Gaussian Processes

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1 Random vector

2 Brownian motion

$$\text{Brownian motion}(B_t) = \text{Wiener process } (W_t)$$

Proposition.

$$\exists \lim_{n\to\infty} P_{ij}(n) = \pi_j^* > 0$$

$$\sum_{j=1}^{M} \pi_j^* = 1$$

Quizzes

(Quiz 1). Let X_t be a Brownian motion. Find

$$K(t,s) - Var(X_{min(t,s)})$$

(Answer) Trivially 0, since

$$Var(X_{min(t,s)}) = min(t,s)$$

(Quiz 2). Let $Y_{n+1} := aY_n + X_n$, where $n = 0, 1, \cdots$, $Y_0 := 0$, $|a| < 1$, $X_0, X_1, \cdots \overset{iid}{\sim} N(0,1)$. Find $cov(Y_4, Y_3)$.

(Answer) Note that

$$\begin{aligned} Cov(Y_4, Y_3) &= cov(a^3X_0 + a^2X_1 + aX_2 + X_3, a^2X_0 + aX_1 + X_2) \\ &= a^5 + a^3 + a \end{aligned}$$

(Quiz 3).

$$K(t,s) - Var(X_{min(t,s)})$$

(Answer) Trivially 0, since

$$Var(X_{min(t,s)}) = min(t,s)$$