Q1. [5 marks] Plot the following signals:

1 (a) $\sin\left(\frac{\pi}{2}\delta[n]\right)$

1 (b) $u[n] * (\delta[n-2] - \delta[n+1])$

1.5 (c) $\left(\sum_{k=-N}^{N}\delta[n-k]\right)\cos\left(\frac{\pi}{2}n\right)$

1.5 (d) $\delta[n] * \delta[n-1] * \cdots * \delta[n-N]$ $= \delta[n - \sum n]$

$\sin(\omega_0(n-\Delta)) - \sin(\omega_0 n - \omega_0\Delta)$

$\boxed{\omega_0\Delta = \pi/2}\ \forall \omega_0$

Q2. [5 marks] Consider the following description of an LTI system – when the input signal is $\sin(\omega_0 n)$, the output of the system is $\cos(\omega_0 n)$ for all frequencies $\omega_0 \in (-\pi, \pi]$. $(\omega_0 \neq 0, \pi)$

1 (a) What are eigensignals of an LTI system?

4 (b) Using the given information, find and plot the frequency response of this system.

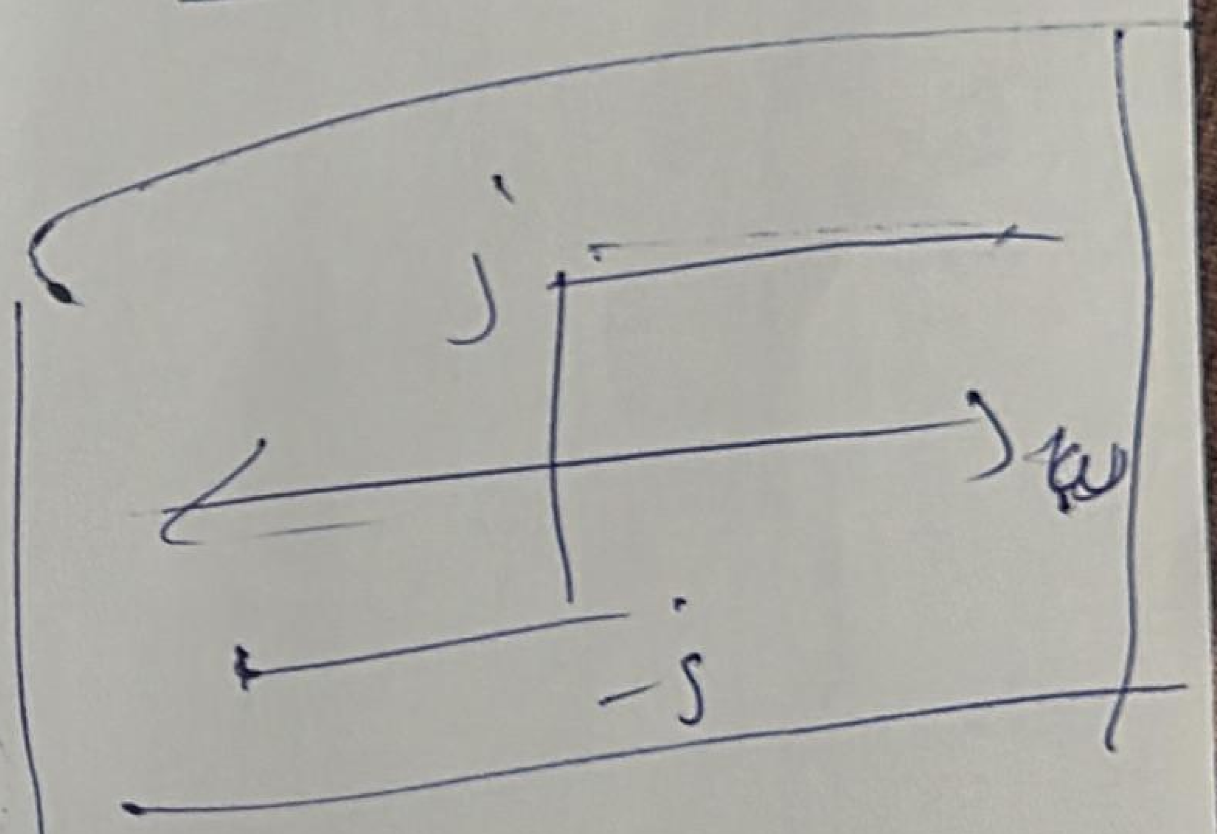
$$\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \longrightarrow \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$\omega_0 > 0$

ω_0: $\frac{1}{2j} \to \frac{1}{2}$

$H(\omega) = j$

$-\omega_0$: $-\frac{1}{2j} \to \frac{1}{2}$



$-\sin(\omega_0 n) \to \cos(\omega_0 n)$

$\omega_0 < 0$

Q3. [8 marks] An LTI system is given by the following difference equation:

$$y[n] = x[n] - \alpha y[n-1],\ \alpha \in \mathbb{R}.$$

3 (a) Assuming the condition of initial rest, find the impulse response of this system.

2 (b) Find range of α such that this system is stable.

3 (c) Analyze this stable system in terms of its frequency selectivity, i.e., low pass, high pass, etc. and its dependence on α.

Q4. [2 marks]

When a continuous-time periodic square wave with period T is given as input to some systems, the following signals are observed at the output.

(a) Output of system A – sinusoid with period T

(b) Output of system B – sinusoid with period $T/2$

(c) Output of system C – sinusoid with period $2T$.

Which of these systems may not be an LTI system? Justify your answer.

$a_k \rightarrow \boxed{k\omega_0}$ $\quad k \in \mathbb{Z}$

$k \cdot \frac{2\pi}{T}$

(1) $\omega = \boxed{\frac{2\pi}{T}}$

(2) $\frac{2\pi}{T/2} = 2 \boxed{\frac{2\pi}{T}}$

(3) $\frac{2\pi}{2T} = \frac{1}{2} \boxed{\frac{2\pi}{T}}$