Symmetric Ciphers

CLASSICAL ENCRYPTION TECHNIQUES

AES - Finite Field Algorithm

In AES, all operations are performed on 8-bit bytes

the arithmetic operations of addition, multiplication, and division are performed over the finite Field GF(2^8)

$$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 = \sum_{i=0}^{n-1} a_i x^i$$

There are a total of 2^n different polynomials

For n=3, 2^3=8the polynomials in the set are

$$\begin{array}{llll} 0 & x & x^2 & x^2 + x \\ 1 & x + 1 & x^2 + 1 & x^2 + x + 1 \end{array}$$

AES - Finite Field Algorithm

Arithmetic follows the ordinary rules of polynomial arithmetic using the basic rules of algebra with the following two refinements.

Arithmetic on the coefficients is performed modulo 2. This is the same as the XOR operation

If multiplication results in a polynomial of degree greater than n-1, then then the polynomial is reduced modulo some irreducible polynomial m(x) of degree n

The polynomial[9] $f(x) = x^4 + 1$ over GF(2) is reducible, because

$$x^4 + 1 = (x + 1)(x^3 + x^2 + x + 1).$$

Ex: finite field GF(2^3)

- x^3+x^2+1
- x^3+x+1

AES => GF(2^8)

- $x^8+x^4+x^3+x^1+1$

Group: (A1)–(A4)

Abelian group: (A1)–(A5)

Ring: (A1)–(M3)

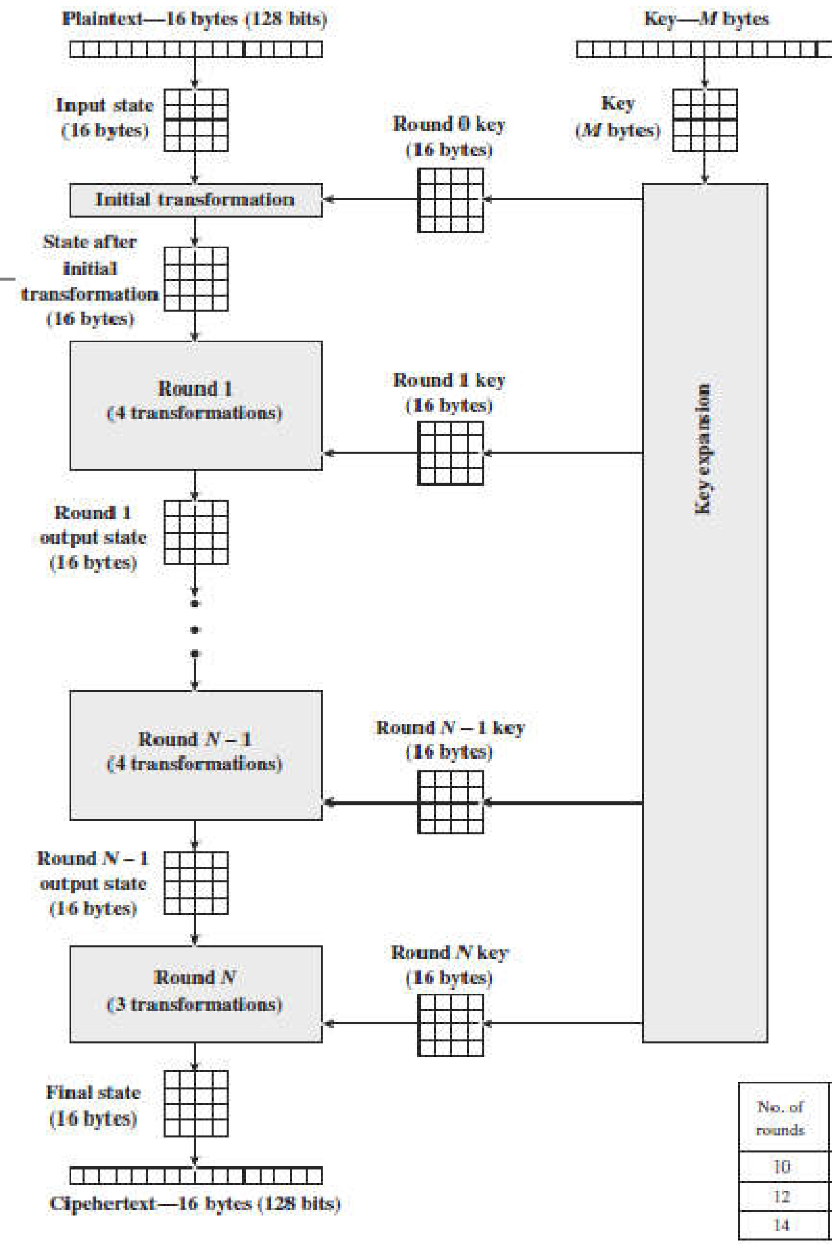
Commutative ring: (A1)–(M4)

Integral domain: (A1)–(M6)

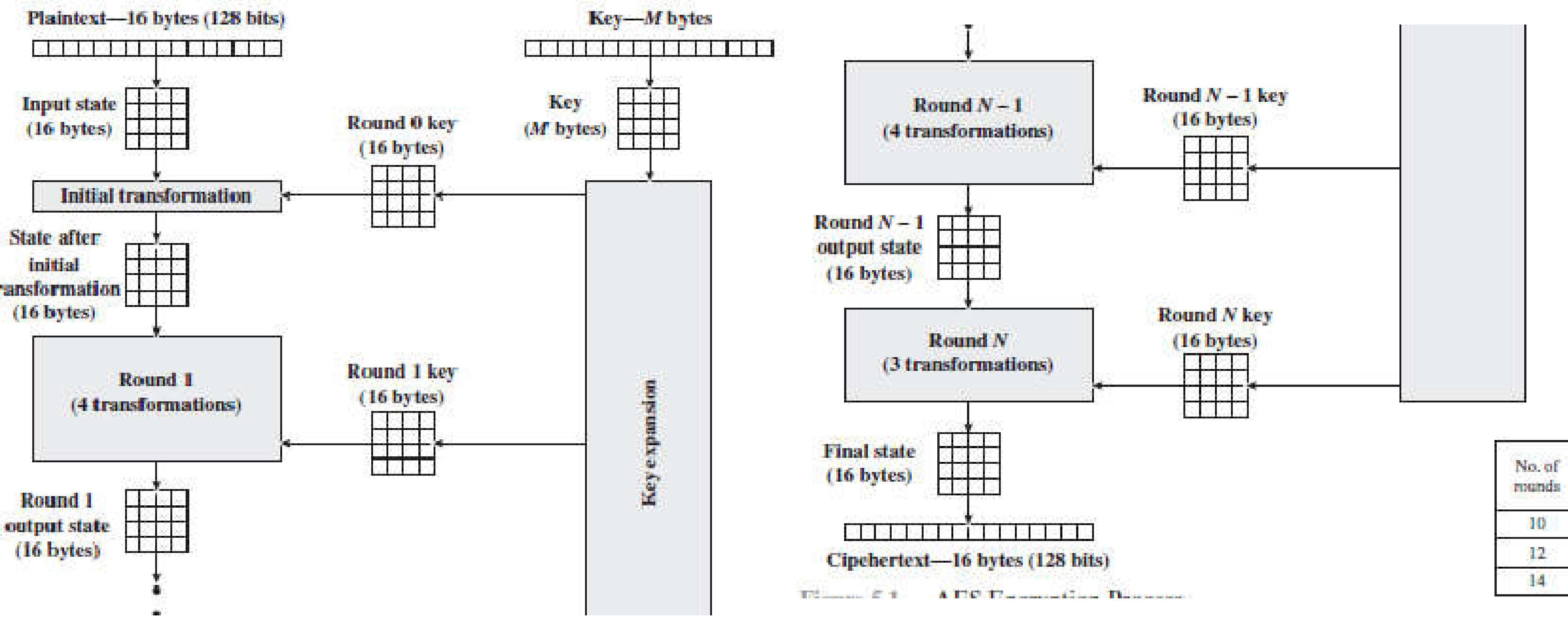
Field: (A1)–(M7)

(A1) Closure under addition:	If a and b belong to S, then $a + b$ is also in S
(A2) Associativity of addition:	$a + (b + c) = (a + b) + c$ for all a, b, c in S
(A3) Additive identity:	There is an element 0 in R such that $a + 0 = 0 + a = a$ for all a in S
(A4) Additive inverse:	For each a in S there is an element $-a$ in S such that $a + (-a) = (-a) + a = 0$
(A5) Commutativity of addition	$a + b = b + a$ for all a, b in S
(M1) Closure under multiplication:	If a and b belong to S, then ab is also in S
(M2) Associativity of multiplication:	$a(bc) = (ab)c$ for all a, b, c in S
(M3) Distributive laws:	$a(b + c) = ab + ac$ for all a, b, c in S $(a + b)c = ac + bc$ for all a, b, c in S
(M4) Commutativity of multiplication:	$ab = ba$ for all a, b in S
(M5) Multiplicative identity:	There is an element 1 in S such that $a1 = 1a = a$ for all a in S
(M6) No zero divisors:	If a, b in S and $ab = 0$, then either $a = 0$ or $b = 0$
(M7) Multiplicative inverse:	If a belongs to S and a 0, there is an element a^{-1} in S such that $aa^{-1} = a^{-1}a = 1$

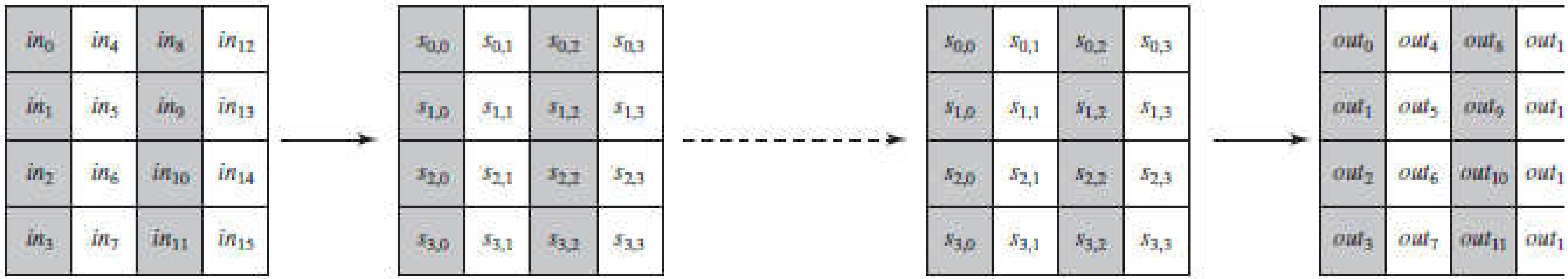
AES Encryption Process



AES Encryption Process



AES Data Structure

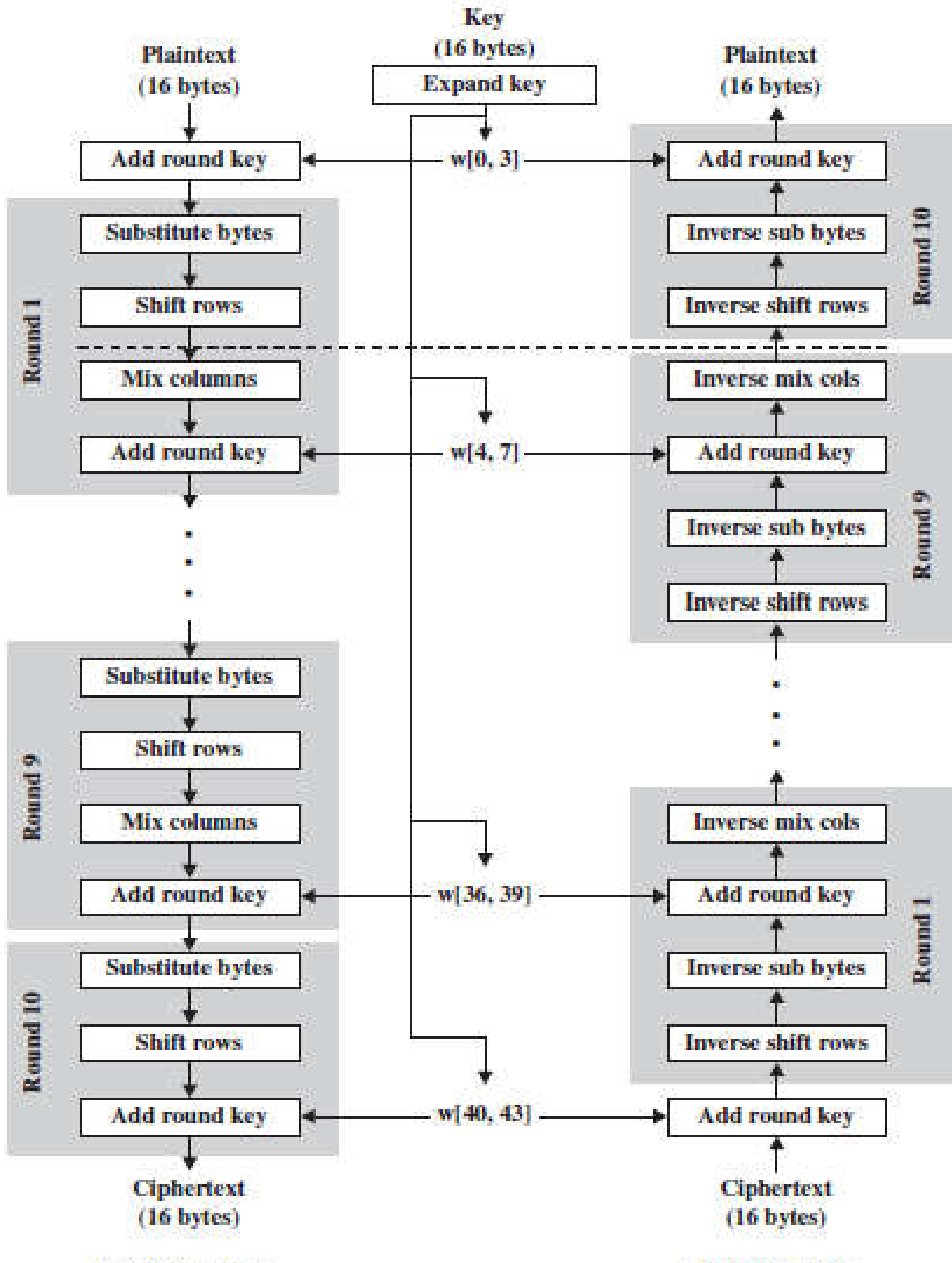


(a) Input, state array, and output

(b) Key and expanded key

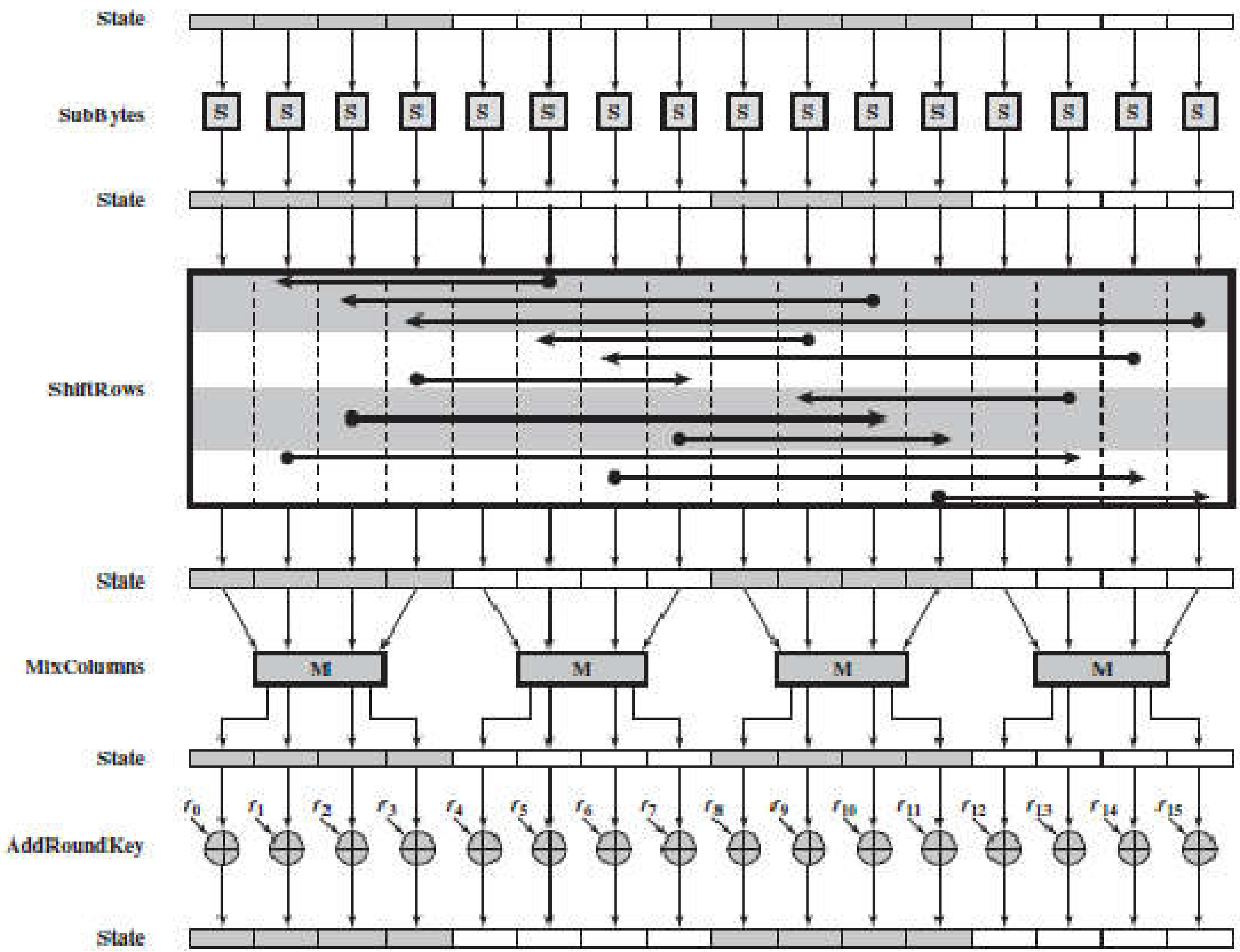
AES Data Structure

Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240



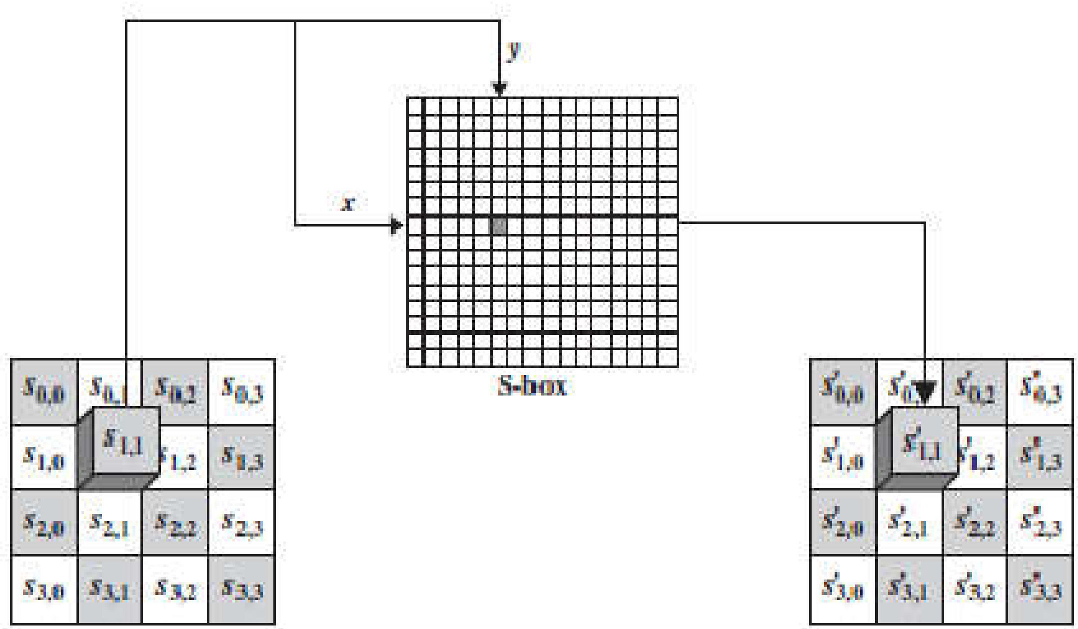
Key
(16 bytes)
Expand key
Plaintext
(16 bytes)
Add round key
w[0, 3]
Substitute bytes
Shift rows
Mix columns
Add round key
w[4, 7]
Round 1
Substitute bytes
Shift rows
Mix columns
Add round key
w[36, 39]
Round 9
Substitute bytes
Shift rows
Add round key
w[40, 43]
Round 10
Ciphertext
(16 bytes)
Plaintext
(16 bytes)
Add round key
Inverse sub bytes
Inverse shift rows
Round 10
Inverse mix cols
Add round key
Inverse sub bytes
Inverse shift rows
Round 9
Inverse mix cols
Add round key
Inverse sub bytes
Inverse shift rows
Round 1
Add round key
Ciphertext
(16 bytes)

Ecryption



State
SubBytes
S S S S S S S S S S S S S S S S
State
ShiftRows
State
MixColumns
M M M M
State
r_0 r_1 r_2 r_3 r_4 r_5 r_6 r_7 r_8 r_9 r_{10} r_{11} r_{12} r_{13} r_{14} r_{15}
AddRoundKey
State

AES - Substitution



AES – S-BOX

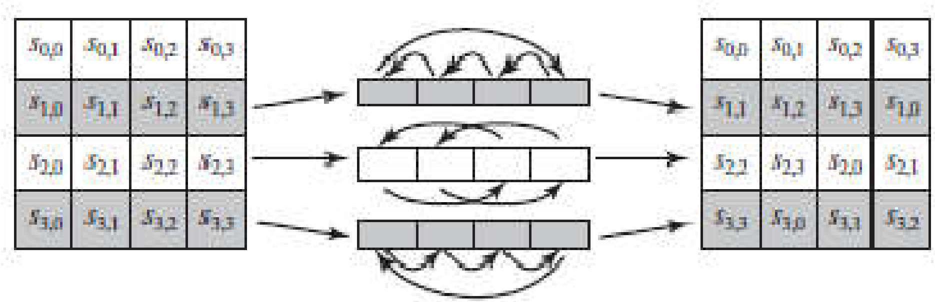
		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

→

87	F2	4
EC	6E	4
4A	C3	4
8C	D8	9

AES – ShiftRows Transformation

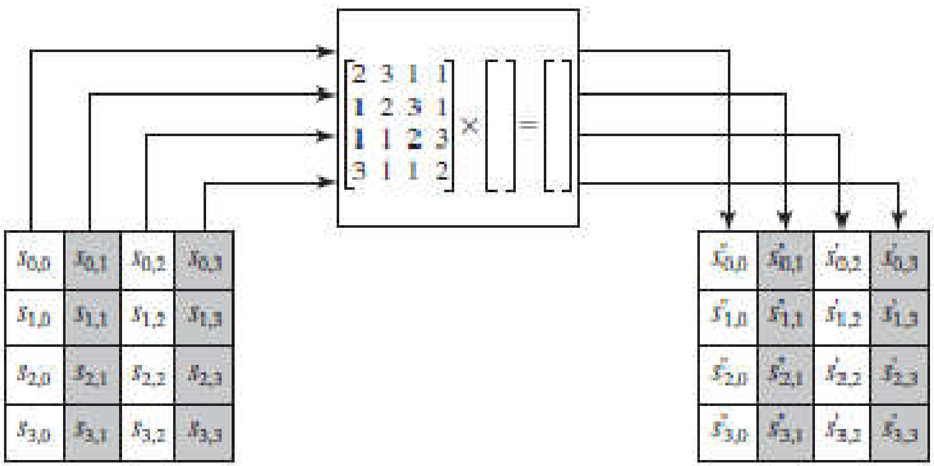


87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

MixColumns Transformation



$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s \end{bmatrix}$$

MixColumns Transformation

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

$\rightarrow$

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

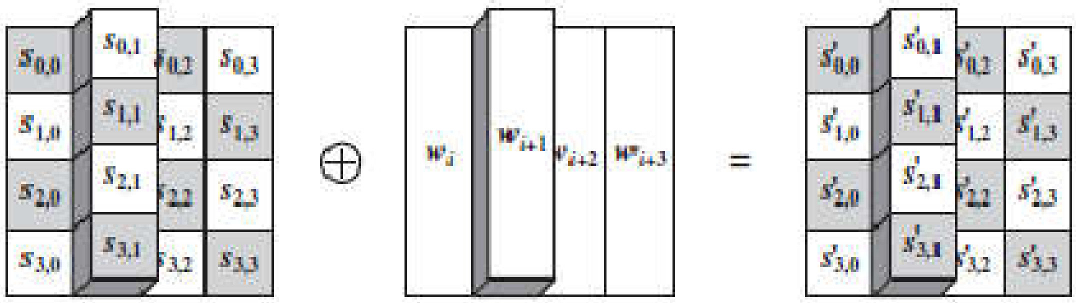
$$
\begin{aligned}
(\{02\}\cdot\{87\}) \oplus (\{03\}\cdot\{6E\}) \oplus \{46\} \oplus \{A6\} &= \{47\}\\
\{87\} \oplus (\{02\}\cdot\{6E\}) \oplus (\{03\}\cdot\{46\}) \oplus \{A6\} &= \{37\}\\
\{87\} \oplus \{6E\} \oplus (\{02\}\cdot\{46\}) \oplus (\{03\}\cdot\{A6\}) &= \{94\}\\
(\{03\}\cdot\{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\}\cdot\{A6\}) &= \{ED\}
\end{aligned}
$$

$$\{02\}\cdot\{87\} = (0000\ 1110) \oplus (0001\ 1011) = (0001\ 0101)$$

$$\{03\}\cdot\{6E\} = \{6E\} \oplus (\{02\}\cdot\{6E\})$$

$$
\begin{aligned}
\{02\}\cdot\{87\} &= 0001\ 0101\\
\{03\}\cdot\{6E\} &= 1011\ 0010\\
\{46\} &= 0100\ 0110\\
\{A6\} &= \underline{1010\ 0110}\\
& \ 0100\ 0111
\end{aligned}
$$

AES - AddRoundKey



$$\begin{array}{|c|c|c|c|}\hline 47 & 40 & A3 & 4C \\ \hline 37 & D4 & 70 & 9F \\ \hline 94 & E4 & 3A & 42 \\ \hline ED & A5 & A6 & BC \\ \hline \end{array} \oplus \begin{array}{|c|c|c|c|}\hline AC & 19 & 28 & 57 \\ \hline 77 & FA & D1 & 5C \\ \hline 66 & DC & 29 & 00 \\ \hline F3 & 21 & 41 & 6A \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline EB & 59 & 8B & 1B \\ \hline 40 & 2E & A1 & C3 \\ \hline F2 & 38 & 13 & 42 \\ \hline 1E & 84 & E7 & D6 \\ \hline \end{array}$$

Inputs for Single AES Round

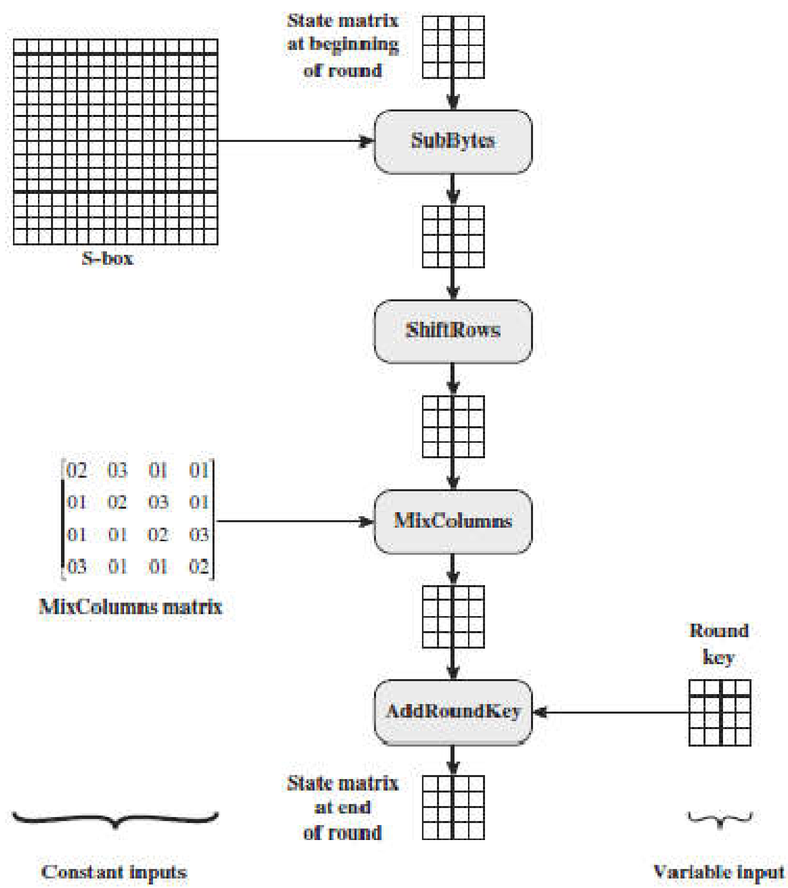
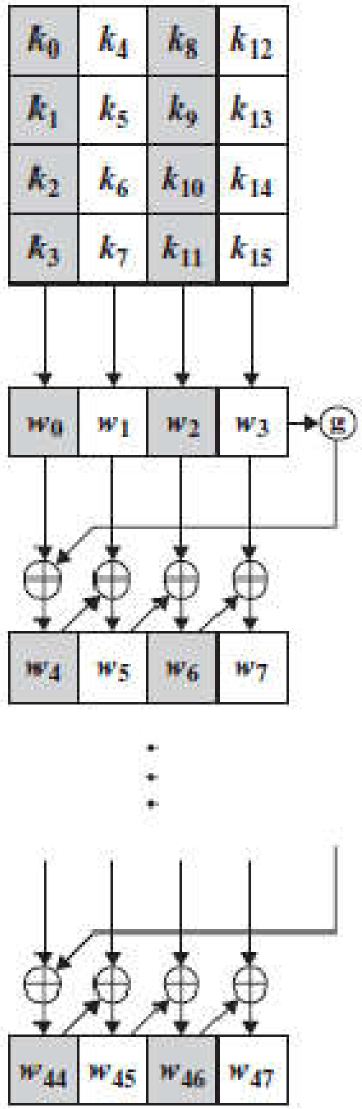
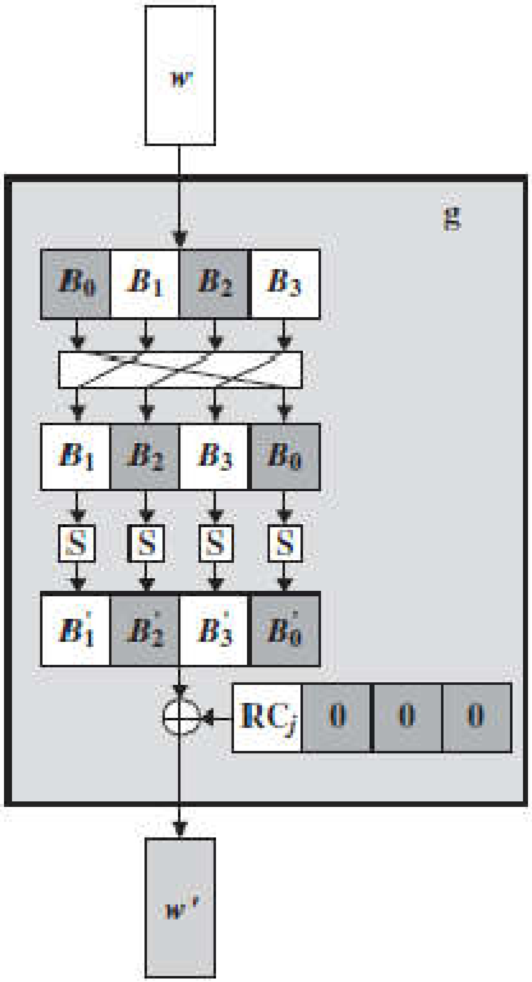


Figure 5.8 Inputs for Single AES Round



(a) Overall algorithm



(b) Function g

AES Example

Plaintext:	0123456789abcdeffedcba9876543210
Key:	0f1571c947d9e8590cb7add6af7f6798
Ciphertext:	ff0b844a0853bf7c6934ab4364148fb9

AES Example

Table 5.4 AES Example

Start of Round	After SubBytes	After ShiftRows	After MixColumns	Round K
01 89 fe 76 23 ab dc 54 45 cd ba 32 67 ef 98 10				0f 47 0c 15 d9 b7 71 e8 ad c9 59 d6
0e ce f2 d9 36 72 6b 2b 34 25 17 55 ae b6 4e 88	ab 8b 89 35 05 40 7f f1 18 3f f0 fc e4 4e 2f c4	ab 8b 89 35 40 7f f1 05 f0 fc 18 3f c4 e4 4e 2f	b9 94 57 75 e4 8e 16 51 47 20 9a 3f c5 d6 f5 3b	dc 9b 97 90 49 fe 37 df 72 b0 e9 3f
65 0f c0 4d 74 c7 e8 d0 70 ff e8 2a 75 3f ca 9c	4d 76 ba e3 92 c6 9b 70 51 16 9b e5 9d 75 74 de	4d 76 ba e3 c6 9b 70 92 9b e5 51 16 de 9d 75 74	8e 22 db 12 b2 f2 dc 92 df 80 f7 c1 2d c5 1e 52	d2 49 de c9 80 7e 6b b4 c6 b7 5e 61
5c 6b 05 f4 7b 72 a2 6d b4 34 31 12 9a 9b 7f 94	4a 7f 6b bf 21 40 3a 3c 8d 18 c7 c9 b8 14 d2 22	4a 7f 6b bf 40 3a 3c 21 c7 c9 8d 18 22 b8 14 d2	b1 c1 0b cc ba f3 8b 07 f9 1f 6a c3 1d 19 24 5c	c0 89 57 af 2f 51 df 6b ad 39 67 06
71 48 5c 7d 15 dc da a9 26 74 c7 bd 24 7e 22 9c	a3 52 4a ff 59 86 57 d3 f7 92 c6 7a 36 f3 93 de	a3 52 4a ff 86 57 d3 59 c6 7a f7 92 de 36 f3 93	d4 11 fe 0f 3b 44 06 73 cb ab 62 37 19 b7 07 ec	2c a5 f2 5c 73 22 65 0e a3 f1 96 90
f8 b4 0c 4c 67 37 24 ff ae a5 c1 ea e8 21 97 bc	41 8d fe 29 85 9a 36 16 e4 06 78 87 9b fd 88 65	41 8d fe 29 9a 36 16 85 78 87 e4 06 65 9b fd 88	2a 47 c4 48 83 e8 18 ba 84 18 27 23 eb 10 0a f3	58 fd 0f 9d ee cc 36 38 9b eb 7d ed
72 ba cb 04 1e 06 d4 fa b2 20 bc 65 00 6d e7 4e	40 f4 1f f2 72 6f 48 2d 37 b7 65 4d 63 3c 94 2f	40 f4 1f f2 6f 48 2d 72 65 4d 37 b7 2f 63 3c 94	7b 05 42 4a 1e d0 20 40 94 83 18 52 94 c4 43 fb	71 8c 83 c7 29 e5 4c 74 ef c2 bf 52
0a 89 c1 85 d9 f9 c5 e5 d8 f7 f7 fb 56 7b 11 14	67 a7 78 97 35 99 a6 d9 61 68 68 0f b1 21 82 fa	67 a7 78 97 99 a6 d9 35 68 0f 61 68 fa b1 21 82	ec 1a c0 80 0c 50 53 c7 3b d7 00 ef b7 22 72 e0	37 bb 38 14 3d d8 93 e7 08 48 f7 a5
db a1 f8 77 18 6d 8b ba a8 30 08 4e ff d5 d7 aa	b9 32 41 f5 ad 3c 3d f4 c2 04 30 2f 16 03 0e ac	b9 32 41 f5 3c 3d f4 ad 30 2f c2 04 ac 16 03 0e	b1 1a 44 17 3d 2f ec b6 0a 6b 2f 42 9f 68 f3 b1	48 f3 cb 26 1b c3 45 a2 aa 20 d7 72
f9 e9 8f 2b 1b 34 2f 08 4f c9 85 49 bf bf 81 89	99 1e 73 f1 af 18 15 30 84 dd 97 3b 08 08 0c a7	99 1e 73 f1 18 15 30 af 97 3b 84 dd a7 08 08 0c	31 30 3a c2 ac 71 8c c4 46 65 48 eb 6a 1c 31 62	fd 0e c5 0d 16 d5 42 e0 4a cb 1c 6e
cc 3e ff 3b a1 67 59 af 04 85 02 aa a1 00 5f 34	4b b2 16 e2 32 85 cb 79 f2 97 77 ac 32 63 cf 18	4b b2 16 e2 85 cb 79 32 77 ac f2 97 18 32 63 cf	4b 86 8a 36 b1 cb 27 5a fb f2 f2 af cc 5a 5b cf	b4 ba 7f 8e 98 4d f3 13 59 52 4e 20
ff 08 69 64 0b 53 34 14 84 bf ab 8f 4a 7c 43 b9				

Reference books

Cryptography and Network Security Principles and Practices

- William Stallings

Network Security PRIVATE Communication in a PUBLIC World

- Chalie Kaufman, Radia Perlman, Mike Speciner