

1. Purposes of Modelling



Inference, Prediction.

Show a pic of real examples, explain these terms.

2. Data structure and Basic Model Assumptions

Data table.

Assumptions: two approaches

1. Underlying model style
2. Bayesian style

3. LASSO outline:

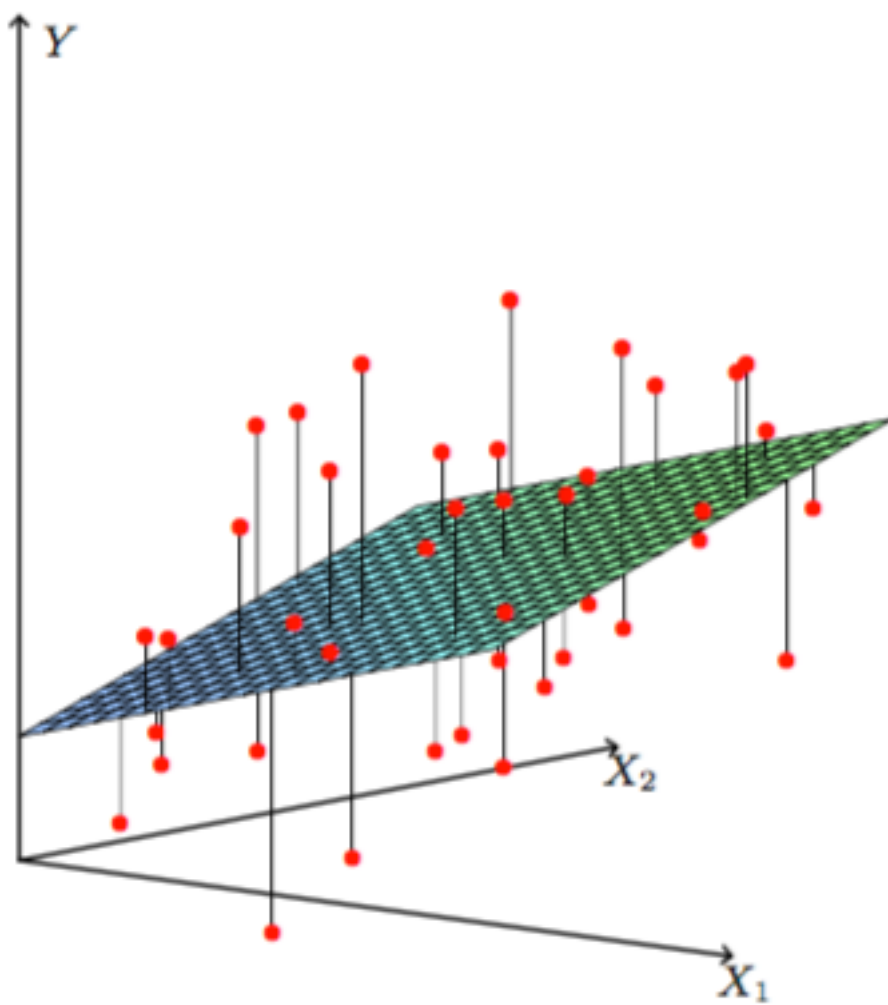
Linear regression

$$f(X) = \beta_0 + \sum_{j=1}^p X_j \beta_j$$

$$\text{RSS}(\lambda) = (\mathbf{y} - \mathbf{X}\beta)^T (\mathbf{y} - \mathbf{X}\beta) + \lambda \beta^T \beta$$

$$\text{RSS}(\beta) = (\mathbf{y} - \mathbf{X}\beta)^T (\mathbf{y} - \mathbf{X}\beta)$$

$$\frac{\partial \text{RSS}}{\partial \beta} = -2\mathbf{X}^T (\mathbf{y} - \mathbf{X}\beta)$$

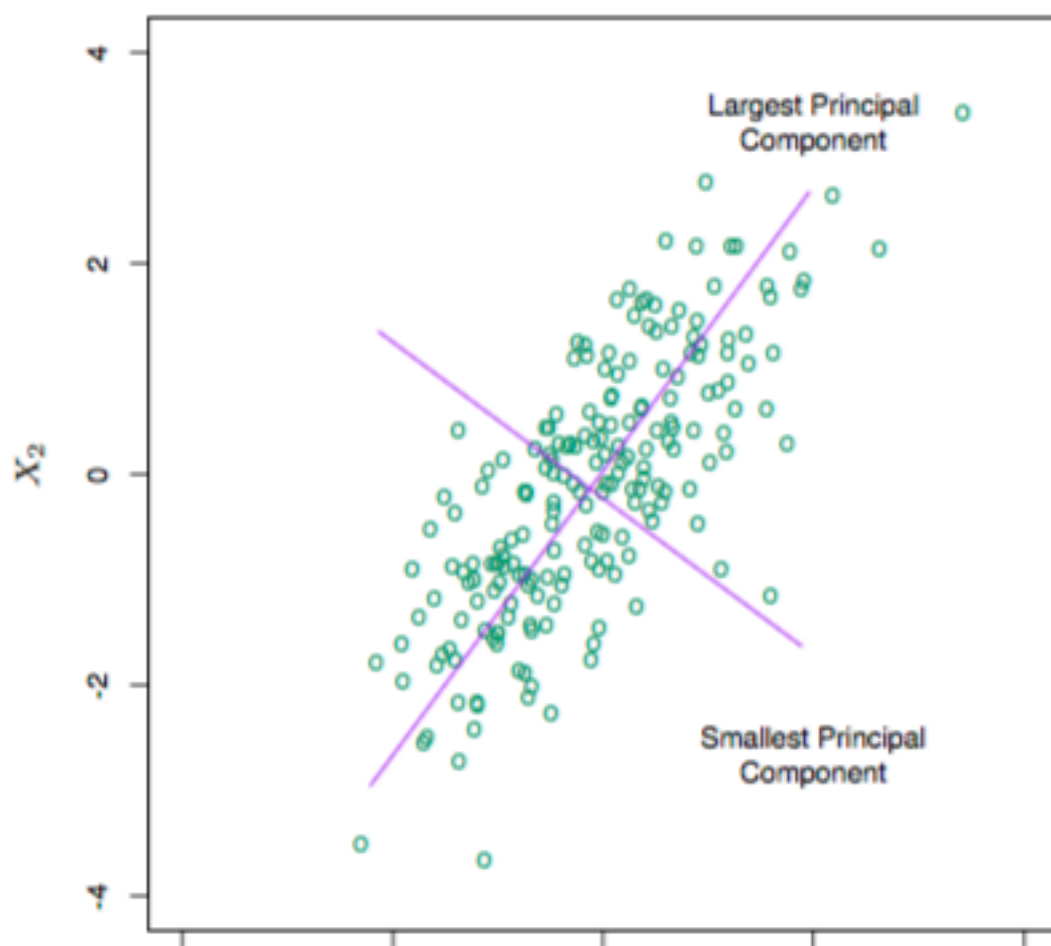
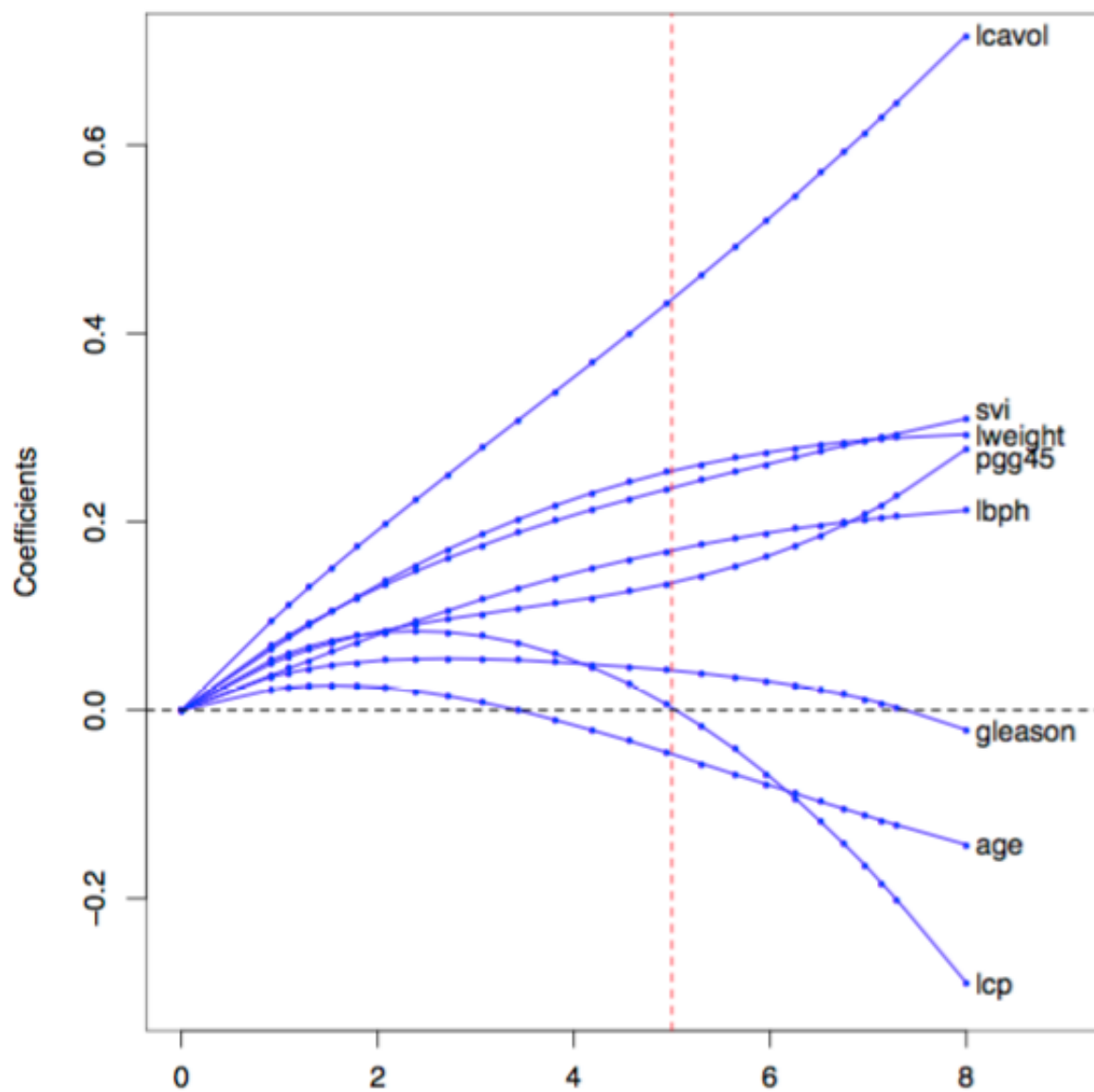


4. Ridge regression

$$\hat{\beta}^{\text{ridge}} = \underset{\beta}{\operatorname{argmin}} \left\{ \sum_{i=1}^N (y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j)^2 + \lambda \sum_{j=1}^p \beta_j^2 \right\}$$

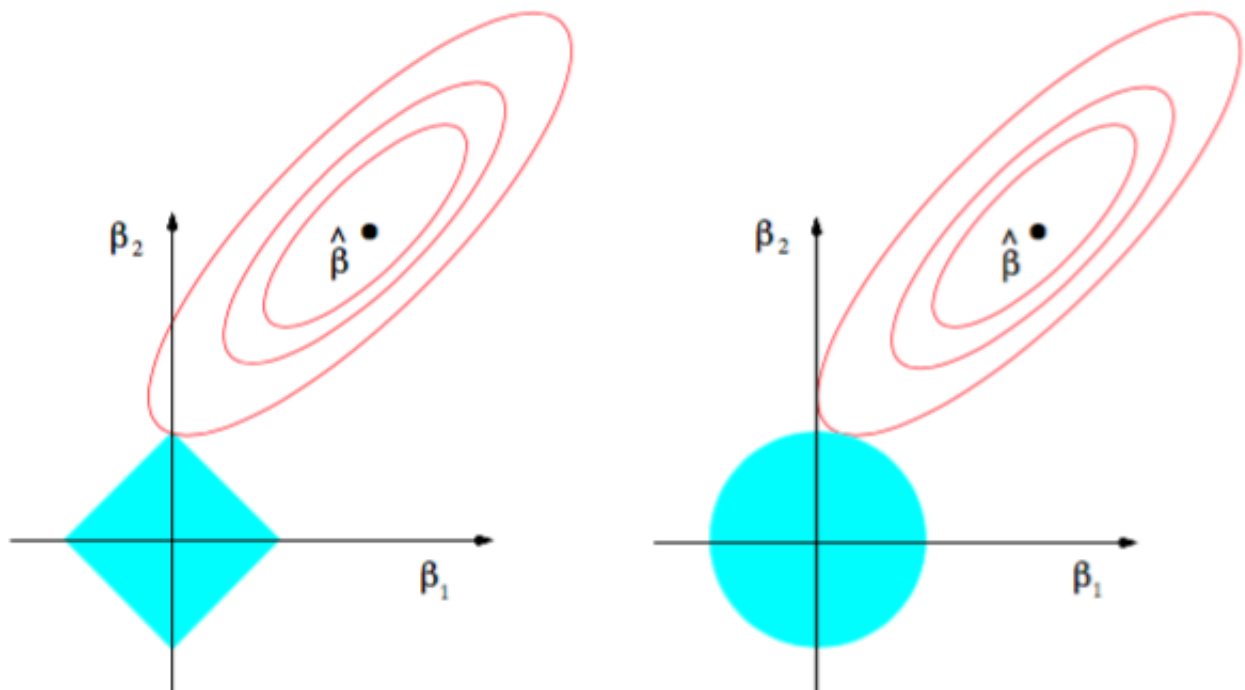
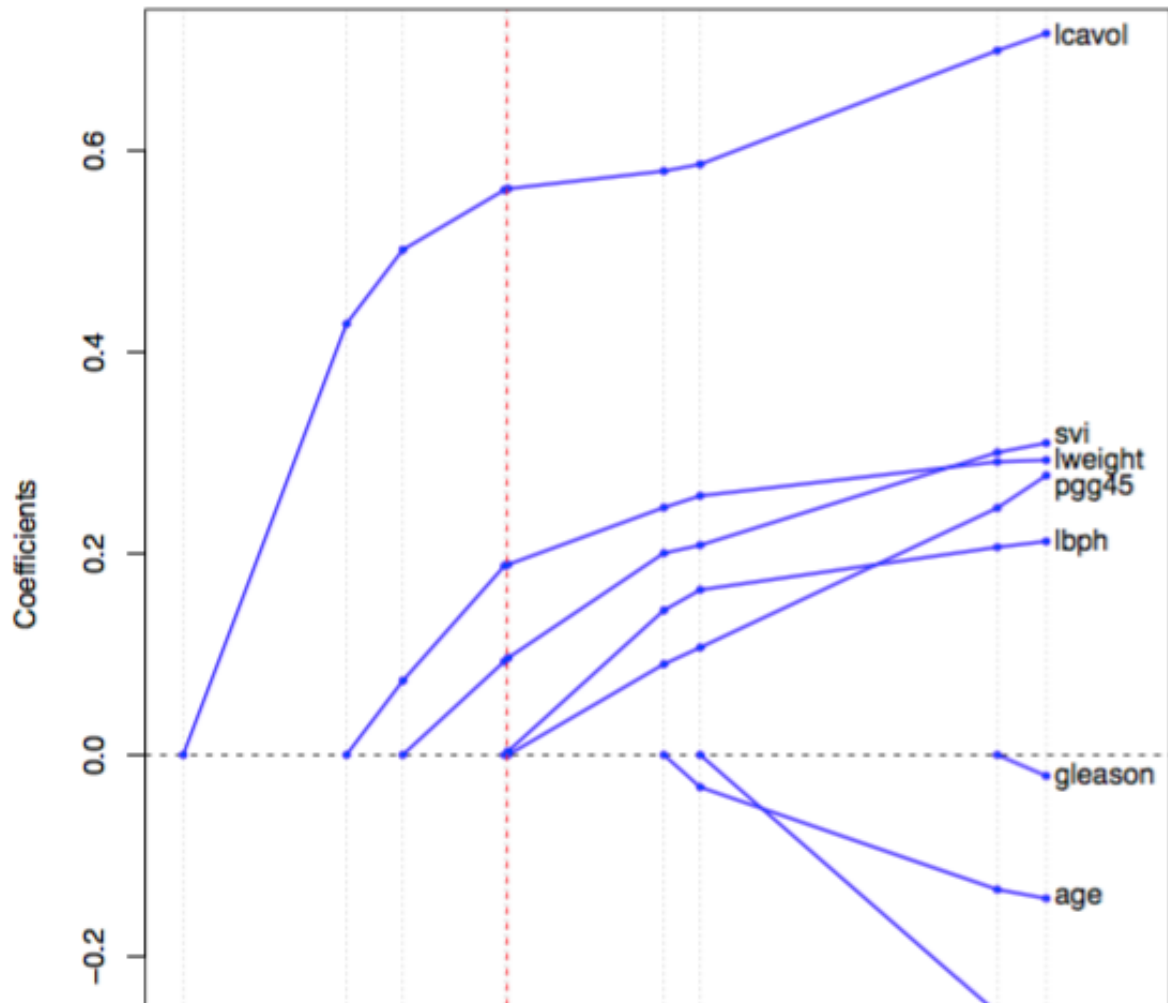
$$\hat{\beta}^{\text{ridge}} = (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{I})^{-1} \mathbf{X}^T \mathbf{y}$$

make the problem non-singular.
SVD -> Principle components.



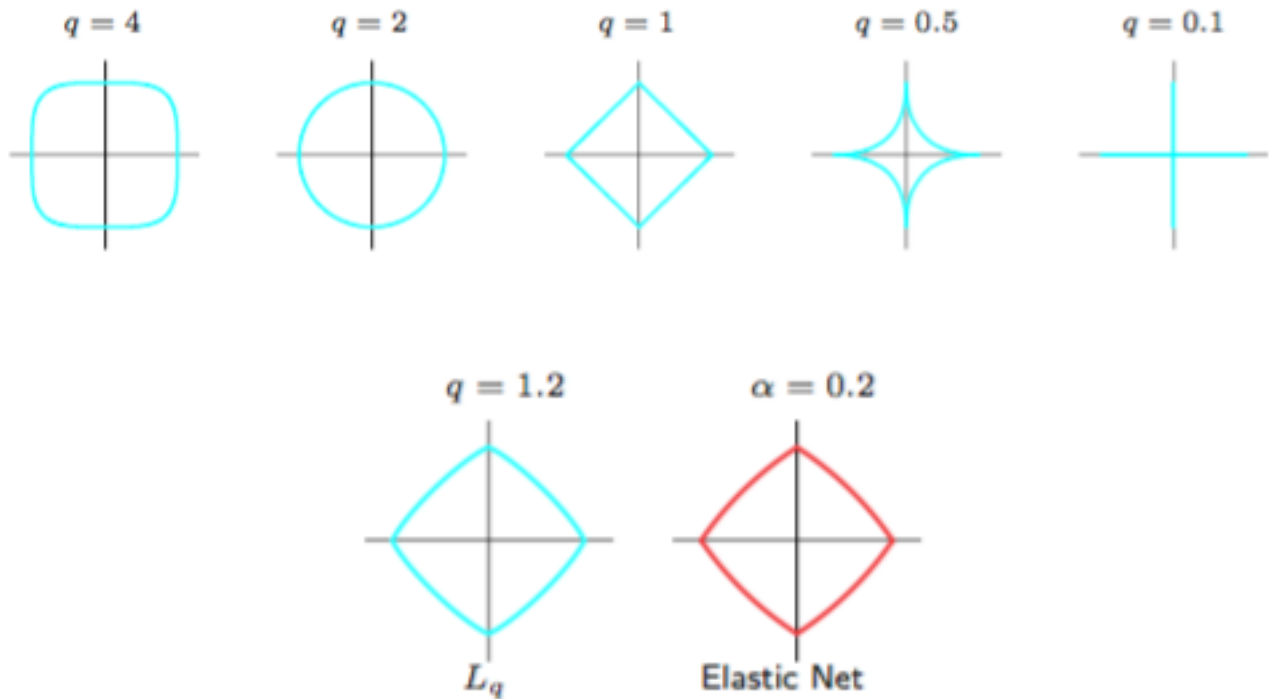
5. Lasso

$$\hat{\beta}^{\text{lasso}} = \underset{\beta}{\operatorname{argmin}} \left\{ \frac{1}{2} \sum_{i=1}^N (y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\}$$



6. General

$$\tilde{\beta} = \operatorname{argmin}_{\beta} \left\{ \sum_{i=1}^N (y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j)^2 + \lambda \sum_{j=1}^p |\beta_j|^q \right\}$$



Which lambda to choose when building model? Cross-validation.

7. Random forest

Decision Tree

Bagging

Random forest

<https://lagunita.stanford.edu/c4x/HumanitiesScience/StatLearning/asset/trees.pdf>