29. 04. 19

* Smallest component of 2d is triangle.

A triangle of 3 points is always 2d

but a quadrilateral of 4 points is not always 3d

So, triangle is used to model.

- Generate 2d image from 3d model
- Simulate
- Animate
- Model
- Light real world

Two approaches to generate image :

- Rasterization
  - Efficient, not high quality — for real time app.
- Light transport / ray tracing
  - High quality, not efficient

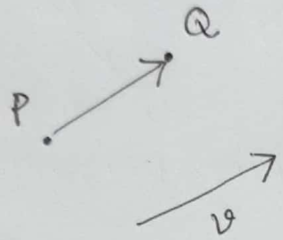
04.05.19

Vector tools for computer graphics

· Difference between point & vector.

⇒ point specifies a location

vector has magnitude & direction.



$$Q = P + v$$

· Normalizing : Process of calculating unit vector.

Unit vector makes calcalation easier.

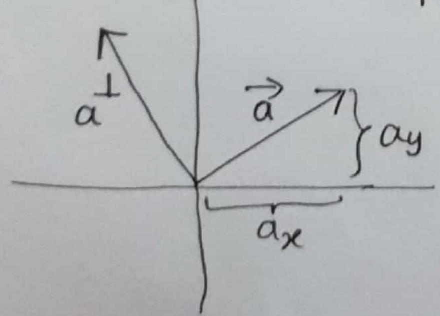
e.g. $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

for unit vector, the formula becomes $\cos\theta = \vec{a}\cdot\vec{b}$

· 2D perp vector

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

perp $\vec{a}^{\perp} = x\hat{i} + y\hat{j}$



$$a_x x + a_y y = 0 \quad \text{— ①}$$

$$a_x^2 + a_y^2 = x^2 + y^2 \quad \text{— ②}$$

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Easier way to rotating a 2D vector counterclockwise.

$a = a_x + i a_y$

$a^{\perp} = \cancel{x+i} \; i a_x + i^2 a_y$

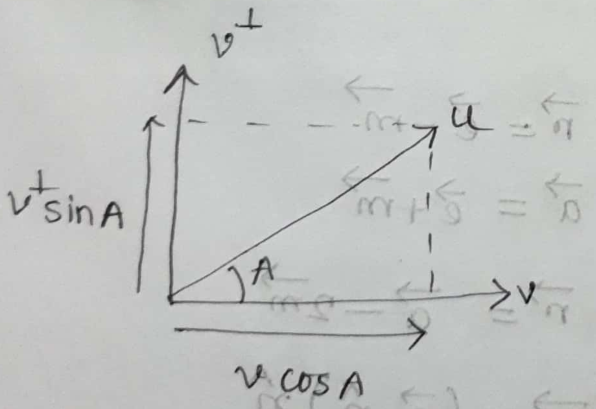
$= -a_y + i a_x$

for a 3D vector, to find perp,

we cross product the vector with an arbitrary 3D vector.

** Arbitrary vector should <u>NOT</u> be same or parallel to the vector.

· Rotation in 2D

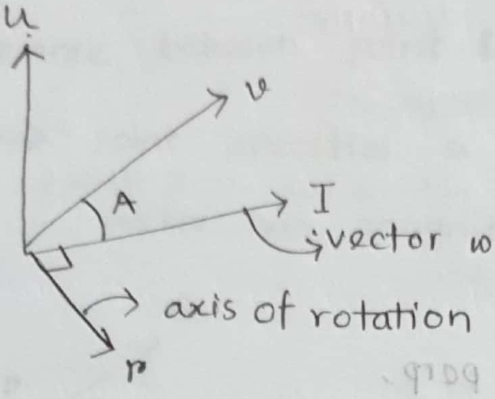


$\vec{u} = v\cos A\,\hat{i} + v^{\perp}\sin A\,\hat{j}$

· Rotation in 3D

Thumb rule

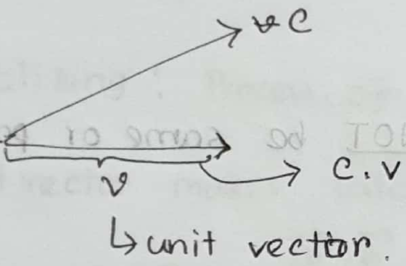
* when axis of rotation and vector are perpendicular to each other,



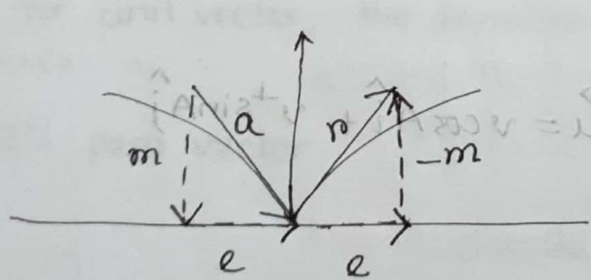
u is perpendicular to both I & r, $u = I \times r$.

$\vec{v} = I \cos A \cdot \hat{i} + u \sin A \hat{j}$.

- Orthogonal projection $\longrightarrow (c.v)\vec{v}$.



- Reflection .



$\vec{r} = \vec{e} - \vec{m}$

$\vec{a} = \vec{e} + \vec{m}$

$\vec{r} = \vec{a} - 2\vec{m}$

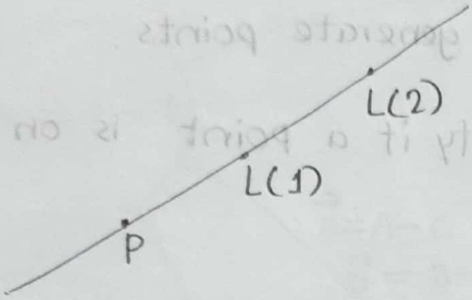
$\vec{m} = (\vec{a}.\hat{n})\hat{n}$

** unit vector.

- Representing lines.

  - Parametric representation

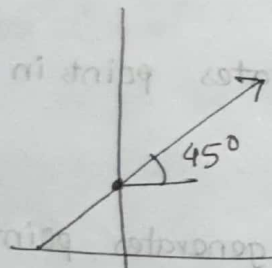
    → Easier to transform from 2d to 3d.



$L(t) = P + t\vec{v}$, $t \in R$.

$L(1) = P + \vec{v}$

$L(2) = P + \vec{2v}$



$y = x + 1$

To express $y = x+1$ into parametric form.

Let's find a point and a vector

$P = (0, 1)$

$\vec{v} = \hat{i} + \hat{j} = (1, 1)$.

$L(t) = (0, 1) + t(1, 1)$, $L(t)$ can vary

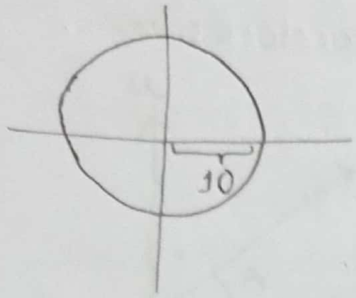
for $P = (1, 2)$, $\vec{v} = (2, 2)$

$L(t) = (1, 2) + t(2, 2)$.

* We can write the component of a dimension from the parametric form.

e.g., $L(t) = (9, 10) + t(5, 5)$

$\cancel{x(t)} = 9 + 5\cancel{(t)}t$, $y(t) = 10 + 5t$



$x(t) = 10 \cos t$

$y(t) = 10 \sin t$

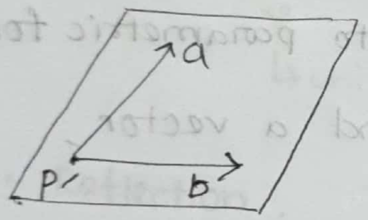
$x^2 + y^2 = 100$

* Parametric form is useful to generate points.

* Equation is useful to verify if a point is on a locus.

· Planes in 3d.

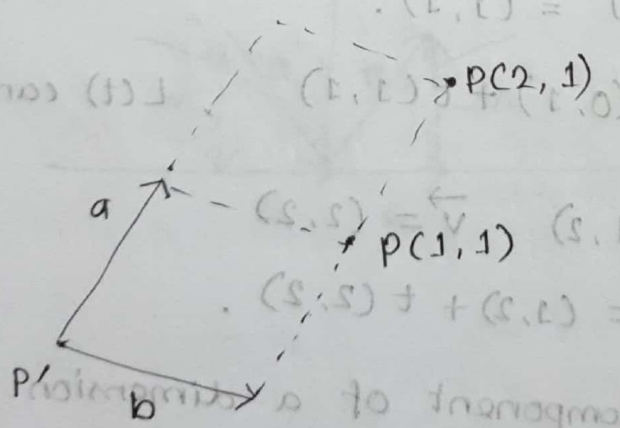
· parametric form



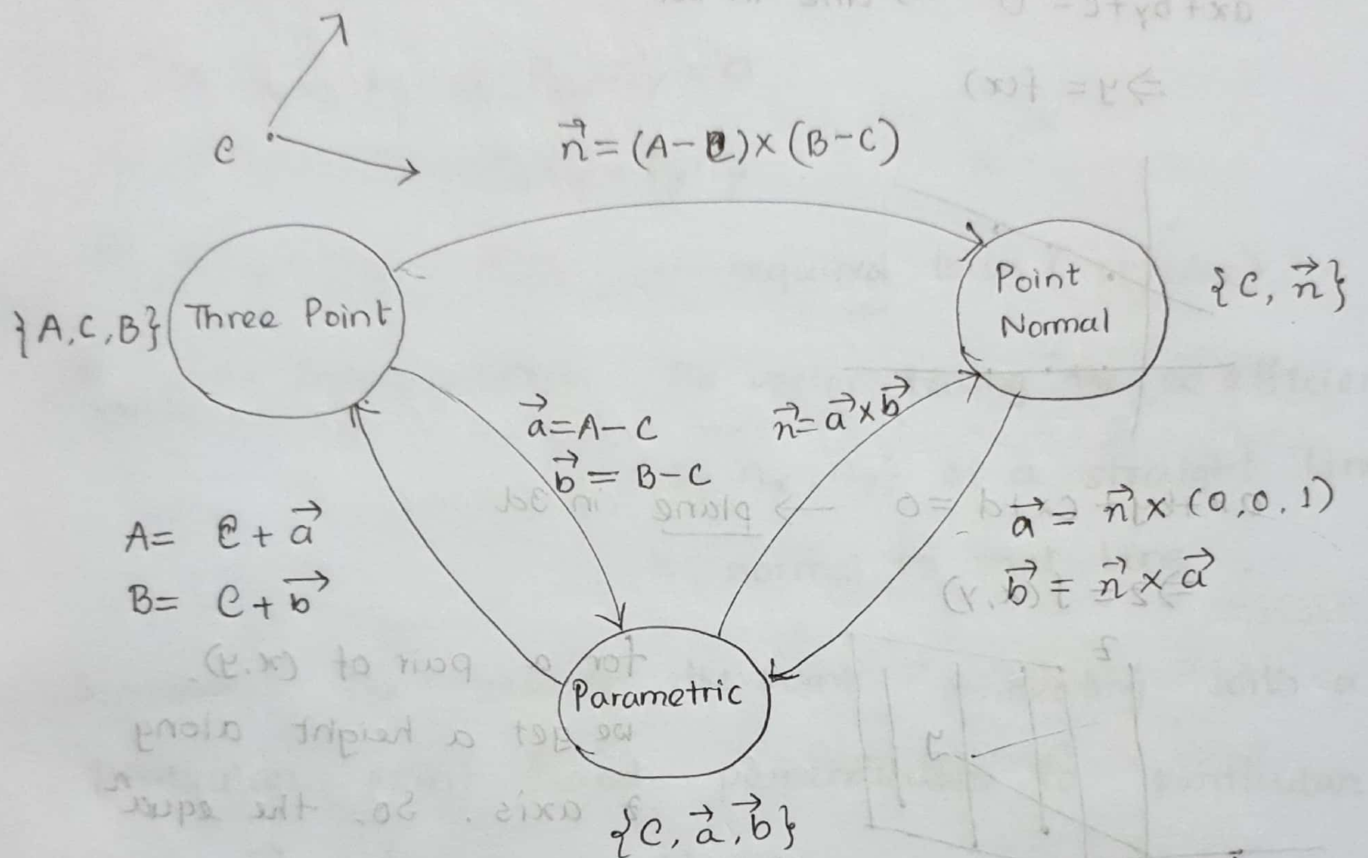
$L(t) = P + t\vec{v}$ ← generates points in 3d line

~~$P(t) =$~~

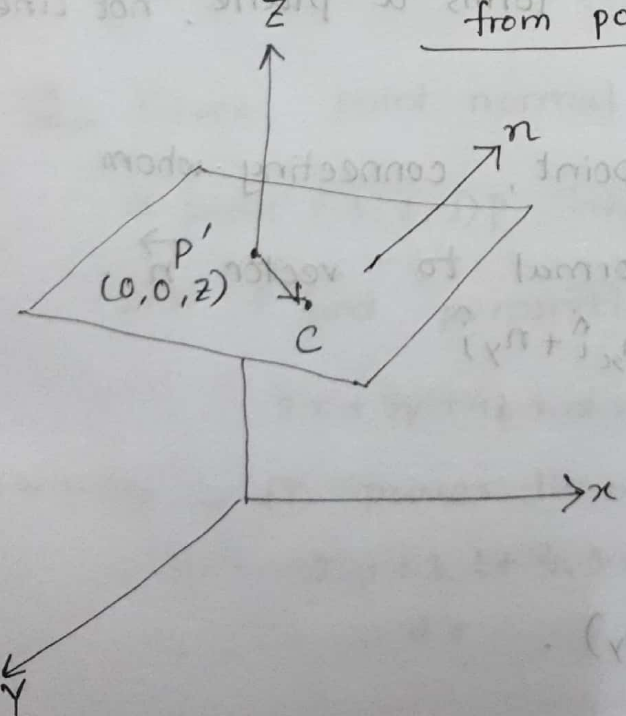
$\boxed{P(s,t) = P' + s\vec{a} + t\vec{b}}$ ← generates points in 3d plane



· Conversion of different representations.



from point normal to three point



$$\vec{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$$

(1) We first find the intersection point of the plane and one axis, P'

(2) Then,

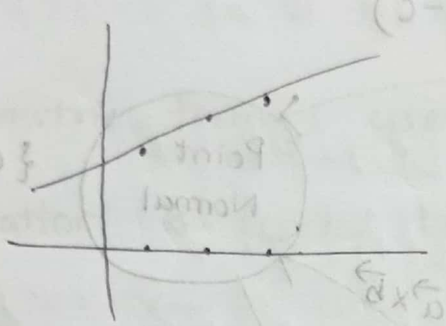
$$(C - P')\cdot n = 0$$
$$\Rightarrow Cn = P'n$$
$$\Rightarrow P'n = C\cdot n$$
$$\Rightarrow z\, n_z = Cn$$
$$\therefore z = \frac{C.n}{n_z}$$

06.05.19

· Equation of a plane.

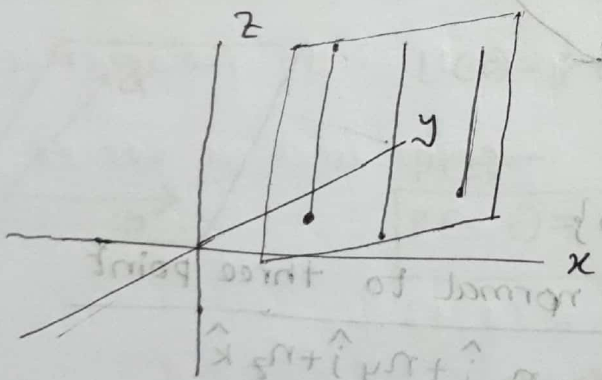
$ax+by+c=0 \rightarrow$ Line in 2d

$\Rightarrow y = f(x)$



$ax+by+cz+d=0 \rightarrow$ plane in 3d

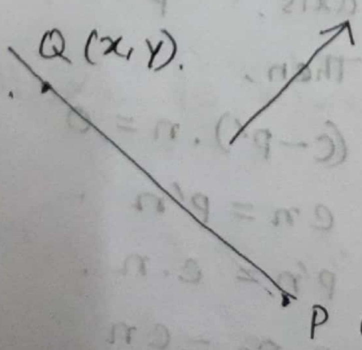
$\Rightarrow z = f(x,y)$



: for a pair of (x,y), we get a height along z axis. So, the equa^n forms a plane, not line.

· Find the locus of those point, connecting whom with P, the locus becomes normal to vector $\vec{n}$.

$\vec{n} = n_x\hat{i} + n_y\hat{j}$



Let Q (x, y) be a point on the required locus.

$\vec{QP} \cdot \vec{n} = 0$

$\Rightarrow (x - P_x) n_x + (y - P_y) n_y = 0$

$\Rightarrow x n_x + y n_y = P_x n_x + P_y n_y$.

$\Rightarrow x n_x + y n_y = P \cdot n$ ← required locus (st. line).

* ↳ Interpretation : The vector taking the co-efficients (n_x, n_y) of a straight line is normal to that line.

Similarly, the locus of the points connecting with a particular point P and perpendicular to particular vector $\vec{n}$. forms a plane.

Q Given, point normal form of a vector $\vec{n} = (2, 3, 4)$.

a point (1, 1, 1) P. Find the equa^n of plane connecting with P and perpendicular to $\vec{n}$.

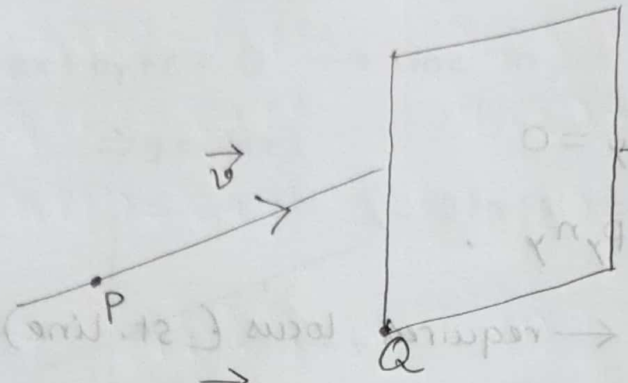
$2x + 3y + 4z + d = 0$, — ①

① passes through P.

$2.1 + 3.1 + 4.1 + d = 0$

$\therefore d = \ldots$

Line - Plane instruction



$L(t) = P + t\vec{v}$

$L(t') = Q$. $\therefore$ At Q, value of x, y, z satisfies the equa^n of plane.

Example

$L(t) = (1,2,3) + t(2,3,4)$

$P: \; x + 3y + 2z + 5 = 0$.

$x(t') = 1 + 2t'$

$y(t') = 2 + 3t'$

$z(t') = \cancel{4+} \; 3 + 4t'$

Let, $L(t)$ passes through P.

$1 + 2t' + 3(2+3t') + 2(3+4t') + 5 = 0$ — (1)

If solving (1) yields something like

$18 = 0$, it means, $L(t)$ & P are parallel.

If we get, $18 = 18$, it means, $L(t)$ is on P.

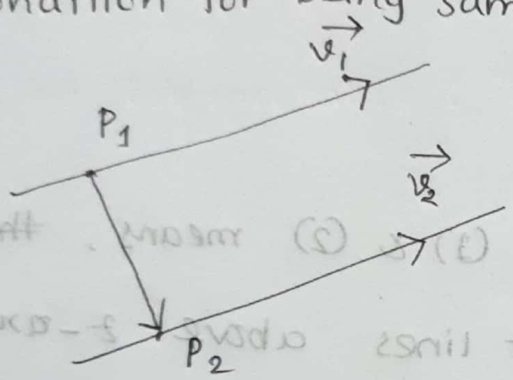
· Line- line intersection.

$L_1 : P_1 + tV_1$

$L_2 : P_2 + sV_2$

If $V_1 \times V_2 = 0$, L_1 & L_2 are parallel.

· Condition for being same line :

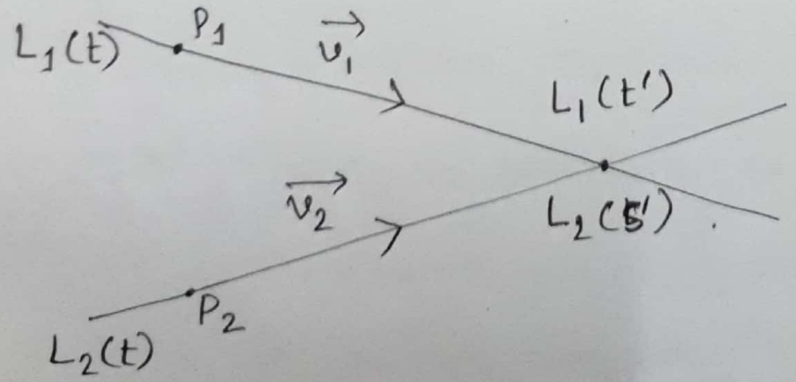


If the vector $\overrightarrow{P_1P_2}$ is at the same direction of $\vec{v_1}$, $\vec{v_1}$ the two lines are same.

· Condition for checking intersection

$L_1(t) = (1,2,3) + t(1,1,1)$

$L_2(s) = (1,1,1) + \cancel{t}\, s(1,2,3)$.



* If $L_1(t)$ & $L_2(t)$ intersect, we get a t' for which $L_1(t')$ & $L_2(s')$ would be the same point.

L_1	L_2
$x_1(t') = 1 + t'$	$x_2(s') = 1 + s'$
$y_1(t') = 2 + t'$	$y_2(s') = 1 + 2s'$
$z_1(t') = 3 + t'$	$z_2(s') = 1 + 3s'$

$X \longrightarrow \quad 1 + t' = 1 + s' \quad — (1)$

$Y \longrightarrow \quad 2 + t' = 1 + 2s' \quad — (2)$

$Z \longrightarrow \quad 3 + t' = 1 + 3s' \quad — (3)$

3d interpretation ; Solving (1) & (2) means, ~~the~~ ~~shadow~~ if we see ~~a~~2 straight lines above z-axis, we can see their shadows intersecting .

Solving (3) means, if those st. lines are upon each other or not ?

11.05.19

- Plane-Plane Intersection.

* How to find a point on the line of intersection of two planes?

$x+y+z=6$ $(1,1,1)$ ← Normal direcⁿ.

$x+2y+3z=12$ $(1,2,3)$

Let's ~~take~~ another plane not parallel to previous two planes.

$\cancel{x+3y+5z=20}$ $(1,0,0)$

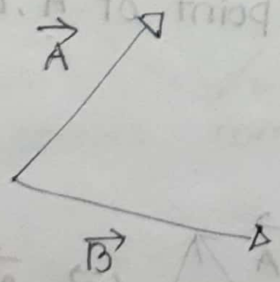
$x=0$

Now, solving these 3 equⁿs we can find an intersecting ~~fi~~ point of 3 planes.

* How to get direction of the ~~to~~ intersecting line?

$u = \vec{n_1} \times \vec{n_2}$

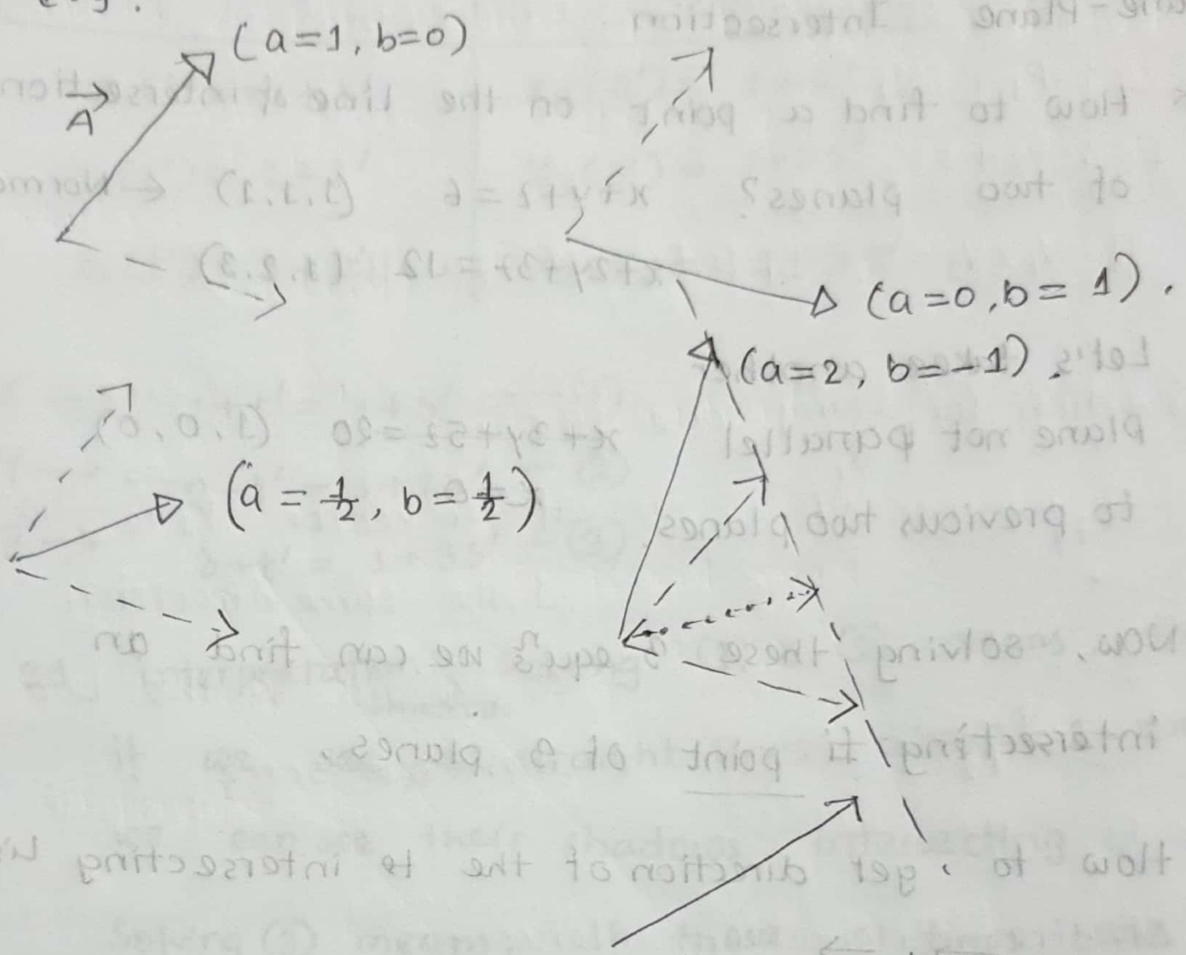
- Linear combination of Vectors.



$a\vec{A} + b\vec{B}$

If sum of weights, $a+b=1$ → affine combination.

e.g.



** See, the affine combinations always stay upon this line joining the end point of $\vec{A}, \vec{B}$.

<u>Q</u> Why it occurs?

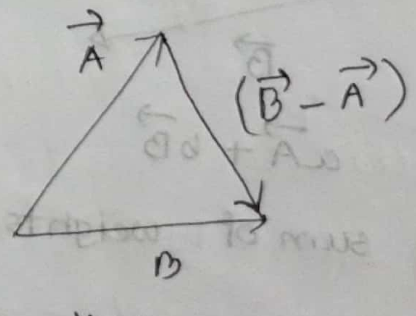
$\Rightarrow$ Since, $a+b=1$,

$a = 1-b$.

$a\vec{A} + b\vec{B}$

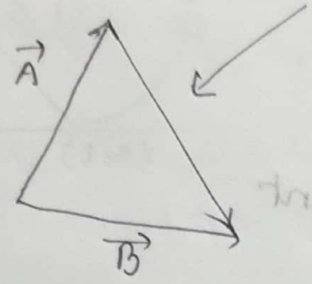
$= (1-b)\,\vec{A} + b\,\vec{B}$

$= \vec{A} + (\vec{B} - \vec{A})\,b$.



$\therefore$ thus . . .

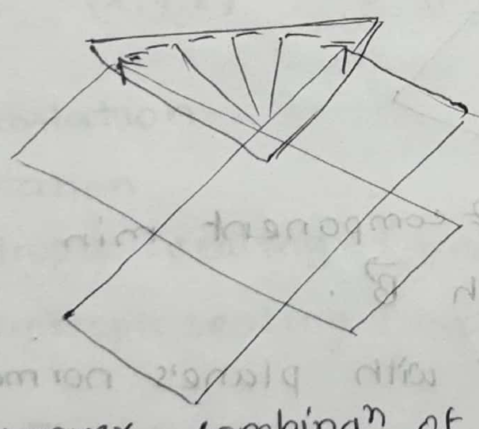
• If sum of weights, $a+b=1$, $a,b>0$, convex combination. In this case it lies on the line,



~~Convex com~~

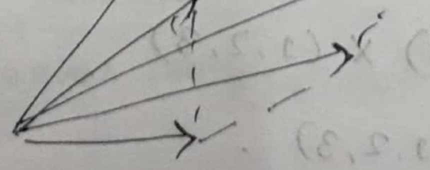
Affine combinaⁿ of two 3d vectors is a plane.

Convex " " " " " " " triangle.



convex

The convex combinaⁿ of 10 3d vectors, is a polygon.



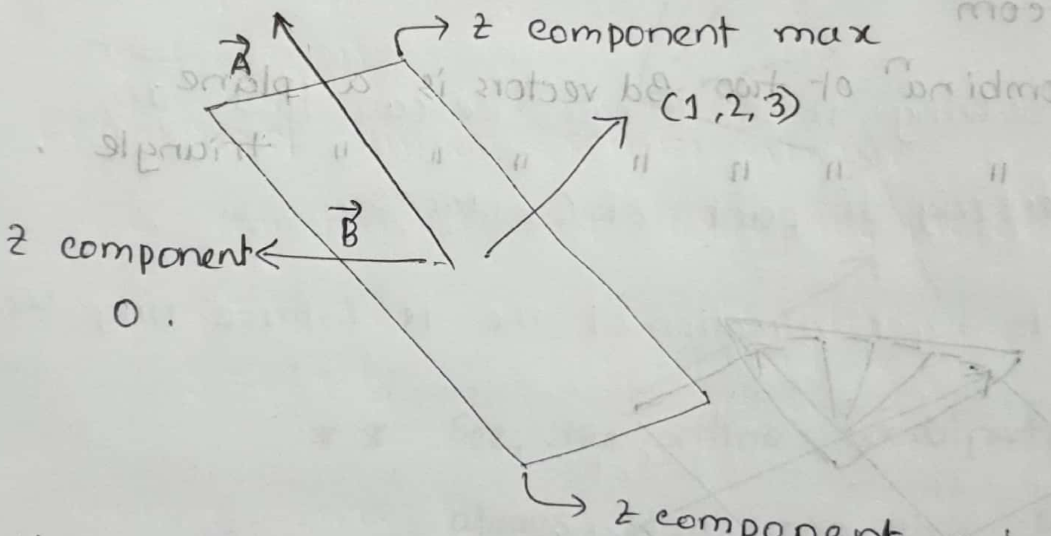
of 2d vectors is $\boxed{\text{convex hull}}$

Problem : What is highest / steepest point along z-axis of the plane $x+2y+3z=1$.

Solution :

$$z = f(x, y)$$

$$\left(\frac{\partial z}{\partial x}, \frac{\partial y}{\partial y}\right)$$ is ~~di~~ gradient



Now, $\vec{A}$ is at 90° with $\vec{B}$.

$\vec{A}$ is also at 90° with plane's normal vector (1, 2, 3)

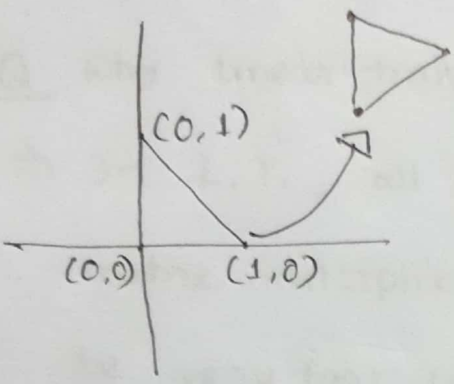
$\vec{B}$ is at 90° with both (0, 0, 1) & (1, 2, 3).

$$\therefore \vec{B} = (0, 0, 1) \times (1, 2, 3)$$

$$\vec{A} = \vec{B} \times (1, 2, 3).$$

12.05.19

Modelling Transformation



gltrirotate (2, 1)
glrotate (45°)
drawtriangle ()

Transformation: Mapping points of one space into points of another space.

$$R^3 \xrightarrow{T} R^3$$

$$(x, y, z) \xrightarrow{T} (x', y', z')$$

→ Translation
→ Rotation
→ Isotropic scaling (uniform scaling of all axes).
→ Heterotropic scaling (non-uniform " " any ")

- Rigid Transforms.
  - Preserves distance, angle
- Similarity Transforms
  - Preserves angle.

* Linear transformation
- → Scaling
- → Reflection
- → Shearing.

$$R^2 \xrightarrow{T} R^2$$

$$(x,y) \xrightarrow{T} (x',y')$$

$$x' = f(x,y)$$
$$y' = g(x,y).$$

Definition of linear transformation:

the transformed point holds the property

$$\boxed{\begin{aligned} x' &= ax+by \\ y' &= cx+dy \end{aligned}}$$

(no constant term)

- Another property

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

** Linear transformation excludes only translation.

translation (2,3), $\left.\begin{aligned} x' &= x+2 \\ y' &= y+3 \end{aligned}\right\}$ ← Not in the form of L.T. point.

This can't be modelled as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$. like L.T.

* **Q** Why linear transformation is important?

⇒ In L.T., all points can be transformed by matrix multiplication. Matrix multiplication can be very fast using special hardwares.

- Translation (2,3).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Here, we transformed in homogeneous co-ordinate.

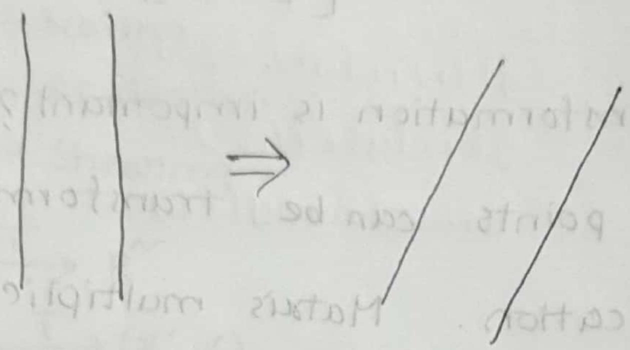
Co-ordinate system

Cartesian	Homogeneous
x, y	x, y, w
x, y, z	x, y, z, w

2d transformation is expressed by [3×3] matrix in homogeneous co-ordinate.

(Similarly 3d by [4×4] matrix)

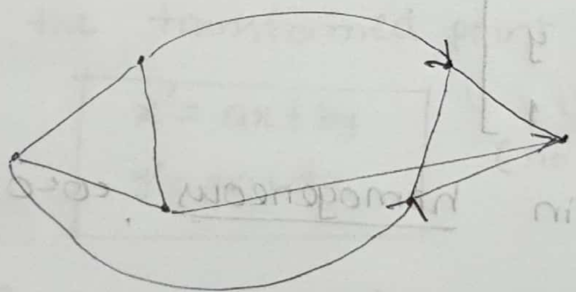
• Affine transformation



preserves parallelity.

• Projective transformation.

preserves line



*** Since, line is preserved, only knowing three transformed points of three edges of triangle is enough.

Otherwise we had to define every point on the triangle.

• Translation – 2D

$x' = 2x$

$y' = y$

$(x, y) \xrightarrow[(2,1)]{\text{2D scale}} (x', y')$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

23.06.19

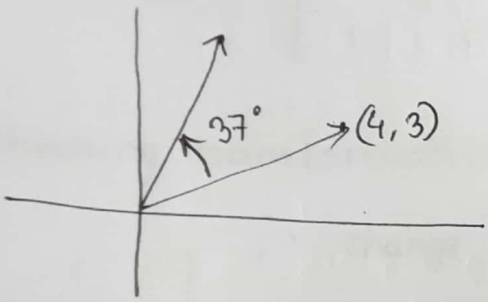
- Transformation

2d Scale (2, 3)

If we want to transform (2,2)

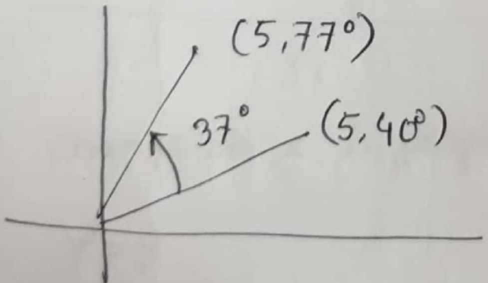
$$\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

- Rotation – 2D



37° rotation.

* The problem is easier if we solve it using polar co-ordinate instead of cartesian co-ordinate.



$(x, y) \rightarrow (r, \theta)$

$x = r\cos \cancel{\theta}\ \phi$

$y = r\sin \cancel{\theta}\ \phi$

$(x, y) \xrightarrow{R_\theta} (x', y')$

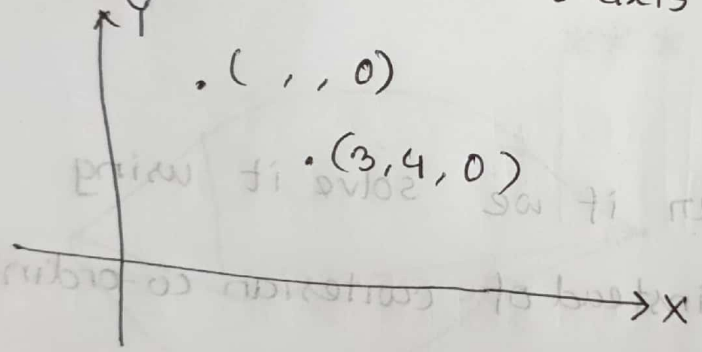
$(r, \phi) \xrightarrow{R_\theta} (r, \theta + \phi)$

* Now if we want to rotate a point (4,3) by 37°

$$\begin{bmatrix} \cos 37° & -\sin 37° \\ \sin 37° & \cos 37° \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

↑ rotation matrix for 2D.

- Rotation – 3D.
  - For rotation about z-axis



$z' = z$

$x' = x\cos\theta - y\sin\theta$

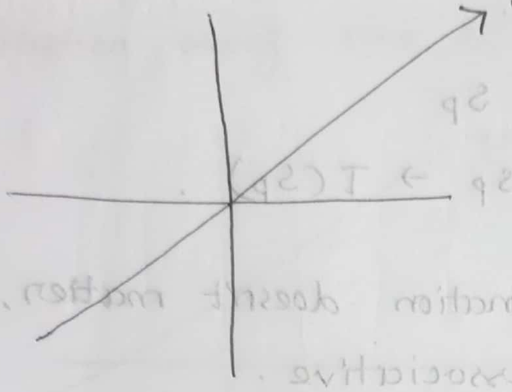
$y' = x\sin\theta + y\cos\theta$

- Mirror reflection
  - Reflection about x-axis.

$x' = x,$

$y' = -y$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



If we reflect a point along the line $y=x$,

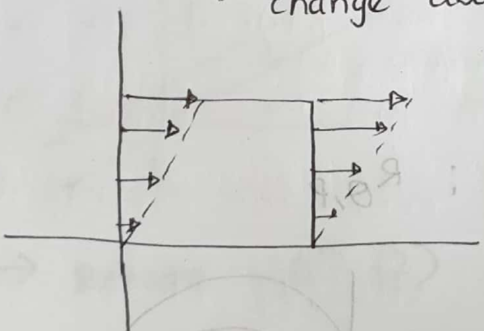
$x' = y$

$y' = x$.

$$matrix = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Shearing transformation

  - change about x-axis



* Change of x is proportional to y.

$y' = y$

$(x'-x) \propto y$

$x' - x = ay$

$x' = x + ay$

* If a is increased, shearing is increased

" " " decreased, " " decreased.

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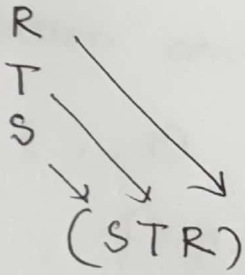
· Combination of transforms.

Scale on point p $\rightarrow$ Sp

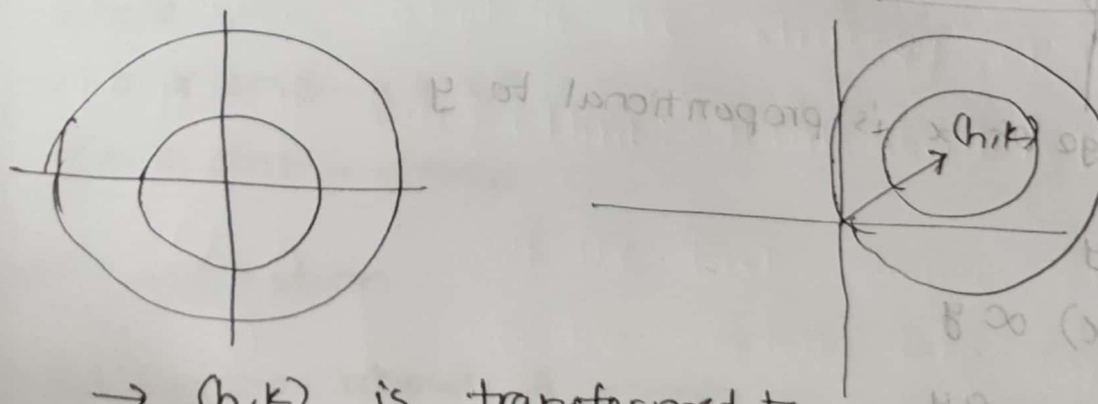
Then apply translate on Sp $\rightarrow$ T(Sp).

* If the order of transformation doesn't matter, the combination is <u>associative</u>.

e.g. $P' = (TS)_P = T(Sp)$



· Rotation of θ about P(h,k) : $R_{\theta,P}$

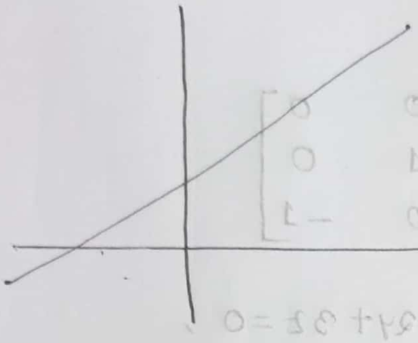


$\rightarrow$ (h,k) is transformed to center

$\rightarrow$ Then rotated ~~along~~ about θ

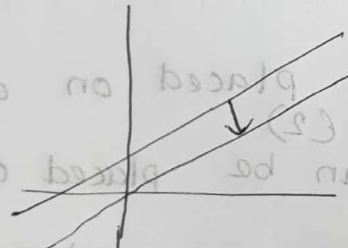
$\rightarrow$ Then, again center is placed on origin.

• Reflection about line L, M_L

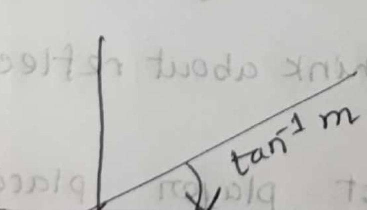


→ we align the line $y = mx + b$ along x-axis.

→ Translate to origin (1)



→ Rotate $-\theta°$ (2)



→ Mirror reflect about x-axis (3)

→ vice versa of (2) (4)

→ " " of (1) (5)

· Reflection about plane,

about ~~xP~~ XY plane,

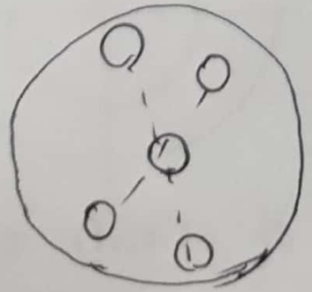
$$x' = x \qquad y' = y \qquad z' = -z$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

about any plane $x + 2y + 3z = 0$,

→ the plane is first aligned to XY.

A game; Coins have to be placed on a round table by multiple players (2). No coin can be placed on the boundary and no coin can overlap with another. What is the strategy of the first player to win? (Hint: think about reflection).



If the first player places coin on the center, then any place of the table has its mirror image along the center. So, after the 2nd player places another coin, the place against its mirror image alway remains unoccupied, where the first player can place coin.

24. 06. 19

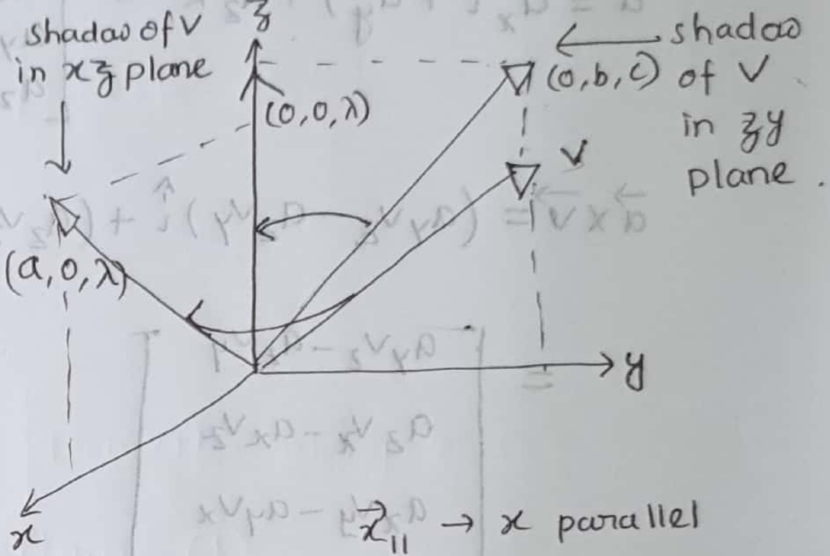
- Aligning vector V with K

$$V = a\hat{i} + b\hat{j} + c\hat{k}$$

θ

$$\lambda = \sqrt{b^2 + c^2}$$

$$\tan^{-1}\frac{b}{c}$$

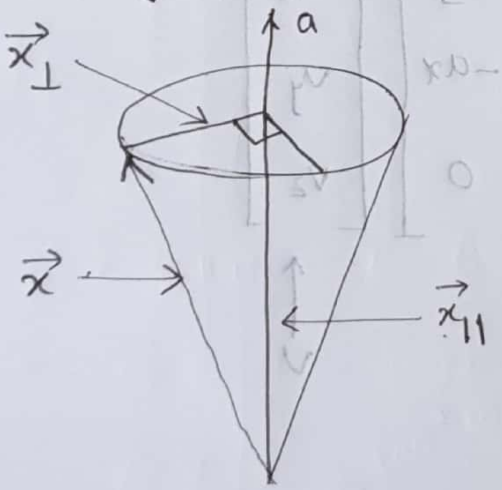


$\vec{x}_{\parallel} \rightarrow x$ parallel

$\vec{x}_{\perp} \rightarrow x$ perpendicular

$R(\vec{x}) \rightarrow$ Rotation of x

- Geometry of a Rotation.



$$\vec{x}_{\parallel} = (\vec{a}\cdot\vec{x})\,\vec{a}$$

$$\vec{x}_{\perp} = \vec{x} - \vec{x}_{\parallel}$$

$$= \vec{x} - (\vec{a}\cdot\vec{x})\,\vec{a}$$

$$R(\vec{x}) = R(\vec{x}_{\parallel}) + R(\vec{x}_{\perp})$$

($R(\vec{x}_{\parallel})$ is as same as $\vec{x}_{\parallel}$)

- Expressing rotation matrices. $T(\vec{v}) = \vec{a} \times \vec{v}$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\vec{a} \times \vec{v} = (a_y v_z - a_z v_y)\hat{i} + (a_z v_x - a_x v_z)\hat{j} + (a_x v_y - a_y v_x)\hat{k}$$

$$= \begin{bmatrix} a_y v_z - a_z v_y \\ a_z v_x - a_x v_z \\ a_x v_y - a_y v_x \end{bmatrix}$$

$$\vec{a} \times \vec{v} = \underbrace{\begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}}_{\vec{a} \times \vec{x}} \underbrace{\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}}_{V}$$

Rodrigues formula

29.06.19

· Finding rotations from a rotation matrix

Axis of rotation $\hat{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$

Angle of rotation θ.

$\cos\theta = c$

$\sin\theta = s$

Q A rotation matrix is given. Find its axis of rotation and angle of rotation.

⇒ At first, find the trace of rotation matrix R

$\text{Trace}(R) = (k_x^2 + k_y^2 + k_z^2)(1-c) + 3c$

$= 1 - c + 3c$

$= 1 + 2c$

$$R = \begin{bmatrix} 0.1 & 0.5 & 0.8 \\ -0.7 & 0.2 & -0.4 \\ 0.9 & -0.7 & 0.3 \end{bmatrix}$$

$1 + 2c = 0.1 + 0.2 + 0.3 = 0.6$.

$\Rightarrow c = -0.2$.

$2k_z$ . . .

Q Trace of a rotation matrix, $1+2c=2$.

$c=\frac{1}{2}$, $\cos\theta=\frac{1}{2}$,

$\therefore \underbrace{\theta=60^\circ,-60^\circ}$.

which angle of rotation will we accept?

$\Rightarrow$

$$k_2 = \frac{R_{2.3}-R_{3.2}}{2\sin\theta} ?$$

It doesn't matter whether we take 60° or -60°. If we encounter -60°, then k_2 becomes negative. Rotating is a vector in negative direction of an axis's negative direction is as same as rotating the vector in positive direction of that axis.

$$R = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$\vec{c_1} = a\hat{i} + d\hat{j} + g\hat{k}$ $\quad\quad$ $\vec{r_1} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{c_2} = b\hat{i} + e\hat{j} + h\hat{k}$ $\quad\quad$ $\vec{r_2} = d\hat{i} + e\hat{j} + f\hat{k}$

$\vec{c_3} = c\hat{i} + f\hat{j} + i\hat{k}$ $\quad\quad$ $\vec{r_3} = g\hat{i} + h\hat{j} + i\hat{k}$.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = R \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ R doesn't change the position of origin.

$$\begin{bmatrix} a \\ d \\ g \end{bmatrix} = R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

R is such a vector that transforms ~~alone~~ the unit vector along x to $a\hat{i} + d\hat{j} + g\hat{k}$ $(\vec{c_1})$

This is the physical <u>significance of column vectors</u> $\vec{c_1}, \vec{c_2}, \vec{c_3}$.

$$R^T R = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= \begin{bmatrix} \vec{c_1}\cdot\vec{c_1} & \vec{c_1}\cdot\vec{c_2} & \vec{c_1}\cdot\vec{c_3} \\ \vec{c_2}\cdot\vec{c_1} & \vec{c_2}\cdot\vec{c_2} & \vec{c_2}\cdot\vec{c_3} \\ \vec{c_3}\cdot\vec{c_1} & \vec{c_3}\cdot\vec{c_2} & \vec{c_3}\cdot\vec{c_3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ ← Identity matrix.

(Since, $\vec{c_1}, \vec{c_2}, \vec{c_3}$ are unit vectors perpendicular to each other)

$R^T . R = I$

$\Rightarrow R . R^T = I$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\vec{r_1} \cdot \vec{r_1} = 1$, $\vec{r_1} \cdot \vec{r_2} = 0$,

$\Rightarrow |\vec{r_1}|^2 = 1$

$\Rightarrow |\vec{r_1}| = 1$

$\therefore \vec{r_1}, \vec{r_2}, \vec{r_3}$ are ~~row vector~~ unit vectors and perpendicular to each other.

$$R.\vec{r_1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$= \begin{bmatrix} \vec{r_1} \cdot \vec{r_1} \\ \vec{r_1} \cdot \vec{r_2} \\ \vec{r_1} \cdot \vec{r_3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\vec{r_1}$ is such a vector that aligns to unit vector along x after rotation by the matrix R.

* * *

~~Rotating column vectors ($\vec{c_1}, \vec{c_2}, \vec{c_3}$)~~

* * *

Rotating unit vectors along x or y or z axis with respect to rotation matrix R, yields to column vector $\vec{c_1}$ or $\vec{c_2}$ or $\vec{c_3}$ respectively.

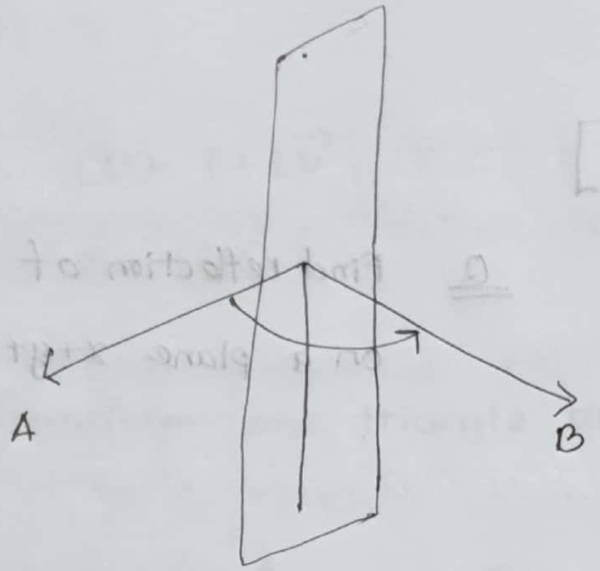
* * *

Rotating row vectors $\vec{r_1}$ or $\vec{r_2}$ or $\vec{r_3}$ with respect to rotation matrix R, yields to unit vector along x or y or z axis respectively.

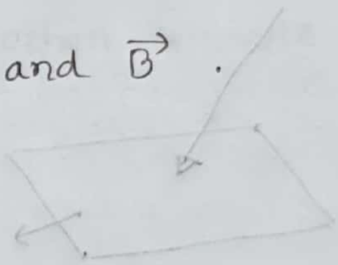
Q Rotation matrix is given

$$\begin{bmatrix} 0.8 & 0.6 & 0 \\ -0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Find rotation angle and axis?



$\vec{A}$ is rotated to $\vec{B}$.
The probable axis of rotation is on the plane that is in ~~betw~~ middle of $\vec{A}$ and $\vec{B}$.

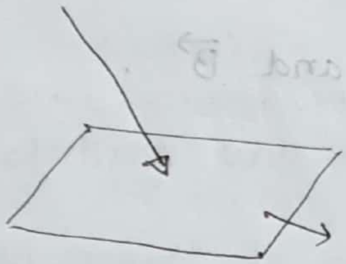


$$\vec{r_3} = \vec{r_1} \times \vec{r_2}$$

CT solve

1 $\hat{i}+\hat{j} = (1,1,0)$

Q Find reflection of $\hat{i}+\hat{j}$ on a plane $x+y+z+10=0$.



$x+y+z+10=0$.

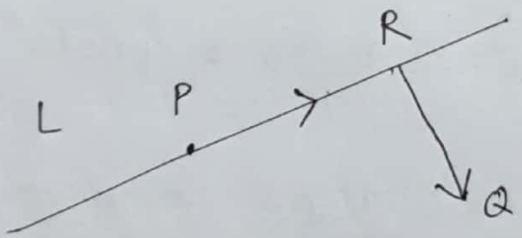
$(1,1,1)$

$\hat{n} = \frac{1}{\sqrt{3}}(1,1,1)$

$\vec{a} = (1,1,0)$.

reflection, $\vec{r} = \vec{a} - (\vec{a}\cdot\hat{n})\hat{n}$.

2



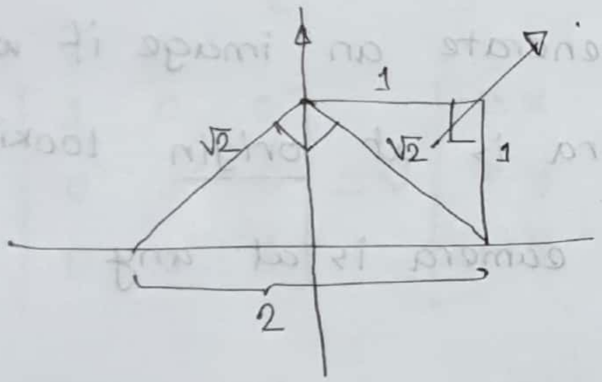
We have to find a point R such that RQ is perpendicular to L.

$\vec{L}\cdot\vec{RQ} = 0$.

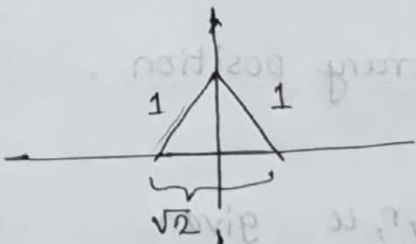
$L(t) = P + t\vec{v}$

$R = L(t')$.

3 Transform one triangle to another triangle.

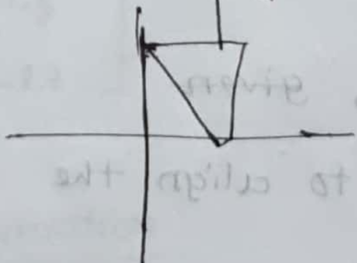


Scale $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\Rightarrow$



Rotation $(-45°)$ $\Rightarrow$

Translate $\left(\frac{1}{2}, \frac{1}{2}\right)$ $\Rightarrow$



30.06.19

View Transformation

↳ Next phase of modelling transformation.

* View transformation makes calculation easier.

~~e.g., we need to calculate distance between~~ two

e.g., it's easier to generate an image if we know that the camera is at origin looking at z axis than camera is at any arbitrary position.

- pos, l, r, u given.

$l \rightarrow -z$

$r \rightarrow x$

$u \rightarrow y$, ~~given~~

we have to align the camera this way.

⇒ Apply translation to move the camera at origin

⇒ Apply rotation to align l, r, u.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = V \begin{bmatrix} r.x \\ r.y \\ r.z \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = V \begin{bmatrix} u.x \\ u.y \\ u.z \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = V \begin{bmatrix} -l.x \\ -l.y \\ -l.z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = V \begin{bmatrix} r.x & u.x & -l.x \\ r.y & u.y & -l.y \\ r.z & u.z & -l.z \end{bmatrix}$$

$$\Rightarrow I = VU$$

$$\Rightarrow V = U^{-1} = U^T$$

$$\therefore V = \begin{bmatrix} r.x & r.y & r.z \\ u.x & u.y & u.z \\ -l.x & -l.y & -l.z \end{bmatrix}$$

Open GL : View Transformation

```
gluLookAt ( eye.x, eye.y. eye.z,
            look.x, look.y, look.z,
            up.x, up.y, up.z )
```

$l =$ look $-$ eye

l. normalize()

$r = l \times u$

r. normalize()

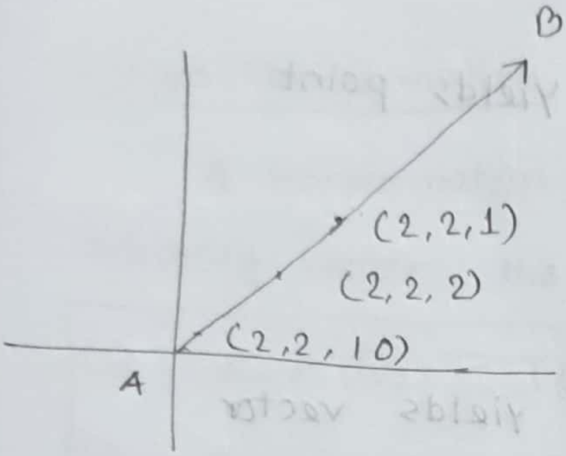
$u = r \times l$.

- Projection transformation

We need to know some theories before.

- Homogeneous Coordinates.

Cartesian	$\rightarrow$	Homogeneous
2D point, (x, y)	$\rightarrow$	(x, y, w)
3D point, (x, y, z)	$\rightarrow$	(x, y, z, w)
(x, y)	$\rightarrow$	$(x, y, 1)$
$\left(\frac{x}{w}, \frac{y}{w}\right)$	$\leftarrow$	(x, y, w)



(2,2,ω)

(2,2,0) means, the point is in the direction of AB at infinity.

** Interpretation of (x,y,ω) in homogeneous coordinate :

If ω=0, (x,y,ω) is a vector

If ω≠0, (x,y,ω) represents a point.

** That's why we calle it homogeneous system as it treats point and vector homogeneously.

* If after any operation, (x,y,ω) has ω≠1 and ω≠0, we have to transform it as

$\left(\frac{x}{\omega}, \frac{y}{\omega}, 1\right)$ instantaneously **.

- Adding vector with point yields point

$$\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

- Adding vector with vector yields vector

$$\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

- Adding point with point yields * <u>middle point of them</u>

***

$$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 2 \\ 1 \end{bmatrix}$$

*** In cartesian coordinate system, there is no meaning of adding two points.

In homogeneous co-ordinate system, adding two points means finding the middle point of them.

01.07.19

Linear Transformation

A transformation will be linear if for any two arbitrary vector, the following two conditions hold ;

(I) $T(v_1) + T(v_2) = T(v_1 + v_2)$

(II) $\alpha T(v_1) = T(\alpha v_1)$

$T : R^3 \to R^3$

$v_1, v_2 \in R^3$.

Example : $T(x) = x^k$,

Let's take ~~two~~ x_1, x_2 .

$T(x_1) + T(x_2) = x_1^k + x_2^k$

$T(x_1 + x_2) = (x_1 + x_2)^k$

$\therefore T(x) = x^k$ will be linear transformation only when $k = 1$.

* If a transformation is linear, we can express it as a product of a matrix .

Q 90° rotation in 2D is linear or not ?

⇒ We'll answer the question by drawing some graphs .

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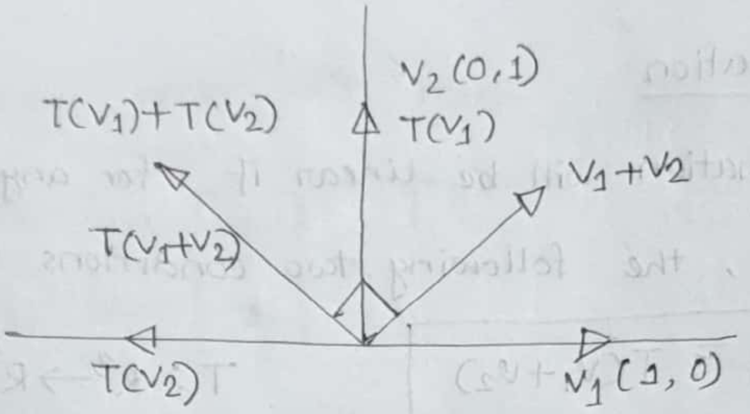
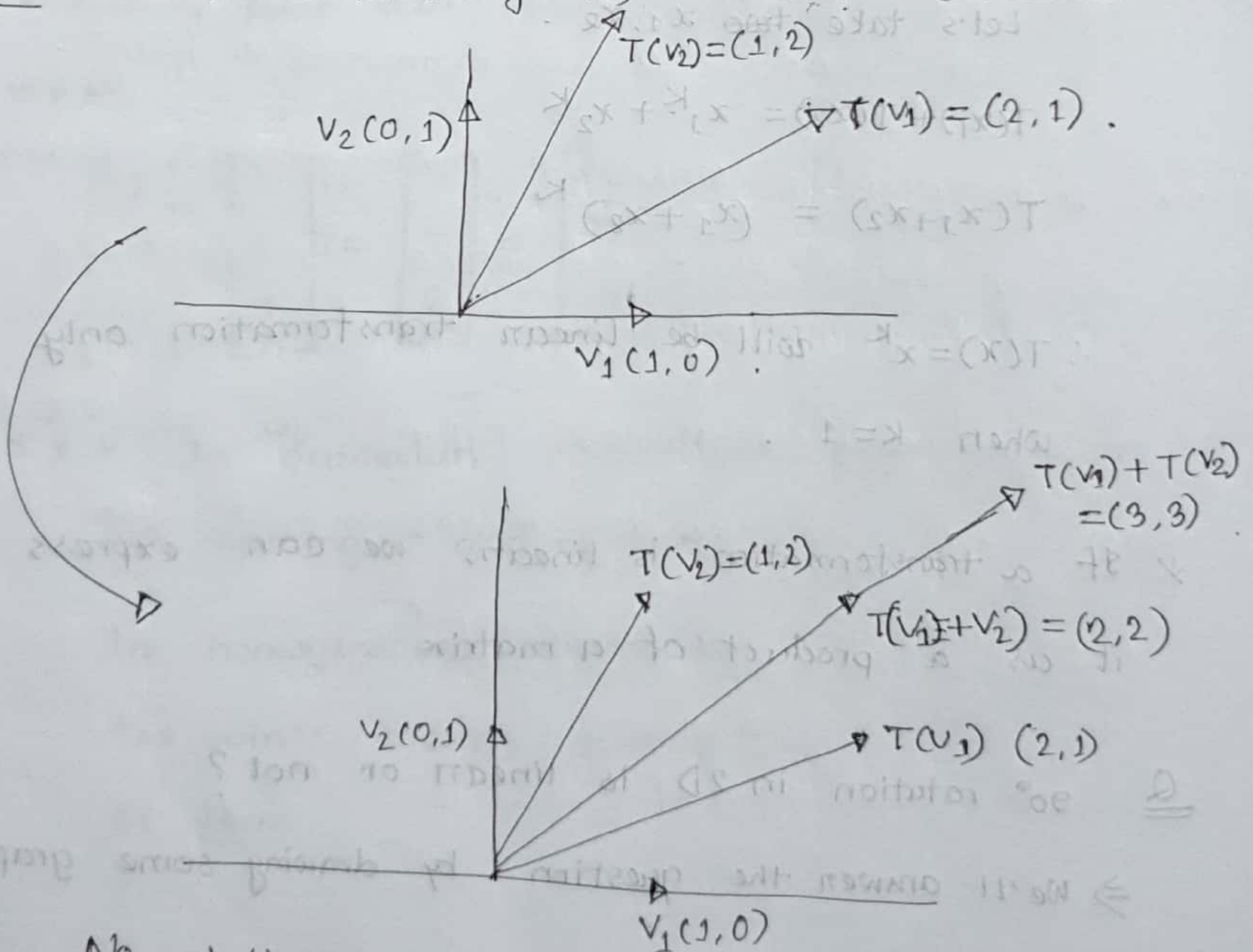


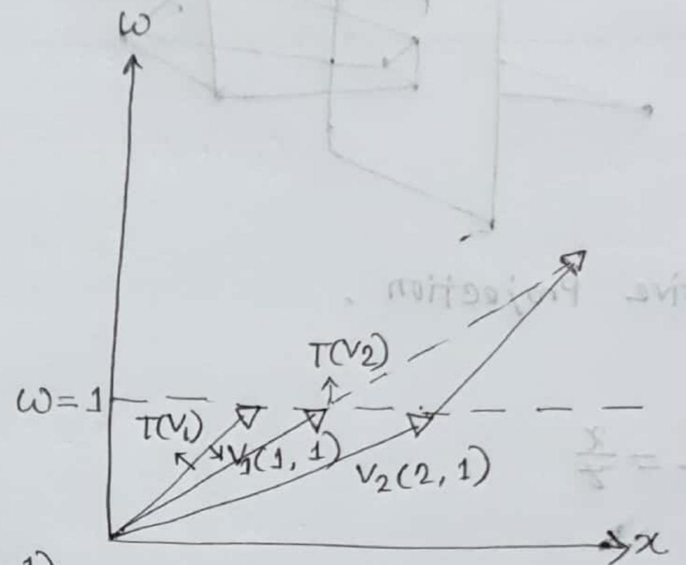
$\therefore 90°$ rotation in 2D is linear.

<u>Q</u> Is 2D translation by $(1,1)$ linear ? in <u>Cartesian</u> system?



<u>No</u>, not linear.

Q Is 1D translation by $(1,1)$ in homogeneous system linear?



$V_1 = (1,1)$

$T(V_1) = (2,2)$

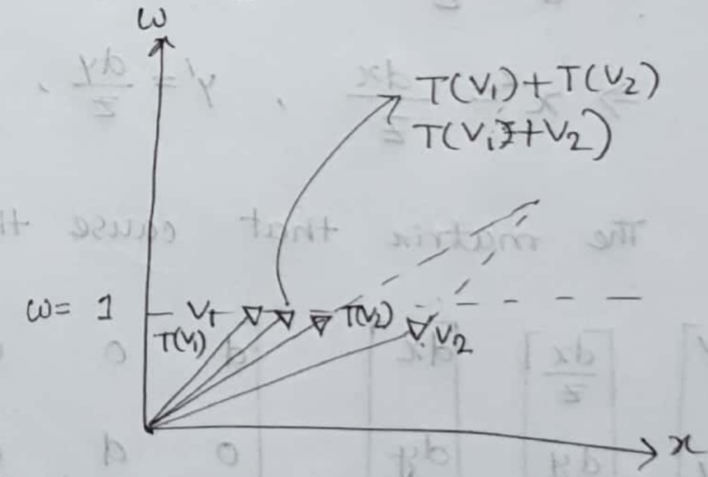
$= (1,1)$

$= V_1$

$V_2 = (2,1)$

$T(V_2) = (3,2)$

$= \left(\frac{3}{2}, 1\right)$



$T(V_1)+T(V_2) = (1,1)+\left(\frac{3}{2},1\right)$

$= \left(\frac{5}{2}, 2\right)$

$= \left(\frac{5}{4}, 1\right)$

$V_1 + V_2 = (1,1)+(2,1)$

$= (3,2) = \left(\frac{3}{2}, 1\right)$

$T(V_1+V_2) = \left(\frac{5}{2}, 2\right)$

$= \left(\frac{5}{4}, 1\right)$

Yes, linear.

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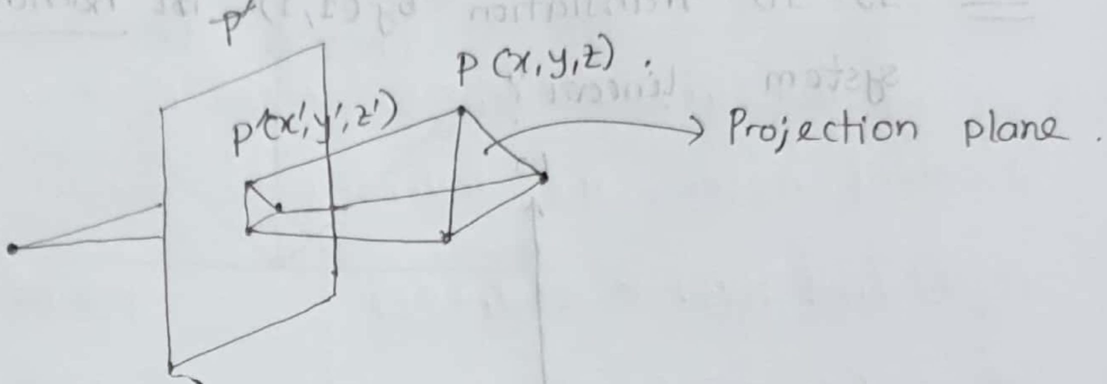

· Perspective Projection,

$$\frac{x'}{z'} = \frac{x}{z}$$

$$\Rightarrow \frac{x'}{d} = \frac{x}{z}$$

$$\Rightarrow x' = \frac{dx}{z}, \quad y' = \frac{dy}{z}, \quad z' = \frac{dz}{z}.$$

The matrix that cause this transformation is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{dx}{z} \\ \frac{dy}{z} \\ \frac{dz}{z} \\ 1 \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ dz \\ z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2nd assignment will be assigned on next saturday.

during theory class