Pokémon Go Raids: Maximizing your Candies

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1 Introduction

The introduction of trading provided players a new way of gathering candies. Trainers can now get between 1 and 3 candies for a trade and I will make the assumption that an event could potentially double these numbers. Moreover, trainers can get more balls thanks to the friendship system. This small article will give you all the (realistic) assumptions that I do and the results on the optimal strategy to choose in order to maximize your candies according to these assumptions.

2 Notations and assumptions

Let's consider the case where one player can use Pinap and Golden Razz Berries. This player has enough berries of each type and his only goal is to maximize his number of candies (i.e. there is no cost at using any berry). This player always throws the same way.

Let C be the random variable referring to the number of candies you will get from one raid and c be the raw number of candies that Pokémon species gives (e.g. 10 for Tyranitar, 3 for Snorlax, etc.).

Let n be the number of Premier balls received, p the maximum number of Pinap Berries used and r the maximum number of Golden Razz Berries used. We have $n = p + r$. Given a number of pinap berries and a number of (golden) razz berries, one will always use his Pinap berries first (it is the optimal strategy).

Each Pokéball thrown is modeled as a random variable following a Bernoulli distribution. Let $X_1, \ldots, X_n$ be these random variables. We have:

$$X_1, \ldots, X_p \overset{\text{iid}}{\sim} \mathcal{B}(\lambda_p)$$
$$X_{p+1}, \ldots, X_n \overset{\text{iid}}{\sim} \mathcal{B}(\lambda_r)$$
$$X_1, \ldots, X_n \text{ independent}$$

λ_p and λ_r can be computed using the Grand Unified Catch Theory:

$$\lambda = 1 - \left(1 - \frac{\text{BCR}}{2\,\text{CPM}}\right)^{\text{multipliers}}$$

where multipliers = Ball × Curve × Berry × Throw × Medal.

Finally, let T be the number of Pokéballs thrown before you catch the Pokémon.

Now we can dive into our problematic.

3 Problematic

C is a random variable that can take 3 different values:

$$C = \begin{cases} 0 & \text{with probability } \mathbb{P}(C = 0) \\ c + k & \text{with probability } \mathbb{P}(C = c) \\ 2c + k & \text{with probability } \mathbb{P}(C = 2c) \end{cases}$$

where k is the number of "extra" candies that one gets for trading and/or transferring that Pokémon. Let's assume that k ranges from 1 to 8, with 8 corresponding to a double candy bonus for both trading and transferring while getting the same species when trading.

The quantity that one wants to maximize is the expected value of C:

$$\mathbb{E}[C] = \sum_{t \in \{0, c+k, 2c+K\}} t \, \mathbb{P}(C = t) = (c + k) \, \mathbb{P}(C = c) + (2c + k) \, \mathbb{P}(C = 2c)$$

One needs to know the distribution of C. Let's derive it.

$$\mathbb{P}(C = 2c + k) = \mathbb{P}(T \le p)$$
$$= \sum_{t=1}^{p} \mathbb{P}(T = t)$$
$$= \lambda_p \sum_{t=1}^{p} (1 - \lambda_p)^{t-1}$$
$$\mathbb{P}(C = 2c + k) = 1 - (1 - \lambda_p)^p$$

$$\mathbb{P}(C = c + k) = \mathbb{P}(p < T \le n)$$
$$= \sum_{t=p+1}^{n} \mathbb{P}(T = t)$$
$$= (1 - \lambda_p)^p \lambda_r \sum_{t=1}^{n-p} (1 - \lambda_r)^{t-1}$$
$$\mathbb{P}(C = c + k) = (1 - \lambda_p)^p \left[1 - (1 - \lambda_r)^{n-p} \right]$$

Put altogether, one gets:

$$\mathbb{E}[C] = (2c + k) \left[1 - (1 - \lambda_p)^p \right] + (c + k)(1 - \lambda_p)^p \left[1 - (1 - \lambda_r)^{n-p} \right]$$
$$= 2c - 2c(1 - \lambda_p)^p + c(1 - \lambda_p)^p - c(1 - \lambda_p)^p (1 - \lambda_r)^{n-p} + k \left[1 - (1 - \lambda_p)^p (1 - \lambda_r)^{n-p} \right]$$
$$\mathbb{E}[C] = 2c - c(1 - \lambda_p)^p \left[1 + (1 - \lambda_r)^{n-p} \right] + k \left[1 - (1 - \lambda_p)^p (1 - \lambda_r)^{n-p} \right]$$

Thus, the optimization problem is the following one:

$$\max_{p \in \{0, \dots, n\}} 2c - c(1 - \lambda_p)^p \left[1 + (1 - \lambda_r)^{n-p} \right] + k \left[1 - (1 - \lambda_p)^p (1 - \lambda_r)^{n-p} \right]$$

This optimization problem is not easy to solve theoretically (to my knowledge) (I'm a bit lazy to check if the objective function is concave and even if it is, KKT conditions may not look nice). But p can take only a small number of values, which means that we can just compute the expected value for each $p \in \{0, \dots, n\}$ and find the optimal p.

4 Many variables

One issue with this very general formulation is the very high number of variables which leads to many scenarios. Let's list all the variables and their possibles values:

- BCR: depends on the Raid Boss (0.1, 0.05, 0.03 and 0.02 are the most common values);
- CPM: two values depending on the presence or absence of weather boost;
- Curve: two values (1 and 1.7);

- Throw: infinite number of values between 1 and 2;
- Medal: 7 values (1, 1.05, 1.1, 1.15, 1.2, 1.25, 1.3);
- c (i.e. base candies): depends on the Raid Boss (3, 5 and 10):
- k (i.e. bonus candies): ranging from 1 to 8 depending on your patience/bag space and what your trading partners have.

I will set some values to make the analysis easier and focus on the impact of the remaining features:

- curved throws only (Curve = 1.7);
- gold medal(s) for the type(s) of the Raid Boss (Medal = 1.3);
- no weather boost;
- an "average" great throw (Throw = 1.5).

I will analyze the impact of the "extra" candies for trading and/or transferring in this scenario for several Raid Bosses.

5   Kirlia (BCR = 0.1, 5 base candies)

5.1   1 extra candy

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	5.92	5.96	5.98	5.99	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0
1	7.14	7.2	7.23	7.24	7.25	7.26	7.26	7.26	7.26	7.26	7.26	7.26	7.26
2	8.02	8.11	8.16	8.18	8.19	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2
3	8.62	8.77	8.84	8.87	8.89	8.9	8.9	8.9	8.9	8.9	8.9	8.9	8.9
4	8.99	9.22	9.33	9.38	9.41	9.42	9.43	9.43	9.43	9.43	9.43	9.43	9.43
5	9.15	9.5	9.67	9.75	9.79	9.81	9.82	9.82	9.82	9.83	9.83	9.83	9.83
6	9.07	9.61	9.88	10.0	10.06	10.09	10.11	10.12	10.12	10.12	10.12	10.12	10.12
7		9.55	9.96	10.16	10.25	10.3	10.32	10.33	10.34	10.34	10.34	10.34	10.34
8			9.92	10.22	10.37	10.44	10.48	10.49	10.5	10.5	10.51	10.51	10.51
9				10.19	10.42	10.53	10.58	10.61	10.62	10.63	10.63	10.63	10.63
10					10.39	10.56	10.65	10.69	10.71	10.72	10.72	10.72	10.72
11						10.55	10.67	10.74	10.77	10.78	10.79	10.79	10.79
12							10.66	10.76	10.8	10.82	10.84	10.84	10.84
13								10.75	10.82	10.85	10.87	10.88	10.88
14									10.81	10.86	10.89	10.9	10.91
15										10.86	10.9	10.92	10.93
16											10.89	10.92	10.94
17												10.92	10.94
18													10.94

Table 1 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

5.2   8 extra candies

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	12.83	12.92	12.96	12.98	12.99	13.0	13.0	13.0	13.0	13.0	13.0	13.0	13.0
1	14.0	14.13	14.2	14.23	14.24	14.25	14.26	14.26	14.26	14.26	14.26	14.26	14.26
2	14.8	15.01	15.11	15.15	15.18	15.19	15.2	15.2	15.2	15.2	15.2	15.2	15.2
3	15.29	15.61	15.76	15.83	15.87	15.89	15.9	15.9	15.9	15.9	15.9	15.9	15.9
4	15.48	15.97	16.21	16.32	16.38	16.41	16.42	16.43	16.43	16.43	16.43	16.43	16.43
5	15.35	16.11	16.48	16.66	16.75	16.79	16.81	16.82	16.82	16.82	16.83	16.83	16.83
6	14.84	16.02	16.59	16.86	17.0	17.06	17.09	17.11	17.12	17.12	17.12	17.12	17.12
7		15.64	16.52	16.94	17.15	17.25	17.3	17.32	17.33	17.34	17.34	17.34	17.34
8			16.23	16.89	17.21	17.36	17.44	17.47	17.49	17.5	17.5	17.51	17.51
9				16.68	17.17	17.41	17.52	17.58	17.61	17.62	17.63	17.63	17.63
10					17.01	17.38	17.56	17.64	17.69	17.71	17.72	17.72	17.72
11						17.26	17.53	17.67	17.73	17.76	17.78	17.79	17.79
12							17.45	17.65	17.75	17.8	17.82	17.84	17.84
13								17.58	17.74	17.81	17.85	17.87	17.88
14									17.69	17.81	17.86	17.89	17.9
15										17.77	17.85	17.9	17.92
16											17.83	17.89	17.92
17												17.87	17.92
18													17.9

Table 2 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

6 Tyranitar (BCR = 0.05, 10 base candies)

6.1 1 extra candy

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	9.69	10.08	10.35	10.55	10.68	10.78	10.84	10.89	10.92	10.95	10.96	10.97	10.98
1	10.7	11.18	11.52	11.76	11.93	12.05	12.13	12.19	12.23	12.26	12.28	12.29	12.3
2	11.46	12.06	12.48	12.78	12.98	13.13	13.23	13.3	13.35	13.39	13.41	13.43	13.44
3	11.98	12.72	13.24	13.61	13.86	14.04	14.17	14.26	14.32	14.36	14.39	14.41	14.43
4	12.26	13.18	13.82	14.27	14.59	14.81	14.96	15.07	15.15	15.2	15.24	15.27	15.29
5	12.28	13.41	14.21	14.77	15.16	15.43	15.63	15.76	15.86	15.92	15.97	16.0	16.02
6	12.03	13.43	14.42	15.11	15.59	15.93	16.17	16.34	16.45	16.54	16.59	16.63	16.66
7		13.22	14.43	15.29	15.89	16.31	16.6	16.81	16.95	17.06	17.13	17.18	17.21
8			14.25	15.3	16.04	16.56	16.93	17.18	17.36	17.49	17.58	17.64	17.68
9				15.14	16.06	16.7	17.15	17.47	17.69	17.84	17.95	18.03	18.08
10					15.91	16.71	17.27	17.66	17.93	18.13	18.26	18.36	18.42
11						16.59	17.28	17.76	18.1	18.34	18.51	18.62	18.71
12							17.17	17.77	18.19	18.48	18.69	18.84	18.94
13								17.68	18.2	18.56	18.82	19.0	19.12
14									18.12	18.57	18.88	19.11	19.26
15										18.5	18.89	19.16	19.36
16											18.83	19.17	19.41
17												19.12	19.41
18													19.36

Table 3 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

6.2 8 extra candies

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	15.85	16.5	16.94	17.26	17.48	17.64	17.74	17.82	17.87	17.91	17.94	17.96	17.97
1	16.67	17.46	18.02	18.41	18.68	18.87	19.01	19.1	19.17	19.21	19.25	19.27	19.28
2	17.19	18.17	18.85	19.34	19.67	19.91	20.08	20.2	20.28	20.33	20.37	20.4	20.42
3	17.4	18.62	19.47	20.06	20.48	20.78	20.98	21.13	21.23	21.3	21.35	21.38	21.41
4	17.31	18.81	19.86	20.59	21.11	21.48	21.73	21.91	22.03	22.12	22.18	22.23	22.26
5	16.86	18.72	20.02	20.93	21.57	22.02	22.34	22.56	22.71	22.82	22.9	22.95	22.99
6	16.04	18.34	19.95	21.08	21.87	22.42	22.81	23.09	23.28	23.41	23.51	23.57	23.62
7		17.62	19.61	21.01	21.99	22.68	23.16	23.5	23.74	23.9	24.02	24.1	24.16
8			19.0	20.72	21.94	22.79	23.38	23.8	24.09	24.3	24.44	24.55	24.62
9				20.19	21.69	22.74	23.48	23.99	24.36	24.61	24.79	24.91	25.0
10					21.22	22.52	23.43	24.07	24.52	24.84	25.06	25.21	25.32
11						22.12	23.24	24.04	24.59	24.98	25.26	25.45	25.58
12							22.89	23.87	24.56	25.04	25.38	25.62	25.79
13								23.57	24.42	25.02	25.43	25.73	25.93
14									24.15	24.89	25.41	25.77	26.03
15										24.66	25.3	25.75	26.07
16											25.1	25.66	26.05
17												25.49	25.97
18													25.82

Table 4 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

7 Legendary (BCR = 0.03, 3 base candies)

7.1 1 extra candy

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	2.87	3.09	3.26	3.4	3.51	3.61	3.68	3.74	3.79	3.83	3.86	3.89	3.91
1	2.96	3.21	3.4	3.56	3.69	3.8	3.88	3.95	4.01	4.05	4.09	4.12	4.14
2	3.01	3.29	3.51	3.69	3.84	3.96	4.06	4.13	4.2	4.25	4.29	4.32	4.35
3	3.02	3.33	3.59	3.79	3.96	4.1	4.21	4.29	4.37	4.42	4.47	4.51	4.54
4	2.99	3.34	3.63	3.86	4.05	4.21	4.33	4.43	4.51	4.58	4.63	4.68	4.71
5	2.91	3.31	3.64	3.9	4.12	4.29	4.43	4.55	4.64	4.71	4.77	4.82	4.86
6	2.78	3.24	3.61	3.91	4.15	4.35	4.51	4.64	4.75	4.83	4.9	4.95	5.0
7		3.12	3.54	3.88	4.16	4.38	4.57	4.71	4.83	4.93	5.01	5.07	5.12
8			3.44	3.82	4.14	4.39	4.6	4.76	4.9	5.01	5.1	5.17	5.23
9				3.72	4.08	4.37	4.6	4.79	4.94	5.07	5.17	5.25	5.32
10					3.99	4.32	4.58	4.8	4.97	5.11	5.22	5.32	5.39
11						4.23	4.53	4.78	4.97	5.13	5.26	5.37	5.45
12							4.46	4.73	4.96	5.14	5.28	5.4	5.5
13								4.66	4.92	5.12	5.29	5.42	5.53
14									4.85	5.08	5.27	5.43	5.55
15										5.02	5.24	5.41	5.55
16											5.18	5.38	5.54
17												5.33	5.51
18													5.47

Table 5 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

7.2 8 extra candies

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	7.9	8.49	8.96	9.35	9.66	9.92	10.12	10.29	10.43	10.53	10.62	10.69	10.75
1	7.72	8.39	8.93	9.37	9.73	10.02	10.25	10.44	10.59	10.71	10.82	10.9	10.96
2	7.47	8.23	8.84	9.34	9.75	10.07	10.34	10.55	10.73	10.87	10.98	11.07	11.15
3	7.13	8.0	8.7	9.26	9.72	10.09	10.39	10.63	10.83	10.99	11.12	11.22	11.31
4	6.71	7.69	8.48	9.12	9.64	10.07	10.41	10.68	10.91	11.09	11.23	11.35	11.45
5	6.19	7.3	8.2	8.93	9.52	10.0	10.38	10.7	10.95	11.16	11.32	11.46	11.57
6	5.56	6.82	7.84	8.67	9.34	9.88	10.32	10.68	10.96	11.2	11.39	11.54	11.66
7		6.24	7.4	8.34	9.1	9.72	10.21	10.62	10.95	11.21	11.42	11.6	11.74
8			6.87	7.94	8.8	9.5	10.06	10.52	10.89	11.19	11.44	11.63	11.79
9				7.45	8.43	9.22	9.86	10.38	10.8	11.14	11.42	11.64	11.82
10					7.98	8.88	9.61	10.2	10.67	11.06	11.37	11.63	11.83
11						8.46	9.29	9.96	10.5	10.94	11.3	11.59	11.82
12							8.91	9.67	10.29	10.79	11.19	11.52	11.78
13								9.32	10.02	10.59	11.05	11.42	11.72
14									9.7	10.34	10.86	11.29	11.63
15										10.05	10.64	11.12	11.51
16											10.37	10.91	11.35
17												10.66	11.16
18													10.93

Table 6 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

8 Legendary (BCR = 0.02, 3 base candies)

8.1 1 extra candy

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	2.27	2.5	2.69	2.87	3.01	3.14	3.25	3.35	3.44	3.51	3.57	3.63	3.68
1	2.29	2.53	2.74	2.93	3.09	3.23	3.35	3.46	3.55	3.63	3.7	3.76	3.81
2	2.28	2.54	2.77	2.98	3.15	3.3	3.44	3.55	3.65	3.74	3.81	3.88	3.94
3	2.24	2.53	2.78	3.0	3.19	3.36	3.5	3.63	3.74	3.83	3.92	3.99	4.05
4	2.19	2.5	2.78	3.01	3.22	3.4	3.56	3.7	3.81	3.92	4.01	4.08	4.15
5	2.1	2.45	2.75	3.01	3.23	3.43	3.6	3.75	3.88	3.99	4.08	4.17	4.24
6	2.0	2.37	2.7	2.98	3.22	3.44	3.62	3.78	3.92	4.05	4.15	4.24	4.32
7		2.27	2.62	2.93	3.2	3.43	3.63	3.81	3.96	4.09	4.21	4.31	4.39
8			2.53	2.86	3.15	3.4	3.62	3.81	3.98	4.12	4.25	4.36	4.45
9				2.77	3.09	3.36	3.6	3.81	3.99	4.14	4.28	4.4	4.5
10					3.0	3.3	3.56	3.79	3.98	4.15	4.3	4.43	4.54
11						3.22	3.5	3.75	3.96	4.15	4.31	4.45	4.57
12							3.43	3.69	3.92	4.13	4.3	4.45	4.59
13								3.62	3.87	4.09	4.28	4.45	4.59
14									3.8	4.04	4.25	4.43	4.59
15										3.98	4.2	4.4	4.57
16											4.14	4.36	4.54
17												4.3	4.5
18													4.45

Table 7 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

8.2 8 extra candies

	6	7	8	9	10	11	12	13	14	15	16	17	18
0	6.25	6.87	7.41	7.88	8.29	8.64	8.95	9.22	9.45	9.65	9.83	9.98	10.11
1	6.0	6.67	7.26	7.77	8.21	8.6	8.93	9.22	9.48	9.7	9.89	10.06	10.2
2	5.7	6.43	7.07	7.63	8.11	8.53	8.89	9.21	9.48	9.72	9.93	10.11	10.27
3	5.35	6.15	6.85	7.45	7.97	8.43	8.83	9.17	9.47	9.73	9.96	10.15	10.33
4	4.96	5.82	6.58	7.24	7.81	8.3	8.73	9.11	9.43	9.72	9.96	10.18	10.36
5	4.5	5.45	6.27	6.98	7.6	8.14	8.61	9.02	9.37	9.68	9.95	10.18	10.39
6	4.0	5.02	5.91	6.69	7.37	7.95	8.46	8.91	9.29	9.63	9.92	10.17	10.39
7		4.54	5.51	6.36	7.09	7.73	8.28	8.76	9.18	9.55	9.86	10.14	10.38
8			5.06	5.97	6.77	7.47	8.07	8.59	9.05	9.45	9.79	10.09	10.35
9				5.54	6.41	7.17	7.82	8.39	8.89	9.32	9.69	10.02	10.3
10					6.0	6.82	7.54	8.16	8.7	9.17	9.57	9.93	10.24
11						6.44	7.21	7.89	8.48	8.99	9.43	9.81	10.15
12							6.85	7.58	8.22	8.78	9.26	9.68	10.04
13								7.24	7.93	8.54	9.06	9.52	9.91
14									7.61	8.26	8.83	9.33	9.76
15										7.96	8.58	9.12	9.58
16											8.29	8.87	9.38
17												8.6	9.15
18													8.89

Table 8 – Pinap vs Golden Razz Berries: Expected number of candies for each scenario. Each column is the number of Premier Balls and each row is the number of Pinap Berries used before switching to Golden Razz Berries.

9 Conclusion

With a simple probabilistic model, it is possible to get a good idea about the best strategy in order to maximize your number of candies.

This analysis shows that, **in some scenarios, the number of extra candies has a big impact on the optimal strategy to adopt** (i.e. when to switch from Pinap Berries to Golden Razz Berries). These scenarios correspond to Raid Bosses with low BCR and few base candies: the number of extra candies become so important that, for these Raid Bossed that are hard to catch, the most important thing is to catch them if you are aiming for 8 extra candies (unless if you have a high amount of Premier Balls and you can afford some Pinap Berries first).