into a different type of neutrino as it propagates through space.

Neutrino oscillations have been depicted in the FIG. 2. The figure shows three different neutrinos travelling through space. Each is distinct (with differing frequencies) because each has a different superposition of the mass eigenstates. Different flavors of a neutrino similarly correspond to different superpositions of mass states that inter convert into one another as the neutrinos propagate due to phase changes.

Neutrino oscillations are periodic, however. That is, the change in the superposition of the mass states is periodic in time. This results in the superposition state going back to the initial superposition state after a specific amount of time. Therefore, a neutrino that starts off in one particular flavor state, changes into different flavor states as it propagates, but also returns to the original state after a certain interval of time.

The flavor state of the neutrino will, thus, continuously oscillate back and forth, that is as long as the neutrino can maintain it's quantum mechanical behaviour. Since the mass differences between the different mass states of the neutrinos are quite small, this sort of quantum mechanical behaviour can be maintained over a long time, and these oscillations are observable over macroscopic distances.

Neutrino oscillations have been subsequently observed and established as verified phenomenon. The first experimental evidence for neutrino oscillations came from Japan's Super Kamiokande neutrino observatory in 1998 which studied atmospheric neutrinos.

2. The Mikheyev–Smirnov–Wolfenstein effects (MSW Effects)

The Mikheyev–Smirnov–Wolfenstein effects (MSW Effects) are another phenomenon associated with neutrinos. The MSW effect is responsible for modifying neutrino oscillations in matter.

MSW effects are mostly isolated flips or conversions of the flavor states rather than constant neutrino oscillation [1]. The presence of electron in matter can affect the energy levels of the mass states of propagating neutrinos.

This effect is analogous to the refraction of light waves passing from one medium to another with differing optical densities. As the neutrinos travel through matter, their effective mass becomes different than the initial state they were in resulting in a difference in neutrino oscillations in matter (since the flavor oscillations depend on the mass difference squared).

MSW effects are important in regions of high electron densities, for example the interior of the sun. As stated earlier, MSW effects are like isolated flips. For the solar neutrinos, one flip may occur in the interior of the sun and another may take place as the beam of neutrinos passes through the earth.

The MSW effect has also been experimentally verified. The experimental evidence came from observing and studying solar neutrinos in different facilities like the Sudbury Neutrino Observatory, Canada, and earlier evidence (with lower confidence) came from the Kamiokande and the Super Kamiokande, Japan.

III. THE STANDARD SOLAR MODEL (SSM)

The Standard Solar Model is a theory of stellar evolution that attempts to describe our sun as a spherically symmetric, quasi-static ball of gas with various ionisation levels and completely ionised plasma within the deep interior. The stellar structure is described by a set of differential equations and constrained by boundary conditions such as the luminosity, the age, the radius and so on.

The crucial requirements for a mathematical treatment of a solar model are as follows [1]:

- Hydrostatic equilibrium (stationary)
- Thermal equilibrium (radiation-dominated)
- Energy production
- Equation of state (ideal gas)

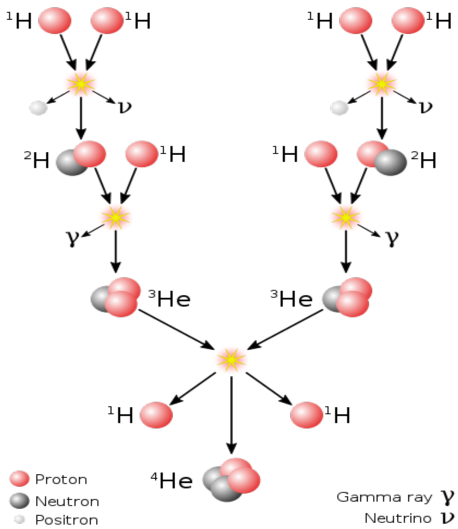

FIG. 3 A schematic for the PP I Cycle
(Image source: [9])

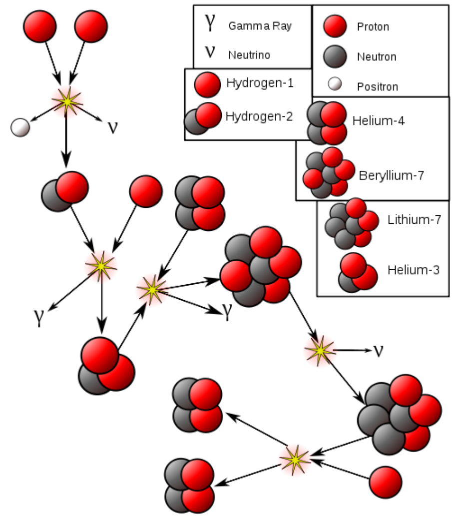

FIG. 4 A schematic for the PP II Cycle
(Image source: [9])

A. Production of solar neutrinos

As mentioned earlier, energy production is an important part of any viable stellar model. The luminosity of our sun is about 3.8×10^{26} W corresponding to a huge amount of energy being generated in it's core.

Just to get some perspective about the amount of energy the sun generates, consider a ton of coal. One ton of coal generates about 2,460 kWh of electricity, or, 2.46×10^6 W *per hour*, compared to the sun's 1.4×10^{30} W *per hour*. The difference is staggering; about 1.75×10^{24} *tonnes* of coal is required to match the energy generated each hour by the sun. This clearly shows that the energy generated in the sun is beyond imagination.

The next question obviously concerns the mechanism by which such an amount of energy is continuously generated in the sun. Through the 19th century, the source of the sun's energy was a major puzzle and several ideas were put forward [15]. For example, Lord Kelvin and Hermann Von Helmholtz provided a gravitational contraction mechanism to explain the energy output, but the proposal could not properly account for the age of the sun. Ernest Rutherford, in 1904, suggested radioactive decay in the interior of the sun. In 1919, Arthur Eddington put forward his suggestion of nuclear fusion in the sun's interior, and this was later verified by Cecilia Payne, who confirmed that hydrogen is a major element in the sun, based on Saha's Ionisation Equation, which should be the case if indeed there was nuclear fusion going on. This idea was worked upon and given a shape by Chandrashekhar and Hans Bethe. Today, there is no doubt that it is indeed nuclear fusion that is responsible for the generation of energy in the sun and other stars.

The set of reactions in the sun's core responsible for energy generation are also responsible for producing the solar neutrinos. The principal reactions believed to be responsible for neutrino production are collectively called the pp-chain, or the proton-proton chain.

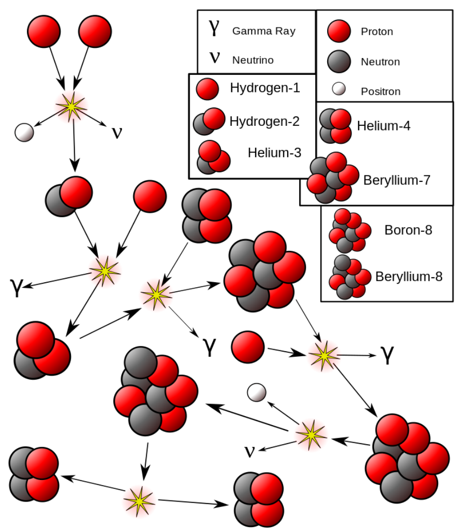

FIG. 5 A schematic for the PP III Cycle
(Image source: [9])

The pp-chain consists of the following reactions [1–3]:

- PP I Cycle (FIG. 3):
  - $p + p \rightarrow d + e^+ + \nu_e$ (pp-neutrinos)
  - $p + p + e \rightarrow d + \nu_e$ (pep-neutrinos)
- PP II Cycle (FIG. 4):

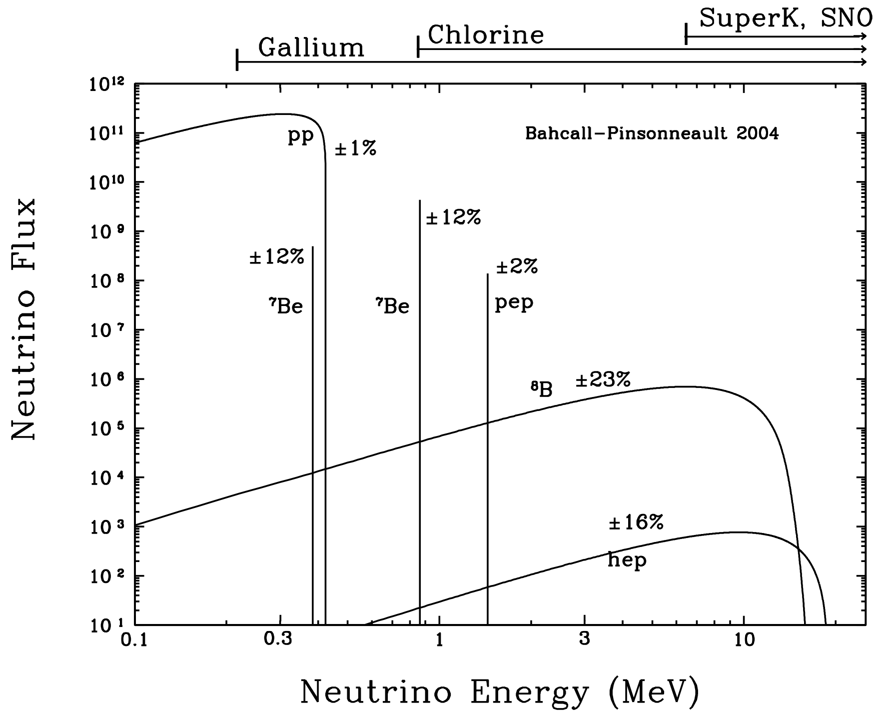

FIG. 6 The solar neutrino energy spectrum as predicted by the SSM put forward by Bahcall and Pinsonneault. The figure also shows the various neutrino observatories and the neutrinos they can detect.
(Image source: [10])

- $^{7}Be\ +\ e \rightarrow\ ^{7}Li\ +\ \nu_e$
  (^{7}Be-neutrinos)

- PP III Cycle (FIG. 5):
  - $^{8}B\ \rightarrow\ ^{8}Be\ +\ e^{+}\ +\ \nu_e$
    (^{8}B-neutrinos)
- PP IV Cycle:
  - $^{3}He\ +\ p\ \rightarrow\ ^{4}He\ +\ e^{+}\ +\ \nu_e$
    (hep-neutrinos)

Most of the neutrinos produced by the sun are the pp-neutrinos ($\sim$ 91%). The high energy ^{8}B-neutrinos have a relatively smaller flux, while the hep-neutrinos, which are the highest energy neutrinos, have a flux that is three times smaller than the ^{8}B-neutrino flux.
All the fusion reactions can be summarised as,

$$4p\ \rightarrow\ ^{4}He\ +\ 2e^{+}\ +\ 2\nu_e$$

The total energy released in this entire process is about 26.731 MeV. The energy released in the sun can be realted to the neutrino flux since solar luminosity calculations derive from this. The solar luminosity constrains the solar neutrino flux.

B. Viability of the Standard Solar Model

An important question can be raised at this juncture about the credibility and validity of the Standard Solar Model. The neutrino deficit that makes up the solar neutrino problem may as well be a failure of the Standard Solar Model, rather than there being something else. In other words, it may seem possible that the neutrino deficit results from the Standard Solar Model being wrong since all of the flux calculations are based on it. Possibly there is no deficit at all, and it is the model that needs improvement.
The first point to note about the SSM and the solar neutrino problem is that it is the solar interior that generates the solar neutrinos. For example, the ^{8}B-neutrinos and the ^{7}Be-neutrinos are mostly created at a depth between 0.04 $R_\odot$ and 0.06 $R_\odot$, where $R_\odot$ is the radius of the sun [3]. Now, the interaction between the solar plasma and the neutrinos is very weak because the solar plasma is far colder than the neutrinos. The predicted temperature of the solar core is about $1.5 \times 10^{7}\ K$ which corresponds to an energy of nearly 1 keV. This value is far lower than the energy of even the least energetic neutrinos, as such even if the Standard Model of the sun was wrong about the sun's core, the shape of the solar neu-

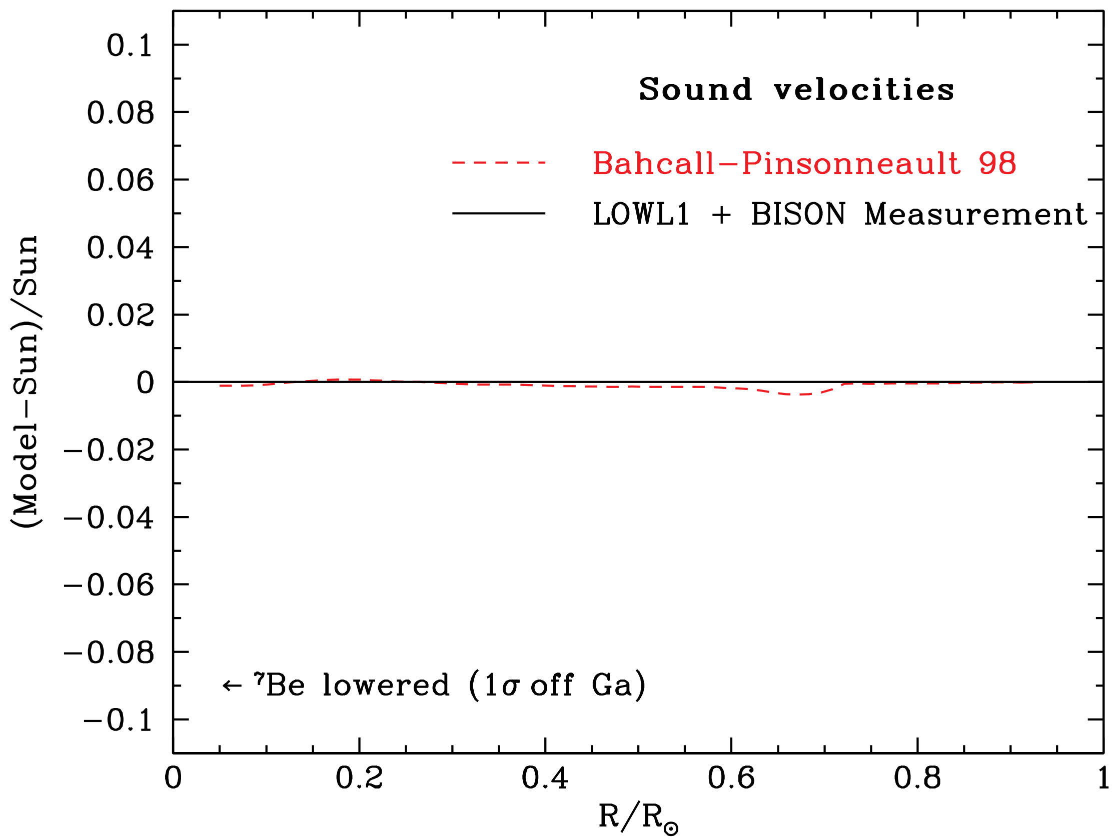

FIG. 7 Fractional difference between solar velocities, one calculated on the basis of the SSM, and the other, measured by helioseismology. As can be clearly seen, the prediction and the actual experimental data match with each other to a stunning degree making a strong case for the credibility of the Standard Solar Model.
(Image source: [10])

trino energy spectrum would still be approximately the same (depicted in FIG. 6) [2].

The second point to take into account is the spectacular success of the Standard Solar Model, and the fact that it's predicted values about certain other factors match closely with the values measured using helioseismology. For example, helioseismology can make quite an accurate determination of the sound velocities in the sun based on a large number of measurements of the solar frequencies (p-modes). The values calculated on the basis of the Standard Solar Model and those obtained from helioseismology agree quite well; the fractional difference, as represented in FIG. 7, is quite low.

From the striking similarity of the experimental values and the theoretical data, it can be assumed that even at the depths at which solar neutrinos are generated, the Standard Solar Model approximately holds. A pretty impressive recent development that vindicates faith in the Standard Solar Model is that the solar sound velocity calculated agrees to better than 0.2% even at depths of $d < 0.1\ R_{\odot}$, that is nearly the centre of the sun's core. This clearly shows that the model is quite applicable even at the depths the neutrinos are produced.[3]

The most important takeaway from this, is the fact that if the Standard Solar Model's predictions of sound velocities fit so well at the required depths, then one can take the predictions the theory makes for the temperature profile of the sun's interior at face value. This is particularly important because the neutrino generating reactions in the sun's core depend significantly on the core temperature. For instance, the rate for the ^7Be-neutrino generation reaction (PP II Cycle) is predicted to depend upon the core temperature T , like T^{10}, while that for the ^8B-neutrino generating reaction (PP III Cycle) varies as T^{25}. Evidently, even a very small deviation from the calculated temperatures will lead to a huge change in the solar neutrino flux [2].

IV. NEUTRINO DETECTORS AND THEIR WORKING PRINCIPLES

Neutrinos are quite elusive particles. They have a very small mass, and they can interact only through gravity and the weak interaction.
The gravitational force does not affect neutrinos much since their mass is too low. Gravity depends on the masses of the interacting particles, and as such due to the very low rest mass of the neutrinos, the gravitational interaction with neutrinos is negligible.
On the other hand, there is the weak force which interacts with neutrinos in two different manners:

- Neutral current interactions
- Charged current interactions

Each of these is considered below.

A. Neutrinos and the Weak Interaction

1. Neutral Current Interaction

One of the ways the weak force interacts is the neutral current interaction. The neutral current interaction is mediated by the transfer of Z bosons. The Z boson is an electrically neutral carrier particle, the exchange of Z boson transfers momentum, energy, spin but the quantum numbers of the interacting particles (the flavor, the charge etc) remain unaffected. The neutral current interaction owes its name to the fact that the Z boson carries no electric charge and there is no charge transfer involved.
Detectors built on the neutral current interaction of neutrinos can be those based on the Cherenkov radiation. A neutrino will enter the detector, transfer some of it's momentum and energy to a target particle, and then leave the detector. If the target particle is lightweight and charged (like an electron, e), it may reach relativistic speeds and emit radiation which may be detected directly. This sort of radiation emitted when a charged particle in a particular medium travels faster than the speed of light in that particular medium is termed as Cherenkov radiation.
While neutrinos of all three flavor states can be detected through this, no distinction can be made between them, that is, no information about the flavor of the neutrinos is left behind.

2. Charged Current Interaction

Another way in which the weak force may interact is through the charged current interaction. This two bosons mediating this particular interaction are the W^+ and the W^- bosons. This interaction is the most easily detected weak interaction in nature. The W bosons, as mentioned earlier, are of two types, with the W^+ carrying a positive charge, while the W^- carries a negative charge. The W bosons mediate neutrino absorption or emission, and these processes are accompanied by absorption or emission of either an electron or a positron or a change in the flavor and charge of the neutrino, such as in K-capture or β decay.
Detectors based on charged current interactions make advantage of high energy neutrinos. In these interactions, a neutrino with high energy will transform into its leptonic partner (electron, muon or tau) and these particles can then be detected. A point to note is that the neutrino must have sufficient energy to create it's more massive partner lepton's mass, otherwise it cannot interact via this method.
Detectors that can detect and differentiate between the three different types of leptons produced in charged current interactions (electron, muon or tau) can determine the flavor of the incident neutrino. Since the interaction involves transfer of electric charge, the target particle also changes character (for example, a neutron might change into a proton).

V. DETECTOR PRINCIPLES AND TYPES

Neutrino detectors can be broadly classified into two types [1]:

1. Radiochemical detectors
2. Real time detectors

Each of these is taken up in details below.

A. Radiochemical detectors

For radiochemical detectors, one has a huge amount of an element, say ^{A}Z. The solar neutrinos enter the detector, and occasionally interacts via charged current interactions with this element, converting it to $^{A}(Z+1)$, which can be later detected and counted to get a rough estimate of the solar neutrino flux.
Two important example of such radiochemical detectors are as follows. One is based on a Cl to Ar conversion, and another on Ga to Ge conversion [1, 3].

1. A crucial example of a detector based on this technique is the Homestake chlorine detector. It consists of a tank filled with tetrachloroethylene, or C_2Cl_4.
   It utilises the reaction,
   $$\nu_e \ + \ ^{37}Cl \ \rightarrow \ ^{37}Ar \ + \ e$$
   The threshold energy for this reaction is, $E_{th}^{Cl|Ar} = 0.814 \ MeV$.

A chlorine-37 atom is occasionally converted through the neutrino interactions, giving us an argon-37 atom. He gas is used to purge the fluid (tetrachloroethylene) on a periodic basis (usually every two or three months), and this, in turn, removes the argon present. The He is cooled and separated from the argon. A rough estimate of the solar neutrino flux can then be obtained by counting the number of argon atoms. The Homestake detector was the first solar neutrino detector, and had a major hand behind the recognition of the solar neutrino problem as an area of modern physics that need redressing.

2. A similar sort of radiochemical detector is based on a gallium-71 to germanium-71 (an unstable isotope of germanium) conversion via neutrino interaction. The reaction involved here is,

$$\nu_e \;+\; {}^{71}Ga \;\rightarrow\; {}^{71}Ge \;+\; e$$

The threshold energy for this reaction is, $E_{th}^{Ga|Ge} = 0.233\ MeV$.
The germanium is extracted and concentrated and the number of neutrinos determined. The SAGE detector, Russia and the GALLEX/GNO experiments were gallium based.

The advantage of gallium based detectors is the threshold energy of the neutrinos needed for the conversion. The Homestakes detector has a threshold of 0.814 MeV, while the gallium based detectors, in comparison, have a threshold energy of 0.233 MeV.

B. Real Time detectors

Real time detectors usually record the neutrinos as a result of a scintillation or flash of light that is detected by some sort of photo-multiplier arrangements. Unlike radiochemical experiments, which only measure a rate i.e. the number of solar neutrinos in a given span of time, real time detectors can yield a lot of information about the neutrinos detected, including event time, the direction of individual events, energy and sometimes, even flavor information of the incident neutrinos.
In real time Cherenkov detectors, a high energy neutrino is allowed to interact with atomic nuclei producing charged leptons, like an electron or muon. If a charged lepton moves in a given detector medium with a speed that is higher than the speed of light for that particular medium, then a kind of visible optical shock wave is produced. This sort of radiation is named Cherenkov radiation, and when this Cherenkov radiation is detected, it can be observed as a typical "ring-like" pattern in an array of the photo-multipliers detecting it. [13]
Examples for detectors or experiments based on this principle are as follows.

1. Two important examples of Cherenkov detectors are the Kamiokande and the Super Kamiokande, both in Japan. Both of these are Cherenkov detectors and use pure water, H_2O, for the detection of neutrinos.
The reaction (neutrino-electron scattering) utilised for detection in these two detectors is,

$$\nu_e \;+\; e \;\rightarrow\; \nu_e \;+\; e$$

The threshold for the Kamiokande is about 7.0 MeV, while the Super Kamiokande has a threshold of about 5.5 MeV.
The Kamiokande detector, in 1988, actually provided the first evidence that neutrinos do arrive on the earth from the sun's direction, that is, it provided the first direct evidence for the production of solar neutrinos. It also detected a neutrino burst from the supernova SN 1987A, out of the huge number of neutrinos emitted ($\sim 10^{57}$); only about 19 neutrinos were detected by scientists. The Kamiokande terminated it's activity in 1995. On the other hand, the Super Kamiokande is the largest water filled Cherenkov detector with 11,000 photo-multiplier tubes, buried a kilometre underground, aurrounding about 50,000 tonnes of pure water and has played a crucial role in verifying neutrino oscillations and in the Solar Neutrino Problem.

2. Another example of a Cherenkov detector is the Sudbury Neutrino Detector, SNO, in Canada. Unlike the Kamiokande and the Super Kamiokande, the SNO utilises, instead of pure water, ultrapure heavy water, D_2O. The SNO utilises about 1000 tonnes of heavy water in a vessel of diameter of about 12 metre.
The specific reaction taking place in the SNO is,

$$\nu_e \;+\; d \;\rightarrow\; p \;+\; p \;+\; e$$

This is a charged current interaction with a Q value of $-1.44\ MeV$.
On top of the neutrino interactions detected in the pure water based detectors, the Kamiokande and the Super Kamiokande, there is a specific interaction that can be observed in the heavy water, that is by the SNO detector. A neutrino can occasionally cause the deuterium in the heavy water to break up, resulting in a gamma ray burst that can be detected. The reaction is as follows,

$$\nu_x \;+\; d \;\rightarrow\; p \;+\; n \;+\; \nu_x$$

The ν_x represents the fact that neutrinos of all three flavors can participate in this reaction. The Q value of this interaction, which is a neutral current interaction, is about $-2.2\ MeV$.
In addition, the SNO can also detect elastic scattering of neutrinos on electrons.

C. Other detector methods

There are detectors set up on related principles for the detection of neutrinos. Some examples are:

1. The Radio Ice Cherenkov detects Cherenkov radiation emitted due to neutrinos travelling through the Antartic ice. A similar experiment detecting Cherenkov radiation due to neutrinos in Antarctica is the IceCube Neutrino observatory in Antarctica.

2. ANITA (Antarctic Impulse Transient Antenna) is another detector in the Antarctic that detects Askaryan radiation due to ultra high energy neutrinos travelling through the Antartic ice. The Askaryan radiation is similar to the Cherenkov effect. It occurs when a particle travels through a dense dielectric (like ice) with a velocity that is greater than the phase velocity of light. This produces a large number of secondary charged particles, and results in the emission of a cone of radiation with wavelength belonging to the radio or microwave part of the electromagnetic spectrum.

3. Scintillators are another class of real time detectors that detect scintillations produced due to neutrinos in a particular medium. For example, MiniBooNE is a neutrino detector that utilises mineral oil as a detector medium. Even if Cherenkov radiation is not emitted by the neutrinos passing through the detector, scintillation light will still be emitted as mineral oil is a natural scintillator and this can be detected. The Cowan-Reines neutrino experiment also utilised scintillation based detection techniques to prove the existence of antineutrinos back in 1956. A similar technique is used at the KamLAND detector to study antineutrino oscillations.

VI. THE MATHEMATICS OF NEUTRINO OSCILLATIONS

A. Flavor mixing and neutrino oscillations

Now, neutrino oscillations play a crucial part in the Solar Neutrino Problem, and hence, should be looked into with a bit more detail.
Say there are neutrinos, ν_i with distinct mass eigenstates, m_i, where $i = 1, 2, 3, ...$ The neutrinos can be of three different flavors, e, μ, τ with the neutrino in the flavor state, x, defined by ν_x, where $x = e, \mu, \tau$. Consider the following decay of a W^+ boson,

$$W^+ \ \to \ l_x^+ \ + \ \nu_i$$

Here, l_x^+ is the charged lepton produced, while ν_i is the associated neutrino in the decay. Leptonic mixing tells us that each time such a decay occurs, a particular charged lepton l_x^+ is created along with a neutrino ν_i, but the neutrino produced in the decay is not unique and may be different during different runs of the decay.
Therefore, one can define a neutrino in the flavor state x produced in the above bosonic decay by the following,

$$|\nu_x\rangle \ = \ \sum_i U_{xi}^* \, |\nu_i\rangle \tag{1}$$

Here U_{xi} is a unitary transformation, as required by quantum mechanics, and is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, or the PMNS matrix. It is commonly of the form (if $i = 1, 2, 3$ and with $x = e, \mu, \tau$),

$$U \ = \ \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Since U is unitary, one can invert it and write the mass eigenstates in terms of a flavor basis as,

$$|\nu_i\rangle \ = \ \sum_y U_{yi} \, |\nu_y\rangle \tag{2}$$

Now, applying the Schrodinger equation to each of these $|\nu_i\rangle$, one can describe them by plane wave solutions of the form [6] (considering natural units $c = 1, \hbar = 1$),

$$|\nu_i(T_i)\rangle \ = \ e^{-im_iT_i} \, |\nu_i(0)\rangle \ = \ e^{-i(E_it - p_iL)} \, |\nu_i(0)\rangle \tag{3}$$

Here, m_i refers to mass and T_i the time in the frame of ν_i. Due to Lorentz invariance, the phase factor m_iT_i was expressed as $E_it - p_iL$. Now the energy-mass equivalence relation in natural units, ($c = 1, \hbar = 1$), is

$$E^2 \ = \ m^2c^4 \ + \ p^2c^2 \ = \ m^2 \ + \ p^2$$

Taking into consideration those cases where $t \approx L$, the above equation and assuming that the neutrino has a definite momentum, p,

$$E_i \ = \ \sqrt{m^2 \ + \ p^2} \ \approx \ p \ + \ \frac{m_i^2}{2p}$$

Here, since the neutrinos are extremely relativistic, the momentum is considered to be much higher than the mass.
Therefore, combining all of this,

$$|\nu_i(T_i)\rangle \ = \ e^{\frac{-im_i^2L}{2p}} \, |\nu_i(0)\rangle \tag{4}$$

Or,

$$|\nu_i(T_i)\rangle \;=\; e^{\frac{-im_i^2 L}{2E}}\,|\nu_i(0)\rangle \qquad (5)$$

Here, $E \simeq p$ denotes the average energy of the neutrinos in different mass states.
The above equation makes it clear that neutrino eigenstates having different masses, m_i, travel through a medium with different frequencies. As can be seen, the heavier ones oscillate at a faster rate than the lighter ones.
Summarising (x, y are indices for the flavor eigenstates, i is the index for the mass eigenstates),

$$|\nu_x\rangle \;=\; \sum_i U_{xi}^*\,|\nu_i\rangle \qquad (1)$$

$$|\nu_i\rangle \;=\; \sum_y U_{yi}\,|\nu_y\rangle \qquad (2)$$

$$|\nu_i(T_i)\rangle \;=\; e^{\frac{-im_i^2 L}{2p}}\,|\nu_i(0)\rangle \qquad (4)$$

Substituting (4) into (1), for a neutrino ν_x travelling distance L in time t,

$$|\nu_x(L)\rangle \;=\; \sum_i U_{xi}^* e^{\frac{-im_i^2 L}{2E}}\,|\nu_i\rangle \qquad (6)$$

As mentioned earlier, here, $E \simeq p$ denotes the average energy of the neutrinos in different mass eigenstates.
From (2),

$$|\nu_i\rangle \;=\; \sum_y U_{yi}\,|\nu_y\rangle$$

Substituting (2) into (6),

$$|\nu_x(L)\rangle \;=\; \sum_i U_{xi}^* e^{\frac{-im_i^2 L}{2E}} \sum_y U_{yi}\,|\nu_y\rangle$$

$$\Rightarrow\; |\nu_x(L)\rangle \;=\; \sum_y \left(\sum_i U_{xi}^* U_{yi} e^{\frac{-im_i^2 L}{2E}}\right)|\nu_y\rangle \qquad (7)$$

From the above equation, it is evident that the original neutrino, ν_x, behaves as a superposition of neutrinos of all the different flavors, $\nu_y,\; y = e, \mu, \tau$, as it propagates through the distance L [6]. In other words, the original neutrino is produced along with the charged lepton, l_x^+, at the neutrino generation site by the decay of a W^+ boson, while the neutrino detector at the detector site (a distance L away) detects a charged lepton of a different flavor, say l_β^+ (which shows that the neutrino flavor state has changed). This equation, therefore, captures the idea of neutrino oscillations. Another important point to note here is that the mass eigenstates have to be necessarily non zero for neutrino oscillations to take place, which shows that neutrinos do have masses, since neutrino oscillations have been experimentally verified.

B. The probabilities associated with neutrino oscillations

From the last subsection, in particular equation (7),

$$|\nu_x(L)\rangle \;=\; \sum_y \left(\sum_i U_{xi}^* U_{yi} e^{\frac{-im_i^2 L}{2E}}\right)|\nu_y\rangle$$

Now, the probability that a neutrino, ν_x, in a given flavor state changes into a different flavor state, say it becomes ν_y, is given by $P(\nu_x \to \nu_y)$ such that,

$$P(\nu_x \to \nu_y) \;=\; |\langle \nu_y | \nu_x(L)\rangle|^2 \qquad (8)$$

Or,

$$P(\nu_x \to \nu_y) \;=\; \left|\sum_i U_{xi}^* U_{yi} e^{-i\frac{m_i^2 L}{2E}}\right|^2 \qquad (9)$$

Analysing the above equation and simplifying it greatly, $P(\nu_x \to \nu_y) \;=$

$$\delta_{xy} \;-\; 4\sum_{i>j} \mathrm{Re}\left(U_{xi}^* U_{yi} U_{xj} U_{yj}^*\right) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$

$$+\; 2\sum_{i>j} \mathrm{Im}\left(U_{xi}^* U_{yi} U_{xj} U_{yj}^*\right) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right) \qquad (10)$$

Here, δ_{xy} is the Kronecker Delta, and $\Delta m_{ij}^2 \;=\; m_i^2 - m_j^2$. This equation the desired probability for flavor mixing in neutrinos.

C. Flavor mixing in the two neutrinos case

For only two neutrinos, say ν_α converts to ν_β, the PMNS matrix takes the form,

$$U \;=\; \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad (11)$$

Here, θ is the mixing angle, that is, it is a parameter for this case. Now, the probability $P(\nu_\alpha \to \nu_\beta)$ is,

$$P(\alpha \to \beta) \;=\; \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \text{ (natural units)} \qquad (12)$$

$(\beta \neq \alpha)$

Converting from natural to SI units, and $\beta \neq \alpha$, the probability $P(\nu_\alpha \to \nu_\beta)$ is,

$$P(\nu_\alpha \to \nu_\beta) \;=\; \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m^2 L}{E}\frac{[eV^2]km}{GeV}\right) \qquad (13)$$

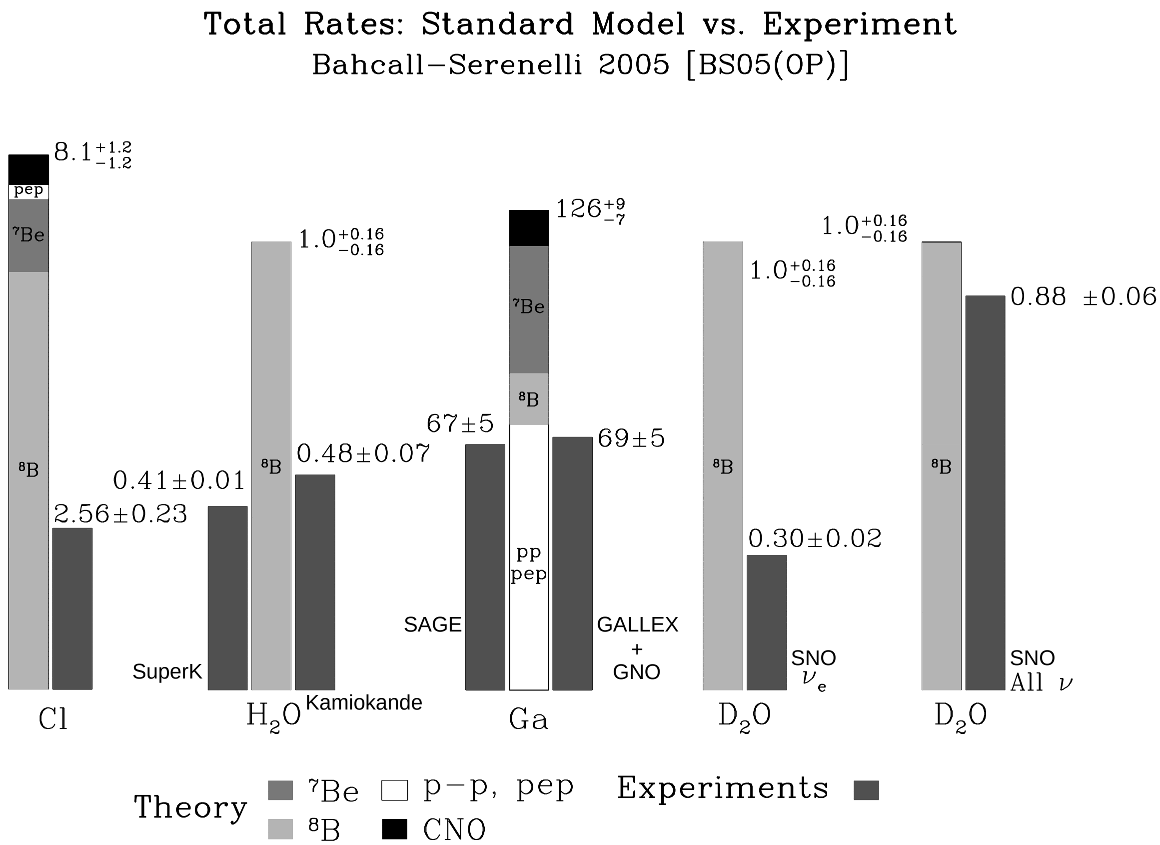

FIG. 8 A plot comparing the neutrino flux of the sun calculated on the basis of the SSM versus the experimental flux measured. It is evident that there is a large solar neutrino flux deficit in the experimental data (the darker bars) as compared to the theoretical predictions (the lighter bars) made on the basis of the Standard Solar Model.
(Image source: [10])

Using these expressions many solar neutrino detectors have analysed data [3]. The two neutrino case is the case considered by most neutrino detectors to derive the theoretical flux values. In a lot of these cases, the actual scenario may be a quasi two neutrino case, different from a genuine two neutrino flavor mixing case [6]. Atmospheric neutrino experiments have also been carried out using these expressions.

VII. THE SOLAR NEUTRINO PROBLEM AND IT'S RESOLUTION

The Solar Neutrino Problem revolves around the flux deficit of the solar neutrinos (which are mostly electron neutrinos, ν_e) detected on the earth compared to the expected solar neutrino fluxes as calculated on the basis of the Standard Solar Model which, incidentally, was established by J. Bahcall, who also helped in the experiment at the Homestake neutrino detector which first detected and recognised that there was a deficit. The flux differences between the calculated and the theoretical figures are summarised in the FIG. 8

A. Proposed solutions

Most of the early proposals or ideas to resolve the Solar Neutrino Problem centred around modifying the existing solar models in some way or the other. These included suggestions that the the parameters inside the sun, the temperature and the pressure, were different than what was predicted based on the Standard Solar Model.
As previously mentioned in the subsection titled "Viability of the Standard Solar Model", even if the model was wrong about the structure of the internal core of the sun, the shape of the neutrino energy spectrum (FIG. 6) remains more or less the same [2]. This point is important

because if the flux deficit is to be explained by the Standard Solar Model as it stands, then the solar core temperature should be lower than what had been predicted. On the other hand, a reduction in the solar core temperature would cause other discrepancies; the reactions of the pp-chain will be affected. As mentioned previously in the said subsection, the reactions for the generation of the solar neutrinos depend quite significantly on the temperatures in the solar core; even the slightest of deviations can cause a huge change. This change in the reactions of the pp-chain will, in turn, lead to a change in the details of the neutrino energy spectrum, which, therefore, will create a different and new discrepancy. In a similar fashion, other modifications to the Standard Solar Model that may help in explaining the solar neutrino flux deficit lead to new discrepancies which will then need to be explained.

B. The resolution of the problem

1. A brief history of the resolution

The solar neutrino flux deficit first came to light in the mid 1960s, and it went on garnering interest for quite a long time until it was finally resolved in the 2000s thanks to the work carried out by the Sudbury Neutrino Observatory and the Super Kamiokande.

The Kamiokande and another detector, the IMB, detected a neutrino burst from the supernova SN 1987A in 1987. These experimental results were the first to hint at the possibility that neutrinos were probably not massless as the Standard Model of particle physics assumes, but rather, had finite, small, but non-zero, masses. This was because of a difference in the time of arrival of the neutrinos at the Kamiokande and the IMB. However, as mentioned earlier, neutrinos are weakly interacting particles and as such the detection events were quite small in number. Hence, the conclusions about the mass of the neutrinos was not yet certain. On top of that, neither the Kamiokande nor the IMB were detectors built to study supernova neutrinos, and as such, the question of whether the neutrinos have mass could not be settled definitively.

In 1998, the first strong piece of evidence for neutrino oscillations came from the experimental observations carried out by the Super Kamiokande neutrino detector in Japan. The Super Kamiokande detector observed a difference in the atmospheric neutrino flux for neutrinos coming from directly above the detector and those coming through the earth. In particular, this experiment was detecting only one type of neutrino, the muon neutrino, ν_μ. Muon neutrinos are created in the upper levels of the atmosphere of the earth by cosmic rays and these were the ones being detected by the detector. It was observed that the number of muon neutrinos coming through the earth was lower in number compared to the ones coming from directly above the detector. This result was consistent with the phenomenon of neutrino oscillations, that is the muon neutrinos, ν_μ coming through the earth were getting converted into tau neutrinos, ν_τ, due to neutrino oscillations which led to a change in the flavor of the neutrino. This result made the phenomenon of neutrino oscillation a feasible solution to the deficit in the solar electron neutrino flux and was a first nudge in this direction.

In 1999, the Sudbury Neutrino Observatory begun it's operations and started collecting data.

The evidence for flavor mixing in neutrinos was finally confirmed by the Sudbury Neutrino Observatory and the Super Kamiokande detector. This was found out by measuring the differences between the electron neutrino flux measured by the Sudbury Neutrino Observatory and the flux measured at the Super Kamiokande detector. If the Standard Model was correct in it's assumption of massless neutrinos, the electron neutrino fraction measured by the two detectors would have been the same, but in reality the measured fluxes were different, finally proving that, indeed, neutrinos have finite, non zero masses.

The SNO was more concerned with detecting the high energy ^8B-neutrinos, generated in the PP III Cycle, which were not much affected by the oscillations in either the sun or the earth owing to their large energy ($\sim 10\ MeV$). Even then, a large discrepancy was expected in the ^8B-neutrino flux due to the Mikheyev–Smirnov–Wolfenstein effects (MSW effects). While the SNO was more directed towards studying the electron neutrinos from the sun, it did observe neutrinos of all the three different flavors collectively and was, thus, able to determine the fraction of electron neutrinos. After extensive analysis, the fraction came about to be 34%, and this figure matched perfectly with the figure put forward by the Standard Solar Model. Hence, the problem was taken to be resolved at last.

2. Resolving the Solar Neutrino Problem

Most physicists currently believe that neutrino oscillations and the Mikheyev–Smirnov–Wolfenstein effects (MSW effects) provide the most elegant solution to the Solar Neutrino Problem.

According to the currently accepted solution, the neutrino oscillations play a major role in the observed flux deficit of the solar neutrinos. As the solar neutrinos travel from the solar core to the earth, they undergo flavor mixing and neutrino oscillations.

From (7),

$$|\nu_x(L)\rangle \;=\; \sum_y \left(\sum_i U^*_{xi} U_{yi} e^{\frac{-im_i^2 L}{2E}} \right) |\nu_y\rangle$$

This equation shows that a neutrino, in flight (covering a distance, L), can be written down in form of

a superposition of different flavor states. Neutrino oscillations lead to flavor mixing, and result in one flavor of neutrino changing into a different flavor of neutrino. Since solar neutrinos are mostly electron neutrinos, ν_e, most detectors measure the solar electron neutrino flux, rather than just counting the number of neutrinos of all flavors originating from the sun. The neutrino oscillations lead to a number of electron neutrinos changing flavor and becoming muon or tau neutrinos, ν_μ and ν_τ respectively. Since most detectors are aimed at observing the electron neutrino flux, the converted solar neutrinos are not counted or detected and go unaccounted for, resulting in an apparent deficit in the solar neutrino flux. There is actually no deficit in the number of solar neutrinos but in the number of solar electron neutrinos, and this deficit can be accounted for when one considers the electron neutrinos that changed flavours due to neutrino oscillations on the way to earth.
The second effect to consider here is the Mikheyev–Smirnov–Wolfenstein effect (MSW effect). The MSW effects matter for the high energy solar neutrinos in two ways. In the sun's core, the MSW effects will tend to lead to flavor mixing, once again leading to a change in the number of electron neutrinos which change flavor to become muon or tau neutrinos, ν_μ and ν_τ respectively. In this way, the MSW effects will have a hand in reducing the observed high energy solar electron neutrino flux. On the other hand, the MSW effects become important again once these neutrinos reach the earth. In this case, some of the previously converted neutrinos may change back to electron neutrinos once again, and this particular change will increase the solar electron neutrino flux.
On the basis of these two points, one can claim that there is no actual deficit of solar neutrinos, but due to neutrino oscillations leading to flavor mixing and the MSW effects in matter, some of the electron neutrinos emerging from the solar interior get converted to other flavors of neutrinos which go undetected. However, the Sudbury Neutrino Observatory collected extensive data on all the flavors of neutrinos coming from the sun so as to estimate the fraction of solar electron neutrinos reaching the earth, and as mentioned earlier, the fraction came to 34% in perfect agreement with the values derived from theoretical considerations of quantum mechanics and the Standard Solar Model.
This is the reasoning believed in by most physicists, and this resolution is considered widely to be thee actual solution to the Solar Neutrino Problem.

3. Other exotic proposals

There are other exotic proposals to solve the Solar Neutrino Problem [4], for example, resonant spin-flavor phenomenon which bears a resemblance to the way the MSW effect leads to a lower than expected experimental solar neutrino flux. This particular phenomenon leads to conversion of solar neutrinos into active antineutrinos or sterile neutrinos. In both cases, the antineutrinos or neutrinos formed are non electron flavors and this leads to a reduction of the observed solar neutrino flux and can help in resolving the problem.
Another alternative idea exists in this context. This particular idea involves a violation of the Equivalence Principle. It may be the case that flavor changes and neutrino oscillations may be induced through gravitational forces. This may lead to the decrease in the observed solar neutrino flux and account for the problem at hand.
Another idea which accounts for the problem is the Non Standard Neutrino Interactions (NSNI) with matter that can also be a source of neutrino oscillations. This may be relevant when considering neutrino interactions with the dense solar matter and may account for part of the neutrino oscillations leading to the decrease in the solar neutrino flux observed.
However, for most part, these solutions and proposals hinge significantly on the phenomenon of neutrino oscillations and flavor mixing. While these exotic alternative solutions are interesting and are worked on by scientists, most people are of the opinion that it is the simpler explanation discussed in the previous section which plays the most crucial role in accounting for the deficit in the electron neutrino flux from the sun.

VIII. CONCLUSION AND OUTLOOK

The Solar Neutrino Problem presents a fascinating example of how experimental research work and theoretical research work go hand in hand in science to advance the current scopes of human knowledge. The golden years of this particular problem were between 2001 to 2003 as per John Bahcall, a pioneer physicist who established the Standard Solar Model on the basis of which all theoretical calculations were made [11]. It was in these years that the Solar Neutrino Problem was most worked on and it all culminated into fruition with the resolution of the problem at the very end.
In total there have been 4 Nobel Laureates connected with the Solar Neutrino Problem. In 2002, the Nobel Committee awarded part of the Nobel Prize in Physics to Masatoshi Koshiba and Ray Davis. John Bahcall had collaborated with Ray Davis on the Homestake chlorine experiment which identified and recognised the deficit in the solar neutrino flux. The second Nobel Laureate, Masatoshi Koshiba, on the other hand, headed the Kamiokande detector which verified and confirmed the fact that the solar neutrino deficit as measured by the Homestake detector was indeed very much real. The Kamiokande (along with the IMB) also provided the first indications of the fact that neutrinos were actually not

massless particles as had been assumed, but they did, in fact, have a finite, non zero mass.

After this, in 2015, once again the Nobel Committee awarded part of the Nobel Prize in Physics to Arthur McDonald and Takaaki Kajita. Takaaki Kajita headed the team that detected and confirmed neutrino oscillations at the Super Kamiokande detector. On the other hand, Arthur McDonald headed the research team at the Sudbury Neutrino Observatory, and had also provided evidence for neutrino oscillations (in actuality, the SNO team had investigated high energy neutrinos and as such it were the MSW effects that were more important). The two experiments had collected extensive data, opened up new pathways and had, finally, resolved the longstanding Solar Neutrino Problem.

For the time being, the Solar Neutrino Problem is widely considered to be solved. Unless new evidence emerges to the contrary, this current proposal of neutrino oscillations, MSW effects and flavor mixing provides the best solution. On the other hand, research work on the elusive neutrinos is still going strong. A lot of things about neutrinos are a mystery, and there are new experiments being set up to study and learn more about neutrinos in general.

The Solar Neutrino Problem stood open for nearly four decades, it was four decades of relentless experimental work, four decades of extensive theoretical work that finally managed to crack the problem and resolve it with some certainty. There may have only been four Nobel Laureates in this particular area of research, but behind those four Nobel Prizes lies the hard work, dedication and passion of so many researchers who worked day in and day out for that one elusive answer. Above all, the Solar Neutrino Problem and it's fantastic resolution make us appreciate the spirit of science, relentless and ever flowing.

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