Q1. [10 marks]
Consider the following transformation of points in s-plane to z-plane to design an IIR digital filter, $s = \frac{z}{1+\alpha z}$ where $\alpha > 0$ is some real number.

(a) [3] Find the range of α such that this transformation maps all the points on the imaginary axis in the s-plane to points inside or on the unit circle in the z-plane.

(b) [3] For the above range of α, show that points on the imaginary axis map to a circle in the z-plane. Find the center and radius of this circle as function of α.

(c) [4] Consider the analog filter with transfer function $H(s) = \frac{1}{s+1}$. Find the transfer function of the corresponding digital filter obtained using the above method and identify its poles and zeros. Comment on the frequency selective nature of this digital filter and analyze its behavior as a function of α using geometrical arguments.

Q2. [10 marks] An LTI system H has the following behaviour – when the input signal is $(\frac{1}{2})^n u[n]$, the output is $u[n]$. last.

(a) [4] Find and plot the impulse response of this system.

(b) [2] Analyze causality and stability of this system.

(c) [4] Find the output of this system if the input signal is $2^n u[n]$.

(-1)

Q3. [10 marks] Siva wishes to design a length $L = 3$ causal FIR digital filter with symmetric structure. Answer the following questions to help his design process.

$\frac{6\pi}{2\times 2} - \frac{\pi}{4}$

(a) [4] Find the filter coefficients if the filter has linear phase and the desired gains (magnitude response) at frequencies $\omega = 0$ and $\omega = \frac{\pi}{2}$ are 2 and 1 respectively.

$-\frac{7\pi}{4}$

$-1)$

(b) [2] Find and sketch the magnitude and phase spectrum plots for this filter.

(c) [4] Let the designed filter have z-transform $H(z)$. Madhuri is trying to implement a causal inverse for this system, i.e., the filter $\frac{1}{H(z)}$. Find the pole-zero plot and write down the difference equation corresponding to this system. Investigate its stability and find its output when the input is $\delta[n] + \delta[n-1]$.

$\frac{\pi}{2} + \tan^{-1}(1)$

$\pi/4$

$|e^{j\omega}| \leq 1$

$\cos\omega - 2\cos^2\omega$

Q4. [10 marks] Describe the decimation-in-time radix-2 FFT algorithm along with a diagram. Explain with calculations how it leads to computational gains when finding the DFT of a signal of length $N = 2^r$. Apply the algorithm to compute the 8-length DFT of the signal $\{1, 1, 1, 1, 0, 0, 0, 0\}$.

$x[-n] = \{ 0$

Q5. [10 marks] Consider the continuous-time signal $e^{j2\pi t}$.

(a) [2] Plot samples of this signal when sampled at 1 Hz, 2 Hz, and 4 Hz respectively.

(b) [3] Find and plot the DTFT of these three sampled signals.

(c) [5] **If ideal reconstruction is performed, find the continuous-time signals obtained from each of the sampled signals. Explain your analysis using appropriate frequency domain plots. Assume that the low pass filter used during ideal reconstruction has cut-off frequency equal to 49% of the corresponding sampling frequency.**

$\cos(2\pi t) + j\sin(2\pi t)$

$t = 1/4$

Q6. [10 marks] Let the N-point DFT of the N-length signal $x[n]$ be denoted by $X[k]$. Consider the $2N$-length signal $y[n] = [x[n], x[-n]]$ obtained by concatenating $x[n]$ and $x[-n]$. Let the $2N$-point DFT of $y[n]$ be denoted $Y[k]$.

(a) [6] Given $Y[k]$, explain how you can obtain $X[k]$. Provide relevant equations and simplify them.

(b) [4] Verify your algorithm using the 2-length signal $x[n] = \{1, 0\}$.

$1 \cdot e^{-j\frac{2\pi}{2}}$