

Contents

1 Basic	1
1.1 Shell Script	1
1.2 Debug Macro*	1
1.3 Pragma / FastIO	1
1.4 Divide	1
2 Data Structure	1
2.1 Leftist Tree	1
2.2 Splay Tree	1
2.3 Link Cut Tree	2
2.4 Treap	3
2.5 vEB Tree	3
3 Flow / Matching	3
3.1 Dinic	3
3.2 Min Cost Max Flow	4
3.3 Kuhn Munkres	4
3.4 Hopcroft Karp	5
3.5 SW Min Cut	5
3.6 Gomory Hu Tree	5
3.7 Blossom	5
3.8 Min Cost Circulation	6
3.9 Weighted Blossom	6
3.10 Flow Model	7
4 Graph	7
4.1 Heavy-Light Decomposition	7
4.2 Centroid Decomposition	8
4.3 Edge BCC	8
4.4 Vertex BCC / Round Square Tree	8
4.5 SCC	8
4.6 2SAT	8
4.7 Virtual Tree	9
4.8 Directed MST	9
4.9 Dominator Tree	9
4.10 Bipartite Edge Coloring	9
4.11 Edge Coloring	10
4.12 Maximum Clique	10
5 String	10
5.1 Aho-Corasick Automaton	10
5.2 KMP Algorithm	11
5.3 Z Algorithm	11
5.4 Manacher	11
5.5 Suffix Array	11
5.6 Suffix Automaton	11
5.7 Minimum Rotation	12
5.8 Palindrome Tree	12
5.9 Lyndon Factorization	12
5.10 Main Lorentz	12
6 Math	12
6.1 Miller Rabin / Pollard Rho	12
6.2 Ext GCD	13
6.3 Chinese Remainder Theorem	13
6.4 PiCount	13
6.5 Linear Function Mod Min	13
6.6 Gauss Elimination	13
6.7 Floor Sum	14
6.8 Quadratic Residue	14
6.9 Discrete Log	14
6.10 Factorial without Prime Factor	14
6.11 Berlekamp Massey	14
6.12 Simplex	14
6.13 Euclidean	15
6.14 Linear Programming Construction	15
6.15 Theorem	15
6.16 Estimation	16
6.17 General Purpose Numbers	16
6.18 Calculus	16
7 Polynomial	16
7.1 Number Theoretic Transform	16
7.2 Fast Fourier Transform	16
7.3 Primes	16
7.4 Polynomial Operations	16
7.5 Fast Linear Recursion	18
7.6 Fast Walsh Transform	18
8 Geometry	18
8.1 Basic	18
8.2 SVG Writer	18
8.3 Sort	19
8.4 Intersections	19
8.5 Point Inside Check	19
8.6 Convex Hull	20
8.7 Point Segment Distance	20
8.8 Vector In Polygon	20
8.9 Minkowski Sum	20
8.10 Rotating SweepLine	20
8.11 Half Plane Intersection	20
8.12 Minimum Enclosing Circle	20
8.13 Point Inside Triangle	21
8.14 Heart	21
8.15 Tangents	21
8.16 Convex Cut	21
8.17 Union of Circles	21
8.18 Union of Polygons	22
8.19 Delaunay Triangulation	22
8.20 Triangulation Voronoi	22
8.21 External Bisector	22
8.22 Intersection Area of Polygon and Circle	23
8.23 3D Point	23
8.24 3D Convex Hull	23
9 Else	23
9.1 Pbds	23
9.2 Bit Hack	23
9.3 Dynamic Programming Condition	24
9.3.1 Totally Monotone (Concave/Convex)	24
9.3.2 Monge Condition (Concave/Convex)	24
9.3.3 Optimal Split Point	24
9.4 Smawk Algorithm	24
9.5 Slope Trick	24
9.6 ALL LCS	24
9.7 Hilbert Curve	24
9.8 Line Container	25
9.9 Min Plus Convolution	25
9.10 Matroid Intersection	25
9.11 Simulated Annealing	25
9.12 Bitset LCS	25
9.13 Binary Search On Fraction	25
9.14 Cyclic Ternary Search	25
9.15 Tree Hash	25
9.16 Python Misc	25

1 Basic

1.1 Shell Script

```
cpp hash.cpp -dD -P -fpreprocessed | tr -d "[:space:]"
| md5sum | cut -c -6
```

1.2 Debug Macro* [2e0e48]

```
#ifndef ABS
template <typename T>
ostream& operator << (ostream &o, vector <T> vec) {
    o << "{"; int f = 0;
    for (T i : vec) o << (f++ ? " " : "") << i;
    return o << "}";
}

void bug_(int c, auto ...a) {
    cerr << "\e[1;" << c << "m";
    (... , (cerr << a << " "));
    cerr << "\e[0m" << endl;
}

#define bug_(c, x...) bug_(c, __LINE__, "[" + string(#
x) + "]", x)
#define bug(x...) bug_(32, x)
```

```
#define bugv(x...) bug_(36, vector(x))
#define safe bug_(33, "safe")
#else
#define bug(x...) void(0)
#define bugv(x...) void(0)
#define safe void(0)
#endif
```

1.3 Pragma / FastIO

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
#include <unistd.h>
char OB[65536]; int OP;
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)
        ) == buf ? -1 : *p++;
}
inline int R() {
    static char c;
    while((c = RC()) < '0'); int a = c ^ '0';
    while((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.4 Divide

```
ll floor(ll a, ll b) {return a / b - (a < 0 && a % b);}
ll ceil(ll a, ll b) {return a / b + (a > 0 && a % b);}
a / b < x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x < a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)
```

2 Data Structure

2.1 Leftist Tree [75d338]

```
// max heap
struct node {
    ll rk, data, size, sum;
    node *l, *r;
    node(ll k) : rk(0), data(k), size(1), sum(k), l(0), r(0) {}
};
#undef sz
ll sz(node *p) { return p ? p->size : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1;
    a->size = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}

void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}
```

2.2 Splay Tree [21142b]

```
struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    ll v[N];
    Splay() {}
    void init() {
```

```

    rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
    sz[0] = 1, v[0] = inf;
}
int newnode(int p, int x) {
    int id = _id++;
    v[id] = x, pa[id] = p;
    ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    return id;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int gp = pa[p], c = ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
}
void splay(int i) {
    while (~pa[i]) {
        int p = pa[i];
        if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
            == i ? i : p);
        rotate(i);
    }
    rt = i;
}
int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
        if (v[i] == x) return splay(i), i;
        if (v[i] > x) {
            last = i;
            if (ch[i][0] == -1) break;
            i = ch[i][0];
        }
        else {
            if (ch[i][1] == -1) break;
            i = ch[i][1];
        }
    }
    splay(i);
    return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};

```

2.3 Link Cut Tree [d01a7d]

```

// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N];
    int sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
    // virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
    }
};

```

```

    return x;
}
void pull(int i) {
    sz[i] = v[i] + sz2[i];
    mx[i] = w[i];
    if (ch[i][0]) {
        sz[i] += sz[ch[i][0]];
        mx[i] = max(mx[i], mx[ch[i][0]]);
    }
    if (ch[i][1]) {
        sz[i] += sz[ch[i][1]];
        mx[i] = max(mx[i], mx[ch[i][1]]);
    }
}
void push(int i) {
    if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
        rev[i] = false;
}
void reverse(int i) {
    if (!i) return;
    swap(ch[i][0], ch[i][1]);
    rev[i] ^= true;
}
bool isrt(int i) { // rt of splay
    if (!pa[i]) return true;
    return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int c = ch[i][!x], gp = pa[p];
    if (ch[gp][0] == p) ch[gp][0] = i;
    else if (ch[gp][1] == p) ch[gp][1] = i;
    pa[i] = gp, ch[i][!x] = p, pa[p] = i;
    ch[p][x] = c, pa[c] = p;
    pull(p), pull(i);
}
void splay(int i) {
    vector<int> anc;
    anc.pb(i);
    while (!isrt(anc.back()))
        anc.pb(pa[anc.back()]);
    while (!anc.empty())
        push(anc.back()), anc.pop_back();
    while (!isrt(i)) {
        int p = pa[i];
        if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
            == p ? i : p);
        rotate(i);
    }
}
void access(int i) {
    int last = 0;
    while (i) {
        splay(i);
        if (ch[i][1])
            sz2[i] += sz[ch[i][1]];
        sz2[i] -= sz[last];
        ch[i][1] = last;
        pull(i), last = i, i = pa[i];
    }
}
void makert(int i) {
    access(i), splay(i), reverse(i);
}
void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
}
void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
}
int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
}

```

```
}
};
```

2.4 Treap [fbf3b7]

```
struct node {
    int data, size;
    node *l, *r;
    node(int k) : data(k), size(1), l(0), r(0) {}
    void up() {
        size = 1;
        if (l) size += l->size;
        if (r) size += r->size;
    }
    void down() {}
};
#undef sz
int sz(node *a) { return a ? a->size : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    o->down(), split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
    o->up();
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c; // [l, r)
    o->down();
    split2(o, a, b, l), split2(b, b, c, r - l);
    // operate
    o = merge(a, merge(b, c)), o->up();
}
```

2.5 vEB Tree [087d11]

```
using u64 = uint64_t;
constexpr int lsb(u64 x)
{ return x ? __builtin_ctzll(x) : 1 << 30; }
constexpr int msb(u64 x)
{ return x ? 63 - __builtin_clzll(x) : -1; }
```

```
template<int N, class T = void>
struct veb {
    static const int M = N >> 1;
    veb<M> ch[1 << N - M];
    veb<N - M> aux;
    int mn, mx;
    veb() : mn(1 << 30), mx(-1) {}
    constexpr int mask(int x) { return x & ((1 << M) - 1); }
    bool empty() { return mx == -1; }
    int min() { return mn; }
    int max() { return mx; }
    bool have(int x)
    { return x == mn ? true : ch[x >> M].have(mask(x)); }
    void insert_in(int x) {
        if (empty()) return mn = mx = x, void();
        if (x < mn) swap(x, mn);
        if (x > mx) mx = x;
        if (ch[x >> M].empty()) aux.insert_in(x >> M);
        ch[x >> M].insert_in(mask(x));
    }
    void erase_in(int x) {
        if (mn == mx) return mn = 1 << 30, mx = -1, void();
        if (x == mn)
            mn = x = (aux.min() << M) ^ ch[aux.min()].min();
        ch[x >> M].erase_in(mask(x));
        if (ch[x >> M].empty()) aux.erase_in(x >> M);
        if (x == mx) {
            if (aux.empty()) mx = mn;
            else mx = (aux.max() << M) ^ ch[aux.max()].max();
        }
    } // 06a669
    void insert(int x) {
        if (!have(x)) insert_in(x);
    }
    void erase(int x) {
        if (have(x)) erase_in(x);
    }
    int next(int x) { // >= x
        if (x > mx) return 1 << 30;
        if (x <= mn) return mn;
        if (mask(x) <= ch[x >> M].max()) return ((x >> M)
            << M) ^ ch[x >> M].next(mask(x));
        int y = aux.next((x >> M) + 1);
        return (y << M) ^ ch[y].min();
    }
    int prev(int x) { // < x
        if (x <= mn) return -1;
        if (x > mx) return mx;
        if (x <= (aux.min() << M) + ch[aux.min()].min())
            return mn;
        if (mask(x) > ch[x >> M].min()) return ((x >> M) <<
            M) ^ ch[x >> M].prev(mask(x));
        int y = aux.prev(x >> M);
        return (y << M) ^ ch[y].max();
    }
};
template <int N>
struct veb <N, typename enable_if<N <= 6>::type> {
    u64 a;
    veb() : a(0) {}
    void insert_in(int x) { a |= 1ull << x; }
    void insert(int x) { a |= 1ull << x; }
    void erase_in(int x) { a &= ~(1ull << x); }
    void erase(int x) { a &= ~(1ull << x); }
    bool have(int x) { return a >> x & 1; }
    bool empty() { return a == 0; }
    int min() { return lsb(a); }
    int max() { return msb(a); }
    int next(int x) { return lsb(a & ~((1ull << x) - 1)); }
    int prev(int x) { return msb(a & ((1ull << x) - 1)); }
}; // e36c96
```

3 Flow / Matching

3.1 Dinic [b68676]

```
template <typename T>
struct Dinic { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    struct edge { int to, rev; T cap, flow; };
    int n, s, t;
    vector <vector <edge>> g;
    vector <int> dis, cur;
```

```

T dfs(int u, T cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < sz(g[u]); ++i) {
        edge &e = g[u][i];
        if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
            T df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df;
                g[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}

bool bfs() {
    dis.assign(n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (auto &u : g[v])
            if (dis[u.to] == -1 && u.flow != u.cap) {
                q.push(u.to);
                dis[u.to] = dis[v] + 1;
            }
    }
    return dis[t] != -1;
}

T solve(int _s, int _t) {
    s = _s, t = _t;
    T flow = 0, df;
    while (bfs()) {
        cur.assign(n, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}

void add_edge(int u, int v, T cap) {
    g[u].pb(edge{v, sz(g[v]), cap, 0});
    g[v].pb(edge{u, sz(g[u]) - 1, 0, 0});
}

Dinic (int _n) : n(_n), g(n) {}
//void reset() {
//    for (int i = 0; i < n; ++i)
//        for (auto &j : g[i]) j.flow = 0;
//}
};

```

3.2 Min Cost Max Flow [92f08e]

```

template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
    const T1 INF1 = numeric_limits<T1>::max() / 2;
    const T2 INF2 = numeric_limits<T2>::max() / 2;
    struct edge { int v; T1 f; T2 c; };
    int n, s, t;
    vector<vector<int>> g;
    vector<edge> e;
    vector<T2> dis, pot;
    vector<int> rt, vis;
    // bool DAG()...
    bool SPFA() {
        rt.assign(n, -1), dis.assign(n, INF2);
        vis.assign(n, false);
        queue<int> q;
        q.push(s), dis[s] = 0, vis[s] = true;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            vis[v] = false;
            for (int id : g[v]) {
                auto [u, f, c] = e[id];
                T2 ndis = dis[v] + c + pot[v] - pot[u];
                if (f > 0 && dis[u] > ndis) {
                    dis[u] = ndis, rt[u] = id;
                    if (!vis[u]) vis[u] = true, q.push(u);
                }
            }
        }
        return dis[t] != INF2;
    }
}; // df1862

```

```

bool dijkstra() {
    rt.assign(n, -1), dis.assign(n, INF2);
    priority_queue<pair<T2, int>, vector<pair<T2, int>>, greater<pair<T2, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
        auto [d, v] = pq.top(); pq.pop();
        if (dis[v] < d) continue;
        for (int id : g[v]) {
            auto [u, f, c] = e[id];
            T2 ndis = dis[v] + c + pot[v] - pot[u];
            if (f > 0 && dis[u] > ndis) {
                dis[u] = ndis, rt[u] = id;
                pq.emplace(ndis, u);
            }
        }
    }
    return dis[t] != INF2;
} // d46baf

vector<pair<T1, T2>> solve(int _s, int _t) {
    s = _s, t = _t, pot.assign(n, 0);
    vector<pair<T1, T2>> ans; bool fr = true;
    while ((fr ? SPFA() : SPFA())) {
        for (int i = 0; i < n; ++i)
            dis[i] += pot[i] - pot[s];
        T1 add = INF1;
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            add = min(add, e[rt[i]].f);
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            e[rt[i]].f -= add, e[rt[i] ^ 1].f += add;
        ans.emplace_back(add, dis[t]), fr = false;
        for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return ans;
}

void add_edge(int u, int v, T1 f, T2 c) {
    g[u].pb(sz(e)), e.pb({v, f, c});
    g[v].pb(sz(e)), e.pb({u, 0, -c});
}

MCMF (int _n) : n(_n), g(n), e() {}
//void reset() {
//    for (int i = 0; i < sz(e); ++i) e[i].f = 0;
//}
}; // 383274

```

3.3 Kuhn Munkres [7f3209]

```

template <typename T> // maximum perfect matching
struct KM { // 0-based, remember to init edge weight
    const T INF = numeric_limits<T>::max() / 2;
    int n; vector<vector<T>> w;
    vector<T> hl, hr, slk;
    vector<int> fl, fr, vl, vr, pre;
    queue<int> q;
    bool check(int x) {
        if (vl[x] == 1, ~fl[x])
            return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }

    void bfs(int s) {
        vl.assign(n, 0), vr.assign(n, 0);
        slk.assign(n, INF), pre.assign(n, 0);
        while (!q.empty()) q.pop();
        q.push(s), vr[s] = 1;
        while (true) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x) {
                    d = hl[x] + hr[y] - w[x][y];
                    if (!vl[x] && slk[x] >= d) {
                        if (pre[x] == y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
                }
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;

```

```

        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
}
T solve() {
    fl.assign(n, -1), fr.assign(n, -1);
    hl.assign(n, 0), hr.assign(n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(all(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
void add_edge(int a, int b, T wei) { w[a][b] = wei; }
KM (int _n) : n(_n), w(n, vector<T>(n, -INF)) {}
};

```

3.4 Hopcroft Karp [372c8b]

```

struct HopcroftKarp { // 0-based
    int n, m;
    vector<vector<int>> g;
    vector<int> l, r, d;
    bool dfs(int x) {
        for (int y : g[x]) if (r[y] == -1 ||
            (d[r[y]] == d[x] + 1 && dfs(r[y])))
            return l[x] = y, r[y] = x, d[x] = -1, true;
        return d[x] = -1, false;
    }
    bool bfs() {
        d.assign(n, -1);
        queue<int> q;
        for (int x = 0; x < n; ++x) if (l[x] == -1)
            d[x] = 0, q.push(x);
        bool good = false;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int y : g[x])
                if (r[y] == -1) good = true;
                else if (d[r[y]] == -1)
                    d[r[y]] = d[x] + 1, q.push(r[y]);
        }
        return good;
    }
    int solve() {
        int res = 0;
        l.assign(n, -1), r.assign(m, -1);
        while (bfs())
            for (int x = 0; x < n; ++x) if (l[x] == -1)
                res += dfs(x);
        return res;
    }
    void add_edge(int x, int y) { g[x].pb(y); }
    HopcroftKarp (int _n, int _m) : n(_n), m(_m), g(n) {}
};

```

3.5 SW Min Cut [f7fc17]

```

template <typename T>
struct SW { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    vector<vector<T>> g;
    vector<T> sum;
    vector<bool> vis, dead;
    int n;
    T solve() {
        T ans = INF;
        for (int r = 0; r + 1 < n; ++r) {
            vis.assign(n, 0), sum.assign(n, 0);
            int num = 0, s = -1, t = -1;
            while (num < n - r) {
                int now = -1;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i] &&
                        (now == -1 || sum[now] > sum[i])) now = i;
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i)

```

```

                    if (!vis[i] && !dead[i]) sum[i] += g[now][i];
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i)
                g[i][s] += g[i][t], g[s][i] += g[t][i];
            dead[t] = true;
        }
        return ans;
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w; }
    SW (int _n) : n(_n), g(n, vector<T>(n)), dead(n) {}
};

```

3.6 Gomory Hu Tree [90ead2]

```

vector<array<int, 3>> GomoryHu(Dinic<int> flow) {
    // Tree edge min = mcut (0-based)
    int n = flow.n;
    vector<array<int, 3>> ans;
    vector<int> rt(n);
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset();
        ans.pb({i, t, flow.solve(i, t)});
        flow.bfs();
        for (int j = i + 1; j < n; ++j)
            if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
    }
    return ans;
}

```

3.7 Blossom [cbc9d3]

```

struct Matching { // 0-based
    int n, tk;
    vector<vector<int>> g;
    vector<int> fa, pre, match, s, t;
    queue<int> q;
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++, x = Find(x), y = Find(y);
        for (; ; swap(x, y)) if (x != n) {
            if (t[x] == tk) return x;
            t[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool bfs(int r) {
        iota(all(fa), 0), fill(all(s), -1);
        while (!q.empty()) q.pop();
        q.push(r), s[r] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int u : g[x]) {
                if (s[u] == -1) {
                    pre[u] = x, s[u] = 1;
                    if (match[u] == n) {
                        for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;
                        return true;
                    }
                    q.push(match[u]);
                    s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = lca(u, x);
                    blossom(x, u, l), blossom(u, x, l);
                }
            }
        }
        return false;
    }
};

```



```

int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) if (match[x] == n)
        res += bfs(x);
    return res;
}
void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u);
}
Matching (int _n) : n(_n), tk(0), g(n), fa(n + 1),
    pre(n + 1, n), match(n + 1, n), s(n + 1), t(n) {}
};

```

3.8 Min Cost Circulation [53a447]

```

struct MinCostCirculation { // 0-base
    struct Edge {
        ll from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    ll dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
        void try_edge(Edge &cur) {
            if (cur.cap > cur.flow) return cur.cap++, void();
            BellmanFord(cur.to);
            if (dis[cur.from] + cur.cost < 0) {
                cur.flow++, G[cur.to][cur.rev].flow--;
                for (int i = cur.from; past[i]; i = past[i]->from) {
                    auto &e = *past[i];
                    e.flow++, G[e.to][e.rev].flow--;
                }
            }
            cur.cap++;
        }
        void solve(int mxlg) {
            for (int b = mxlg; b >= 0; --b) {
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])
                        e.cap *= 2, e.flow *= 2;
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])
                        if (e.fcap >> b & 1)
                            try_edge(e);
            }
        }
        void init(int _n) { n = _n;
            for (int i = 0; i < n; ++i) G[i].clear();
        }
        void add_edge(ll a, ll b, ll cap, ll cost) {
            G[a].pb(Edge{a, b, 0, cap, 0, cost, sz(G[b]) + (a
                == b)});
            G[b].pb(Edge{b, a, 0, 0, 0, -cost, sz(G[a]) - 1});
        }
    } mcmf; // O(VE * ELogC)
};

```

3.9 Weighted Blossom [dc42e4]

```

#define pb emplace_back
#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slack, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
};

```

```

WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
    g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
    flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
}
int ED(edge e) {
    return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
}
void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
}
void set_slack(int x) {
    slack[x] = 0;
    REP(u, 1, n)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x, slack[x]);
}
void q_push(int x) {
    if (x <= n) q.push(x);
    else for (int y : flo[x]) q_push(y);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (int y : flo[x]) set_st(y, b);
} // ae3b3a
vector<int> split_flo(auto &f, int xr) {
    auto it = find(all(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
        reverse(1 + all(f), it = f.end() - pr);
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    REP(i, 0, int(z.size())-1) set_match(z[i], z[i ^
        1]);
    set_match(xr, v); f.insert(f.end(), all(z));
}
void augment(int u, int v) {
    for (;;) {
        int xnv = st[match[u]]; set_match(u, v);
        if (!xnv) return;
        set_match(v = xnv, u = st[pa[xnv]]);
    }
}
int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u) {
        if (vis[u] == t) return u;
        vis[u] = t; u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
} // 0569c4
void add_blossom(int u, int o, int v) {
    int b = int(find(n + 1 + all(st), 0) - begin(st));
    lab[b] = 0, S[b] = 0; match[b] = match[o];
    vector<int> f = {o};
    for (int x : {u, v}) {
        for (int y; x != o; x = st[pa[y]])
            f.pb(x), f.pb(y = st[match[x]]), q_push(y);
        reverse(1 + all(f));
    }
    flo[b] = f; set_st(b, b);
    REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
    REP(x, 1, n) flo_from[b][x] = 0;
    for (int xs : flo[b]) {
        REP(x, 1, nx)
            if (g[b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]
                ))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        REP(x, 1, n)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {

```

```

    if (xs == -1) { xs = x; continue; }
    pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
    slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
}
for (int x : flo[b])
    if (x == xr) S[x] = 1, pa[x] = pa[b];
    else S[x] = -1, set_slack(x);
st[b] = 0;
}
bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    }
    else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
} // 61368c
bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    REP(x, 1, nx) if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            REP(v, 1, n)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (ED(g[u][v]) != 0)
                        update_slack(u, st[v], slack[st[v]]);
                    else if (on_found_edge(g[u][v])) return true;
                }
        }
        int d = inf;
        REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
            d = min(d, lab[b] / 2);
        REP(x, 1, nx)
            if (int s = slack[x]; st[x] == x && s && S[x] <= 0)
                d = min(d, ED(g[s][x]) / (S[x] + 2));
        REP(u, 1, n)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {
                if (lab[u] <= d) return false;
                lab[u] -= d;
            }
        REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
            lab[b] += d * (2 - 4 * S[b]);
        REP(x, 1, nx)
            if (int s = slack[x]; st[x] == x &&
                s && st[s] != x && ED(g[s][x]) == 0)
                if (on_found_edge(g[s][x])) return true;
        REP(b, n + 1, nx)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)
                expand_blossom(b);
    }
    return false;
} // 61b100
pair<ll, int> solve() {
    fill(all(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    REP(u, 1, n) lab[u] = w_max;
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void set_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w;
} // f1e757

```

3.10 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - Dfs from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - Change the weight of each edge to $\mu(u) + \mu(v) - w(u, v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Let the maximum weight matching of the graph be x , the answer will be $\sum \mu(v) - x$.

4 Graph

4.1 Heavy-Light Decomposition [9ec77f]

```

struct HLD { // 0-based, remember to build
    int n, _id;
    vector<vector<int>> g;
    vector<int> dep, pa, tsz, ch, hd, id;
    void dfs(int v, int p) {
        dep[v] = ~p ? dep[p] + 1 : 0;
        pa[v] = p, tsz[v] = 1, ch[v] = -1;
        for (int u : g[v]) if (u != p) {
            dfs(u, v);
            if (ch[v] == -1 || tsz[ch[v]] < tsz[u])
                ch[v] = u;
            tsz[v] += tsz[u];
        }
    }
    void hld(int v, int p, int h) {
        hd[v] = h, id[v] = _id++;
        if (~ch[v]) hld(ch[v], v, h);
        for (int u : g[v]) if (u != p && u != ch[v])
            hld(u, v, u);
    }
    vector<pii> query(int u, int v) {
        vector<pii> ans;
        while (hd[u] != hd[v]) {
            if (dep[hd[u]] > dep[hd[v]]) swap(u, v);
            ans.emplace_back(id[hd[v]], id[v] + 1);
            v = pa[hd[v]];
        }
        if (dep[u] > dep[v]) swap(u, v);
        ans.emplace_back(id[u], id[v] + 1);
        return ans;
    }
};

```

```

}
void build() {
    for (int i = 0; i < n; ++i) if (id[i] == -1)
        dfs(i, -1), hld(i, -1, i);
}
void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
HLD (int _n) : n(_n), _id(0), g(n), dep(n), pa(n),
    tsz(n), ch(n), hd(n), id(n, -1) {}
};

```

4.2 Centroid Decomposition [28b80a]

```

struct CD { // 0-based, remember to build
    int n, lg; // pa, dep are centroid tree attributes
    vector<vector<int>> g, dis;
    vector<int> pa, tsz, dep, vis;
    void dfs1(int v, int p) {
        tsz[v] = 1;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs1(u, v), tsz[v] += tsz[u];
    }
    int dfs2(int v, int p, int _n) {
        for (int u : g[v])
            if (u != p && !vis[u] && tsz[u] > _n / 2)
                return dfs2(u, v, _n);
        return v;
    }
    void dfs3(int v, int p, int d) {
        dis[v][d] = ~p ? dis[p][d] + 1 : 0;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs3(u, v, d);
    }
    void cd(int v, int p, int d) {
        dfs1(v, -1), v = dfs2(v, -1, tsz[v]);
        vis[v] = true, pa[v] = p, dep[v] = d;
        dfs3(v, -1, d);
        for (int u : g[v]) if (!vis[u])
            cd(u, v, d + 1);
    }
    void build() { cd(0, -1, 0); }
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u); }
    CD (int _n) : n(_n), lg(_lg(n) + 1), g(n),
        dis(n, vector<int>(lg)), pa(n), tsz(n),
        dep(n), vis(n) {}
};

```

4.3 Edge BCC [cf5e55]

```

struct EBCC { // 0-based, remember to build
    int n, m, nbcc;
    vector<vector<pii>> g;
    vector<int> pa, low, dep, bcc_id, stk, is_bridge;
    void dfs(int v, int p, int f) {
        low[v] = dep[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (auto [u, e] : g[v]) {
            if (low[u] == -1)
                dfs(u, v, e), low[v] = min(low[v], low[u]);
            else if (e != f)
                low[v] = min(low[v], dep[u]);
        }
        if (low[v] == dep[v]) {
            if (~f) is_bridge[f] = true;
            int id = nbcc++, x;
            do {
                x = stk.back(), stk.pop_back();
                bcc_id[x] = id;
            } while (x != v);
        }
    }
    void build() {
        is_bridge.assign(m, 0);
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i, -1, -1);
    }
    void add_edge(int u, int v) {
        g[u].emplace_back(v, m), g[v].emplace_back(u, m++);
    }
    EBCC (int _n) : n(_n), m(0), nbcc(0), g(n), pa(n),
        low(n, -1), dep(n), bcc_id(n), stk() {}
};

```

4.4 Vertex BCC / Round Square Tree [66d85d]

```

struct BCC { // 0-based, remember to build
    int n, nbcc; // note for isolated point
    vector<vector<int>> g, _g; // id >= n: bcc
    vector<int> pa, dep, low, stk, pa2, dep2;
    void dfs(int v, int p) {
        dep[v] = low[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (int u : g[v]) if (u != p) {
            if (low[u] == -1) {
                dfs(u, v), low[v] = min(low[v], low[u]);
                if (low[u] >= dep[v]) {
                    int id = nbcc++, x;
                    do {
                        x = stk.back(), stk.pop_back();
                        _g[id + n].pb(x), _g[x].pb(id + n);
                    } while (x != u);
                    _g[id + n].pb(v), _g[v].pb(id + n);
                } else low[v] = min(low[v], dep[u]);
            }
        }
        bool is_cut(int x) { return sz(_g[x]) != 1; }
        vector<int> bcc(int id) { return _g[id + n]; }
        int bcc_id(int u, int v) {
            return pa2[dep2[u] < dep2[v] ? v : u] - n;
        }
        void dfs2(int v, int p) {
            dep2[v] = ~p ? dep2[p] + 1 : 0, pa2[v] = p;
            for (int u : _g[v]) if (u != p) dfs2(u, v);
        }
        void build() {
            low.assign(n, -1);
            for (int i = 0; i < n; ++i) if (low[i] == -1)
                dfs(i, -1), dfs2(i, -1);
        }
        void add_edge(int u, int v) {
            g[u].pb(v), g[v].pb(u); }
        BCC (int _n) : n(_n), nbcc(0), g(n), _g(2 * n),
            pa(n), dep(n), low(n), stk(), pa2(n * 2),
            dep2(n * 2) {}
};

```

4.5 SCC [9bee8c]

```

struct SCC {
    int n, nscc, _id;
    vector<vector<int>> g;
    vector<int> dep, low, scc_id, stk;
    void dfs(int v) {
        dep[v] = low[v] = _id++, stk.pb(v);
        for (int u : g[v]) if (scc_id[u] == -1) {
            if (low[u] == -1) dfs(u);
            low[v] = min(low[v], low[u]);
        }
        if (low[v] == dep[v]) {
            int id = nscc++, x;
            do {
                x = stk.back(), stk.pop_back(), scc_id[x] = id;
            } while (x != v);
        }
    }
    void build() {
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i);
    }
    void add_edge(int u, int v) { g[u].pb(v); }
    SCC (int _n) : n(_n), nscc(0), _id(0), g(n), dep(n),
        low(n, -1), scc_id(n, -1), stk() {}
};

```

4.6 2SAT [938072]

```

struct SAT { // 0-based, need SCC
    int n; vector<pii> edge; vector<int> is;
    int rev(int x) { return x < n ? x + n : x - n; }
    void add_ifthen(int x, int y) {
        add_clause(rev(x), y); }
    void add_clause(int x, int y) {
        edge.emplace_back(rev(x), y);
        edge.emplace_back(rev(y), x); }
    bool solve() {
        // is[i] = true -> i, is[i] = false -> -i
    }
};

```



```

    SCC scc(2 * n);
    for (auto [u, v] : edge) scc.add_edge(u, v);
    scc.build();
    for (int i = 0; i < n; ++i) {
        if (scc.scc_id[i] == scc.scc_id[i + n])
            return false;
        is[i] = scc.scc_id[i] < scc.scc_id[i + n];
    }
    return true;
}
SAT (int _n) : n(_n), edge(), is(n) {}
};

```

4.7 Virtual Tree [f7650b]

```

// need lca, in, out
vector<pii> virtual_tree(vector<int> &v) {
    auto cmp = [&](int x, int y) {return in[x] < in[y];};
    sort(all(v), cmp);
    for (int i = 1; i < sz(v); ++i)
        v.pb(lca(v[i - 1], v[i]));
    sort(all(v), cmp);
    v.resize(unique(all(v)) - v.begin());
    vector<int> stk(1, v[0]);
    vector<pii> res;
    for (int i = 1; i < sz(v); ++i) {
        int x = v[i];
        while (out[stk.back()] < out[x]) stk.pop_back();
        res.emplace_back(stk.back(), x), stk.pb(x);
    }
    return res;
}

```

4.8 Directed MST [a2498b]

```

using D = int;
struct edge { int u, v; D w; };
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
    using T = pair<D, int>;
    using PQ = pair<priority_queue<T, vector<T>,
        greater<T>>, D>;
    auto push = [&](PQ &pq, T v) {
        pq.first.emplace(v.first - pq.second, v.second);
    };
    auto top = [&](const PQ &pq) -> T {
        auto r = pq.first.top();
        return {r.first + pq.second, r.second};
    };
    auto join = [&push, &top](PQ &a, PQ &b) {
        if (sz(a.first) < sz(b.first)) swap(a, b);
        while (!b.first.empty())
            push(a, top(b)), b.first.pop();
    };
    vector<PQ> h(n * 2);
    for (int i = 0; i < sz(e); ++i)
        push(h[e[i].v], {e[i].w, i});
    vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
        n * 2);
    iota(all(a), 0);
    auto o = [&](int x) { int y;
        for (y = x; a[y] != y; y = a[y]);
        for (int ox = x; x != y; ox = x)
            x = a[x], a[ox] = y;
        return y;
    };
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
            p]].u)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].second = -h[q].first.top().first;
                    join(h[pa[q] = a[q] = p], h[q]);
                } while ((q = o(e[r[q]].u)) != p);
            }
            v[p] = i;
            while (!h[p].first.empty() && o(e[top(h[p]).
                second].u) == p)
                h[p].first.pop();
        }
    }
}

```

```

        r[p] = top(h[p]).second;
    }
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--)
    if (i != root && v[i] != n) {
        for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
            pa[f]) v[f] = n;
        ans.pb(r[i]);
    }
return ans;
}

```

4.9 Dominator Tree [7eadea]

```

struct DominatorTree {
    int n, id;
    vector<vector<int>> g, rg, bucket;
    vector<int> sdom, dom, vis, rev, pa, rt, mn, res;
    // dom[s] = s, dom[v] = -1 if s -> v not exists
    int query(int v, int x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], 1);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]])
            mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : g[v]) {
            if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];
            rg[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : rg[i])
                sdom[i] = min(sdom[i], sdom[query(u, 0)]);
            if (i) bucket[sdom[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, 0);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = pa[i];
        }
        fill(all(res), -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i)
            res[rev[i]] = rev[dom[i]];
        res[s] = s;
        for (int i = 0; i < n; ++i) dom[i] = res[i];
    }
    void add_edge(int u, int v) { g[u].pb(v); }
    DominatorTree (int _n) : n(_n), id(0), g(n), rg(n),
        bucket(n), sdom(n), dom(n, -1), vis(n, -1),
        rev(n), pa(n), rt(n), mn(n), res(n) {}
};

```

4.10 Bipartite Edge Coloring [a22d96]

```

struct BipartiteEdgeColoring { // 1-based
    // returns edge coloring in adjacent matrix G
    int n, m;
    vector<vector<int>> col, G;
    int find_col(int x) {
        int c = 1;
        while (col[x][c] c++;
        return c;
    }
    void dfs(int v, int c1, int c2) {
        if (!col[v][c1]) return col[v][c2] = 0, void(0);
        int u = col[v][c1];
        dfs(u, c2, c1);
        col[v][c1] = 0, col[v][c2] = u, col[u][c2] = v;
    }
    void solve() {

```

```

    for (int i = 1; i <= n + m; ++i)
        for (int j = 1; j <= max(n, m); ++j)
            if (col[i][j])
                G[i][col[i][j]] = G[col[i][j]][i] = j;
} // u = left index, v = right index
void add_edge(int u, int v) {
    int c1 = find_col(u), c2 = find_col(v + n);
    dfs(u, c2, c1);
    col[u][c2] = v + n, col[v + n][c2] = u;
}
BipartiteEdgeColoring (int _n, int _m) : n(_n),
    m(_m), col(n + m + 1, vector<int>(max(n, m) + 1)),
    G(n + m + 1, vector<int>(n + m + 1)) {}
};

```

4.11 Edge Coloring [5b1e8f]

```

struct Vizing { // 1-based
    // returns edge coloring in adjacent matrix G
    int n;
    vector<vector<int>> C, G;
    vector<int> X, vst;
    vector<pii> E;
    void solve() {
        auto update = [&](int u)
        { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
            return p;
        };
        auto flip = [&](int u, int c1, int c2) {
            int p = C[u][c1];
            swap(C[u][c1], C[u][c2]);
            if (p) G[u][p] = G[p][u] = c2;
            if (!C[u][c1]) X[u] = c1;
            if (!C[u][c2]) X[u] = c2;
            return p;
        };
        fill(1 + all(X), 1);
        for (int t = 0; t < sz(E); ++t) {
            auto [u, v0] = E[t];
            int v = v0, c0 = X[u], c = c0, d;
            vector<pii> L;
            fill(1 + all(vst), 0);
            while (!G[u][v0]) {
                L.emplace_back(v, d = X[v]);
                if (!C[v][c]) {
                    for (int a = sz(L) - 1; a >= 0; --a)
                        c = color(u, L[a].first, c);
                } else if (!C[u][d]) {
                    for (int a = sz(L) - 1; a >= 0; --a)
                        color(u, L[a].first, L[a].second);
                } else if (vst[d]) break;
                else vst[d] = 1, v = C[u][d];
            }
            if (!G[u][v0]) {
                for (; v; v = flip(v, c, d), swap(c, d));
                if (int a; C[u][c0]) {
                    for (a = sz(L) - 2;
                        a >= 0 && L[a].second != c; --a);
                    for (; a >= 0; --a)
                        color(u, L[a].first, L[a].second);
                }
                else --t;
            }
        }
        void add_edge(int u, int v) { E.emplace_back(u, v); }
        Vizing(int _n) : n(_n), C(n + 1, vector<int>(n + 1)),
            G(n + 1, vector<int>(n + 1)), X(n + 1), vst(n + 1) {}
};

```

4.12 Maximum Clique [5ed877]

```

struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;

```

```

    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) a[i].reset();
    }
    void add_edge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0, m = sz
            (r);
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; ++i) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; ++k)
            for (int p = cs[k].Find_first(); p < N;
                p = cs[k].Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<N> nmask = mask & a[p];
            for (int i : r)
                if (a[p][i]) nr.pb(i);
            if (!nr.empty()) {
                if (1 < 4) {
                    for (int i : nr)
                        d[i] = (a[i] & nmask).count();
                    sort(nr.begin(), nr.end(),
                        [&](int x, int y) { return d[x] > d[y]; });
                }
                csort(nr, nc), dfs(nr, nc, l + 1, nmask);
            } else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), q--;
        }
    }
    int solve(bitset<N> mask = bitset<N>(),
        string(N, '1')) { // vertex mask
        vector<int> r, c;
        ans = q = 0;
        for (int i = 0; i < n; ++i)
            if (mask[i]) r.pb(i);
        for (int i = 0; i < n; ++i)
            d[i] = (a[i] & mask).count();
        sort(r.begin(), r.end(),
            [&](int i, int j) { return d[i] > d[j]; });
        csort(r, c), dfs(r, c, 1, mask);
        return ans; // sol[0 ~ ans-1]
    }
};

```

5 String

5.1 Aho-Corasick Automaton [77096b]

```

struct AC { // remember to build_fail!!!
    int ch[N][C], to[N][C], fail[N], cnt[N], _id;
    // fail link tree: fail[i] -> i
    AC() { reset(); }
    int newnode() {
        fill_n(ch[_id], C, 0), fill_n(to[_id], C, 0);
        fail[_id] = cnt[_id] = 0; return _id++;
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) {
            if (!ch[now][c - 'a'])
                ch[now][c - 'a'] = newnode();
            now = ch[now][c - 'a'];
        }
        cnt[now]++; return now;
    }
};

```

```

void build_fail() {
    queue<int> q;
    for (int i = 0; i < C; ++i) if (ch[0][i])
        q.push(ch[0][i]), to[0][i] = ch[0][i];
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (int i = 0; i < C; ++i) {
            if (!ch[v][i]) to[v][i] = to[fail[v]][i];
            else {
                int u = ch[v][i], k = fail[v];
                while (k && !ch[k][i]) k = fail[k];
                if (ch[k][i]) k = ch[k][i];
                fail[u] = k, cnt[u] += cnt[k], to[v][i] = u;
                q.push(u);
            }
        }
    }
}
// int match(string &s) {
//     int now = 0, ans = 0;
//     for (char c : s) {
//         now = to[now][c - 'a'];
//         ans += cnt[now];
//     }
//     return ans;
// }
void reset() { _id = 0, newnode(); }
} ac;

```

5.2 KMP Algorithm [9f8819]

```

auto build_fail(auto s) {
    vector<int> f(sz(s) + 1, 0);
    int k = 0;
    for (int i = 1; i < sz(s); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}
int match(auto s, auto t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < sz(s); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == sz(t)) ans++, k = f[k];
    }
    return ans;
}

```

5.3 Z Algorithm [e028f9]

```

auto buildZ(auto s) {
    int n = sz(s), l = 0, r = 0;
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]])
            l = i, r = i + Z[i], Z[i]++;
    }
    return Z;
}

```

5.4 Manacher [4e2fd6]

```

// return value only consider string tmp, not s
// return array length = 2N - 1
auto manacher(string tmp) {
    string s = "&";
    for (char c : tmp) s.pb(c), s.pb('%');
    int l = 0, r = 0, n = sz(s);
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
        while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
        if (Z[i] + i > r) l = i, r = Z[i] + i;
    }
    for (int i = 0; i < n; ++i)
        Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
    return vector<int>(1 + all(Z) - 1);
}

```

5.5 Suffix Array [58ed43]

```

auto sais(const auto &s) {
    const int n = sz(s), z = ranges::max(s) + 1;
    if (n == 1) return vector{0};
    vector<int> c(z); for (int x : s) c[x]++;
    partial_sum(all(c), c.begin());
    vector<int> sa(n); auto I = views::iota(0, n);
    vector<bool> t(n, true);
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && t[x] && !t[x - 1];
    });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y-- if (!t[y]) sa[x[s[y] - 1]++] = y;
        for (auto x = c; int y : sa | views::reverse)
            if (y-- if (t[y]) sa[--x[s[y]]] = y;
    };
    vector<int> lms, q(n); lms.reserve(n);
    for (auto x = c; int i : I | is_lms)
        q[i] = sz(lms), lms.pb(sa[--x[s[i]]] = i);
    induce(); vector<int> ns(sz(lms));
    for (int j = -1, nz = 0; int i : sa | is_lms) {
        if (j >= 0) {
            int len = min({n - i, n - j, lms[q[i] + 1] - i});
            ns[q[i]] = nz += lexicographical_compare(
                s.begin() + j, s.begin() + j + len,
                s.begin() + i, s.begin() + i + len);
        }
        j = i;
    }
    fill(all(sa), 0); auto nsa = sais(ns);
    for (auto x = c; int y : nsa | views::reverse)
        y = lms[y], sa[--x[s[y]]] = y;
    return induce(), sa;
} // 0eb2d2
struct Suffix {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    int n; vector<int> sa, lcp, rk;
    Suffix(auto _s) : n(sz(_s)), lcp(n), rk(n) {
        vector<int> s(n + 1); // s[n] = 0;
        for (int i = 0; i < n; ++i) s[i] = _s[i];
        // _s shouldn't contain 0
        sa = sais(s), sa.erase(sa.begin());
        for (int i = 0; i < n; ++i) rk[sa[i]] = i;
        for (int i = 0, h = 0; i < n; ++i) {
            if (!rk[i]) { h = 0; continue; }
            for (int j = sa[rk[i] - 1]; max(i, j) + h < n &&
                s[i + h] == s[j + h];) ++h;
            lcp[rk[i]] = h ? h - 1 : 0;
        }
    }
    //int queryLCP(int i, int j) {
    //    auto [l, r] = minmax({rk[i] + 1, rk[j] + 1});
    //    return lcp_range_min(l, r);
    //}
}; // 4422fa

```

5.6 Suffix Automaton [12d8e9]

```

struct SAM {
    int ch[2 * N][C], len[2 * N], link[2 * N], pos[2 * N],
        cnt[2 * N], _id;
    // node -> strings with the same endpos set
    // Length in range [len(link) + 1, len]
    // node's endpos set -> pos in the subtree of node
    // Link -> longest suffix with different endpos set
    // Len -> longest suffix
    // pos -> end position
    // cnt -> size of endpos set
    SAM() { reset(); }
    int newnode() {
        fill_n(ch[_id], C, 0);
        len[_id] = link[_id] = pos[_id] = cnt[_id] = 0;
        return _id++;
    }
    void build(string s) {
        int lst = 0;
        for (int i = 0; i < sz(s); ++i) {
            char c = s[i];

```

```

int cur = newnode();
len[cur] = len[lst] + 1, pos[cur] = i + 1;
int p = lst;
while (~p && !ch[p][c - 'a'])
    ch[p][c - 'a'] = cur, p = link[p];
if (p == -1) link[cur] = 0;
else {
    int q = ch[p][c - 'a'];
    if (len[p] + 1 == len[q]) {
        link[cur] = q;
    } else {
        int nxt = newnode();
        len[nxt] = len[p] + 1, link[nxt] = link[q];
        pos[nxt] = 0;
        for (int j = 0; j < C; ++j)
            ch[nxt][j] = ch[q][j];
        while (~p && ch[p][c - 'a'] == q)
            ch[p][c - 'a'] = nxt, p = link[p];
        link[q] = link[cur] = nxt;
    }
}
cnt[cur]++, lst = cur;
}
}
// void build_count() {
//     vector<int> p(_id);
//     iota(all(p), 0);
//     sort(all(p),
//          [&](int i, int j) {return len[i] > len[j];});
//     for (int i = 0; i < _id; ++i) if (~link[p[i]])
//         cnt[link[p[i]]] += cnt[p[i]];
// }
void reset() { _id = 0, newnode(), link[0] = -1; }
} sam;

```

5.7 Minimum Rotation [561109]

```

string rotate(const string &s) {
    int n = sz(s), i = 0, j = 1;
    string t = s + s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

5.8 Palindrome Tree [f67ae4]

```

struct PAM {
    int ch[N][C], cnt[N], fail[N], len[N], _id;
    // 0 -> even root, 1 -> odd root
    PAM () { reset(); }
    int newnode() {
        fill_n(ch[_id], C, 0);
        cnt[_id] = fail[_id] = len[_id] = 0;
        return _id++;
    }
    void build(string s) {
        int lst = 1;
        for (int i = 0; i < sz(s); ++i) {
            while (s[i - len[lst] - 1] != s[i])
                lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = newnode();
                len[idx] = len[lst] + 2;
                int now = fail[lst];
                while (s[i - len[now] - 1] != s[i])
                    now = fail[now];
                fail[idx] = ch[now][s[i] - 'a'];
                ch[lst][s[i] - 'a'] = idx;
            }
            lst = ch[lst][s[i] - 'a'], cnt[lst]++;
        }
    }
    void build_count() {
        for (int i = _id - 1; i > 1; --i)
            cnt[fail[i]] += cnt[i];
    }
}

```

```

}
void reset() { _id = 0, newnode(), newnode(),
    len[0] = 0, fail[0] = 1, len[1] = -1; }
} pam;

```

5.9 Lyndon Factorization [5e52cf]

```

// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
vector<string> duval(const string &s) {
    vector<string> ans;
    for (int n = sz(s), i = 0, j, k; i < n; ) {
        for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
            k = (s[k] < s[j] ? i : k + 1);
        for (; i <= k; i += j - k)
            ans.pb(s.substr(i, j - k)); // s.substr(L, Len)
    }
    return ans;
}

```

5.10 Main Lorentz [b38f07]

```

// [L, r, Len]: p in [L, r] => s[p, p + Len * 2] tandem
// you might need to compress manually
auto main_lorentz(string _s) {
    vector<array<int, 3>> rep;
    auto dfs = [&](auto self, string s, int sft) -> void
    {
        int n = sz(s);
        if (n == 1) return;
        int nu = n / 2, nv = n - nu;
        string u = s.substr(0, nu), v = s.substr(nu),
            ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.
                rend());
        self(self, u, sft), self(self, v, sft + nu);
        auto z1 = buildZ(ru), z2 = buildZ(v + '#' + u),
            z3 = buildZ(ru + '#' + rv), z4 = buildZ(v
                );
        auto get_z = [](vector<int> &z, int i) {
            return 0 <= i && i < sz(z) ? z[i] : 0; };
        auto add_rep = [&](bool left, int c, int l, int k1,
            int k2) {
            int L = max(1, 1 - k2), R = min(1 - left, k1);
            if (L > R) return;
            if (left) rep.pb({sft + c - R, sft + c - L, 1});
            else rep.pb({sft + c - R - 1 + 1, sft + c - L - 1
                + 1, 1});
        };
        for (int cntr = 0; cntr < n; cntr++) {
            int l, k1, k2;
            if (cntr < nu) {
                l = nu - cntr;
                k1 = get_z(z1, nu - cntr);
                k2 = get_z(z2, nv + 1 + cntr);
            } else {
                l = cntr - nu + 1;
                k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
                k2 = get_z(z4, (cntr - nu) + 1);
            }
            if (k1 + k2 >= 1)
                add_rep(cntr < nu, cntr, l, k1, k2);
        }
    };
    dfs(dfs, _s, 0);
    return rep;
}

```

6 Math

6.1 Miller Rabin / Pollard Rho [f7e2bb]

```

ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long
    double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022};
ll Pow(ll a, ll b, ll n) {
    ll res = 1;
    for (; b; b >>= 1, a = mul(a, a, n))
        if (b & 1) res = mul(res, a, n);
    return res;
}
bool check(ll a, ll d, int s, ll n) {

```

```

a = Pow(a, d, n);
if (a <= 1) return 1;
for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
    if (a == 1) return 0;
    if (a == n - 1) return 1;
}
return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, s++;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
} // 5761f3
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, t++;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(ll n) {
    if (n == 1) return;
    if (IsPrime(n)) return res[n]++, void(0);
    ll d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
} // 57e9e3

```

6.2 Ext GCD [a4b22d]

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    if (b == 0) return {1, 0};
    auto [y, x] = extgcd(b, a % b);
    return pair<ll, ll>(x, y - (a / b) * x);
}

```

6.3 Chinese Remainder Theorem [90d2ce]

```

pair<ll, ll> CRT(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return make_pair(-1, -1); // no sol
    m1 /= g, m2 /= g;
    pair<ll, ll> p = extgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return make_pair((res % lcm + lcm) % lcm, lcm);
}

```

6.4 PiCount [1db46f]

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i])
        for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.pb(i);
}
void init(){

```

```

sieve(V);
small_pi[0] = 0;
for(int i = 1; i < V; ++i)
    small_pi[i] = small_pi[i - 1] + isp[i];
for(int i = 0; i < M; ++i) dp[0][i] = i;
for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
    dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i - 1]];
}
ll phi(ll n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(1ll * primes[a - 1] * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
ll PiCount(ll n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
        small_pi[y];
    ll res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n /
        primes[a]) - PiCount(primes[a]) + 1, 0ll);
    return res;
}

```

6.5 Linear Function Mod Min [5552e3]

```

ll topos(ll x, ll m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    Log m)
ll min_rem(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    for (ll g = __gcd(a, m); g > 1; ) return g * min_rem(n
        , m / g, a / g, b / g) + (b % g);
    for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
        = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            ll lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
} // ab2d19
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(Log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
        % (m / g);
    return {mn, x};
} // 017ca5

```

6.6 Gauss Elimination [41dc4e]

```

auto gauss(vector<vector<int>> a, vector<int> b) {
    // solve ax = b
    int n = sz(a), m = sz(a[0]), rk = 0;
    vector<int> depv, free(m, true);
    for (int i = 0; i < m; ++i) {
        int p = -1;
        for (int j = rk; j < n; ++j)
            if (p == -1 || abs(a[j][i]) > abs(a[p][i]))
                p = j;
        if (p == -1 || a[p][i] == 0) continue;
        swap(a[p], a[rk]), swap(b[p], b[rk]);
        int inv = Pow(a[rk][i], mod - 2);
        for (int &x : a[rk]) x = mul(x, inv);
        b[rk] = mul(b[rk], inv);
    }

```



```

    for (int j = 0; j < n; ++j) if (j ^ rk) {
        int x = a[j][i];
        for (int k = 0; k < m; ++k)
            a[j][k] = sub(a[j][k], mul(x, a[rk][k]));
        b[j] = sub(b[j], mul(x, b[rk]));
    }
    depv.pb(i), free[i] = false, rk++;
}
vector<int> x; vector<vector<int>> h;
for (int i = rk; i < n; ++i) if (b[i] != 0)
    return make_pair(x, h); // not consistent
x.resize(m);
for (int i = 0; i < rk; ++i) x[depv[i]] = b[i];
for (int i = 0; i < m; ++i) if (free[i]) {
    h.emplace_back(m), h.back()[i] = 1;
    for (int j = 0; j < rk; ++j)
        h.back()[depv[j]] = sub(0, a[j][i]);
}
return make_pair(x, h); // solution = x + span(h[i])
}

```

6.7 Floor Sum [49de67]

```

// sum_{i=0}^{n-1} floor((a * i + b) / m) in Log(n + m + a + b)
// only works for a, b >= 0!!!
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

6.8 Quadratic Residue [51ec55]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ) {
        b = rand() % p;
        d = (11l * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (p + 1) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
            g1 = (g0 * f1 + g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
        f1 = (2 * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.9 Discrete Log [cf360f]

```

ll DiscreteLog(ll a, ll b, ll m) { // a^x = b (mod m)
    const int B = 35000;
    ll k = 1 % m, ans = 0, g;
    while ((g = gcd(a, m)) > 1) {

```

```

        if (b == k) return ans;
        if (b % g) return -1;
        b /= g, m /= g, ans++, k = (k * a / g) % m;
    }
    if (b == k) return ans;
    unordered_map<ll, int> m1;
    ll tot = 1;
    for (int i = 0; i < B; ++i)
        m1[tot * b % m] = i, tot = tot * a % m;
    ll cur = k * tot % m;
    for (int i = 1; i <= B; ++i, cur = cur * tot % m)
        if (m1.count(cur))
            return 11l * i * B - m1[cur] + ans;
    return -1;
}

```

6.10 Factorial without Prime Factor [c324f3]

```

// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k

```

6.11 Berlekamp Massey [3a60a6]

```

// need add, sub, mul
// find min |c| such that a_n = sum c_j * a_{n-j-1}, 0-based
// O(N^2), if |c| = k, |a| >= 2k sure correct
vector<int> BerlekampMassey(vector<int> a) {
    auto f = [&](vector<int> v, ll c) {
        for (int &x : v) x = mul(x, c); return v;
    };
    vector<int> c, best;
    int pos = 0, n = sz(a);
    for (int i = 0; i < n; ++i) {
        int error = a[i];
        for (int j = 0; j < sz(c); ++j)
            error = sub(error, mul(c[j], a[i - 1 - j]));
        if (error == 0) continue;
        int inv = Pow(error, mod - 2);
        if (c.empty()) {
            c.resize(i + 1), pos = i, best.pb(inv);
        } else {
            vector<int> fix = f(best, error);
            fix.insert(fix.begin(), i - pos - 1, 0);
            if (sz(fix) >= sz(c)) {
                best = f(c, sub(0, inv));
                best.insert(best.begin(), inv);
                pos = i, c.resize(sz(fix));
            }
            for (int j = 0; j < sz(fix); ++j)
                c[j] = add(c[j], fix[j]);
        }
    }
    return c;
}

```

6.12 Simplex [e34964]

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
    bool ls(T a, T b) {return a < b && !eq(a, b);}
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;

```

```

for (int i = 0; i < m; ++i)
    for (int j = 0; j < n; ++j) a[i][j] = 0;
for (int i = 0; i < m; ++i) b[i] = 0;
for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
}
void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector<int> nz;
    for (int i = 0; i < n; ++i) {
        a[x][i] /= k;
        if (!eq(a[x][i], 0)) nz.pb(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
        if (i == x || eq(a[i][y], 0)) continue;
        k = a[i][y], a[i][y] = 0;
        b[i] -= k * b[x];
        for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
}
// 0: found solution, 1: no feasible solution, 2:
// unbounded
int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (true) {
        int x = -1, y = -1;
        for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
        if (x == -1) break;
        for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
        if (y == -1) return 1;
        pivot(x, y);
    }
    while (true) {
        int x = -1, y = -1;
        for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
        if (y == -1) break;
        for (int i = 0; i < m; ++i)
            if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
        if (x == -1) return 2;
        pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n)
        sol[Left[i]] = b[i];
    return 0;
}
};

```

6.13 Euclidean

$$m = \lfloor \frac{an+b}{c} \rfloor$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) + h(a \bmod c, b \bmod c, c, n) + 2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) + 2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

6.14 Linear Programming Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.
 Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.
 \bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$

$$\bullet \sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$$

$$\bullet \sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$$

4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.15 Theorem

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Erdős-Gallai Theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

- Burnside's Lemma

Let X be a set and G be a group that acts on X . For $g \in G$, denote by X^g the elements fixed by g :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called bigraphic if there is a labeled simple bipartite graph such that a and b is the degree sequence of this bipartite graph.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called digraphic if there is a labeled simple directed graph such that each vertex v_i has indegree a_i and outdegree b_i .

- Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume $= \pi h^2 (3r - h) / 3 = \pi h (3a^2 + h^2) / 6 = \pi r^3 (2 + \cos \theta) (1 - \cos \theta)^2 / 3$.
- Area $= 2\pi r h = \pi (a^2 + h^2) = 2\pi r^2 (1 - \cos \theta)$.

6.16 Estimation

n	2	3	4	5	6	7	8	9	20	30	40	50	100		
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8		
n	100	1e3	1e6				1e9		1e12		1e15		1e18		
$d(i)$	12	32	240				1344		6720		26880		103680		
arg	60	840	720720	735134400	963761198400	866421317361600	8976124847866								
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8
n	2	3	4	5	6	7	8	9	10	11	12	13			
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7			

6.17 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. $k, j: s.t. \pi(j) > \pi(j+1), k+1, j: s.t. \pi(j) \geq j, k, j: s.t. \pi(j) > j.$

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.18 Calculus

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x \quad \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \quad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \quad \int x e^{ax} = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int \sin^2(x) = \frac{x}{2} - \frac{1}{4} \sin 2x \quad \int \sin^3 x = \frac{1}{12} \cos 3x - \frac{3}{4} \cos x$$

$$\int \cos^2(x) = \frac{x}{2} + \frac{1}{4} \sin 2x \quad \int \cos^3 x = \frac{1}{12} \sin 3x + \frac{3}{4} \sin x$$

$$\int \sin ax \cos ax = \frac{1}{2a} \sin^2(ax) \quad \int x \sin x \cos x = -\frac{x}{4} \cos 2x + \frac{1}{8} \sin 2x$$

$$\int x \sin x = \sin x - x \cos x \quad \int x \cos x = \cos x + x \sin x$$

$$\int x e^x = e^x (x - 1) \quad \int x^2 e^x = e^x (x^2 - 2x + 2)$$

$$\int x^2 \sin x = 2x \sin x - (x^2 - 2) \cos x$$

$$\int x^2 \cos x = 2x \cos x + (x^2 - 2) \sin x$$

$$\int e^x \sin x = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\int e^x \cos x = \frac{1}{2} e^x (\sin x + \cos x)$$

$$\int x e^x \sin x = \frac{1}{2} e^x (x \sin x - x \cos x + \cos x)$$

$$\int x e^x \cos x = \frac{1}{2} e^x (x \sin x + x \cos x - \sin x)$$

7 Polynomial

7.1 Number Theoretic Transform [36b2ce]

```
// mul, add, sub, Pow
struct NTT {
    int w[N];
    NTT() {
        int dw = Pow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i)
            w[i] = mul(w[i - 1], dw);
    } // 0 <= a[i] < P
    void operator()(vector<int>& a, bool inv = false) {
        int n = sz(a);
        for (int j = 1, x = 0; j < n - 1; ++j) {
            for (int k = n >> 1; (x ^= k) < k; k >>= 1);
            if (j < x) swap(a[j], a[x]);
        }
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    int tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(1 + all(a));
            int invn = Pow(n, mod - 2);
            for (int i = 0; i < n; ++i)
                a[i] = mul(a[i], invn);
        }
    }
} ntt;
```

7.2 Fast Fourier Transform [026694]

```
using T = complex<double>;
const double PI = acos(-1);
struct FFT {
    T w[N];
    FFT() {
        T dw = {cos(2 * PI / N), sin(2 * PI / N)};
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
    }
    void operator()(vector<T>& a, bool inv = false) {
        // see NTT, replace ll with T
        if (inv) {
            reverse(1 + all(a));
            T invn = 1.0 / n;
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
        }
    }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	922337203673735297	3

7.4 Polynomial Operations [e39ccf]

```
typedef vector<int> Poly;
Poly Mul(Poly a, Poly b, int bound = N) {
    int m = sz(a) + sz(b) - 1, n = 1;
    while (n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], b[i]);
    ntt(a, true), a.resize(min(m, bound));
}
```

```

    return a;
} // 8e2e8b
Poly Inverse(Poly a) {
    // O(NLogN), a[0] != 0
    int n = sz(a);
    Poly res(1, Pow(a[0], mod - 2));
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly v1(a.begin(), a.begin() + m * 2), v2 = res;
        v1.resize(m * 4), v2.resize(m * 4);
        ntt(v1), ntt(v2);
        for (int i = 0; i < m * 4; ++i)
            v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
        ntt(v1, true);
        res.resize(m * 2);
        for (int i = 0; i < m; ++i)
            res[i] = add(res[i], res[i]);
        for (int i = 0; i < m * 2; ++i)
            res[i] = sub(res[i], v1[i]);
    }
    res.resize(n);
    return res;
} // 4d79c8
pair<Poly, Poly> Divide(Poly a, Poly b) {
    // a = bQ + R, O(NLogN), b.back() != 0
    int n = sz(a), m = sz(b), k = n - m + 1;
    if (n < m) return {{0}, a};
    Poly ra = a, rb = b;
    reverse(all(ra)), ra.resize(k);
    reverse(all(rb)), rb.resize(k);
    Poly Q = Mul(ra, Inverse(rb), k);
    reverse(all(Q));
    Poly res = Mul(b, Q), R(m - 1);
    for (int i = 0; i < m - 1; ++i)
        R[i] = sub(a[i], res[i]);
    return {Q, R};
} // 7d15e3
Poly SqrtImpl(Poly a) {
    if (a.empty()) return {0};
    int z = QuadraticResidue(a[0], mod), n = sz(a);
    if (z == -1) return {-1};
    Poly q(1, z);
    const int inv2 = (mod + 1) / 2;
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        q.resize(m * 2);
        Poly f2 = Mul(q, q, m * 2);
        for (int i = 0; i < m * 2; ++i)
            f2[i] = sub(f2[i], a[i]);
        f2 = Mul(f2, Inverse(q), m * 2);
        for (int i = 0; i < m * 2; ++i)
            q[i] = sub(q[i], mul(f2[i], inv2));
    }
    q.resize(n);
    return q;
} // 984549
Poly Sqrt(Poly a) {
    // O(NLogN), return {-1} if not exists
    int n = sz(a), m = 0;
    while (m < n && a[m] == 0) m++;
    if (m == n) return Poly(n);
    if (m & 1) return {-1};
    Poly s = SqrtImpl(Poly(a.begin() + m, a.end()));
    if (s[0] == -1) return {-1};
    Poly res(n);
    for (int i = 0; i < sz(s); ++i)
        res[i + m / 2] = s[i];
    return res;
} // d1acd7
Poly Derivative(Poly a) {
    int n = sz(a);
    Poly res(n - 1);
    for (int i = 0; i < n - 1; ++i)
        res[i] = mul(a[i + 1], i + 1);
    return res;
} // 001be0
Poly Integral(Poly a) {
    int n = sz(a);
    Poly res(n + 1);
    for (int i = 0; i < n; ++i)
        res[i + 1] = mul(a[i], Pow(i + 1, mod - 2));
    return res;
}

```

```

} // 6fc53d
Poly Ln(Poly a) {
    // O(NLogN), a[0] = 1
    int n = sz(a);
    if (n == 1) return {0};
    Poly d = Derivative(a);
    a.pop_back();
    return Integral(Mul(d, Inverse(a), n - 1));
} // 377d20
Poly Exp(Poly a) {
    // O(NLogN), a[0] = 0
    int n = sz(a);
    Poly q(1, 1);
    a[0] = add(a[0], 1);
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly g(a.begin(), a.begin() + m * 2), h(all(q));
        h.resize(m * 2), h = Ln(h);
        for (int i = 0; i < m * 2; ++i)
            g[i] = sub(g[i], h[i]);
        q = Mul(g, q, m * 2);
    }
    q.resize(n);
    return q;
} // 525e8f
Poly PolyPow(Poly a, ll k) {
    int n = sz(a), m = 0;
    Poly ans(n, 0);
    while (m < n && a[m] == 0) m++;
    if (k && m && (k >= n || k * m >= n)) return ans;
    if (m == n) return ans[0] = 1, ans;
    int lead = m * k;
    Poly b(a.begin() + m, a.end());
    int base = Pow(b[0], k), inv = Pow(b[0], mod - 2);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], inv);
    b = Ln(b);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], k % mod);
    b = Exp(b);
    for (int i = lead; i < n; ++i)
        ans[i] = mul(b[i - lead], base);
    return ans;
} // 7d695a
vector<int> Evaluate(Poly a, vector<int> x) {
    if (x.empty()) return {};
    int n = sz(x);
    vector<Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector<Poly> down(n * 2);
    down[1] = Divide(a, up[1]).second;
    for (int i = 2; i < n * 2; ++i)
        down[i] = Divide(down[i >> 1], up[i]).second;
    Poly y(n);
    for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
    return y;
} // bff354
Poly Interpolate(vector<int> x, vector<int> y) {
    int n = sz(x);
    vector<Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    Poly a = Evaluate(Derivative(up[1]), x);
    for (int i = 0; i < n; ++i)
        a[i] = mul(y[i], Pow(a[i], mod - 2));
    vector<Poly> down(n * 2);
    for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
    for (int i = n - 1; i > 0; --i) {
        Poly lhs = Mul(down[i * 2], up[i * 2 + 1]);
        Poly rhs = Mul(down[i * 2 + 1], up[i * 2]);
        down[i].resize(sz(lhs));
        for (int j = 0; j < sz(lhs); ++j)
            down[i][j] = add(lhs[j], rhs[j]);
    }
    return down[1];
} // af80e7
Poly TaylorShift(Poly a, int c) {

```

```

// return sum a_i(x + c)^i;
// fac[i] = i!, facp[i] = inv(i!)
int n = sz(a);
for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
reverse(all(a));
Poly b(n);
int w = 1;
for (int i = 0; i < n; ++i)
    b[i] = mul(facp[i], w), w = mul(w, c);
a = Mul(a, b, n), reverse(all(a));
for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
return a;
} // 3a3763
vector<int> SamplingShift(vector<int> a, int c, int m){
// given f(0), f(1), ..., f(n - 1)
// return f(c), f(c + 1), ..., f(c + m - 1)
int n = sz(a); // 4d649d
for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
Poly b(n);
for (int i = 0; i < n; ++i) {
    b[i] = facp[i];
    if (i & 1) b[i] = sub(0, b[i]);
}
a = Mul(a, b, n);
for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
reverse(all(a));
int w = 1;
for (int i = 0; i < n; ++i)
    b[i] = mul(facp[i], w), w = mul(w, sub(c, i));
a = Mul(a, b, n);
reverse(all(a));
for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
a.resize(m), b.resize(m);
for (int i = 0; i < m; ++i) b[i] = facp[i];
a = Mul(a, b, m);
for (int i = 0; i < m; ++i) a[i] = mul(a[i], fac[i]);
return a;
} // 2e52c1

```

7.5 Fast Linear Recursion [8b69ed]

```

int FastLinearRecursion(vector<int> a, vector<int> c,
    ll k) {
// a_n = sigma c_j * a_{n - j - 1}, 0-based
// O(NlogNlogK), |a| = |c|
int n = sz(a);
if (k < n) return a[k];
vector<int> base(n + 1, 1);
for (int i = 0; i < n; ++i)
    base[i] = sub(0, c[n - i - 1]);
vector<int> poly(n);
(n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
auto calc = [&](vector<int> p1, vector<int> p2) {
// O(n^2) bruteforce or O(nlogn) NTT
return Divide(Mul(p1, p2), base).second; };
vector<int> res(n, 0); res[0] = 1;
for (; k; k >>= 1, poly = calc(poly, poly)) {
    if (k & 1) res = calc(res, poly);
}
int ans = 0;
for (int i = 0; i < n; ++i)
    ans = add(ans, mul(res[i], a[i]));
return ans;
}

```

7.6 Fast Walsh Transform

```

void fwt(vector<int> &a, bool inv = false) {
// and : x += y * (1, -1)
// or : y += x * (1, -1)
// xor : x = (x + y) * (1, 1/2)
//      y = (x - y) * (1, 1/2)
int n = __lg(sz(a));
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
        int x = a[j ^ (1 << i)], y = a[j];
        // do something
    }
}
}
vector<int> subs_conv(vector<int> a, vector<int> b) {
// c_i = sum_{j & k = 0, j | k = i} a_j * b_k

```

```

int n = __lg(sz(a));
vector ha(n + 1, vector<int>(1 << n));
vector hb(n + 1, vector<int>(1 << n));
vector c(n + 1, vector<int>(1 << n));
for (int i = 0; i < 1 << n; ++i) {
    ha[__builtin_popcount(i)][i] = a[i];
    hb[__builtin_popcount(i)][i] = b[i];
}
for (int i = 0; i <= n; ++i)
    or_fwt(ha[i]), or_fwt(hb[i]);
for (int i = 0; i <= n; ++i)
    for (int j = 0; i + j <= n; ++j)
        for (int k = 0; k < 1 << n; ++k)
            c[i + j][k] = add(c[i + j][k],
                mul(ha[i][k], hb[j][k]));
for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
vector<int> ans(1 << n);
for (int i = 0; i < 1 << n; ++i)
    ans[i] = c[__builtin_popcount(i)][i];
return ans;
}

```

8 Geometry

8.1 Basic

```

template<typename T> struct P {};
using Pt = P<ll>;
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
ll abs2(Pt a) { return a * a; }
double abs(Pt a) { return sqrt(abs2(a)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 &&
    sign((c - a) * (c - b)) <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
bool cmp(Pt a, Pt b)
{ return pos(a) == pos(b) ? sign(a ^ b) > 0 :
    pos(a) < pos(b); }
bool same_vec(Pt a, Pt b, int d) // d = 1: check dir
{ return sign(a ^ b) == 0 && sign(a * b) > d * 2 - 2; }
bool same_vec(Line a, Line b, int d)
{ return same_vec(a.b - a.a, b.b - b.a, d); }
Pt perp(Pt a) { return Pt(-a.y, a.x); } // CCW 90 deg
Pt ref(Pt a) { return pos(a) == 1 ? Pt(-a.x, -a.y) : a; }
// double part
double theta(Pt a)
{ return normalize(atan2(a.y, a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) // CCW
{ double c = cos(o), s = sin(o);
    return Pt(c * a.x - s * a.y, s * a.x + c * a.y); }
Pt proj_vec(Pt a, Pt b, Pt c) // vector ac proj to ab
{ return (b - a) * ((c - a) * (b - a)) / (abs2(b - a)); }
Pt proj_pt(Pt a, Pt b, Pt c) // point c proj to ab
{ return proj_vec(a, b, c) + a; }

```

8.2 SVG Writer

```

#ifdef ABS
class SVG { // SVG("test.svg", 0, 0, 10, 10)
    void p(string_view s) { o << s; }
    void p(string_view s, auto v, auto... vs) {
        auto i = s.find('$');
        o << s.substr(0, i) << v, p(s.substr(i + 1), vs...);
    }
    ofstream o; string c = "red";
public:
    SVG(auto f, auto x1, auto y1, auto x2, auto y2) : o(f) {
        p("<svg xmlns='http://www.w3.org/2000/svg' "
            "viewBox='$ $ $ $'>\n"
            "<style>{*stroke-width:0.5%;}</style>\n",
            x1, -y2, x2 - x1, y2 - y1); }
    ~SVG() { p("</svg>\n"); }
    void color(string nc) { c = nc; }
    void line(auto x1, auto y1, auto x2, auto y2) {
        p("<line x1='$' y1='$' x2='$' y2='$' stroke='$' />\n",
            c,

```



```

    x1, -y1, x2, -y2, c); }
void circle(auto x, auto y, auto r) {
    p("<circle cx='$' cy='$' r='$' stroke='$' "
      "fill='none'/>\n", x, -y, r, c); }
void text(auto x, auto y, string s, int w = 12) {
    p("<text x='$' y='$' font-size='$px'>$</text>\n",
      x, -y, w, s); }
}; // write wrapper for complex if use complex
#else
struct SVG { SVG(auto ...) {} }; // you know how to
#endif

```

8.3 Sort

```

// cmp in Basic: polar angle sort
// all points are on line ab. closer to a: front
bool cmp_line(Pt s, Pt t, Pt a, Pt b) {
    Pt v = a - b;
    if (sign(v.x)) return sign(s.x - t.x) == sign(v.x);
    else return sign(s.y - t.y) == sign(v.y);
} // 3dc688
// intersect points polar angle sort, deno: positive
bool cmp_fraction_polar(pair<Pt, ll> o, pair<Pt, ll> s,
    pair<Pt, ll> t) { // C^3 / C^2
    Pt u = s.first * o.second - o.first * s.second; //C^5
    Pt v = t.first * o.second - o.first * t.second; //C^5
    // u /= gcd(u.x, u.y) might lower the range to C
    return cmp(u, v);
} // 2d4450
struct Seg {
    Pt a, b; // a.x < b.x
    bool operator < (const Seg &o) const {
        if (a == o.a) return ori(o.b, a, b) == 1;
        if (a.x <= o.a.x) return ori(o.a, a, b) == 1;
        return ori(a, o.a, o.b) == -1;
    }
};
struct Polar_Seg {
    Pt a, b; // ori(Pt(0, 0), a, b) > 0
    bool operator < (const Polar_Seg &o) const {
        if (a == o.a) return (o.b, a, b) == -1;
        if (btwangle(Pt(0, 0), a, b, o.a, 0))
            return ori(o.a, a, b) == -1;
        return ori(a, o.a, o.b) == 1;
    }
};
struct Arc { // contain(a, b): circle a in circle b?
    Cir c; int s; // 0 -> up, 1 -> down
    bool operator < (const Arc &b) const {
        if (c.id == b.c.id) return s < b.s;
        if (contain(c, b.c)) return b.s == 1;
        else if (contain(b.c, c)) return s == 0;
        else if (c.o.y == b.c.o.y) return c.id < b.c.id;
        else return c.o.y > b.c.o.y;
    }
};

```

8.4 Intersections

```

// m=0: segment, m=1: ray from l.a to l.b, m=2: Line
bool lines_intersect_check(Line l1, int m1, Line l2,
    int m2, int strict) {
    auto on = [&](Line l, int m, Pt p) {
        if (ori(l.a, l.b, p) != 0) return false;
        if (m && abs2(l.a - p) > abs2(l.b - p)) return true;
        return m == 2 || sign((p - l.a) * (p - l.b)) <= -
            strict;
    };
    if (same_vec(l1, l2, 0)) {
        return on(l1, m1, l2.a) || on(l1, m1, l2.b) ||
            on(l2, m2, l1.a) || on(l2, m2, l1.b);
    }
    auto good = [&](Line l, int m, Line o) {
        if (m && abs((l.a - o.a) ^ (l.a - o.b)) > abs((l.b
            - o.a) ^ (l.b - o.b))) return true;
        return m == 2 || ori(l.a, o.a, o.b) * ori(l.b, o.a,
            o.b) == -1;
    };
    if (good(l1, m1, l2) && good(l2, m2, l1)) return 1;
    if (!strict) {
        if (m2 != 2 && on(l1, m1, l2.a)) return 1;
    }
}

```

```

    if (m2 == 0 && on(l1, m1, l2.b)) return 1;
    if (m1 != 2 && on(l2, m2, l1.a)) return 1;
    if (m1 == 0 && on(l2, m2, l1.b)) return 1;
}
return 0;
} // 56cc8d
// notice two lines are parallel
auto lines_intersect(Line a, Line b) {
    auto abc = (a.b - a.a) ^ (b.a - a.a);
    auto abd = (a.b - a.a) ^ (b.b - a.a);
    return make_pair((b.b * abc - b.a * abd), abc - abd);
} // 726acc
// res[0] -> res[1] and l.a -> l.b: same direction
vector<Pt> circle_line_intersect(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
        ) / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
        - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
} // b7bdce
// covered area of c1: arc from res[0] to res[1], CCW
vector<Pt> circles_intersect(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
        + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
        c2.r - c1.r * c1.r) / (2 * d2));
    double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
        * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
    Pt v = perp(c2.o - c1.o) * A / (2 * d2);
    if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
    return {u - v, u + v};
} // 0acf68
// return edge endpoint, at point -> ids are the same
vector<pii> convex_line_intersect(vector<Pt> &C, Line
    la) {
    auto dis = [&](int p)
    { return (la.b - la.a) ^ (C[p] - la.a); };
    auto gao = [&](int s) {
        return cyc_tsearch(sz(C), [&](int i, int j)
            { return sign(dis(i) - dis(j)) == s; });
    };
    int x = gao(1), y = gao(-1), n = sz(C);
    if (sign(dis(x)) < 0 || sign(dis(y)) > 0) return {};
    if (sign(dis(x)) == 0 || sign(dis(y)) == 0) {
        int v = ((sign(dis(x)) == 0 ? x : y) + n - 1) % n;
        vector<pii> vec;
        for (int i = 0; i < 3; ++i, v = (v + 1) % n)
            if (sign(dis(v)) == 0) vec.emplace_back(v, v);
        return vec;
    }
    auto get = [&](int l, int r, int s) {
        while ((l + 1) % n != r) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (sign(dis(m)) == s ? l : r) = m;
        }
        if (sign(dis(r)) == 0) return pii(r, r);
        return pii(l, r);
    };
    return {get(x, y, 1), get(y, x, -1)};
} // 5ddd35

```

8.5 Point Inside Check

```

// get edge index: check (0, a), (0, b) first
// then after binary search, check (a, b)
bool point_in_convex(vector<Pt> &C, Pt p, bool strict =
    true) {
    // only works when no three points are collinear
    int a = 1, b = sz(C) - 1, r = !strict;
    if (sz(C) == 0) return false;
    if (sz(C) < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r) return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
}

```

```

return ori(C[a], C[b], p) < r;
} // 722991
// -1: out, 0: edge, 1: in
int point_in_poly(vector<Pt> poly, Pt o, int strict) {
    int cnt = 0;
    for (int i = 0; i < sz(poly); ++i) {
        Pt a = poly[i], b = poly[(i + 1) % sz(poly)];
        if (btw(o, a, b)) return !strict;
        cnt ^= ((o.y < a.y) - (o.y < b.y)) * ori(o, a, b) > 0;
    }
    return cnt ? 1 : -1;
} // 94b56b
// return q's relation with circumcircle of tri(p[0], p[1], p[2])
bool point_in_cc(array<Pt, 3> p, Pt q) {
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) * ((p[(i + 1) % 3] - q) ^ (p[(i + 2) % 3] - q));
    return det > 0; // in: >0, on: =0, out: <0
} // cc76d3

```

8.6 Convex Hull [d490c0]

```

auto convex_hull(vector<Pt> pts) {
    sort(all(pts), [&](Pt a, Pt b) {
        return a.x == b.x ? a.y < b.y : a.x < b.x; });
    vector<Pt> ans = {pts[0]};
    for (int t = 0; t < 2; ++t, reverse(all(pts))) {
        for (int i = 1, m = sz(ans); i < sz(pts); ++i) {
            while (sz(ans) > m && ori(ans[sz(ans) - 2],
                ans.back(), pts[i]) <= 0) ans.pop_back();
            ans.pb(pts[i]);
        }
    }
    if (sz(ans) > 1) ans.pop_back();
    return ans;
}

```

8.7 Point Segment Distance [4249fd]

```

double point_segment_dist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1) * (p - q1)) >= 0)
        return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

8.8 Vector In Polygon [6dac08]

```

// ori(a, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(Pt a, Pt b, Pt c, Pt p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector{cur, p} in counter-clockwise order prv, cur, nxt
bool inside(Pt prv, Pt cur, Pt nxt, Pt p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}
// call "inside" not btwangle

```

8.9 Minkowski Sum [2ff069]

```

void reorder(vector<Pt> &P) {
    rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b) {
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    })), P.end());
}
auto minkowski(vector<Pt> P, vector<Pt> Q) {
    // P, Q: convex polygon, CCW order
    reorder(P), reorder(Q); int n = sz(P), m = sz(Q);
    P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
    vector<Pt> ans;
    for (int i = 0, j = 0; i < n || j < m; ) {
        ans.pb(P[i] + Q[j]);
        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
        if (val >= 0) i++;
    }
}

```

```

if (val <= 0) j++;
}
return ans;
}

```

8.10 Rotating SweepLine [56f0e2]

```

struct Event {
    Pt d; int u, v;
    bool operator < (const Event &o) {
        return sign(d ^ o.d) > 0;
    }
};
void rotating_sweepline(vector<Pt> pts) {
    int n = sz(pts);
    vector<int> ord(n), pos(n);
    vector<Event> e;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            e.pb({ref(pts[i] - pts[j]), i, j});
    sort(all(e));
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) {
        return (sign(pts[i].y - pts[j].y) == 0 ?
            pts[i].x < pts[j].x : pts[i].y < pts[j].y); });
    for (int i = 0; i < n; ++i) pos[ord[i]] = i;
    auto makeReverse = [&](auto v) {
        sort(all(v)), v.resize(unique(all(v)) - v.begin());
        vector<pii> segs;
        for (int i = 0, j = 0; i < sz(v); i = j) {
            for (; j < sz(v) && v[j].u - v[i].u <= j - i; ++j);
            segs.emplace_back(v[i], v[j - 1] + 1 + 1);
        }
        return segs;
    };
    for (int i = 0, j = 0; i < sz(e); i = j) {
        vector<int> tmp;
        for (; j < sz(e) && !(e[i].u < e[j].u); j++)
            tmp.pb(min(pos[e[j].u], pos[e[j].v]));
        for (auto [l, r] : makeReverse(tmp)) {
            reverse(ord.begin() + l, ord.begin() + r);
            for (int t = l; t < r; ++t) pos[ord[t]] = t;
            // update value here
        }
    }
}

```

8.11 Half Plane Intersection [f6c2b0]

```

/* Having solution, check size > 2 */
/* --^-- Line.a --^-- Line.b --^-- */
auto halfplane_intersection(vector<Line> arr) {
    auto area_pair = [&](Line a, Line b) {
        return make_pair((a.b - a.a) ^ (b.a - a.a),
            (a.b - a.a) ^ (b.b - a.a)); };
    auto isin = [&](Line l0, Line l1, Line l2) {
        // Check inter(l1, l2) strictly in l0
        auto [a02X, a02Y] = area_pair(l0, l2);
        auto [a12X, a12Y] = area_pair(l1, l2);
        if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
        return ((__int128)a02Y * a12X -
            (__int128)a02X * a12Y > 0; // C^4
    };
    sort(all(arr), [&](Line a, Line b) {
        if (same_vec(a, b, 1))
            return ori(a.a, a.b, b.b) < 0;
        return cmp(a.b - a.a, b.b - b.a); });
    deque<Line> dq(1, arr[0]);
    auto pop_back = [&](int t, Line p) {
        while (sz(dq) >= t && !isin(p, dq[sz(dq) - 2], dq.back()))
            dq.pop_back();
    };
    auto pop_front = [&](int t, Line p) {
        while (sz(dq) >= t && !isin(p, dq[0], dq[1]))
            dq.pop_front();
    };
    for (auto p : arr)
        if (!same_vec(dq.back(), p, 1))
            pop_back(2, p), pop_front(2, p), dq.pb(p);
    pop_back(3, dq[0]), pop_front(3, dq.back());
    return vector<Line>(all(dq));
}

```

8.12 Minimum Enclosing Circle [2db817]

```

Cir min_enclosing(vector<Pt> p) {
    random_shuffle(all(p));
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < sz(p); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i], r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}

```

8.13 Point Inside Triangle

```

// number of points p with a < p < b such that ori(p, a, b) < 0
int under(Pt a, Pt b) { }
// number of points with a < p < b and ori(p, a, b) = 0
int edge(Pt a, Pt b) { }
// check if this number is calculated
bool check(Pt p) { }
// number of points that strictly inside the triangle
int in_tri(array<Pt, 3> arr) {
    sort(all(arr), [&](Pt i, Pt j) {
        return i.x == j.x ? i.y < j.y : i.x < j.x; });
    auto [a, b, c] = arr;
    int x = ori(b, a, c);
    if (x == 0) return 0;
    if (x == 1) return under(a, b) + under(b, c) - under(a, c) - edge(a, c);
    return under(a, c) - under(a, b) - under(b, c) - edge(a, b) - edge(b, c) - check(b);
}

```

8.14 Heart [043c0d]

```

Pt circenter(Pt p0, Pt p1, Pt p2) {
    // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    double m = 2. * (x1 * y2 - y1 * x2);
    Pt center(0, 0);
    center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
} // 24710a
Pt incenter(Pt p1, Pt p2, Pt p3) {
    // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (p1 * a + p2 * b + p3 * c) / s;
} // 342b59
Pt masscenter(Pt p1, Pt p2, Pt p3) {
    return (p1 + p2 + p3) / 3;
}
Pt orthocenter(Pt p1, Pt p2, Pt p3) {
    return masscenter(p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2;
}

```

8.15 Tangents [277413]

```

auto circle_point_tangent(Cir c, Pt p) {
    vector<Line> res;
    double d_sq = abs2(p - c.o);
    if (sign(d_sq - c.r * c.r) == 0) {
        res.pb({p, p + perp(p - c.o)});
    } else if (d_sq > c.r * c.r) {
        double s = d_sq - c.r * c.r;
        Pt v = p + (c.o - p) * s / d_sq;
        Pt u = perp(c.o - p) * sqrt(s) * c.r / d_sq;
        res.pb({p, v + u});
        res.pb({p, v - u});
    }
}

```

```

return res;
} // 6af9a8
auto circles_tangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> res;
    double d_sq = abs2(c1.o - c2.o);
    if (sign(d_sq) == 0) return res;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return res;
    double h = sqrt(max((double)0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c + sign2 * h * v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) == 0)
            p2 = p1 + perp(c2.o - c1.o);
        res.pb({p1, p2});
    }
    return res;
} // 956705
/* The point should be strictly out of hull
return arbitrary point on the tangent line */
pii point_convex_tangent(vector<Pt> &C, Pt p) {
    auto gao = [&](int s) {
        return cyc_tsearch(sz(C), [&](int x, int y) {
            return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0, 63a82a

```

8.16 Convex Cut [1f8a27]

```

vector<Pt> cut(vector<Pt> poly, Pt s, Pt e) {
    vector<Pt> res;
    for (int i = 0; i < sz(poly); ++i) {
        Pt cur = poly[i], prv = i ? poly[i - 1] : poly.back();
        bool side = ori(s, e, cur) < 0;
        if (side != (ori(s, e, prv) < 0))
            res.pb(lines_intersect({s, e}, {cur, prv}));
        if (side) res.pb(cur);
    }
    return res;
}

```

8.17 Union of Circles [e5c3ee]

```

// notice identical circles, compare cross -> x if the precision is bad
auto circles_border(vector<Cir> c, int id) {
    vector<pair<Pt, int>> vec;
    int base = 0;
    for (int i = 0; i < sz(c); ++i) if (id != i) {
        if (sign(c[id].r - c[i].r) < 0 && abs2(c[id].o - c[i].o) <= (c[id].r - c[i].r) * (c[id].r - c[i].r)) base++;
        auto tmp = circles_intersect(c[id], c[i]);
        if (sz(tmp) == 2) {
            Pt l = tmp[0] - c[id].o, r = tmp[1] - c[id].o;
            vec.emplace_back(l, 1);
            vec.emplace_back(r, -1);
            if (cmp(r, l)) base++;
        }
    }
    vec.emplace_back(Pt(-c[id].r, 0), 0);
    sort(all(vec), [&](auto i, auto j) {
        return cmp(i.first, j.first);
    });
    vector<pair<Pt, Pt>> seg;
    Pt v = Pt(c[id].r, 0), lst = v;
    for (auto [cur, val] : vec) {
        if (base == 0) seg.emplace_back(lst, cur);
        lst = cur, base += val;
    }
    if (base == 0) seg.emplace_back(lst, v);
    for (auto &[l, r] : seg)
        l = l + c[id].o, r = r + c[id].o;
    return seg;
}

```

```

} // 95d3c6
double circles_union_area(vector<Cir> c) {
    double res = 0;
    for (int i = 0; i < sz(c); ++i) {
        auto seg = circles_border(c, i);
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
        for (auto [l, r] : seg) {
            double tl = theta(l - c[i].o), tr = theta(r - c[i].o);
            if (sign(tl - tr) > 0) tr += PI * 2;
            res += F(tr) - F(tl);
        }
    }
    return res / 2;
} // 22d249

```

8.18 Union of Polygons [c7ddf6]

```

// in CCW order, use index as tiebreaker when collinear
auto polys_border(vector<vector<Pt>> poly, int id) {
    auto get = [&](auto &p, int i) {
        return make_pair(p[i], p[(i + 1) % sz(p)]);
    };
    vector<pair<Pt, Pt>> seg;
    for (int e = 0; e < sz(poly[id]); ++e) {
        auto [s, t] = get(poly[id], e);
        vector<pair<Pt, int>> vec;
        vec.emplace_back(s, -1 << 30);
        vec.emplace_back(t, 1 << 30);
        for (int i = 0; i < sz(poly); ++i) {
            int st = find_if(all(poly[i]), [&](Pt p) {
                return ori(p, s, t) == 1; }) - poly[i].begin();
            if (st == sz(poly[i])) continue;
            for (int j = st; j < st + sz(poly[i]); ++j) {
                auto [a, b] = get(poly[i], j % sz(poly[i]));
                if (same_vec(a - b, s - t, -1)) {
                    if (ori(a, b, s) == 0 && same_vec(a - b, s -
                        t, 1) && i <= id) {
                        vec.emplace_back(a, -1);
                        vec.emplace_back(b, 1);
                    }
                } else {
                    int s1 = ori(a, s, t) == 1, s2 = ori(b, s, t) == 1;
                    if (s1 ^ s2) {
                        auto p = lines_intersect({a, b}, {s, t});
                        vec.emplace_back(p, s1 ? 1 : -1);
                    }
                }
            }
        }
        sort(all(vec), [&](auto i, auto j) {
            return cmp_line(i.first, j.first, s, t); });
        int base = 1 << 30; Pt lst(0, 0);
        for (auto [cur, val] : vec) {
            if (!base) seg.emplace_back(lst, cur);
            lst = cur, base += val;
        }
    }
    return seg;
} // 704477
double polys_union_area(vector<vector<Pt>> poly) {
    double res = 0;
    for (int i = 0; i < sz(poly); ++i) {
        auto seg = polys_border(poly, i);
        for (auto [l, r] : seg) res += 1 ^ r;
    }
    return res / 2;
} // d055fb

```

8.19 Delaunay Triangulation [953c88]

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle. */
struct Edge {
    int id; // oidx[id]
    list<Edge>::iterator twin;
    Edge (int _id = 0) : id(_id) {}
};
struct Delaunay { // 0-base

```

```

int n;
vector<int> oidx;
vector<list<Edge>> head; // result udir. graph
vector<Pt> p;
Delaunay (vector<Pt> _p) : n(sz(_p)), oidx(n), head(n)
    , p(_p) {
    iota(all(oidx), 0);
    sort(all(oidx), [&](int a, int b) {
        return make_pair(_p[a].x, _p[a].y) < make_pair(_p[
            b].x, _p[b].y); });
    for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
    divide(0, n - 1);
}
void add_edge(int u, int v) {
    head[u].push_front(Edge(v));
    head[v].push_front(Edge(u));
    head[u].begin()->twin = head[v].begin();
    head[v].begin()->twin = head[u].begin();
}
void divide(int l, int r) {
    if (l == r) return;
    if (l + 1 == r) return add_edge(l, l + 1);
    int mid = (l + r) >> 1, nw[2] = {l, r};
    divide(l, mid), divide(mid + 1, r);
    auto gao = [&](int t) {
        Pt pts[2] = {p[nw[0]], p[nw[1]]};
        for (auto it : head[nw[t]]) {
            int v = ori(pts[1], pts[0], p[it.id]);
            if (v > 0 || (v == 0 && abs2(pts[t ^ 1] - p[it.
                id]) < abs2(pts[1] - pts[0])))
                return nw[t] = it.id, true;
        }
        return false;
    };
    while (gao(0) || gao(1));
    add_edge(nw[0], nw[1]); // add tangent
    while (true) {
        Pt pts[2] = {p[nw[0]], p[nw[1]]};
        int ch = -1, sd = 0;
        for (int t = 0; t < 2; ++t)
            for (auto it : head[nw[t]])
                if (ori(pts[0], pts[1], p[it.id]) > 0 && (ch
                    == -1 || point_in_cc({pts[0], pts[1], p[
                        ch]}, p[it.id])))
                    ch = it.id, sd = t;
        if (ch == -1) break; // upper common tangent
        for (auto it = head[nw[sd]].begin(); it != head[
            nw[sd]].end(); )
            if (lines_intersect_check({pts[sd], p[it->id]},
                0, {pts[sd ^ 1], p[ch]}, 0, 1))
                head[it->id].erase(it->twin), head[nw[sd]].
                    erase(it++);
        else ++it;
        nw[sd] = ch, add_edge(nw[0], nw[1]);
    }
}

```

8.20 Triangulation Voronoi [46f248]

```

// all coord. is even, half plane intersection
auto build_voronoi_line(vector<Pt> arr) {
    int n = sz(arr);
    Delaunay tool(arr);
    vector<vector<Line>> vec(n);
    for (int i = 0; i < n; ++i)
        for (auto e : tool.head[i]) {
            int u = tool.oidx[i], v = tool.oidx[e.id];
            Pt m = (arr[v] + arr[u]) / 2, d = perp(arr[v] -
                arr[u]);
            vec[u].pb(Line{m, m + d});
        }
    return vec;
}

```

8.21 External Bisector [cafb92]

```

Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
    Pt L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}

```

8.22 Intersection Area of Polygon and Circle [000043]

```
double _area(Pt pa, Pt pb, double r){
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
    double cosC = (pa * pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < PI / 2) S -= (acos(h / r) * r * r
            - h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = PI - B - asin(sin(B) / r * a);
        S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
            * r;
    } else S = 0.5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
    double S = 0; int n = sz(poly);
    for (int i = 0; i < n; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
            ori(O, poly[i], poly[(i + 1) % n]);
    return fabs(S);
}
```

8.23 3D Point

```
struct Pt {
    double x, y, z;
    Pt(double _x = 0, double _y = 0, double _z = 0): x(_x
        ), y(_y), z(_z){}
    Pt operator + (const Pt &o) const
    { return Pt(x + o.x, y + o.y, z + o.z); }
    Pt operator - (const Pt &o) const
    { return Pt(x - o.x, y - o.y, z - o.z); }
    Pt operator * (const double &k) const
    { return Pt(x * k, y * k, z * k); }
    Pt operator / (const double &k) const
    { return Pt(x / k, y / k, z / k); }
    double operator * (const Pt &o) const
    { return x * o.x + y * o.y + z * o.z; }
    Pt operator ^ (const Pt &o) const
    { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x
        * o.y - y * o.x)}; }
};
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
Pt cross3(Pt a, Pt b, Pt c)
{ return (b - a) ^ (c - a); }
double area(Pt a, Pt b, Pt c)
{ return abs(cross3(a, b, c)); }
double volume(Pt a, Pt b, Pt c, Pt d)
{ return cross3(a, b, c) * (d - a); }
bool coplaner(Pt a, Pt b, Pt c, Pt d)
{ return sign(volume(a, b, c, d)) == 0; }
Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
{ Pt n = cross3(a, b, c);
    return o - n * ((o - a) * (n / abs2(n))); }
Pt line_plane_intersect(Pt u, Pt v, Pt a, Pt b, Pt c) {
    // intersection of line uv and plane abc
    Pt n = cross3(a, b, c);
    double s = n * (u - v);
    if (sign(s) == 0) return {-1, -1, -1}; // not found
    return v + (u - v) * ((n * (a - v)) / s); }
```

8.24 3D Convex Hull [cc038d]

```
struct Face {
    int a, b, c;
    Face(int _a, int _b, int _c) : a(_a), b(_b), c(_c) {}
};
auto preprocess(auto pts) {
    auto G = pts.begin();
    vector<int> id;
    auto fail = tuple{-1, -1, -1, id};
    int a = find_if(all(pts), [&](Pt z) {
```

```
    return z != *G; }) - G;
    if (a == sz(pts)) return fail;
    int b = find_if(all(pts), [&](Pt z) {
        return cross3(*G, pts[a], z) != Pt(0, 0, 0); }) - G;
    if (b == sz(pts)) return fail;
    int c = find_if(all(pts), [&](Pt z) {
        return sign(volume(*G, pts[a], pts[b], z)) != 0; })
        - G;
    if (c == sz(pts)) return fail;
    for (int i = 0; i < sz(pts); i++)
        if (i != a && i != b && i != c) id.pb(i);
    return tuple{a, b, c, id};
}
// return the faces with pts indexes
vector<Face> convex_hull_3D(vector<Pt> pts) {
    int n = sz(pts);
    if (n <= 3) return {}; // be careful about edge case
    vector<Face> now;
    vector<vector<int>> z(n, vector<int>(n));
    auto [a, b, c, ord] = preprocess(pts);
    if (a == -1) return {};
    now.emplace_back(a, b, c); now.emplace_back(c, b, a);
    for (auto i : ord) {
        vector<Face> nxt;
        for (auto &f : now) {
            auto v = volume(pts[f.a], pts[f.b], pts[f.c], pts
                [i]);
            if (sign(v) <= 0) nxt.pb(f);
            z[f.a][f.b] = z[f.b][f.c] = z[f.c][f.a] = sign(v)
                ;
        }
        auto F = [&](int x, int y) {
            if (z[x][y] > 0 && z[y][x] <= 0)
                nxt.emplace_back(x, y, i);
        };
        for (auto &f : now)
            F(f.a, f.b), F(f.b, f.c), F(f.c, f.a);
        now = nxt;
    }
    return now;
}
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case
// test @ SPOJ CH3D
// double area = 0, vol = 0; // surface area / volume
// for (auto [a, b, c]: faces)
//     area += abs(ver(p[a], p[b], p[c]))/2.0,
//     vol += volume(P3(0, 0, 0), p[a], p[b], p[c])/6.0;
```

9 Else

9.1 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue<int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1); // 4 0
bitset<100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope<int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```
ll next_perm(ll v) { ll t = v | (v - 1);
    return (t + 1) |
        (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1)); }
```


9.3 Dynamic Programming Condition

9.3.1 Totally Monotone (Concave/Convex)

$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
 $\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

9.3.2 Monge Condition (Concave/Convex)

$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
 $\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

9.3.3 Optimal Split Point

If $B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$

then $H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$

9.4 Smawk Algorithm [6edb6c]

```
ll f(int l, int r) {
bool select(int r, int u, int v) {
    // if f(r, v) is better than f(r, u), return true
    return f(r, u) < f(r, v);
}
// For all 2x2 submatrix: (x < y => y is better than x)
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
vector<int> solve(vector<int> &r, vector<int> &c) {
    const int n = sz(r);
    if (n == 0) return {};
    vector<int> c2;
    for (const int &i : c) {
        while (!c2.empty() && select(r[sz(c2) - 1], c2.back(), i)) c2.pop_back();
        if (sz(c2) < n) c2.pb(i);
    }
    vector<int> r2;
    for (int i = 1; i < n; i += 2) r2.pb(r[i]);
    const auto a2 = solve(r2, c2);
    vector<int> ans(n);
    for (int i = 0; i < sz(a2); i++)
        ans[i * 2 + 1] = a2[i];
    int j = 0;
    for (int i = 0; i < n; i += 2) {
        ans[i] = c2[j];
        const int end = i + 1 == n ? c2.back() : ans[i + 1];
        while (c2[j] != end) {
            j++;
            if (select(r[i], ans[i], c2[j])) ans[i] = c2[j];
        }
    }
    return ans;
}
vector<int> smawk(int n, int m) {
    vector<int> row(n), col(m);
    iota(all(row), 0), iota(all(col), 0);
    return solve(row, col);
}
```

9.5 Slope Trick [4969a5]

```
template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
    slope_trick_convex() {left.push(numeric_limits<T>::min() / 2), right.push(numeric_limits<T>::max() / 2);}
    void push_left(T x) {left.push(x - ground_l);}
    void push_right(T x) {right.push(x - ground_r);}
    //add a line with slope 1 to the right starting from x
    void add_right(T x) {
        T l = left.top() + ground_l;
        if (l <= x) push_right(x);
        else push_left(x), push_right(l), left.pop(), minn += 1 - x;
    }
    //add a line with slope -1 to the left starting from x
}
```

```
void add_left(T x) {
    T r = right.top() + ground_r;
    if (r >= x) push_left(x);
    else push_right(x), push_left(r), right.pop(), minn += x - r;
}
//val[i]=min(val[j]) for all i-l<=j<=i+r
void expand(T l, T r) {ground_l -= 1, ground_r += r;}
void shift_up(T x) {minn += x;}
T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() + ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.pb(left.top());
            left.pop();
            if (left.top() + ground_l <= x) {
                res = cur_val + slope * (1 - x);
                break;
            }
            cur_val += slope * (1 - (left.top() + ground_l));
            l = left.top() + ground_l;
            slope += 1;
        }
        for (auto i : trash) left.push(i);
        return res;
    }
    if (x > r) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.pb(right.top());
            right.pop();
            if (right.top() + ground_r >= x) {
                res = cur_val + slope * (x - r);
                break;
            }
            cur_val += slope * ((right.top() + ground_r) - r);
            r = right.top() + ground_r;
            slope += 1;
        }
        for (auto i : trash) right.push(i);
        return res;
    }
    assert(0);
}
};
```

9.6 ALL LCS [ba9cc9]

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(sz(t));
    iota(all(h), 0);
    for (int a = 0; a < sz(s); ++a) {
        int v = -1;
        for (int c = 0; c < sz(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS[s[0, a], t[b, c]] =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

9.7 Hilbert Curve [1274a3]

```
ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 1ll * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
}
```

```
return res;
} // n = 2^k
```

9.8 Line Container [673ffd]

```
// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a
        % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) { // ax + b
        auto z = insert({a, b, 0}); y = z++; x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

9.9 Min Plus Convolution [57ecb0]

```
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector
<int> &b) {
    int n = sz(a), m = sz(b);
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from);
        Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

9.10 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.11 Simulated Annealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans: answer, nw: current value, rnd(): mt19937 rnd
    ()
    if (exp(-(nw - ans) / factor) >= (double)(rnd() %
        base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

9.12 Bitset LCS

```
cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';
```

9.13 Binary Search On Fraction [765c5a]

```
struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !!len;
    }
    return dir ? hi : lo;
}
```

9.14 Cyclic Ternary Search [9017cc]

```
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(1, r % n) ? l : r % n;
}
```

9.15 Tree Hash [34aae5]

```
ull seed;
ull shift(ull x) { x ^= x << 13; x ^= x >> 7;
    x ^= x << 17; return x; }
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u]) if (i != f)
        sum += shift(dfs(i, u));
    return sum;
}
```

9.16 Python Misc

```
from [decimal, fractions, math, random] import *
arr = list(map(int, input().split())) # input
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
    ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
S = set(), S.add((a, b)), S.remove((a, b)) # set
if not (a, b) in S:
    D = dict(), D[(a, b)] = 1, del D[(a, b)] # dict
for (a, b) in D.items():
    arr = [randint(1, C) for i in range(N)]
    choice([8, 6, 4, 1]) # random pick one
```