The Double Slit Experiment in Classical and Quantum Realms

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The Double Slit experiment has played out as a classic in the history of physics with pivotal roles in both classical physics as well as quantum physics. This term paper takes a look at this very iconic experiment that is said to hold the central mystery in quantum mechanics (according to R. P. Feynman) and which brought out a revolution in the classical theories of light as well. The paper starts with an overview of classical optics, Newton's corpuscular theory, Young's double slit experiment and briefly focuses on interference and diffraction ideas. It then takes a look at the role played by the experiment in different areas of quantum physics, including wave - particle duality, uncertainty, electron diffraction, and modified double slit experiments in quantum physics.

CONTENTS

I. INTRODUCTION

> *"The double-slit experiment has in it the heart of quantum mechanics. In reality, it contains the only mystery."*
>
> ***Richard P. Feynman***
> The Feynman Lectures on Physics, Vol. 3 (10)

The double slit experiment is one of the most iconic experiments in the history of physics. It started off as a classic experiment used to thwart the Newtonian idea of corpuscular light but ended up being applied beyond the realm of classical physics to modern quantum physics.

At it's heart the double slit experiment is as simple as one can imagine - all it takes is a source (of light or other particles like electrons as discussed later), two fine slits and a screen (detector in case of electrons). This simple st up is all it takes to delve deep into the mysteries that haunt modern physics - light, matter, the foundations of quantum physics and the reality that surrounds us.

This term paper starts with the idea of light as a particle and wave, the old debate exemplified by the two greats, Newton and Huygens. It follows on the work of Thomas Young, the mind that came up with this experiment, the beginnings of quantum physics and how the double slit experiment becomes increasingly relevant in the idea of a non deterministic (or is it deterministic, after all?) reality that quantum mechanics (in most parts, the Copenhagen interpretation of it, at least) epitomises.

II. THE DOUBLE SLIT EXPERIMENT IN CLASSICAL PHYSICS

A. Conflicting views on light - Newton and Huygens

1. The Corpuscular Theory:

The most iconic scientist in the history of physics was, in all probabilities, Sir Issac Newton, the man who came up with not only a new mathematics to describe reality but also a new approach to the physics, derived from a mathematical description but guided by physical reality rooted in observations and experiments. Though Newton's most famous work involves his (and Gottfried Wilhelm Leibniz's) theory of calculus, his laws of motion, the basis of the Newtonian paradigm, the first "theory of everything" owing to it's vast applicability, and his law of gravitation. However, even apart from these, Newton's achievements are found scattered in several areas , and one of the other subjects he worked primarily on was optics. In this field, Newton carried

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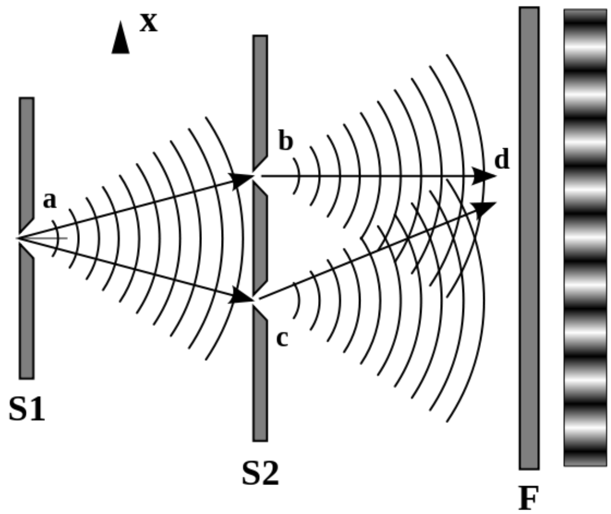

FIG. 1 *Setup for the double slit experiment*
(Image source: (4))

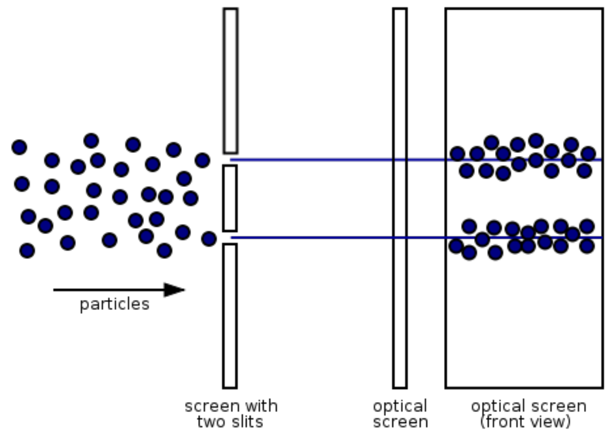

FIG. 2 *Expected density of "markings" if light were corpuscular*
(Image source: (1))

out a large number of experiments, painstakingly noted them down and recorded his observations, reasoned out certain conclusions and put forward his theory of light - the corpuscular theory of light (in actual fact, the theory is credited to Descartes; however the theory was elaborated on and developed to a huge extent by Newton).

According to this theory, light is composed of infinitesimally small "corpuscles" which travel at a finite velocity in straight lines. According to Newton, the phenomenon of refraction and reflection of light and their laws could be explained in a geometric way if such a theory of light holds. While the theory did take care of both reflection and refraction, it could not explain other phenomena, most notably diffraction and interference which the wave theory as put forward by Huygens could explain.

2. Huygens' Wave Theory:

Christiaan Huygens proposed his wave theory of light in parallel to the Newtonian corpuscular theory. For years, this wave theory languished and was lampooned by the esteemed scientists of the day, mostly because it challenged the Newtonian conception and Newton's prestige and fame were far reaching.

Huygens' Principle (also called the Huygens - Fresnel principle) treats light as a wave (with a wavefront) and states that every point on the wavefront is capable of giving rise to secondary wavelets or other wavefronts. According to the theory, light travels in this way, as a wave, represented by a wavefront (the envelope of the particles all oscillating in the same phase) and giving rise to more wavefronts with time. The theory (as developed by Fresnel) was quite successful in explaining reflection and refraction (just like Newton's) but, could also explain diffraction and interference effects that the previous worldview couldn't. After the decisive double sit experiment by Young and the development of the theory by Fresnel, the wave theory replaced the Newtonian corpuscular theory as the theory of light. It was not until the days of quantum mechanics did the idea of a particle theory of light come up again.

B. Young's double slit experiment

The main experiment behind the acceptance of the wave picture of light was an elegant and simple experiment suggested by Thomas Young, a proponent of the wave theory himself.

Young's experiment consisted of passing sun rays through a very small hole, splitting the beam in half by using a thin slit and then observing the resulting pattern. The modern version of the experiment is much the same, but is mostly done with a source, like a laser source, emitting light that passes through two closely placed fine slits (hence, double slit). The resulting pattern is observed on a screen kept on the other side of the slits (Fig. 1).

The interesting feature of the experiment lies in the pattern that is obtained on the screen. If light were to be made of tiny particles, as the corpuscular theory claims, then one could imagine throwing balls (which travel in a straight line in free space) at a wall with 2 very narrow (and closely placed) doors and marking the

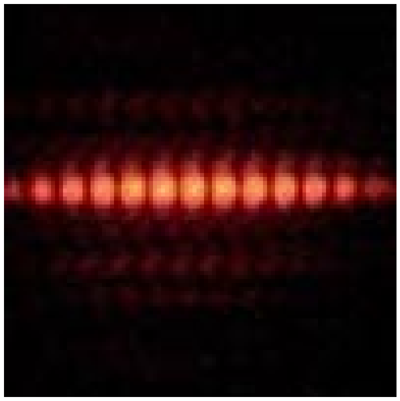

FIG. 3 *The bright and dark fringes observed in the double slit experiment*
(Image source: (7))

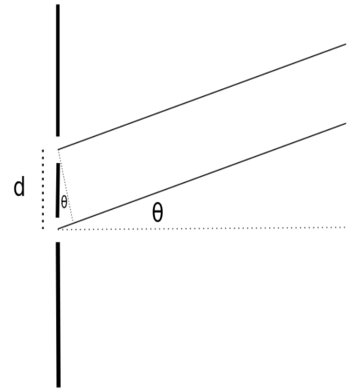

FIG. 4 *A diagram for the interference pattern*
(Image source: (2))

spot where each ball hits on the opposite wall. As one can imagine, owing to the width of the doors, that it will be the areas directly in line with the doors that will bear the most markings. The number of markings will decrease as the distance from the doors increases. The expected density of markings is depicted in Fig. 2.

What Young, and the others after him, observed was very different to this. They observed a sort of periodicity in the distribution, bright and dark "fringes" were formed at regular intervals. These fringes are, in general, called "interference patterns" (or, diffraction, depending on the case) and were a very definitive evidence for the wave nature of light since waves were known to interfere and form such patterns. In fact, Thomas Young had worked on sound waves as well, earlier, and was quite familiar with this sort of patterns and immediately realised that this meant that light travelled as a wave.

What the wave theory says in essence about the phenomena is that since light is a wave, it has phases. One can imagine two waves - a sine and a cosine (always out of phase by $\pi/2$). If we add the these types of waves together which are out of phase, the resulting wave would have different amplitudes to the original ones. For example, consider two waves (of the same amplitude) out of phase by π. Then the crest of one lines up with the trough of the other and vice versa which makes the amplitude of the resulting wave zero. Similarly, waves that are in phase have their amplitudes increased as their crests and troughs line up with the crests and troughs of the other. This phenomena of the superposition of the amplitudes of different waves at a point to give the amplitude of the resulting wave at the same point is called interference. The case in which they are completely out of phase leads to destructive interference and the case where they are perfectly in phase leads to constructive interference. It is this interference patterns that Young observed. Light from the two slits arrived at different points on the screen. At some places they were out of phase (destructive interference) and at others they weren't. This formed the resulting bright and dark fringes on the screen. Such a pattern is depicted in Fig. 3.

C. Interference and Diffraction

In order to analyse the phenomenon of interference, consider Fig. 4.

If the distance between the 2 slits is taken to be d and the width of each slit is considered negligible, then the path difference of the 2 waves is,

$$\Delta r = d \sin \theta$$

A phase difference of $2n\pi$ (for constructive interference) corresponds to a path difference that is an integer multiple of the wavelength, λ. So, for constructive interference,

$$\Delta r = d \sin \theta = m\lambda$$

For destructive interference,

$$\Delta r = d \sin \theta = (m + \frac{1}{2})\lambda$$

An important condition for sources to show interference is coherence, i.e., a constant time independent phase difference in the light waves. Provided that the light waves that arrive at some point are emerging from coherent sources, they will show interference. In fact, the screen with the slits in the set up above makes it so

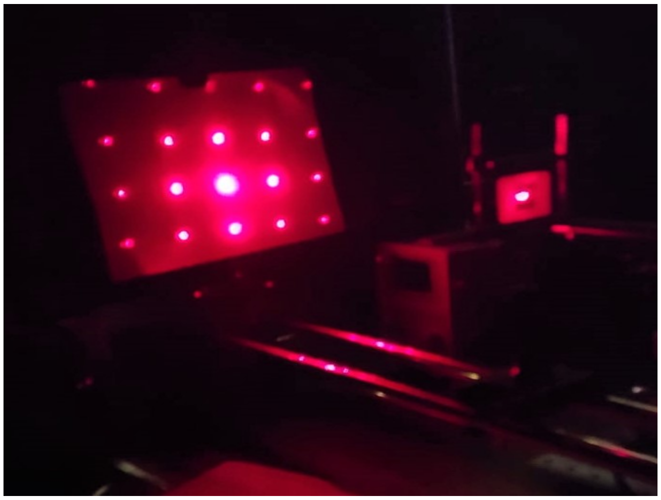

FIG. 5 *Diffraction through a square grating* (Image source: IACS Physics Lab)

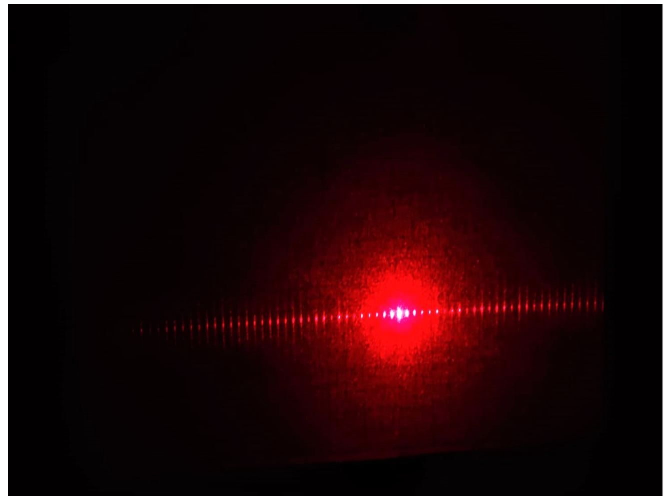

FIG. 6 *Diffraction through a plane grating* (Image source: IACS Physics Lab)

that there are actually 2 sources instead of one, each slit being a source of light, that is. The screen divides the original wavefront so that the 2 new wavefronts have a time independent phase relationship - in principle this is equivalent to using 2 different and independent sources which are coherent and not using any screen or holes.

The reason why normal sources of light around us do not show us any interference patterns is also due to this condition of coherence (11). Since most of the light around us is due to millions and billions of atoms, the phase that they all have is different and keeps changing depending on how long a particular atom remains in an excited state. This is the reason for the sources around us being incoherent and hence, showing no interference pattern.

The above can be shown as follows. Suppose we have light arriving at a point from 2 sources. Since, light is a wave, it can be written down as,

$$y_1 = A\cos\omega t \; ; \; y_2 = A\cos(\omega t + \phi)$$

Here, ϕ represents the phase difference. The resulting wave at the point (using the principle of linear superposition) will be,

$$y = 2A\cos\frac{\phi}{2}\cos(\omega t + \frac{\phi}{2})$$

The intensity will be proportional to the frequency squared and so,

$$I = C\cos^2\frac{\phi}{2}$$

Here, C is some constant of proportionality. Now, if the sources were to be incoherent, the phase would be time dependent. Taking a time average,

$$\langle I\rangle = C\langle\cos^2\frac{\phi}{2}\rangle$$

Now, the RHS varies randomly between 0 and 1; therefore, on an average, it can be approximated as 1/2. This implies,

$$I = \frac{C}{2}$$

The above means that the intensity has no variation at all, but remains constant throughout. This means that there are no places of higher intensity and lower intensity, therefore, no bright or dark fringes. As such, no interference can be observed if the sources used are incoherent.

One thing that can be noticed from Fig. 3 is that the intensity seems to drop as the distance increases. On the other hand, the equation for intensity says something different. A bright fringe occurs if the phase difference, $\phi = 2n\pi$. This means,

$$I = C\cos^2\frac{\phi}{2} = C$$

In other words, the intensity at the bright fringes has to the same regardless of where it is. So, there should be no difference in intensities in the different bright fringes, they should all be the same intensity. Very evidently, something has gone awry then with the experiment or the theoretical model.

The fact is real slits are never really infinitesimally thin as we have assumed in the model above. Real slits have a width, and this width leads to another phenomenon that is similar to interference - diffraction.

Diffraction is the term that is usually used when the light that interferes comes from multiple sources. This view is once again attributed to Feynman who states (chapter 30, volume 1; (10)),

"No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used."

For diffraction, light from different points in the same slit interfere on the screen and produce a pattern of bright and dark fringes. However, in this case, the intensity is not fixed for all bright fringes - the central fringe is the brightest and the ones on either side of it gradually decrease in intensity. Such diffraction patterns are depicted in Fig. 5 and Fig. 6 from an experiment conducted in the IACS physics lab. As one can see from the patterns, the intensity decreases as one goes farther outward from the centre of the screen.

When a normal double slit experiment is carried out, pure interference effects are never really seen exclusively. The pattern due to interference gets modulated by the diffraction patterns (also called the diffraction envelope) giving rise to an intensity that decreases gradually with distance. It is this pattern that one observes on the screen in this case too. This was the problem with the earlier prediction of equal intensities of all bright fringes - it had failed to take into account the non ideal nature of the slits in a real experiment and so, along with it, the phenomenon of diffraction.

III. THE DOUBLE SLIT EXPERIMENT IN QUANTUM PHYSICS

A. The birth of Quantum Mechanics - The Photoelectric Effect

Heinrich Hertz was the first person to observe the photoelectric effect (in 1887) while working with electromagnetic waves. His work was soon followed up by several scientists, most notable, Hallwachs, who observed that the net charge on a zinc plate could be affected by an incident beam of UV light - initially uncharged plates became positively charged and negatively charged ones became neutral. Positively charged ones became more positively charged. Building on these observations, it was concluded that somehow light could cause emission of negatively charged particles from certain metals.

This effect came to be known as the photoelectric effect - the emission of (photo)electrons from a metal surface caused due to the incidence of light. For quite some time, the phenomenon bamboozled physicists - the existing wave theory of light failed spectacularly when it came to explaining the effect.

A setup to observe the phenomena consists of a light source, a vacuum tube with 2 electrodes (one of which is photosensitive) connected to an external voltage supply (in order to manipulate the voltage applied across the electrodes). The observations that were derived from the various experiments were as follows:

1. At a given frequency of light, the photocurrent (the number of photoelectrons per unit time) was proportional to the intensity of light.

2. The emission only takes place if the frequency of light is above a certain, fixed, threshold frequency (different for different materials).

3. When the voltage applied to the electrodes was negative (i.e., the electrons were directed opposite to the anode), the photocurrent drops until it reaches zero at a particular voltage (called the stopping potential). The stopping potential was found to be independent of the intensity if the frequency was kept fixed. In fact, it is found to be directly proportional to the frequency of radiation.

4. The effect is nearly instantaneous - it begins almost immediately after the incidence of light.

For a start, in the wave theory, since light is incident as a wavefront on the metal surface, the energy is distributed all over the wavefront, meaning that individual particles (or electrons) will absorb a lower amount of energy than was provided. Detailed calculations went on to show that in such a case, the emission of a single electron could take several hours or more, but the experimental effect was near instantaneous. On the other hand, since the intensity was supposed to be related to the amplitudes, a higher intensity implies a higher amplitude and hence, more energy for the electrons. Now, when a negative voltage is applied to the electrodes, the electrons and repelled and only the most energetic of them manage to reach the anode. For the stopping potential, even the most energetic of electrons fail to resist the electric field - in other words, the stopping potential is a measure of the energy of the most energetic electrons emitted. But the observations show that the stopping potential is independent of the intensity which implies that the intensity has no effect on the electron energy in stark contradiction to the wave theory predictions. Finally, the wave theory says that no matter what the frequency of the incident light, if the beam is intense enough then over a sufficient amount of time, the electrons should accumulate enough energy needed to escape the metal surface. Once again, this is goes right against the observations made. Obviously, something was very wrong.

The photoelectric effect was explained by Albert Einstein who built on Max Planck's idea of quantized packets of light. What he said was that radiation was not actually a wave, but rather, came in discrete, "quantized" packets, called photons. Now, each photon would have an energy of $h\nu$, where ν was the frequency of the radiation. This means that the intensity only decided the number of such photons passing through an area in a given time, but had nothing to do with the energy of the photons. Further, using the conservation of energy, one could say that (for a single photoelectron with maximum kinetic energy),

$$h\nu \ = \ \phi_0 \ + \ K_{max}$$

Here, the LHS is the energy incident on the electron, while the RHS represents the ways in which it gets distributed after the electron "absorbs" the photon. Some of the energy goes to escape the attractive potential of the metal surface, measured by the term ϕ_0, called the work function, and the rest goes into the kinetic energy of the electrons.

Since the kinetic energy can never be negative, one immediately gets an explanation for the appearance of a threshold frequency as,

$$K_{max} \ \geq \ 0$$

$$\Rightarrow \ h\nu \ \geq \ \phi_0$$

Thus, the threshold frequency is ϕ_0/h. Also, the stopping potential, V_0, must be related as,

$$eV_0 \ = \ K_{max} \ = \ h\nu \ - \ \phi_0$$

As we can see, this explains the dependence of the stopping potential on frequency. Thus, Einstein's explanation though very simple, provides an elegant explanation for the photoelectric effect by describing light as quantized by photons. For this theory, Einstein received the 1923 Physics Nobel Prize.

But a vital question remains at this point - If light is a wave, no one can make sense of the photoelectric effect, but if light is a particle, no one can make sense of the interference and diffraction patterns. Light shows both effects and both the theories - the wave theory and the photon theory - explain the 2 phenomena pretty well in their own areas. But, what, then, is light? Is it a wave or is it a particle? If it is a wave, then how come it shows photoelectric effect? And, if it is a particle, how does one explain interference and diffraction?

To this end, one can once again fall back on Feynman (chapter 1, volume 3; (10)),

"Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behave like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaved like a wave. So it really behaves like neither. Now we have given up. We say: "It is like neither.""

B. Wave - Particle Duality

Building up on the idea that was highlighted in the previous section, one can now claim that maybe light (or, radiation in general) shows both a particle and a wave nature. To certain experiments, like interference and diffraction, it shows it's wave nature and to certain others, like those involving momentum transfer like in the photoelectric effect, it shows it's particle nature. The idea is that we cannot really ask if light is a particle or a wave, it is both and it is neither, it depends on the experiment we are thinking of, there is no one answer. This is what Feynman means when he says light is like "neither". This perspective also goes by a different name - the complementarity principle.

This concept, or idea, that light can show both a particle and a wave nature was extended later by Louis de Broglie to all matter. According to the wave - particle duality, matter and light, both display a dual behaviour - in certain cases, they behave as particles and in others, as waves. In fact, de Broglie hypothesised that the two natures of matter are related as,

$$\lambda \ = \ \frac{h}{p}$$

Here, λ is the wavelength of the "matter wave" and p is it's momentum. This also provides a clear explanation of why such a dual behaviour isn't manifest in our daily lives - the wavelength is inversely proportional to the momentum (which, in case of our daily life objects is related to mass). The Planck constant is already of $\mathcal{O}(10^{-34})$, which shows that the wavelength must be extremely small. This is the reason why we can't observe the wave behaviour of macroscopic matter around us, it's just too small compared to our limits of perception.

What the wave - particle duality implies is that since matter can also show wave-like behaviour, in principle, it is possible to obtain interference and diffraction patterns from matter as well - in other words, the double slit experiment can be used to obtain interference patterns for matter particles at the right scales. Indeed, such diffraction and interference patterns have been observed for electron waves as well ((14)). The challenge with such experiments lies in the scales or dimensions of the apparatus involved.

For example, in the article cited above, the de Broglie wavelength of the electrons involved was about 0.05 Å, a length much smaller than the atomic dimensions itself. Another problem lies in the fact that while some materials are transparent to light, only free space or vacuum is transparent to electrons. The ingenuity of the experimenter lies in overcoming these problems. In this case, a mixture of silver and hydrocarbon polymer was used. The setup was coated with copper electrolytically and there were places were there was only the hydrocarbon and no copper. When the copper layer was peeled away, these areas formed the slits. These slits were then used for the electron diffraction experiment. Such experiments with electron beams have been carried out in different ways using different techniques. In another example, Davisson and Germer carried out a classic experiment wherein they observed electron diffraction patterns after the beam is deflected off a surface of crystal; in the original case, the metal was nickel. In any case, such experiments proved the de Broglie hypothesis and established the duality of waves and particles.

C. The Uncertainty Principle

An important aspect of quantum mechanics is the much famous Heisenberg's Uncertainty Principle. While it is a mathematical statement in itself, and places an almost "unreal" limit to the accuracy of measurements and observations in the quantum realm, a deeper way to view the Uncertainty principle is through the complementarity principle. It is described by John Wheeler (in (18)) as:

"Any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena."

In less clouded terms, this says that certain pairs of "classical" variables or quantities can never be known simultaneously. Examples include the extremely well known x, p variables. What Quantum Mechanics usually means by this sort of a restriction is that these pairs of variables, strictly speaking, operators, do not commute.

To arrive at the Uncertainty principle, we first note that in quantum mechanics observables are represented by Hermitian (i.e., linear, self-adjoint) operators, and the values measured in real life experiments correspond on a one to one basis with the (real) eigenvalues of such operators. Simply put, an operator on a vector space is a mapping from the vector space onto itself (i.e., $\mathcal{O} : \mathcal{H} \to \mathcal{H}$). These operators are, in general, noncommuting, i.e., for 2 Hermitian operators, $\mathcal{A}$ and $\mathcal{B}$, in general,

$$\mathcal{A}\mathcal{B} \neq \mathcal{B}\mathcal{A}$$

Using commutators, this can be written as,

$$[\mathcal{A}, \mathcal{B}] = \mathcal{A}\mathcal{B} - \mathcal{B}\mathcal{A} \neq 0$$

The expectation of a Hermitian operator in the (normalised) state, Ψ, can be defined as,

$$\langle \mathcal{A} \rangle_\Psi = (\Psi, \mathcal{A}\Psi)$$

Here, (,) denotes the inner product. The fluctuation is defined as,

$$\Delta_{\Psi\mathcal{A}} = \sqrt{\langle (\mathcal{A} - \langle \mathcal{A} \rangle_\Psi)^2 \rangle_\Psi} = \sqrt{(\Psi_\mathcal{A}, \Psi_\mathcal{A})}$$

Here, $\Psi_\mathcal{A} = (\mathcal{A} - \langle \mathcal{A} \rangle_\Psi)\Psi$. Now, for 2 Hermitian operators, $\mathcal{A}$ and $\mathcal{B}$, we have,

$$\Delta_{\Psi\mathcal{A}} = \sqrt{(\Psi_\mathcal{A}, \Psi_\mathcal{A})}$$

$$\Delta_{\Psi\mathcal{B}} = \sqrt{(\Psi_\mathcal{B}, \Psi_\mathcal{B})}$$

Using the Cauchy - Schwarz inequality, we can say that,

$$|(\Psi_\mathcal{A}, \Psi_\mathcal{B})|^2 \leq (\Psi_\mathcal{A}, \Psi_\mathcal{A})(\Psi_\mathcal{B}, \Psi_\mathcal{B}) = (\Delta_{\Psi\mathcal{A}}\Delta_{\Psi\mathcal{B}})^2$$

This allows us to get a bound on the fluctuations. The rest of the work remains with the left hand side of the inequality.
Consider,

$$(\Psi_\mathcal{A}, \Psi_\mathcal{B}) = (\Psi, \ (\mathcal{A} - \langle \mathcal{A} \rangle_\Psi)(\mathcal{B} - \langle \mathcal{B} \rangle_\Psi)\Psi)$$

$$= (\Psi, \ (\mathcal{A}\mathcal{B} - \langle \mathcal{A} \rangle_\Psi \langle \mathcal{B} \rangle_\Psi)\Psi)$$

Now, for any $z \in \mathcal{C}$, we have, $|Im(z)| = |(z - z^*)/2i| \leq |z|$.
Now,

$$(\Psi_\mathcal{A}, \Psi_\mathcal{B})^* = (\Psi_\mathcal{B}, \Psi_\mathcal{A}) = (\Psi, \ (\mathcal{B}\mathcal{A} - \langle \mathcal{A} \rangle_\Psi \langle \mathcal{B} \rangle_\Psi)\Psi)$$

This means,

$$|Im((\Psi_\mathcal{A}, \Psi_\mathcal{B}))| = \left| \frac{(\Psi_\mathcal{A}, \Psi_\mathcal{B}) - (\Psi_\mathcal{A}, \Psi_\mathcal{B})^*}{2i} \right|$$

$$= \left| \frac{(\Psi, \ (\mathcal{A}\mathcal{B} - \mathcal{B}\mathcal{A})\Psi)}{2i} \right| = \left| \frac{(\Psi, \ [\mathcal{A}, \mathcal{B}]\Psi)}{2i} \right| = \frac{1}{2} \left| \langle [\mathcal{A}, \mathcal{B}] \rangle_\Psi \right|$$

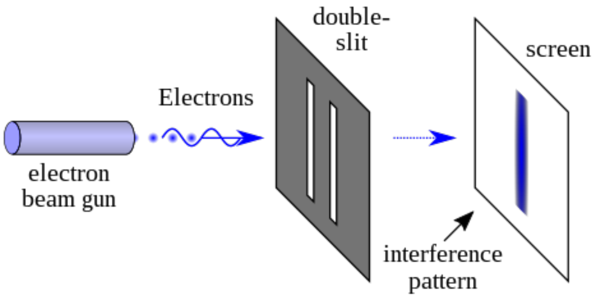

FIG. 7 *The Feynman Double Slit Experiment - At the time when Feynman proposed it, it was just a "gedanken" experiment or thought experiment.*
(Image source:(3))

Finally, putting it all together, we have,

$$\Delta_{\Psi\mathcal{A}}\Delta_{\Psi\mathcal{B}} \;\geq\; \frac{1}{2}\Big|\langle[\mathcal{A},\mathcal{B}]\rangle_{\Psi}\Big|$$

This is the Heisenberg Uncertainty Principle. For $X,\ P$, we have, $[X,P] \;=\; i\hbar$, which gives us,

$$\Delta_{\Psi X}\Delta_{\Psi P} \;\geq\; \frac{\hbar}{2}$$

This is the more familiar form of the Uncertainty Principle.

IV. THE DOUBLE SLIT EXPERIMENT IN MODERN TIMES

A. Feynman's Double Slit experiment

Despite the fact that the wave - particle duality is an extremely exotic and fascinating idea,it takes a while to actually appreciate just how "eccentric" an idea it is.

In order to consider how weird a dual wave - particle behaviour is, we consider electrons. This experiment with electrons was actually first proposed by Feynman (in several lectures) and at the time he came up with it, owing to the complexity of the experimentation involved, it was a thought experiment or "gedanken" experiment. This is why this setup also goes by the name of "Feynman's double slit experiment" (Fig. 7). In particular, we imagine a double slit experiment, except, instead of light, we use electrons. Now, as was discussed previously, these electrons also have a wave nature, and they produce interference and diffraction patterns on our detector/screen. So far, so good.

The truly eccentric nature of the behaviour comes out when we consider the following cases:

1. **Only one slit open -**
   In this case, we close one of the slits and let the other remain open. What could our observations be? Provided the slits are ideal, and infinitesimally small in width, we will not observe the diffraction fringes, but rather just the central maxima. This means that in this case, we'll observe a bell shaped curve centred around the slit that is open ((10)). The same thing occurs for photons as well ((8)).

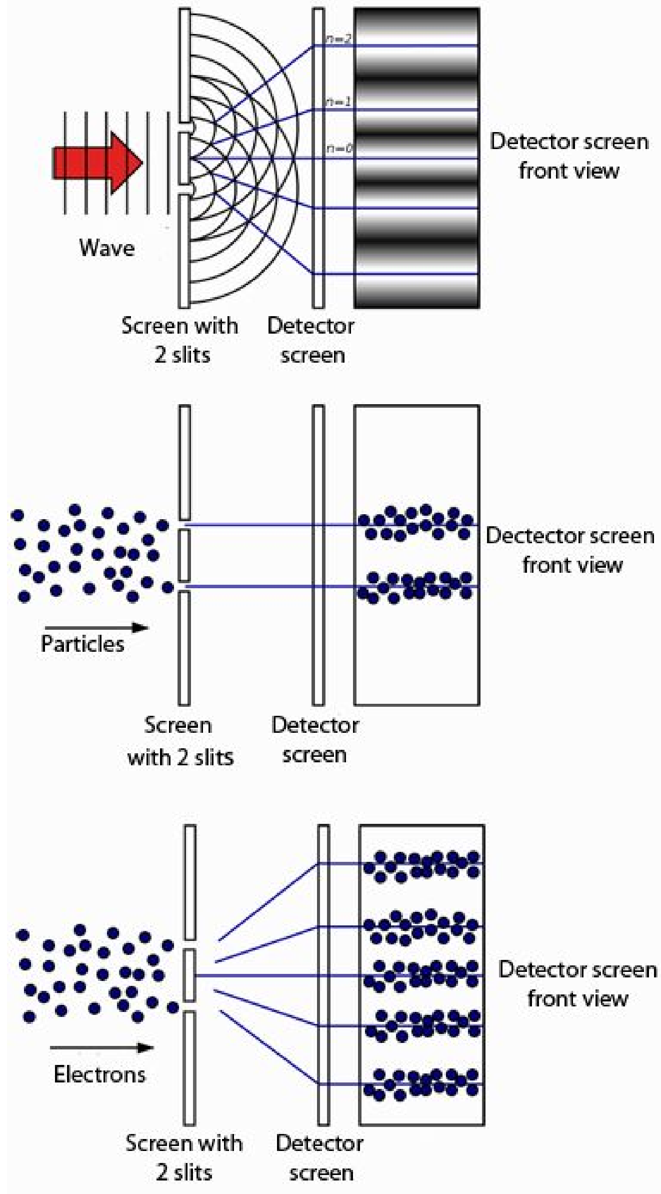

FIG. 8 *Case 2 - Both slits are open*
(Image source:(6))

2. **Both slits open -**
   In this case, as we expect, a pattern of bright and dark fringes form due to interference and diffraction effects that the electron waves undergo (Fig. 8).

3. **Both slits open but with a light source -**
   In this case, as in the second case, both slits are

kept open. But we also introduce a source of light in between the slits and the screen. What this does in essence is to send photons which get scattered by the electrons and hence, we get an idea of which slit the electron is going through. The implicit assumption here is about the discreteness of the electrons - they are assumed (and till date believed to be) indivisible and fundamental. Building on this, we expect them to pass through one slit or the other, because the electron cannot divide into parts and pass through both slits. What do we observe in this case?

Well, in this case, what happens can be seen as follows - this time the detector records each electron and along with it, an accompanying flash of light is also observed, either from near slit 1 or slit 2. It turns out that the final pattern is not an interference pattern - rather 2 peaks form at each slit with decreasing intensity on either side (almost like 2 adjacent bell shaped curves). What kind of a behaviour will show this sort of a pattern? In order to get an idea about it, imagine a horizontal plane with 2 slits and another plane beneath it. Let sand fall on top of the plane with the slits from above (under gravity). The sand grains will pass through either of the 2 slits (either slit 1 or 2, but not both) and form 2 mounds underneath the 2 slits ((8)). But this is exactly what we observe in this particular case as well. This implies that if we start watching the electrons, they begin to show their particle nature.

While this might seem strange at first, a bit of foresight shows us that this must be the case in reality. Photons carry momentum and when electrons scatter photons, there must be some momentum transfer at the very least. This automatically means that we cannot really expect the old interference pattern to show up in this case; the electrons have already been disturbed by the light source.

In fact, there is another way to see this. With the strong light, we expect (as was mentioned) the electrons to choose one of the slits, but not both. Hidden in this very assumption is a big problem - we've started off by assuming *particle* behaviour from the electrons (a wave is under no compulsion to choose only one slit at a time); in hindsight, the fact that they end up behaving as particles ceases to be a big surprise. It is almost as if Nature has played to our expectations.

4. **Both slits open but with a modified light source -**
The natural question at this point is to ask what happens if we use a dimmer source of light? Here, we must keep in mind Einstein's revelations about the particulate nature of the photon. A lower intensity will change only the number of photons going through a particular area per unit time, but not the energy of individual photons themselves. What this means is that while the individual photons still carry the same energy, there will be fewer of them in a dimmer light. This means, that some electrons will go undetected by the light.

The electrons which are disturbed by the light once again form the same pattern as in case 3. On the other hand, the electrons that go without any interaction with light have not been disturbed by any external sources, so they do show the interference pattern.

The way to decrease the energy of the photons (the energy of the photons is related to their momentum by $E = pc$; this means a lower energy implies a lower momentum which would cause less disturbances to the electrons) is by using light of a longer wavelength or a lower frequency. While in principle such a method seems to be viable, in practice there is another problem. This problem is related to the resolving power of light; i.e., light can only resolve 2 objects as separate, distinct objects if the distance between them is greater than the wavelength of the light used. In the case of our modified double slit, when the light wavelength starts to get comparable to the distance between the slits (meaning that now we can't use the flash of light scattered to conclude which slit is being used anymore), only then does the pattern start to deviate back enough to give us the interference pattern again.

Now that we have considered the situation in detail, with the added foresight the results once again become explainable or "clear", so to speak. For this reason, let us consider the Uncertainty Principle. We know that since the X, P operators do not commute and that there is an uncertainty inbuilt in their measurements, it is not possible to measure both simultaneously. This means that when we measure the electron's position (by using our light flash), we must necessarily disturb it's momentum to ensure that there is an uncertainty about it. There is no other way around this. The only cases where we have sent a photon that has not disturbed the momentum, we also do not have the information about the position of the electron.

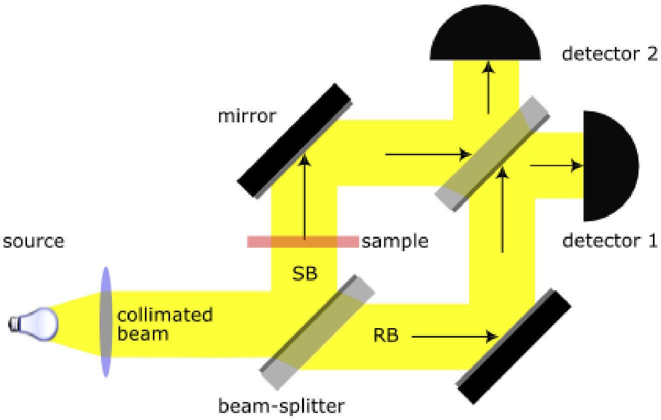

FIG. 9 *A Mach-Zehnder Interferometer*
(Image source:(5))

5. **Single electron through both slits -**
For our final gamble, we imagine sending a single electron through the entire double slit setup. Surely, now it must take one or the other slit? If so, it is displaying it's particle nature and we expect the double bell curve to form as a result.

   Single electron double slit experiments have been performed ((9) and (17)) in laboratory settings and the results that have been observed are nothing short of rattling. It turns out that in this case the pattern obtained on the detector/screen is an interference pattern. At this point, one should pause and think about the absurdity of the matter. If we are sending single particles through the slits, what is it interfering with? With waves, it is easy to see, two waves from to points arrive at a point and interfere. But what is the electron interfering with here? A way to think about this is by considering the wave nature of the electron - maybe the electron arrives as a wave and then interferes with itself? A better way to think about the entire situation is in terms of quantum superposition. It is as if there is an interference of the entire system - the electron having taken one slit is one state for the system, and it having taken the second slit gives us another state. It is the superposition of these two states for the system that provides us with the interference effects ((8)).

Going through all these cases, one feels like Nature is playing a game with human beings, a game mocking our abilities to read it and analyse it, a game that thwarts our every attempt to quantify phenomena, break down Nature and discover it's laws. It's almost like a cat and mouse chase; every time we find ourselves close to a resolution, it appears that we have missed something and Nature has beaten us again.

B. Single Photons

Now that we have considered electrons in detail, we go back to photons. At the time single electron interference experiments were performed, single photon experiments were a faraway dream. This was until the entrance of the French physicist, Alain Aspect, and his team. Aspect and his team used a Mach-Zehnder interferometer (Fig. 9) and a calcium atoms to perform single photon interference experiments. In modern physics, double slit experiments mostly refer to this sort of an equivalent arrangement.

In a Mach-Zehnder interferometer, the photon is sent through a beam splitter (in case of a beam of light, this apparatus transmits half the light and reflects half of it; in case of a single, indivisible photon, the photon has to choose one of the 2 paths). Regardless of the path chosen by the photon, it then is reflected from a mirror, passes through a second beam splitter and is detected at either one of the two detectors. The thing to notice here is that the 2 paths for the photon are indistinguishable; this essentially means that this experiment is akin to the traditional double slit. Once the photon or photons arrive at the detectors, there is no way to tell which path it took. Now, for a Mach-Zehnder interferometer, interference of the photons means that all the photons end up at one of the detectors (detector 1) and none go to the other (detector 2). This means that detector 1 represents constructive interference and 2 represents destructive interference.

What Aspect and his team did was the following: To ensure that they were using single photons, they excited atoms of calcium to a higher energy state using very precise lasers. Such an excited calcium atom returns to the ground state by emitting 2 photons - the first is green and the second, blue (the colours represent the wavelengths). The team used the emission of the green photon as a signal to ready their apparatus. In other words, once the green arrived, they knew that within a few nanoseconds the blue one will arrive too, and when it did, they had the apparatus ready for it. In this way they ensured that at the time of recording their data, only a single, blue photon was present in their apparatus. With the 2 beam splitters in place, the team observed that all the photons ended up at 1 and none at 2 ((8) and (12)).

In usual cases, the beam splitters can introduce the required phase differences in the reflected waves in order to account for the observations. But once again, while it makes sense for a wave to divide into two and interfere, what we have here are single photons. What the photon is really doing is termed as "quantum superposition". As mentioned previously, the two different states of the system interfere and give us an interference pattern. Suffice to say, we still do not have a very nuanced understanding of the effect in terms of physical reality, but only in terms of mathematics.

C. The Delayed Choice experiment and the Quantum Eraser

1. Wheeler's Delayed Choice Experiment

One can modify the previous arrangement for the double slit experiment using a Mach-Zehnder interferometer by bringing in an additional complexity. This complexity was first introduced by John Wheeler and, therefore, is also called Wheeler's delayed choice experiment. The thing to note here is that if the second beam splitter (the one in the upper right) is not in place, then the paths are no longer equal. It is easy to see that in that case, photons going to detector 1 must have been transmitted through the first beam splitter and ones going to 2, must have been reflected. In other words, we know the path each photon takes, and so, the photon will not show interference. Half the time, it'll end up at 1, and the other half, at 2.

Wheeler proposed that the second beam splitter should be put in place after a certain time lapse. Essentially what he proposed was this: give the photon time to enter the setup through the first beam splitter (without any second beam splitter in place). Then, introduce the second beam splitter while the photon is on its way already to either of the detectors. With the first beam splitter in place and no second beam splitter, the paths are no longer equivalent and, therefore, the photon is forced to show its particle nature. But if the second beam splitter is in place, then the photon's wave nature is being probed for because now both paths are equivalent. So, what exactly will the photon do? It cannot go back in time and show interference? Or, can it? In principle, one can also do the opposite. Let the photon enter the apparatus as shown in Fig. 9. Initially, it is the wave nature that we are looking for in the photon. Now, after the photon passes through the first beam splitter, we remove the second one in order to make the paths unequal. In this case, once the paths are unequal, the photon has to show its particle nature. It cannot be in a superposition of going through both the paths, which essentially means it has to go half the time to the detector 1 and half the time to 2. But as before, it cannot go back in time and change its state. So, what does it do?

When Aspect and his team performed the experiment in 2008, they found that the observations were unaffected by when the beam splitter was put into place. This means that if the beam splitter was not it place, no interference was observed even if the beam splitter was removed after the photon's passage through the first one. Similarly, if the beam splitter was in place, interference was observed regardless of when the team placed the beam splitter ((8) and (13)). Something extremely uncanny was taking place, almost as if the photon could reverse it's decisions and prevent us from beating it at the game. Of course, the catch here is in the idea of quantum superposition. The photon does not travel through the setup as either a particle or a wave, but in a quantum superposition. Thought in this way, there is no need for it to go back in time and reverse any decision.

2. The Quantum Eraser

A further extension of this, proposed by Marlan Scully, goes by the "quantum eraser" experiment. In this experiment, the information about the path the photon is taking is recorded, but by a quantum system. The question is the following: the recording of such an information means that the photon will not show interference, but what if such information is "erased" afterwards? Does the interference pattern re-emerge? Scully and Kim ((15)) performed a version of this experiment in 1999 using an arrangement of beam splitters and entangled photon. In essence, two atoms were placed side by side and excited. Both the atoms emitted an "entangled" pair of photons as a result; one of the pair (A1 and A2) travelled towards a screen and the other (say, B1 and B2) in the opposite direction. The entangled B photon could be used to find out which atom the A photon came from. In other words, there was no way of knowing if the photon at the detector, the A photon, came from atom 1 or 2 (was the photon A1 or A2?), but the B photon could be used to determine if the B photon was B1 (from atom 1) or B2 (from atom 2). This meant that the B photon could be used to glean information to distinguish the A photon without disturbing the A photon. The B photons are passed through an arrangement of beam splitters and detectors. Some of the B photons are used to glean information, while the others are "erased".

It was observed that as expected, for the B photons that were measured properly, no interference was seen at the screen but for those that were erased, interference was seen at the screen. The interesting part of the experiment comes from the fact that the time after which the information is erased is irrelevant - in other words, if one detects the A photons and records the pattern much earlier than the information with the B photons is either measured or erased (one easy way to do this is by increasing the distance the B photons travel before being erased or measured), then the patterns will miraculously still correspond perfectly. This means that if the A photons are detected while the B photons are still travelling and the observations analysed, no interference is observed (since, in principle, the information is present). Now, if after this, some B photons are used to glean information and the others erased, and a selective analysis is made (i.e., if the A photons corresponding to the measured B photons are analysed separately from the rest), then, in one case, there is no interference pattern, but somehow, miraculously, in the other case, an interference pattern does appear. This begs the question - was the pattern always there, but hidden, or did it reappear? This experiment has also been successfully conducted by a team led by the Austrian physicist, Anton Zeilinger, and the results have matched with the expectations ((16)). But, in this case, the expectations themselves seem unexpected, as it is with most of quantum physics.

V. CONCLUSION

What started off in the 1800s as a simple and elegant demonstration of the wave nature of light has, as we traced in this term paper, become one of the most iconic and puzzling experiments in all of physics. From the classical to quantum, the double slit experiment has guided many theorists and experimentalists and has led us to new ideas, concepts, theories and paradoxes, not all of them resolved. While the classical double slit experiment is indeed an elegant experiment that spurred much of optics in the 1800s, the true puzzle and value of the experiment comes to the fore when one enters the realm of quantum mechanics. Till date, experimentalists are working on some version or other of this same humble double slit in the hopes of verifying some aspect of the theory or recording some new observations. There is much more to quantum physics and the double slit than what was discussed in the term paper, most notably, the ideas of David Bohm and de Broglie, hidden variables, many worlds interpretation, how Alain Aspect, mentioned already in the text, used the double slit to demonstrate a violation of Bell's inequality, and all the exciting research that is going on in the field of quantum foundations. However, it is not possible to include all of that in a 14 page term paper, so they are best left to standard textbooks and reputed scientific articles. The main point that remains is the enigma that the double slit carries within itself. Even today, it continues to be a guiding light for researchers, both theorists and experimentalists. As the paper started, so shall it end, with a quote (rather, the same quote since it best describes the idea) by the legendary physicist, Richard P. Feynman,

"The double-slit experiment has in it the heart of quantum mechanics. In reality, it contains the only mystery."

Richard P. Feynman

The Feynman Lectures on Physics, Volume 3, (10)

VI. ACKNOWLEDGEMENT

I would like to express my thanks to my physics lab and UG research instructor, Dr. Subham Majumdar, for helping me with the term paper, guiding us on the usage of LaTeX and providing the necessary motivation. I would also like to thank IACS for providing the oppurtunity to explore such a fascinating topic. Lastly, I would like to express my gratitude to my family and friends for their support.

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