Numerical Techiques in Physics

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§ WEEK 1 §

Problem 1: Setting up C++ and Gnuplot

Our very first task would be to setup C++ as it would be our primary coding language.

C++ is a general-purpose, object-oriented programming language that is used to build software for a variety of purposes.

In CS101, we used Simplecpp. But we are grownups now, we will move on to VS Code. Setup cpp and GCC C++ compiler (g++) compiler in VS Code. Follow this tutorial: https://code.visualstudio.com/docs/cpp/config-mingw#_prerequisites

I expect you to know C++, so we are going to skip the tutorial part. But if you want, you can look over some tutorials online.

We will use GNUplot to make plots. Here is the installation tutorial: https://spoken-tutorial.org/media/videos/110/Gnuplot-Installation-Sheet-English.pdf

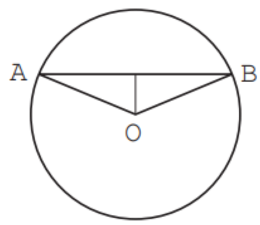
Let's start the problems.

Problem 2: Newton Raphson Method

Resource:

- https://www.geeksforgeeks.org/newton-raphson-method/

1. Find a root of the equation $x^2 - 8x + 11 = 0$ to 5 decimal places using $x_0 = 6$.
2. The circle below has radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to 18 decimal places.



Problem 3: Secant Method

Resources:

- https://www.geeksforgeeks.org/secant-method-of-numerical-analysis/

Use the secant method to find the roots of the function $f(x) = e^{2x} + x - 5$ with $x_0 = 0$ and $x_1 = 1$.

Problem 4: Euler Method

Resources:

- https://tutorial.math.lamar.edu/classes/de/eulersmethod.aspx

Use Euler's method with step sizes $h = 0.1$, $h = 0.05$, and $h = 0.025$ to find approximate values of the solution of the initial value problem

$$y' + 2y = x^3 e^{-2x}, \quad y(0) = 1$$

at $x = 0, 0.1, 0.2, 0.3, \ldots, 1.0$. Compare these approximate values with the values of the exact solution

$$y = \frac{e^{-2x}}{4}\left(x^4 + 4\right).$$

Plot the approximate solutions obtained using Euler's method for each step size h and compare them with the graph of the exact solution.

Problem 5: Runge Kutta 4th order method

Resources:

- https://math.libretexts.org/Courses/Monroe_Community_College/MTH_225_Differential_Equations/03%3A_Numerical_Methods/3.03%3A_The_Runge-Kutta_Method

Solve the above equation using the RK4 method. Plot the RK4 solution and the exact solution and compare your results. *Choose* $h = 0.1$

Problem 6: Solving system of linear equations numerically

Resource:

- https://en.wikipedia.org/wiki/Jacobi_method
- https://en.wikipedia.org/wiki/Gauss%E2%80%93Seidel_method

Solve the following system of linear equations using the Jacobi and Gauss-Seidel methods.

$$4x - y - z = 3 \tag{6.1}$$
$$-2x + 6y + z = 9 \tag{6.2}$$
$$-x + y + 7z = -6 \tag{6.3}$$

Choose the initial guess $\mathbf{x}^{(0)} = (0, 0, 0)$. For convergence use a maximum of 100 iterations or a tolerance of 10^{-5} i.e. $|\mathbf{x}^{(i+1)} - \mathbf{x}^{(i)}| < 10^{-5}$.

Try to make this as general as possible as we will use this for higher dimension matrices and by higher I mean around 10000×10000.