Design and Analysis of Composite Structures - *(AE4ASM109)*
Assignement

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1 Calculation of the elastic properties based on the experimental data

1.1 Elastic properties

For each calculated property, the following methodology was used:

1. Obtain the sample value for each sample and store them in a list.
2. Calculate the mean and standard deviation of each property.
3. Derive the Gaussian distribution from the mean and standard deviation.

All values can be found in Table 1.

At first, to obtain E_1 and E_2 of, both experiments using 90 UD and 0 UD gives E_x and E_y respectively, when loaded by N_x. Since the given values are force and displacement, we can use the elastic relations to obtain the following relation, which calculates Young's modulus as a function of the applied load and displacement, which are given in the MTS folders.

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \{\sigma = \frac{P}{A}\} = \frac{P_x}{A_x \varepsilon_x} = \{\varepsilon = \frac{\delta}{L}\} = \frac{P_x L_x}{A_x \delta_x} \tag{1}$$

Here, P_x denotes the applied force in the x-direction, L_x the length of the sample in the x-direction, A_x the area perpendicular to the x-direction and δ_x the displacement. Note that the calculated Young's modulus is equal to E_1 in the case of the UD 0 degree and E_2 in the case of the UD 90 degree. As the values of E are oscillating much before stabilizing to a more linear relationship before dropping as the laminate fails, the values before linearity are neglected and the values after linearity are ignored to obtain the linear value of E.

The areas and length were calculated as a mean for each sample and the calculation of E was done with each specific sample geometry.

To calculate the Poisson ratio, the DIC folder was used. The major Poisson ratio can be calculated by using the experiments on the UD 0 degree and dividing ε_{xx} with ε_{yy}. The minor Poisson ratio can be done by the same method, but with the UD 90 degree. A quick validation can be done to see if the values calculated are reasonable from theory. This comparison shows that the values are very close, with a relative error of 2.4%.

$$\nu_{21} = \nu_{12} \frac{E_2}{E_1} = 0.0181 \tag{2}$$

Property	Mean	Standard deviation
E_1	165.22 GPa	16.922 GPa
E_2	8.444 GPa	1.077 GPa
ν_{12}	0.35318	0.1809
ν_{21}	0.018489	0.031465
G_{12}	6.4167 GPa	1.1089 GPa
X_t	1923.7 MPa	108.65 MPa
Y_t	107.2 MPa	9.3507 MPa
S_{12}	152.4 MPa	1.7844 MPa

Table 1: Property values obtained by experimental data

For G_{12}, we use the +-45 tests. Some manipulation of the theory of elasticity must be done to obtain the value.

$$\sigma_6 = -mn\sigma_x = \{m = n = \frac{1}{\sqrt{2}}\} = -\frac{\sigma_x}{2} \tag{3}$$

$$\varepsilon_6 = -2mn\varepsilon_x + 2mn\varepsilon_y = \varepsilon_y - \varepsilon_x \tag{4}$$

$$\varepsilon_x = \frac{\sigma_x}{E_x}a \tag{5}$$

From the equations above, the following expression follows:

$$G_{12} = \frac{\sigma_6}{\varepsilon_6} = \frac{-\frac{1}{2}\sigma_x}{\varepsilon_y - \varepsilon_x} = \frac{\sigma_x}{2(\frac{\sigma_x}{E_x} + \frac{\nu_{xy}\sigma_x}{E_x})} = \frac{E_x}{2(1+\nu_{xy})} \tag{6}$$

Here all the values are as defined in the slides. As can be seen from the last expression, G_{12} can be calculated using the global Young's modulus, which we can obtain for the +-45 laminate in the same way we obtained E_1. As for ν_{xy}, we can obtain that in the same way we obtained ν_{12}, but again doing it for the +-45 ply.

To calculate X_t and Y_t, the maximum stress comes directly from the maximum load of each sample. X_t and Y_t are calculated by dividing the maximum load by the area to obtain the stress. The UD 0 sample gives X_t and the UD 90 sample gives Y_t. For S_{12} a similar method is used, but since it is shear, a factor of 2 appears in the denominator.

From the values in the tables, it can be seen that the laminate is a CFRP-laminate. This will be used when setting up the Puck's criterion.

1.2 Quasi-isotropic laminate

For the quasi-isotropic laminate, the same method from finding X_t in Section 1.1 was used to find the maximum load, and the maximum stress follows from the geometry. The resulting values can be seen in Table 2. The Gaussian distribution of the maximum stress can be seen in Figure 1.

The geometry of the laminate gives the thickness of each ply, which will be used for questions 3 and 4.

Property	Mean	Standard deviation
F_{tmax}	41 909 N	837.09 N
X_t	820.86 MPa	16.772 MPa

Table 2: Property values obtained by experimental data

$$t = \frac{h}{N} = 0.1278 \text{ mm} \tag{7}$$

Here, h is the mean thickness of the laminate and N is the number of plies in the laminate.

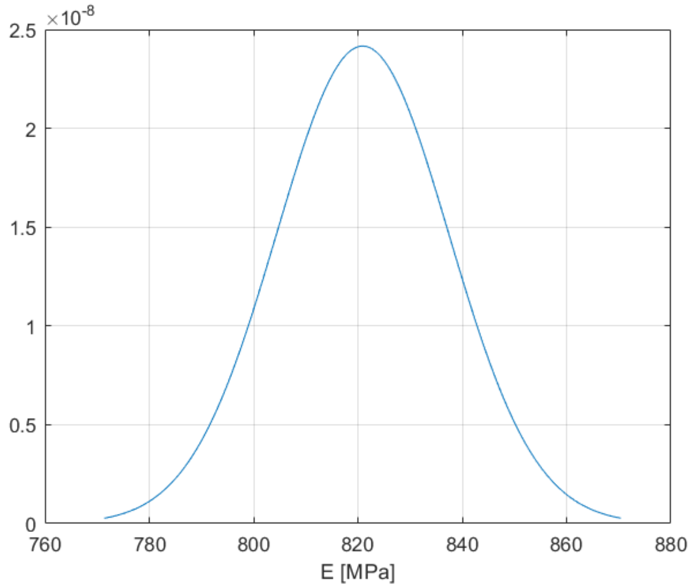

Figure 1: Gaussian distribution of the maximum stress X_t

2 Calculation of the ABD matrix and stress analysis

2.1 Calculation of engineering constants

The engineering constants to be calculated (in-plane and flexural) were: $E_x, E_y, \nu_{xy}, \nu_{yx}, G_{xy}$. The angle θ was selected to be varying between 1 and 90 degrees, with a step of 1 degree. Therefore, for each iteration, for the laminate $[30/\pm\theta/60]_{ns}$, the angle θ has a value between 1 and 90 degrees. At first, the effect of θ variation was studied, so n had a constant value of 1. The in-plane engineering constants were calculated using the following equations:

$$E_x = \frac{A_{xx}A_{yy} - A_{xy}^2}{hA_{yy}} \tag{8}$$

$$E_y = \frac{A_{xx}A_{yy} - A_{xy}^2}{hA_{xx}} \tag{9}$$

$$\nu_{xy} = \frac{A_{xy}}{A_{yy}} \tag{10}$$

$$\nu_{yx} = \frac{A_{xy}}{A_{xx}} \tag{11}$$

$$G_{xy} = \frac{A_{ss}}{h} \tag{12}$$

where A_{xx}, A_{xy}, A_{yy} and A_{ss} are the respective values of the A matrix, and h is the full thickness of the laminate. It is assumed that every ply has a thickness of 0.125 mm, therefore the full thickness is $h = 8 * 0.125 = 1$ mm. The B matrix is zero since the laminate is symmetric.

The plots for the in-plane engineering constants (for $\theta = 1$ to 90 degrees and $n = 1$) are presented below:

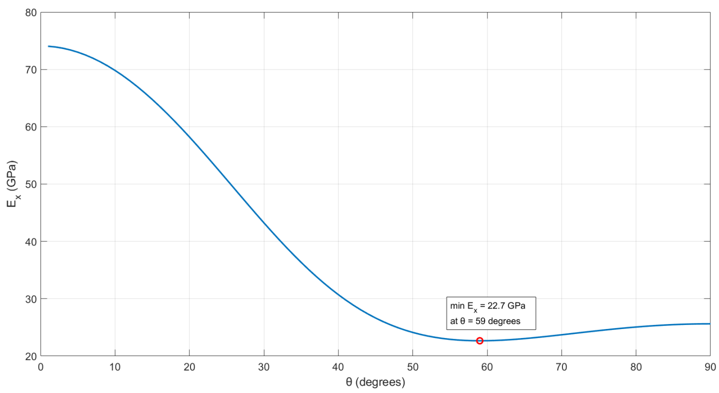

Figure 2: In-plane E_x for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

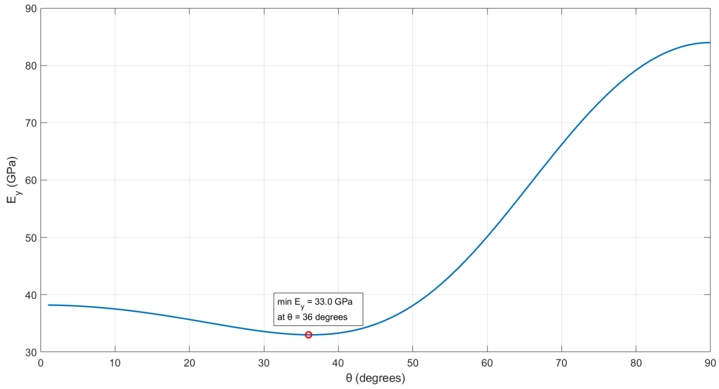

Figure 3: In-plane E_y for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

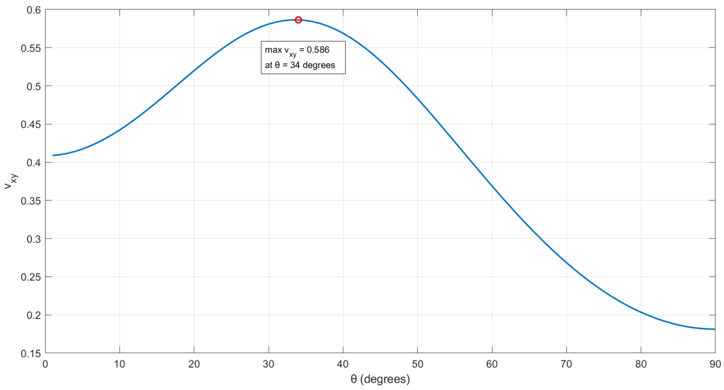

Figure 4: In-plane ν_{xy} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

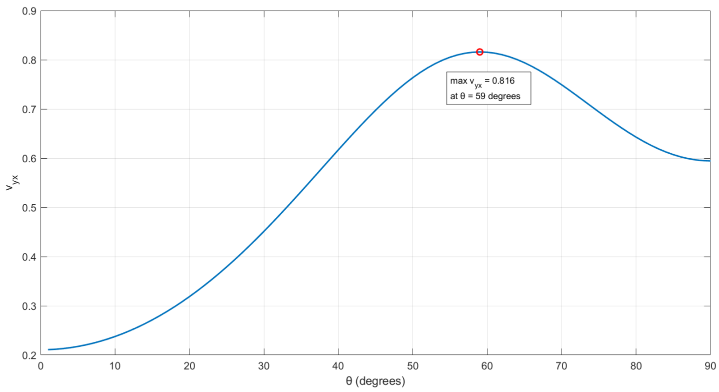

Figure 5: In-plane ν_{yx} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

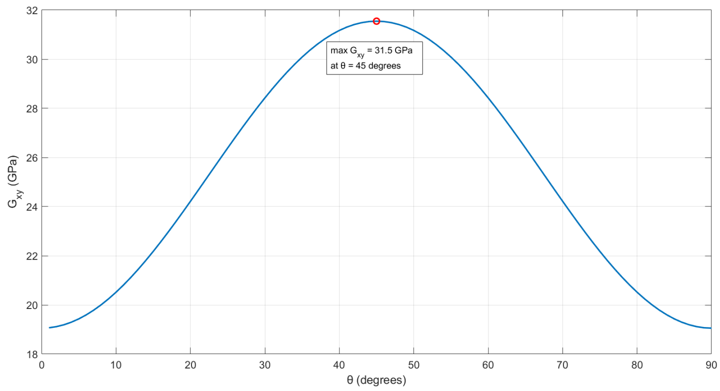

Figure 6: In-plane G_{xy} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

From the figures above, it is clear that E_x follows a general downward trend, with the lowest being $E_x = 22.7$ GPa at $\theta = 59$ degrees, which is reasonable since the highest E_x is expected close to the orientation of 0 degrees, where E_1 is dominant. The transverse Young's modulus, E_y, is between 33 and 38 GPa until $\theta = 36$ degrees, where it gets its lowest value of 33 GPa. After that it starts increasing, a logical behaviour since the closer θ is to 90 degrees, the greater the influence of the transverse modulus. The Poisson's ratios ν_{xy} and ν_{yx} follow a similar trend, with ν_{xy} starting from around 0.41, increasing until $\theta = 34$ degrees and then dropping rapidly to about 0.18 for $\theta = 90$ degrees. The minor Poisson's ratio ν_{yx} starts from a low value of around 0.21, increases all the way to $\nu_{yx} = 0.816$ at $\theta = 59$ degrees and then drops. Lastly, the shear modulus G_{xy} has its largest value of 31.5 GPa at 45 degrees, an expected behaviour since the shear loads are carried more effectively at $\theta = \pm 45$ degrees, from the moment there are plies oriented also at 30 and 60 degrees in the laminate.

Regarding the flexural engineering constants, they are calculated by the following equations:

$$E_x = \frac{12}{h^3 D'_{xx}} \tag{13}$$

$$E_y = \frac{12}{h^3 D'_{yy}} \tag{14}$$

$$\nu_{xy} = -\frac{D'_{xy}}{D'_{xx}} \tag{15}$$

$$\nu_{yx} = -\frac{D'_{xy}}{D'_{yy}} \tag{16}$$

$$G_{xy} = \frac{12}{h^3 D'_{ss}} \tag{17}$$

where $D'_{xx}, D'_{xy}, D'_{yy}$ and D'_{ss} are the respective values of the inverse D matrix and h is the full thickness of the laminate.

The plots for the flexural engineering constants (for $\theta = 1$ to 90 degrees and $n = 1$) are presented in Figures 7 - 11. More or less the same trends can be observed for the flexural constants as well. The values of flexural E_x are slightly higher than those of in-plane E_x, for the same angle θ. The flexural E_y and G_{xy} are generally lower than the in-plane E_y and G_{xy}. Regarding the Poisson's ratios, flexural ν_{xy} and in-plane ν_{yx} are higher. The location (value of θ) of the maximum and minimum values also slightly changes.

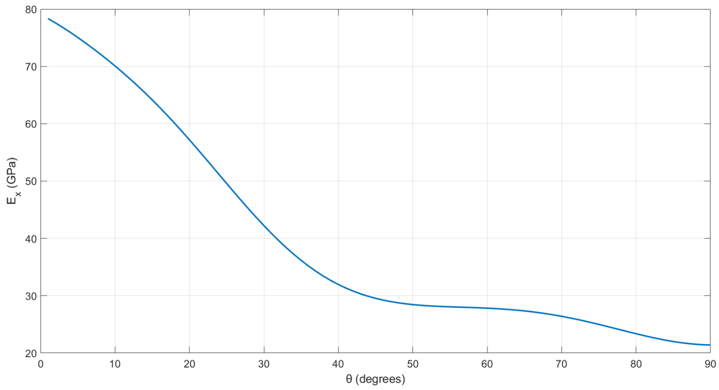

Figure 7: Flexural E_x for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

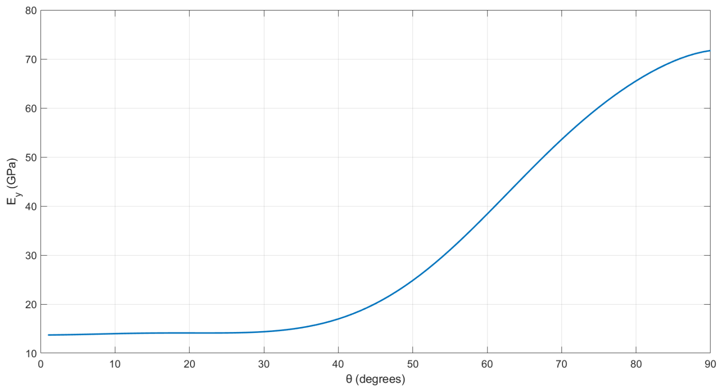

Figure 8: Flexural E_y for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

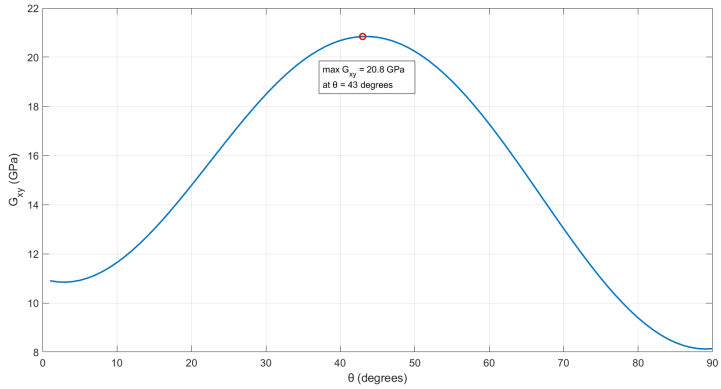

Figure 9: Flexural G_{xy} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

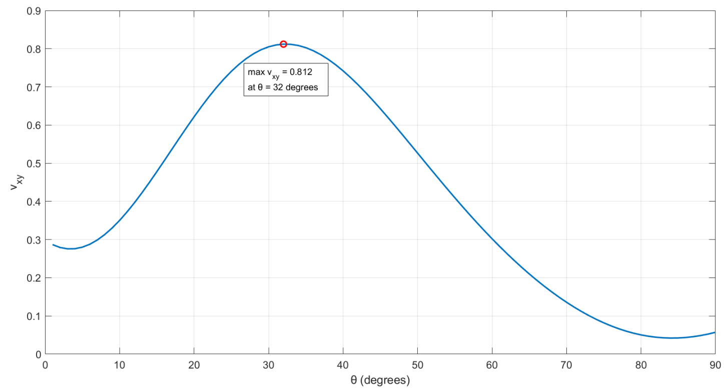

Figure 10: Flexural ν_{xy} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

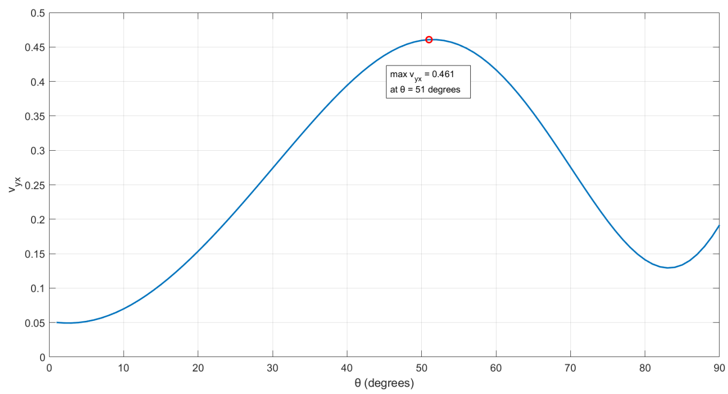

Figure 11: Flexural ν_{yx} for laminate $[30/\pm\theta/60]_{ns}$ as a function of angle θ

Next, the effect of n on the in-plane and flexural engineering constants of the laminate was studied. Two variations of n were studied (apart from $n = 1$), with $n = 2$ and $n = 3$. Figures 12 - 16 show that the in-plane constants are not affected at all by n. This is expected because the in-plane engineering constants depend on the individual ply properties and the ply thickness. As long as these are constant, and the layers are added so that the symmetry is maintained, the in-plane engineering constants will not change.

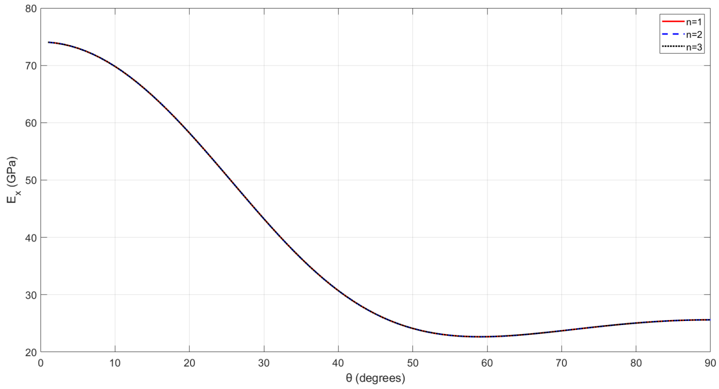

Figure 12: Comparison of in-plane E_x for $n = 1$, $n = 2$ and $n = 3$

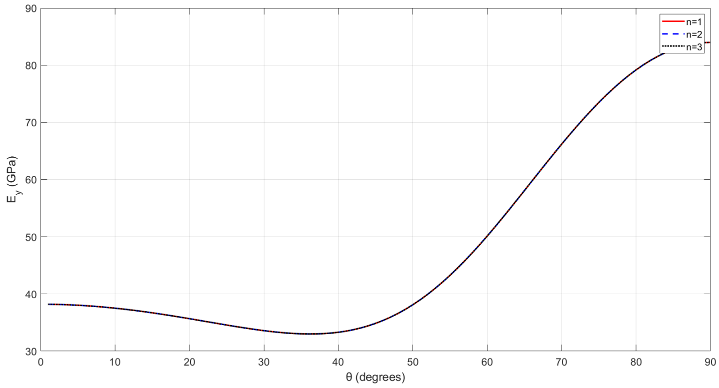

Figure 13: Comparison of in-plane E_y for $n = 1$, $n = 2$ and $n = 3$

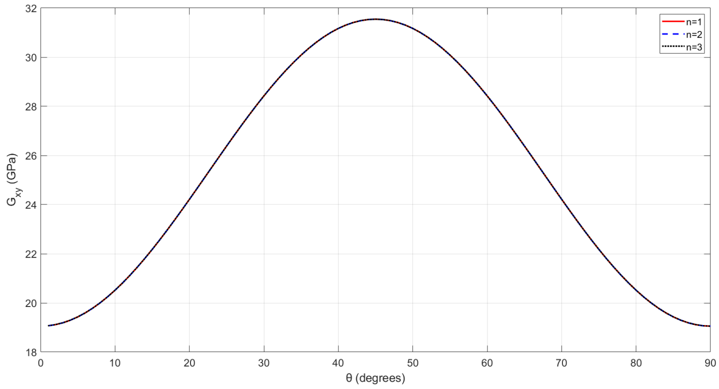

Figure 14: Comparison of in-plane G_{xy} for $n = 1$, $n = 2$ and $n = 3$

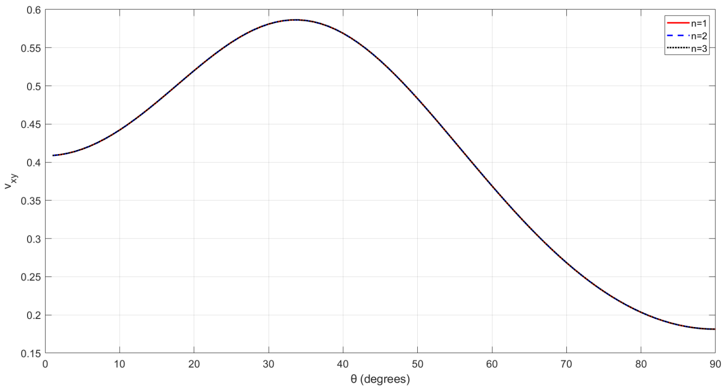

Figure 15: Comparison of in-plane ν_{xy} for $n = 1$, $n = 2$ and $n = 3$

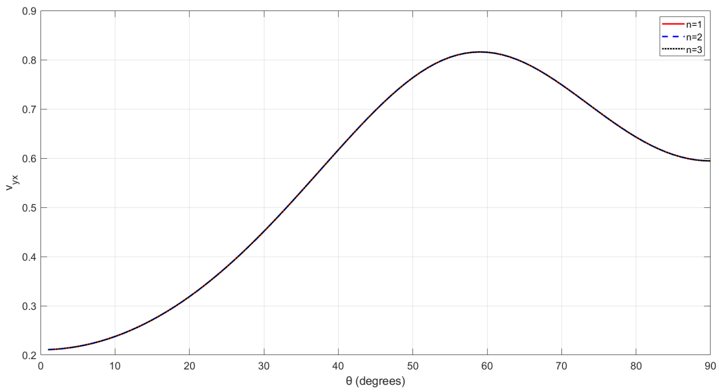

Figure 16: Comparison of in-plane ν_{yx} for $n = 1$, $n = 2$ and $n = 3$

On the other hand, the value of n affects the flexural constants. The relevant comparison is shown in Figures 17 - 21. In general, the more layers are added to the laminate, the higher E_y and G_{xy} become. This is because the thickness of the laminate increases, making the laminate stiffer in bending. However, the effect of n is larger when going from $n = 2$ to $n = 3$ is lower than when going from $n = 1$ to $n = 2$.

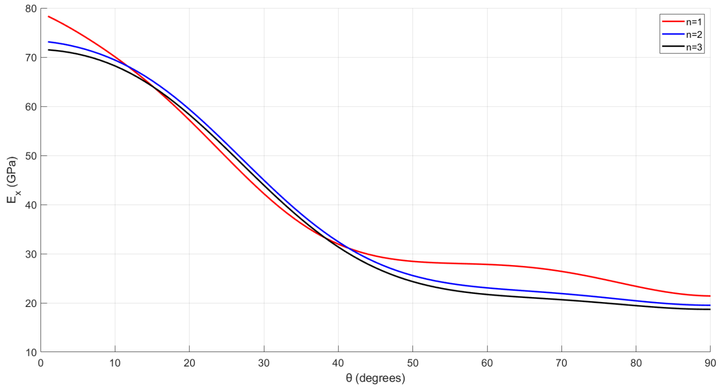

Figure 17: Comparison of flexural E_x for $n = 1$, $n = 2$ and $n = 3$

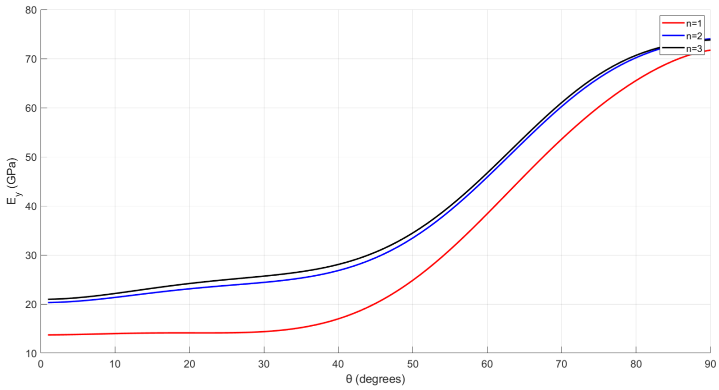

Figure 18: Comparison of flexural E_y for $n = 1$, $n = 2$ and $n = 3$

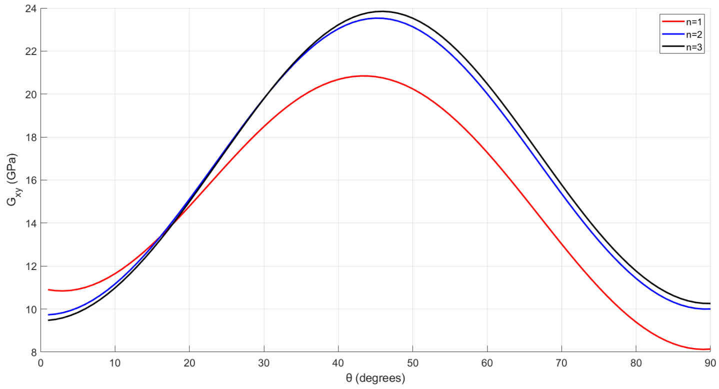

Figure 19: Comparison of flexural G_{xy} for $n = 1$, $n = 2$ and $n = 3$

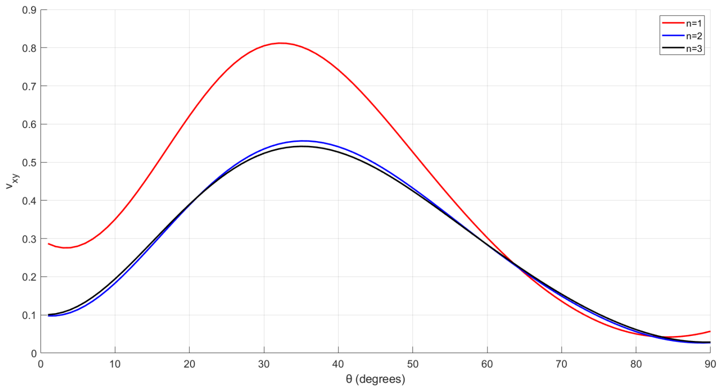

Figure 20: Comparison of flexural ν_{xy} for $n = 1$, $n = 2$ and $n = 3$

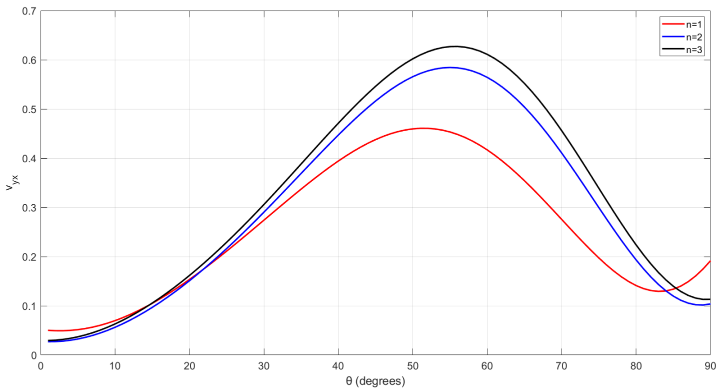

Figure 21: Comparison of flexural ν_{yx} for $n = 1$, $n = 2$ and $n = 3$

2.2 Calculation of stresses and strains

Because the laminate $[0_2/30/60/90]_s$ is symmetric, the B matrix is zero and the global strains are calculated using only the A matrix. The contribution of M_x is zero. Figures 22 - 27 show the stresses and strains for each ply, through the thickness of the laminate. It can be observed that all the figures are symmetric with respect to $z = 0$, which is reasonable since the laminate is symmetric.

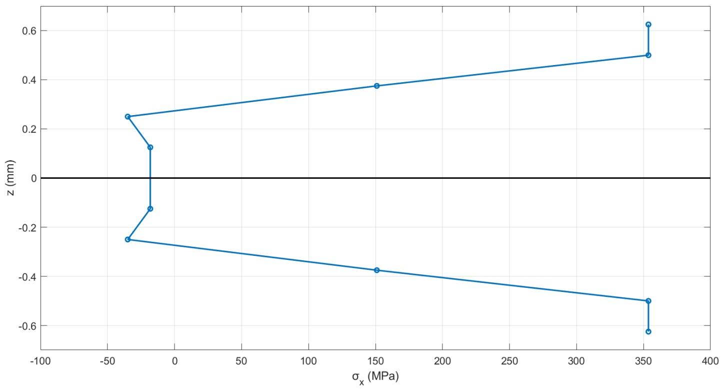

Figure 22: σ_x through the thickness

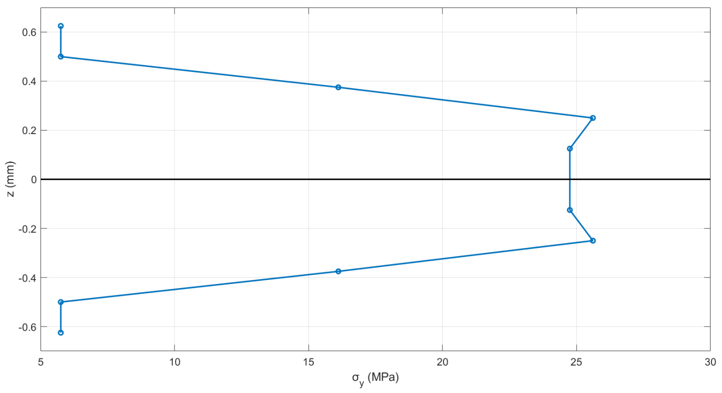

Figure 23: σ_y through the thickness

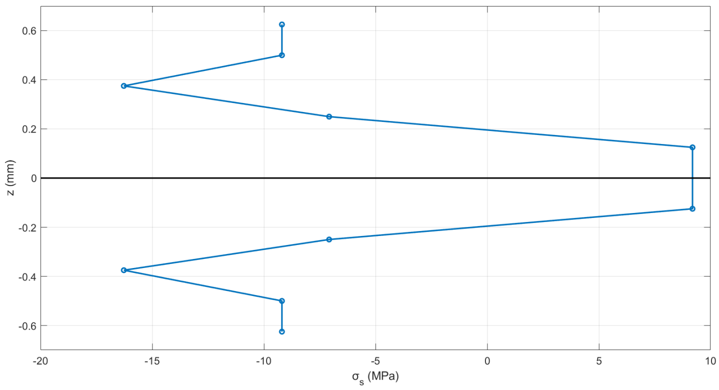

Figure 24: σ_s through the thickness

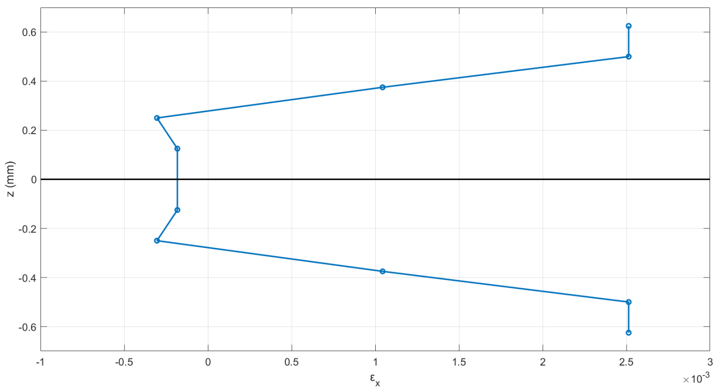

Figure 25: ϵ_x through the thickness

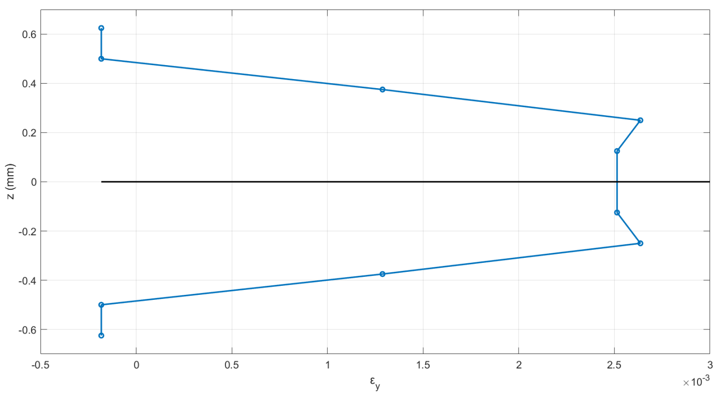

Figure 26: ϵ_y through the thickness

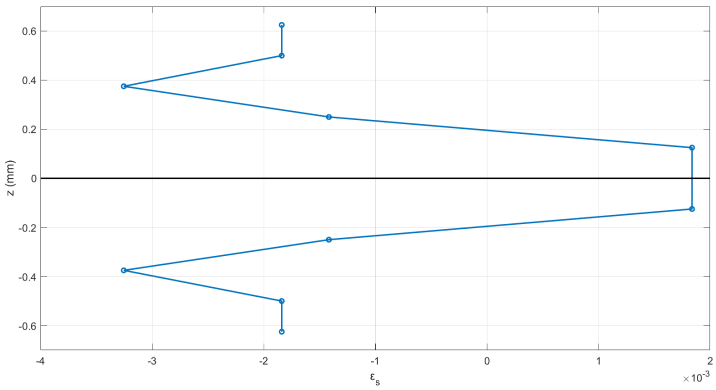

Figure 27: ϵ_s through the thickness

3 Progressive damage analysis

3.1 Analysis of Failure criterion for Quasi-isotropic Laminate

3.1.1 Calculation Layout

Simulating the behavior of a composite laminate under biaxial loading and calculating the failure indices using the Maximum Stress and Puck criteria for each ply.

The composite laminate is defined by its material properties given in table 1, which include the elastic modulus and Poisson's ratio in two directions (E1, E2, nu12, nu21), shear modulus (G12), and the maximum tensile and compressive strengths in the fiber direction (X_T, X_C) and transverse direction (Y_T, Y_C), respectively, these values where calculated in 1 and given in the assignement. The in-plane shear strength (S) is also defined.

The laminate, $[0/90/\pm 45]_{2s}$, consists of 16 plies, with each ply having a specific orientation angle in degrees, from top to bottom, and equal thin ply thickness ($h_{ply} = 0.125[mm]$), which follow from the given geometries. The ABD matrix, which relates the applied loads to the resulting strains and curvatures, is computed based on the Reduced Compliance and Stiffness matrices The global strain and stress states are calculated based on the applied loads and the ABD matrix.

For each ply, the local strains and stresses are calculated and translated from the global and local thickness - this is done due to the fact that the Maximum stress and Puck's criterion on dependent on those parameters.

From the local stress/strains, The failure indices are then calculated using both criterion. The maximum failure index over all plies is determined for each loading ratio ($\frac{\sigma_1}{\sigma_2}$) and for each criterion. From this we can find the loading ratio of First ply failure, in which the initiation force is divided by the Maximum failure index. For last ply failure, we iterate this process by recalculation the relative material properties of the failed laminate by degrading E_1 and E_2, which in turn, recalculated the ABD matrix of our new laminate system. Inside the nested loops, the failure index is calculated by taking the maximum of three types of failure indices - fibre failure under maximum stress criterion and Fiber failure or inter-fiber failure under Puck Criterion. The maximum of the failure indices is used to determine the Maximum failure index for the laminate under a given loading ratio. The maximum stress failure index is then used to determine whether a ply in the laminate has failed, and which type of failure has occurred (Fibre or Matrix). This is repeated until all the plies fail. In the case of Maximum stress, we analyze the maximum of three indices, FI_1 FI_2, FI_3, From this, we can also deduce if the ply fails under fiber (FI_1) In the case of Puck's Criterion, fiber failure under tension or compression is compared to inter-fiber failure. Further information on the calculations are discussed in 3.1.2 The results are plotted as biaxial stress First Fiber failure envelopes for both criteria.

As for the variable parameters of the failure envelope analysis, the loading ratios were varied from $-107[MPa]$ to $220[MPa]$, and were based on the maximum transverse stresses of the laminate (and the limits of Puck criterion), taking 101 points which caused sufficient convergence of the envelopes.

3.1.2 Failure index Calculation

In order to find the failure index, we consider two different criteria, maximum stress, and Puck's Criterion.

Maximum Stress criterion is determined as a function of local stresses $(\sigma_1, \sigma_2, \tau_12)$ in order to find the failure index (FI_i). [1]

$$\text{Maximum Stress} = \begin{cases} FI_1 = \frac{\sigma_1}{X_T}, & \text{for } 0 \leq \sigma_1 \\ FI_1 = \frac{\sigma_1}{X_C}, & \text{for } 0 > \sigma_1 \\ FI_2 = \frac{\sigma_2}{Y_T}, & \text{for } 0 \leq \sigma_2 \\ FI_2 = \frac{\sigma_2}{Y_X}, & \text{for } 0 > \sigma_2 \\ FI_3 = \frac{\tau_{12}}{S}, & \text{for } 0 \leq |\tau_{12}| \end{cases} \tag{18}$$

It is deduced, that due to FI_1 dependence on local stress σ_1, this determined fiber failure. While failure indices FI_2, FI_3 depend on σ_2, and shear stress tau_{12}, indicate Matrix failure

The second criterion is Pucks criterion for predicting the onset of inter-fiber failure in composite laminates subjected to in-plane loading.[2] It relates the strength of a composite laminate to the stress components acting on it and the angle of the laminate's fibers with respect to the loading direction.

In the case of inter-fiber failure, the strength of the laminate is determined by the strength of the fibers and the matrix, as well as the inter-fiber shear strength. Puck's criterion considers the inter-fiber shear strength as a function of the fiber angle and the normal stresses acting on the laminate

The criterion takes in several parameters, including the local lamina stresses $(\sigma_1, \sigma_2, and \tau_{12})$, the material properties, as well as the strength properties $(X_T, X_C, Y_T, Y_C, S, \nu_{12}, E_1)$, the type of fiber used, in this case, the ply analyzed was assumed to be carbon fiber, where $E_f = 270$, calculated from interpolation of common market carbon fibre properties.[3]

In addition to the fiber type, Puck's criterion takes into account inclination parameters which are material properties relating the

The failure factors fiber failure (FF) and the inter-fibre failure (IFF) are based on Puck's criterion. The FF predicts whether the composite material will fail in the direction of the fibers, while the IFF factor predicts whether it will fail due to inter-fiber failure or matrix failure.

Before analyzing the composite under puck's criterion, the type of fiber is determined (in this case carbon fiber), to see what inclination parameters we use for Inter-fiber failure. Then, it calculates the $R_{\perp\perp A}$ (Resistance of the action plane against its fracture due to transverse shear stressing) and tau_{12c} values based on the material properties.

The FF factor is based on whether σ_1 (stresses in the direction of the fibers) is positive or negative, If σ_1 is positive, evaluation is done in the tension failure mode, while if σ_1 is negative, it evaluates the compression failure mode.

$$\text{Fibre Failure} = \begin{cases} FF = \frac{1}{X_T}(\sigma_1 - (\nu_{12} - \frac{\nu_{12} m E_1}{E_f})(\sigma_2 + \tau_{12})), & \text{for } 0 \leq \sigma_1 \\ FF = \frac{1}{-X_C}\left(\sigma_1 - \left(\nu_{12} - \frac{\nu_{12} m E_1}{E_f}\right)(\sigma_2 + \tau_{12})\right), & \text{for } 0 > \sigma_1 \end{cases}, \tag{19}$$

Where it differs from Maximum stress is that is takes into account the fiber loading in the transverse directions with σ_2 and τ_{12}.

The IFF factor is based on the input local stresses. It uses several conditions (Mode A, Mode B, Mode C) to determine whether the composite material will fail due to inter-fiber failure or matrix failure. For each mode, the failures use equations that depend on Y_T, Y_C,S.

More specifically, the 3 different modes happen under 3 different conditions of σ_2 and τ_{12}, with parameters $R_{\perp||}$ (Resistance of the action plane against its fracture due to longitudinal shear stressing).

$$IFF_A = \sqrt{\left(\frac{p_{cT||}}{R_{\perp||}} \cdot \sigma_2\right)^2 + \left(\frac{\sigma_3}{R_{\perp||}}\right)} + \frac{p_{cT||}}{R_{\perp||}} \cdot \sigma_2 \tag{20}$$

for $Y_C > \sigma > 0$ where $R_{\perp||} = S$

$$IFF_B = \sqrt{\left(\frac{p_{cT||}}{R_{\perp||}} \cdot \sigma_2\right)^2 + \left(\frac{\sigma_3}{R_{\perp||}}\right)} + \frac{p_{cT||}}{R_{\perp||}} \cdot \sigma_2 \tag{21}$$

for $-R_{\perp\perp A} <= \sigma_2 < 0$ where $R_{\perp||} = S$

$$IFF_C = \left(\frac{\tau_{12}}{2(1 + pc_{\perp\perp})R_{\perp||}}\right)^2 + \left(\frac{\sigma_2}{R_{\perp||}}\right)^2 \cdot \left(\frac{R_{\perp||}}{-\sigma_2}\right) \tag{22}$$

For $-Y_C < \sigma_2 < -R_{\perp\perp A}$, where $R_{\perp||} = Y_C$

In addition to this, we have to also consider the condition in which the loading ratio is outside the envelope, which exhibits the pure maximum stress criterion

$$\text{IFF} = IFF_{max,C} = \max\left(\frac{\sigma_2}{X_C}, \frac{\tau_{12}}{G_{12}}\right) \tag{23}$$

for $-Y_C > \sigma_2$ or $S < tau_{12}$

$$\text{IFF} = IFF_{max,A} = \max\left(\frac{\sigma_2}{Y_T}, \frac{\tau_{12}}{\tau_{12c}}\right) \tag{24}$$

for $-R_{\perp\perp A} > \sigma_2$ or $\tau_{12c} < \tau_{12}$

All respective equations implemented in the code can be evaluated in the Appendix.

3.2 Analysis of damage envelopes

The following Graphs show the Failure envelopes for a specific iteration of loading ratios.

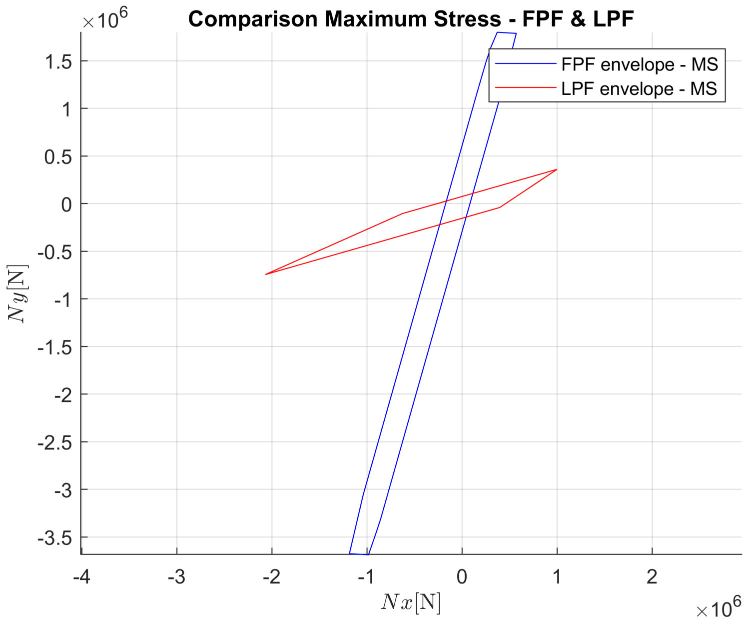

Figure 28: FPF & LPF Maximum Criterion

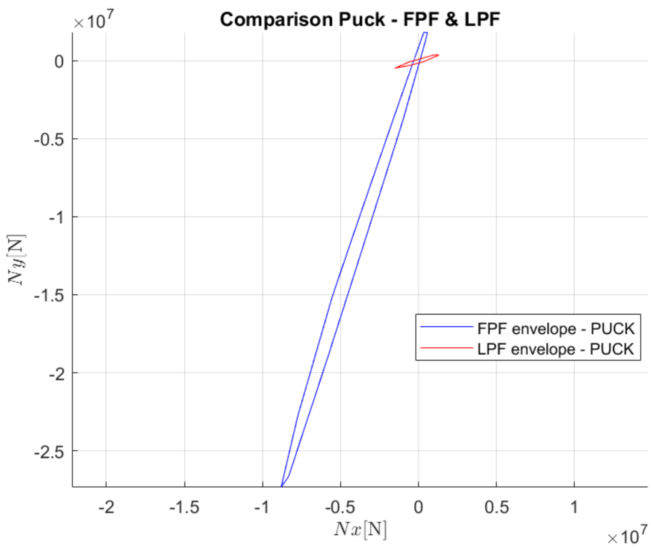

Figure 29: FPF & LPF Puck Criterion

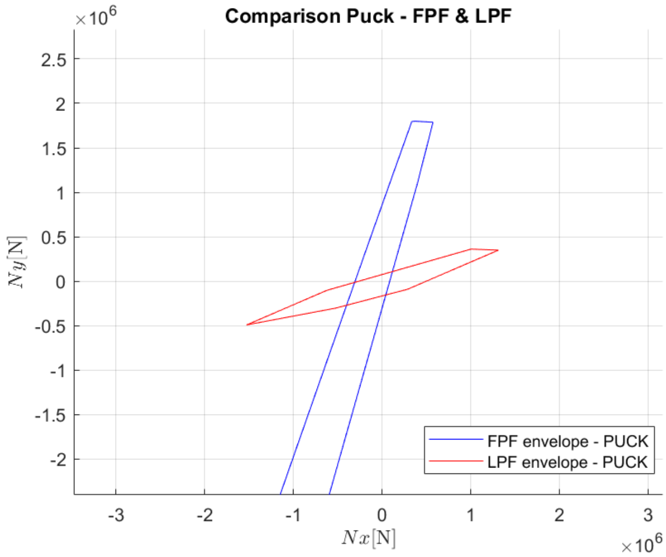

Figure 30: FPF & LPF Puck Criterion

The trends in the graphs 28 and 30 show a clear trend when comparing first-ply failure to last-ply failure. Firstly, it is important to note that the +-45 plies never fail first nor last, it is always either the 0 or 90 UD that fails first and last. This is clear from the inter-fibers/matrix failing before the fibers for all loading ratios, and as the 0 or 90 UD always has a direction that has matrix-dominated properties and a direction that has fiber-dominated properties, compared to the +-45 that has a combination of both. The 0 or 90 UD will fail before the +-45, then the +-45 will fail, and lastly, the alternative UD will fail.

This means that the trends that are shown, where the first ply failure follows a more biaxial load with $N_x = N_y$ (meaning a more shear-like load) and the last ply failure follows a more unidirectional load where one of the load's magnitude is larger than the other.

This means that the first ply does not fail for loading ratios close to one, which is reasonable as the UD plies' matrix experiences the least stress in that case. So to avoid first ply failure, a loading ratio of 1 is preferred for a specified magnitude (so $\vec{N_x} + \vec{N_y} = constant$).

As for the last ply failure, it fails only when all other plies have failed, meaning when only the fiber-dominated property plies are left. This is reasonable, as for the last ply failure, which is always a UD, for the case where the loading ratio is close to one, the local stress on the UD ply is low, but the stresses in the matrix direction are significant, which means it will fail in matrix direction earlier than when the loading ratio is low. When the loading ratio is zero, the full load is applied in the fiber direction and fiber failure occurs.

The reason for the last ply failure to have the trend in small loading ratios is the combination of the two failures explained, where the increase in loading ratio causes lower local stresses but with the risk of inducing matrix failure instead of fiber failure as the local stresses in matrix direction increase.

The maximum stress criterion and Puck's criterion are two commonly used failure criteria in composite materials analysis.

The maximum stress criterion is simple, compared to Puck, which assumes that failure occurs when any of the local stresses exceed the ultimate strength of the material in that direction. This criterion does not take into account any interaction between the stresses and plies, leading to overly conservative designs.

Puck's criterion, on the other hand, takes into account the interaction between the stresses and the material properties such as the fiber and matrix strength. It considers both the fiber failure and the inter-fiber/matrix failure. Puck's criterion is generally considered to be more accurate and less conservative than the maximum stress criterion.

By analyzing the mathematical expressions, in fiber failure (failure in the fiber direction), Maximum stress only takes into account the maximum stresses within the direction of the fiber,[equation 18], whereas Puck's criterion takes a component of the transverse stresses - decreasing the effect of stresses in the fiber direction, [equation 19]. As for Matrix failure or inter-fiber failure, Maximum stress, similarly to fiber failure only takes into account the stresses in the transverse direction,[equation18]. Puck's criterion has additional parameters such as the resistance to transverse shear stressing, and inclination parameters of the fibers which take more physical meaning than just the transverse stresses within the material,[equation 22].

We can see this specifically illustrated in the First ply failure graphs 31 and 32, Where they exhibit similar behaviours in the tensile loading ratio. As mentioned before, for first ply failure, the matrix failures first, meaning under Puck's criterion, we consider that the failure indices of the inter-fiber failure are much larger than those of the fiber failure. This means in the compression loading ratios, we can see that under the puck's criterion, the Inter-fiber failure modes (A, B,C) come into play, in which the plies can sustain more compression loading.

Concerning the Last ply failure, we can refer to graph 33, in which we see that both Puck and maximum stress exhibit similar behaviors (envelopes). This is because, in the last ply failure, the ply revert to fiber failure instead of matrix/inter-fiber failure, under the maximum stress. Deviations in the puck's criterion for last ply failure are due to the additional effect of σ_2.

In terms of damage tolerance, Puck's criterion is expected to have a higher damage tolerance, in first-ply failure, than maximum stress mainly in the compression, which is shown in a graph 31. This is because Puck's criterion considers the interaction between the stresses and the material properties and can tolerate a larger increase in stress before failure occurs. The maximum stress criterion, on the other hand, is more likely to predict failure at lower stresses and is therefore less tolerant of damage.

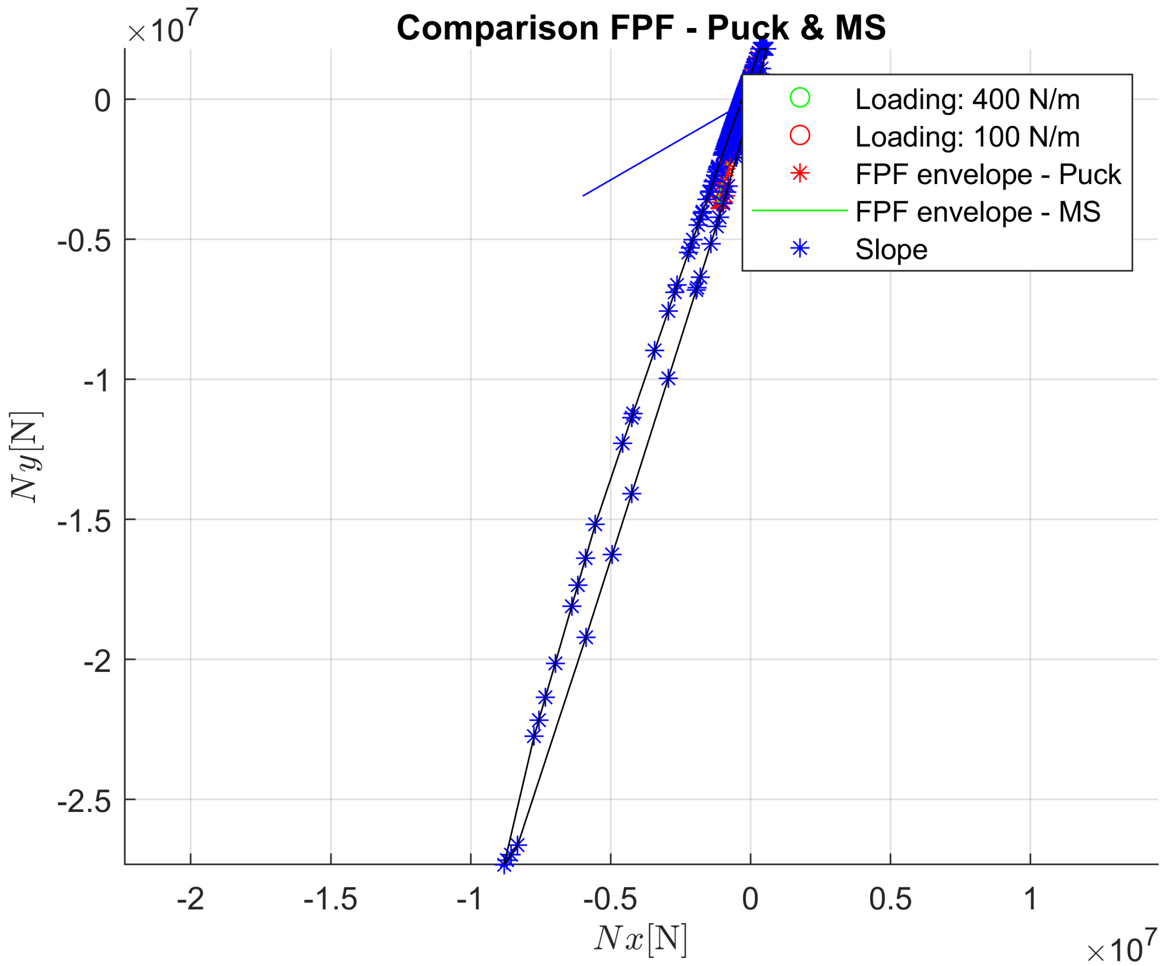

Figure 31: Comparison FPF

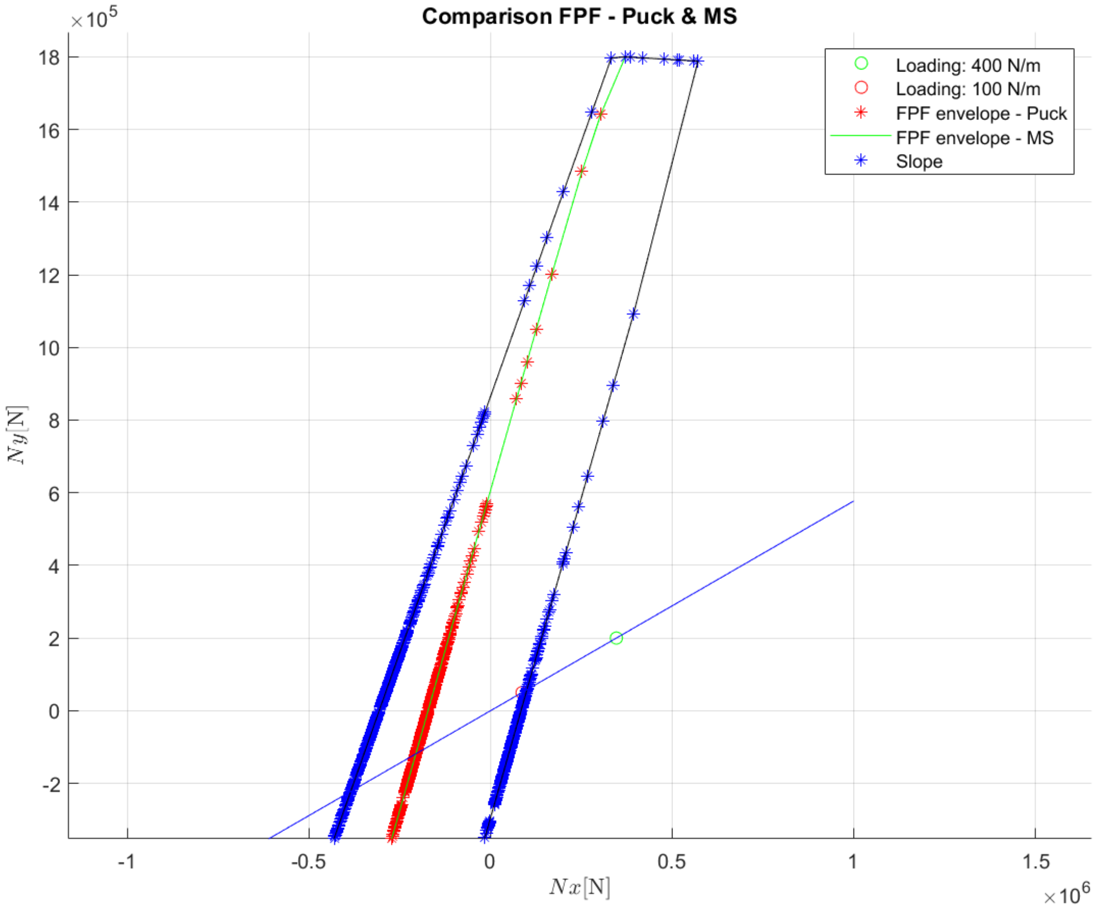

Figure 32: Comparison FPF zoom

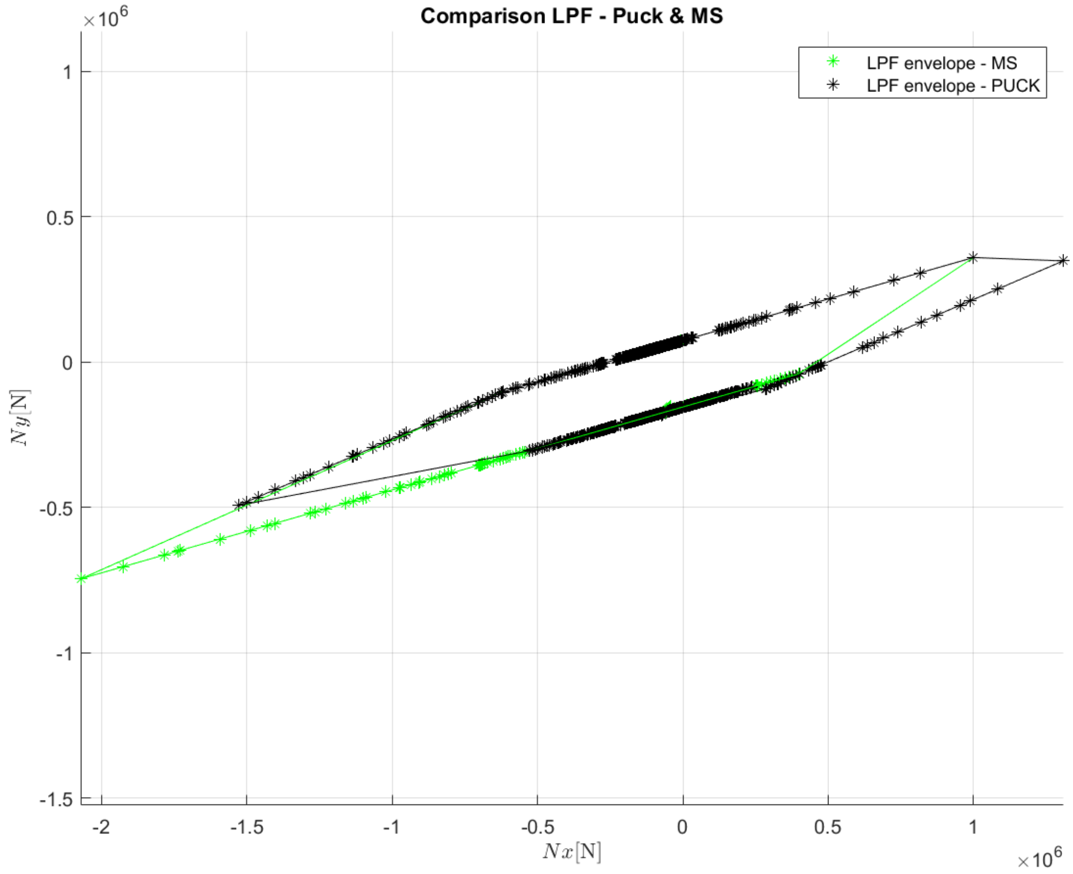

Figure 33: Comparison LPF

However, it's important to note that the accuracy and conservatism of both criteria depend on the placed assumptions and simplifications used in the analysis. These include:

- The material is assumed to be linearly elastic, homogeneous, and isotropic.
- The composite is assumed to be symmetric and layered.
- The fiber is assumed to be continuous and aligned in one direction.
- The failure of the composite is assumed to occur due to the failure of the fibers or the matrix.
- Failure of the matrix and the fibers is assumed to be independent of each other.
- The composite is assumed to be under uniaxial stress.
- The fibers are assumed to be free of initial defects or damage.
- The criterion does not consider the effect of loading rate, temperature, or environmental factors on the composite material behavior.

Therefore, it is always important to carefully evaluate the assumptions and limitations of any failure criterion.

4 Reliability analysis

For the reliability analysis, the elastic and strength values with their corresponding standard deviations from question 1a were used, see Section 1. As for the convergence study, the last slide from Lecture 8 was used [4]. This was chosen due to the ease of programming it and a clear way of seeing the convergence whilst running the rounds of simulations. When using it, N simulations are run where the random variables are assigned, where the variables from Table 1 are seen as random and independent variables, and the program terminates when the first ply failure occurs. For that round, the probability of failure is set to $P_f = \frac{1}{N}$ and is saved. Then a new round is initiated with the same method. Then the mean value of all the probabilities of failure is said to approach the true probability of failure as n approaches ∞, see equation 25.

$$P_f^{predicted} = \lim_{R\to\infty} \frac{P_f^1 + P_f^2 + ... + P_f^R}{R} = \frac{F}{n} = P_f^{true} \tag{25}$$

Here, P_f^i is the probability of failure for round $i \in \{1, 2, ..., R\}$ and R is the total amount of rounds done. The way convergence was checked, is when this sum, for increasing R, approaches a stable value.

4.1 100 N/mm

For N=100 N/mm, the method approached a stable value for R around 10 000, see Figure 34. With this number of rounds, the experiment was repeated 10 times to be able to create a normal distribution from all the independent experiments. This gave a mean value for the probability of failure of 0.243 and a standard deviation of 0.00300. See the distribution in Figure 35. Therefore, the probability of failure can be said to be in the range of 0.239-0.246 with a certainty of 99%.

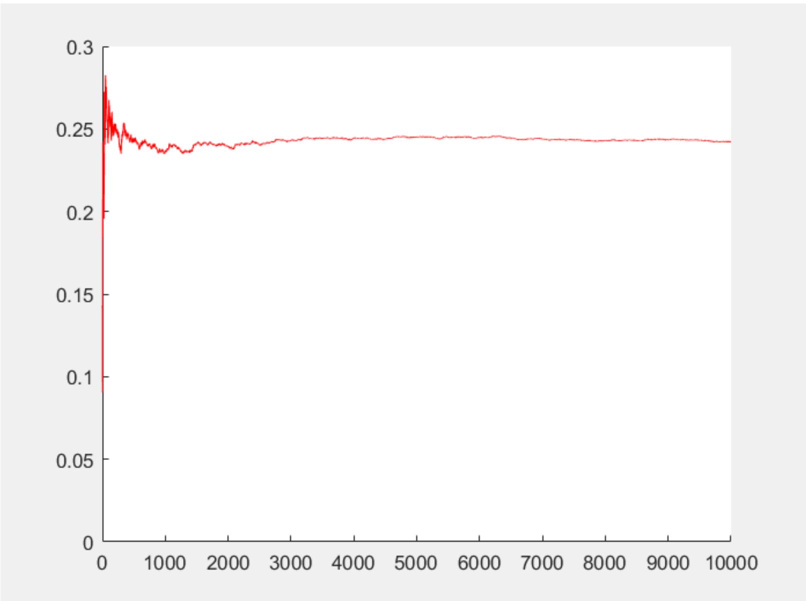

Figure 34: Convergence of the probability of first ply failure

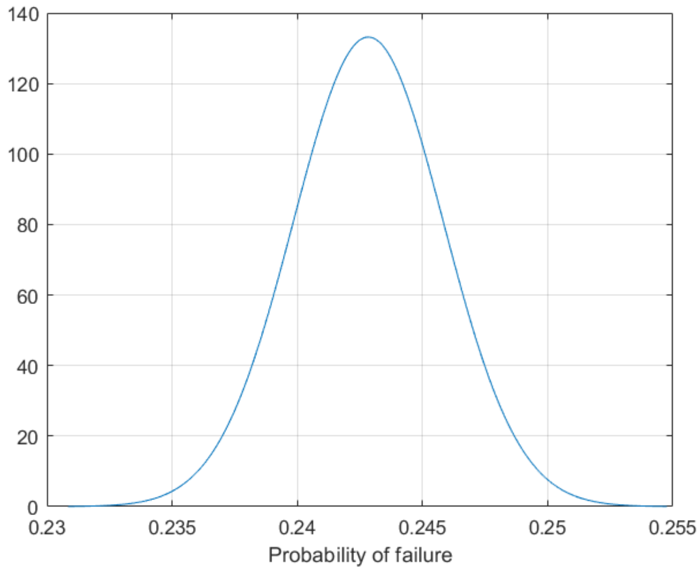

Figure 35: Normal distribution of the probability of first ply failure for N=100 N/mm

4.2 400 N/mm

As for the higher value, there was always a first-ply failure. The simulation ran more than 100 000 times, but every time, a ply failed, see Figure 36. So, to be certain of this, the failure index of each simulation was obtained and a normal distribution of the failure index was created, see Figure 37. From this, it is clear that the failure index is more than 1. The confidence interval for the value being larger than 1 was calculated and with that it is possible to say that the probability of first ply failure is 1 with machine precision certainty, using that machine precision is of the order 1e-14, $1 - 1.9301e - 16 = 1$. This experiment was run 10 times to obtain a normal distribution of the confidence interval to be certain of the calculated confidence interval. The mean of this was calculated and the standard deviation was calculated to be 1.9301e-16, which means, the probability of failure is 1.

As the simulation was run 10 times, equation 25 can still be used to obtain a maximum probability of first-ply failure for the higher load. The way this was done was by solving the probability for no first-ply failure, which is equal to 1 minus the probability of first-ply failure. Assuming the last simulation did not cause failure, and thus $1 - P_f^R = 1 - \frac{1}{1e5} = 0.99999$, doing this for all 10 simulations (where in no simulation, failure occurred), the probability of failure can be said to be higher than 0.99999.

Yet another way to convince ourselves of the result is by looking at the stress envelope for the specified loads. Figure 32 clearly shows that the lower load case is in the vicinity of the failure stress for the mean properties. As the properties change, the stress envelope also changes, and thus can the point be located outside the stress envelope for the lower load. As for the higher load, it is a long distance outside the stress envelope for the mean properties, which again shows that failure occurs.

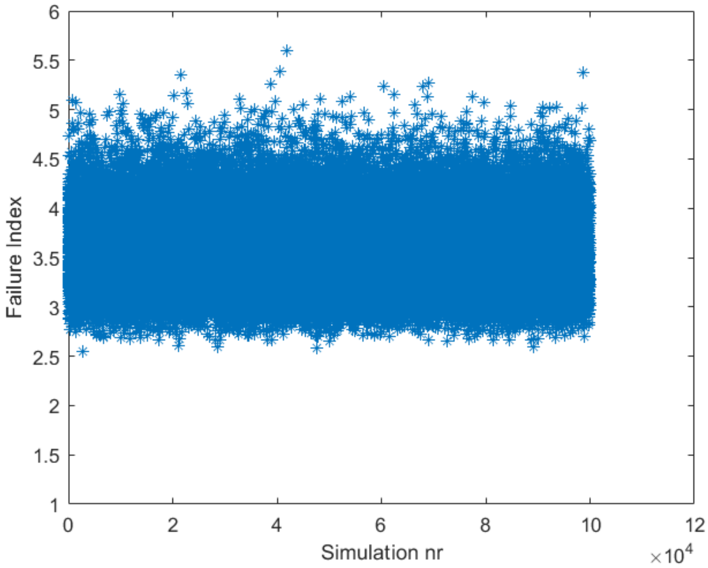

Figure 36: Failure indexes for 100 000 simulations

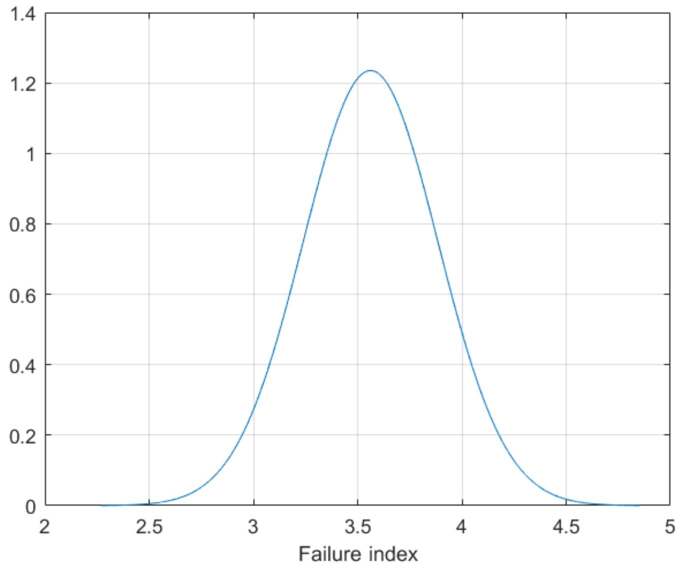

Figure 37: Normal distribution of the failure index for N=400 N/mm

Contributions

All members worked on every question, with one member being responsible for the question, meaning all members contributed to every question. The responsible for Q1 and Q4 was Rasmus van Kerkvoorde. The responsible for Q2 was Thanos Giotas. The responsible for Q3 was Cristophe Hatterer. In the table below, a more detailed list can be seen.

Question	Contributions
Q1a	**Data Analysis/Report:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas, **Review:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas
Q1b	**Calculations/Report:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas, **Review:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas
Q2a	**Calculations/Rational of the selection of θ/Report:** Thanos Giotas + Rasmus van Kerkvoorde + Christophe Hatterer, **Review:** Thanos Giotas + Rasmus van Kerkvoorde + Christophe Hatterer
Q2b	**Calculations/Report/Figures:** Thanos Giotas + Rasmus van Kerkvoorde + Christophe Hatterer, **Review:** Thanos Giotas + Rasmus van Kerkvoorde + Christophe Hatterer
Q3	**Calculations/Report/Figures:** Christophe Hatterer + Rasmus van Kerkvoorde + Thanos Giotas, **Review** Christophe Hatterer + Rasmus van Kerkvoorde + Thanos Giotas
Q4	**Approach/Calculations/Report:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas, **Calculations/Review:** Rasmus van Kerkvoorde + Christophe Hatterer + Thanos Giotas

References

[1] Anthony R. Bunsell and J. Renard. *Fundamentals of Fibre Reinforced Composite Materials.* Institute of Physics Publishing, 2005.

[2] Martin Knops. *Analysis of failure in fiber polymer laminates: The theory of Alfred Puck.* Scholars Portal, 2008.

[3] *Mechanical properties of carbon fibre composite materials, fibre / epoxy resin (120°C cure).* July 2009. URL: `http://www.performance-composites.com/carbonfibre/mechanicalproperties_2.asp`.

[4] Dr. Dimitrios Zarouchas. *Lecture 8, AE4ASM109, part DZ.* 2023.

Appendix

4.2.1 Appendix A

Matlab Functions
Maximum Stress Criterion

```
function [FI_1,FI_2,FI_3]= MaxStress(sigma1,sigma2,sigma3,X_T,X_C,Y_T,Y_C,S_f)

if sigma1 ≥ 0
    FI_1 = sigma1/X_T;
else
    FI_1 = -sigma1/X_C;
end

if sigma2≥0
    FI_2 = sigma2/Y_T;
else
    FI_2 = -sigma2/Y_C;
end
FI_3 = abs(sigma3)/S_f;
end
```

Puck's Criterion

```
function [FF,IFF] = ...
    PuckCriterion(sigma1,sigma2,sigma3,X_T,X_C,Y_T,Y_C,G12_t,nu12,E1,fiber,print)
FF = 0;
IFF = 0;

sigma3 = abs(sigma3);

% Determination of which fiber used
if fiber == 'c'
    m = 1.1;
    % Incline parameters
    ptTll = 0.3;
    pcTll = 0.25;
    ptTT = 0.2;
    pcTT = 0.25;
    Ex = 230e9; % GPa
elseif fiber == 'g'
    m = 1.3;
    % Incline parameters
    ptTll = 0.35;
    pcTll = 0.30;
    ptTT = 0.25;
    pcTT = 0.30;
    Ex = 75e9; % GPa
end

R_TTA = Y_C/(2*(1+pcTT));
tau_12c = G12_t*sqrt(1+2*pcTT);

% Fiber failure (direction of the fibers)
if sigma1 > 0
    FF = 1/X_T*(sigma1 - (nu12-nu12*m*E1/Ex)*(sigma2+sigma3));
    if print == 'y'
        fprintf("Evaluating Fiber Tension\n")
    end
else
```

```
    FF = 1/(-X_C)*(sigma1 - (nu12-nu12*m*E1/Ex)*(sigma2+sigma3));
    if print == 'y'
        fprintf("Evaluating Compression\n")
    end
end

% Matrix Failure
if sigma2 > 0
    if sigma2 > Y_T || sigma3 > G12_t
        IFF = max(abs(sigma2/Y_T),sigma3/G12_t);
    elseif sigma2 < Y_T %Condition Mode A
    R_Tll = G12_t;
    R_Tt = Y_T;
    A_ = (1/R_Tt - ptTll/R_Tll)*sigma2;
    B = sigma3/R_Tll;
    C = ptTll/R_Tll*sigma2;
    IFF = sqrt(A_^2 + B^2) + C;
    if print == 'y'
        fprintf("Interfiber failure Mode A\n")
    end
    end
elseif sigma2 < -Y_C || sigma3 > tau_12c
    IFF =max(abs(sigma2/Y_C),sigma3/tau_12c);
elseif sigma2 ≤ 0 && -R_TTA < sigma2  %  Condition Mode B
    R_Tll = G12_t;
    A_ = (pcTll./R_Tll).*sigma2;
    B = sigma3./R_Tll;
    C = pcTll./R_Tll*sigma2;
    IFF = sqrt(A_.^2 + B.^2) + C;
    if print == 'y'
        fprintf("Interfiber failure Mode B\n")
    end
elseif sigma2 < -R_TTA && -Y_C ≤ sigma2 % Condition Mode C
    R_Tll = Y_C;
    A_ = sigma3./(2*(1+pcTT)*R_Tll);
    B = sigma2./R_Tll;
    C = R_Tll./-sigma2;
    IFF = (A_.^2 + B.^2)* C;
    thetafp = acosd(sqrt(1./2*(1+pcTT)*((R_TTA./R_Tll)*(sigma3./sigma2)+1)));
    if print == 'y'
        fprintf("Interfiber failure Mode C\n")
        disp(thetafp)
    end
else
    if print == 'y'
        fprintf("invalid value of sigma2\n")
    end
end
end
```