Brownian Motions

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1 Joint Probabilities for Brownian Motion

Theorem 2.1. *The conditional density of* $X(t)$ *for* $t_1 < t < t_2$ *given* $X(t_1) = A$, $X(t_2) = B$ *is a normal density with the mean*

$$A + \frac{B - A}{t_2 - t_1}(t - t_1)$$

and the variance

$$\frac{(t_2 - t)(t - t_1)}{t_2 - t - 1}$$

Proof. Let H be the interoccurrence distribution for $N(t)$. Then

1.1 Continuity of Paths and the Maximum Variables

The physical origins of the Brownian motion process suggest that the possible realizations $X(t)$, (i.e. **sample path**) whose movements result from continuous collisions in the surronding medium are continuous functions.

Theorem 3.1. *The probability that $X(t)$ has at least one zero in the interval (t_0, t_1), given $X(0) = 0$, is*

$$\alpha = \frac{2}{\pi} \arccos \sqrt{t_0/t_1}$$

1.2 Variations and Extensions

If $X(t)$ is a standard Brownian motion process, then the processes

$$X_1(t) = cX(t/c^2)$$

$$X_2(t) = \begin{cases} tX(1/t) & t > 0 \\ 0 & t = 0 \end{cases}$$

$$X_3(t) = X(t+h) - X(h)$$

for $c > 0, h > 0$.

1.3 Brownian Motion Absorbed at the Origin