EC5.201 – Signal Processing – Mid Exam

Date: 25th September, 2025
Instructor: Santosh Nannuru

Maximum marks: 50
Exam duration: 90 minutes

Q1. **[10 marks]** Consider the continuous-time signal $x(t)$ given by

$$x(t) = \cos(2\pi 50t) + \cos(2\pi\alpha t)\,,$$

where $\alpha \in (50, 100)$ is a real constant.

(a) What is the Nyquist sampling rate for this signal?

(b) If this signal is sampled at $f_s = 250$ Hz, plot the spectrum of the sampled signal.

(c) For the above signal, Shiva performs sampling and reconstruction using a sampling frequency of f_0. He observes that the reconstructed signal has only 50Hz component present due to aliasing. Derive the relation between f_0 and α.

Q2. **[10 marks]** Prove the general form of the Parseval's relation given by,

$$\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\omega)X_2^*(\omega)d\omega\,.$$

Q3. **[10 marks]** Consider the discrete-time signal,

$$x[n] = u[n] - u[n-L]\,,$$

for some positive integer L.

(a) Find the Fourier transform of $x[n]$.

(b) Plot the magnitude spectrum and phase spectrum of $x[n]$.

(c) Consider the signal $g[n] = \cos\left(\frac{2\pi}{L}n\right)x[n]$. Find and plot the magnitude spectrum and phase spectrum of $g[n]$.

Q4. **[10 marks]** Madhuri is investigating a discrete-time linear and time-invariant system H and makes the following observation: When the input is $\delta[n+1]$, the output signal is $(-1)^n u[n]$.

(a) Find the impulse response of this system. Is this system causal?

(b) Is this system stable? If yes, give proof; else, give an example to support your answer.

Q5. **[10 marks]** We know that for a band-limited signal $x_c(t)$ we can perform an exact reconstruction using the samples $x[n] = x_c(nT_s)$ for appropriate choice of T_s. Show that this result holds even then the samples are offset by a delay $\delta \in (0, T_s)$, i.e., exact reconstruction is possible using the samples $y[n] = x_c(nT_s + \delta)$.